Population growth

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Contents

((use this lecture to discuss the concepts of stable and unstable equilibrium))

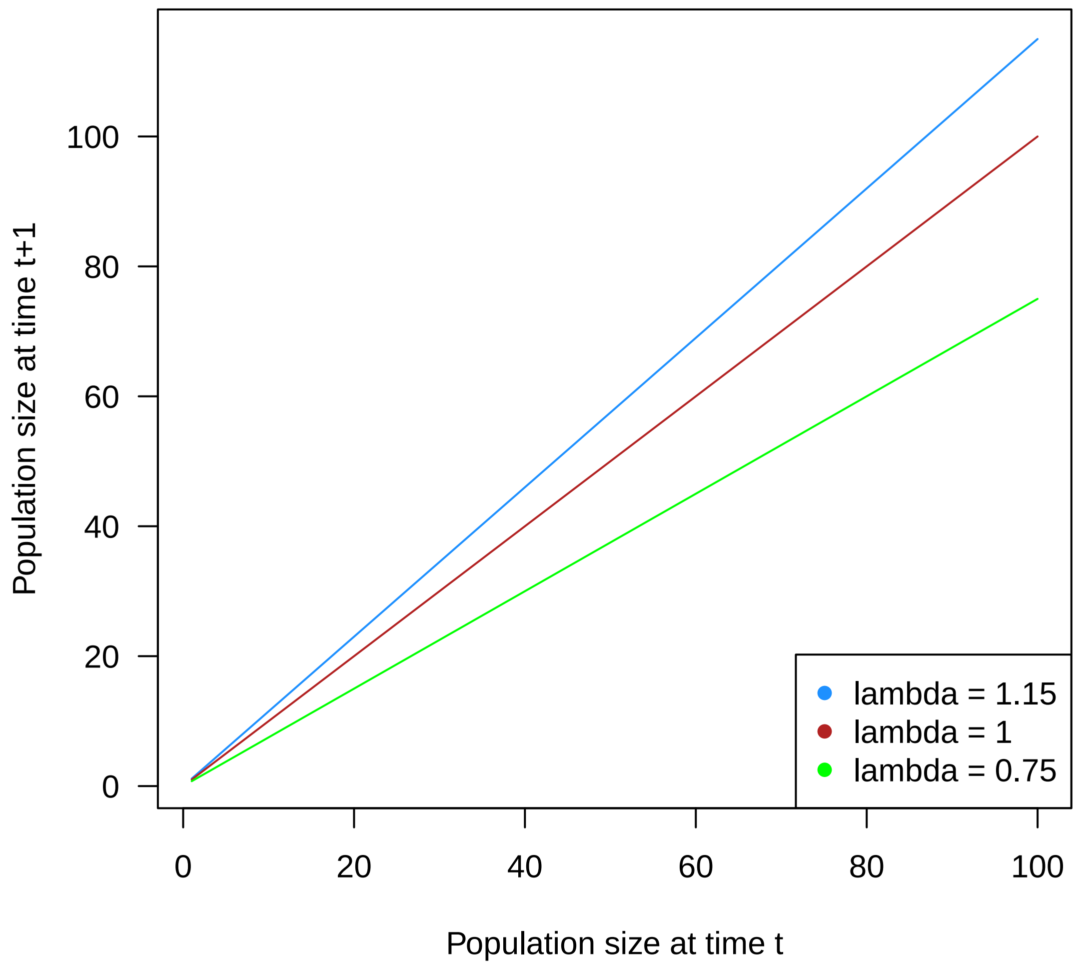
Exponential growth

```
expoGrowth <- function(n, lambda){
  n * lambda
}

n0 <- 1:100

# effect of growth rate
plot(x = n0, y = expoGrowth(n0, lambda = 1.15), type = 'l', las = 1,
  xlab = 'Population size at time t',
  ylab = 'Population size at time t+1', col = 'dodgerblue'
)
lines(n0, expoGrowth(n0, lambda = 1), col = 'firebrick')
lines(n0, expoGrowth(n0, lambda = 0.75), col = 'green')

legend('bottomright',
  paste(expression(lambda), c('= 1.15', '= 1', '= 0.75')),
  pch = 16, col = c('dodgerblue', 'firebrick', 'green')
)
```

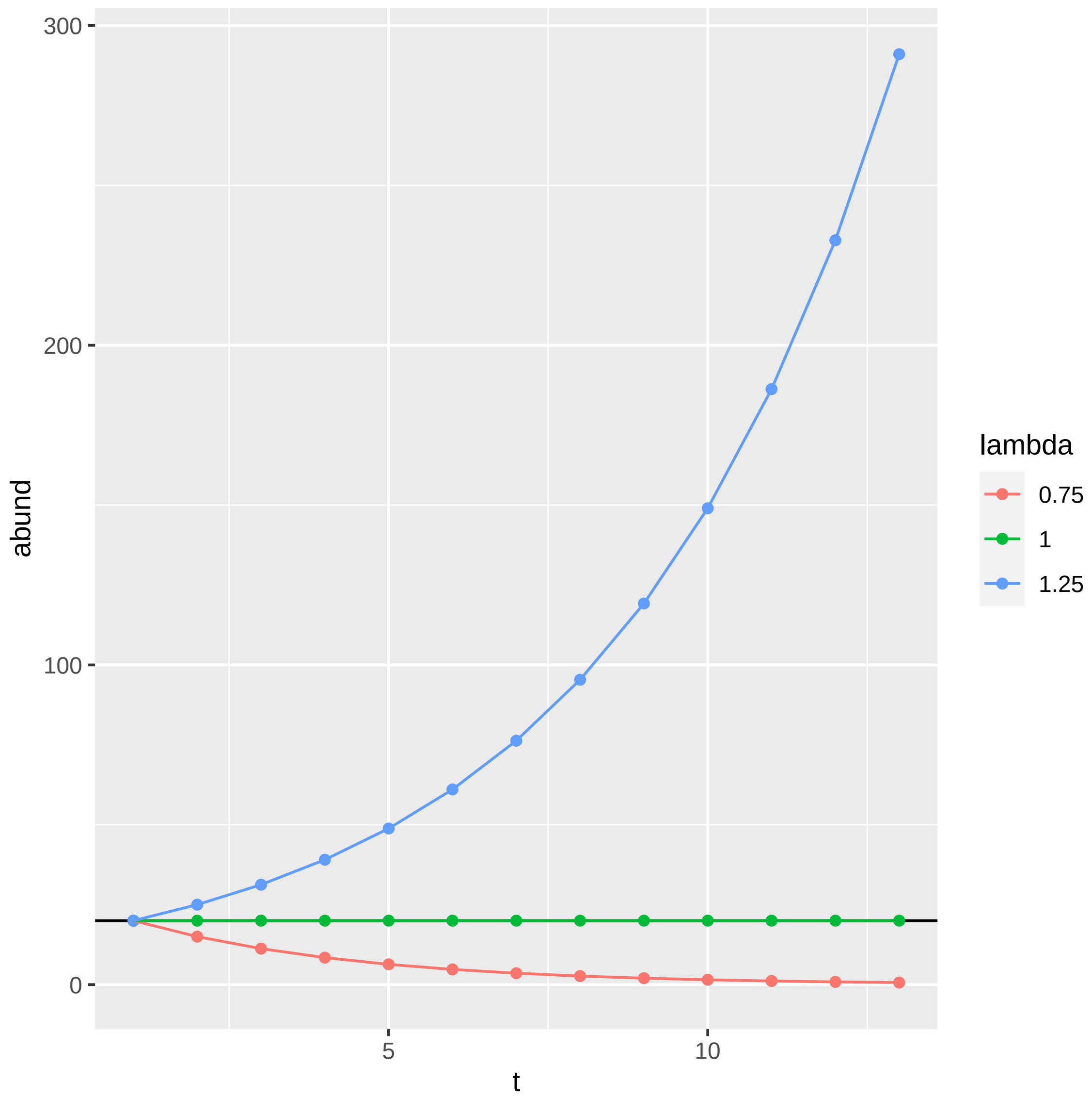



```r
# exponential growth
expoDynamics <- function(n, lambda, steps = 100){
  ret <- c()
  ret[1] <- n
  for(i in 1:steps){
    ret[i+1] <- expoGrowth(ret[i], lambda)
  }
  return(ret)
}
```

```r
library(ggplot2)
t <- 12
n0 <- 20
d <- data.frame(t = rep(1:(t + 1), 3),
           lambda = as.factor(rep(c(0.75, 1, 1.25), each = t + 1)),
           abund = c(
             expoDynamics(n0, lambda = 0.75, steps = t),
```

```r
              expoDynamics(n0, lambda = 1, steps = t),
              expoDynamics(n0, lambda = 1.25, steps = t)
            ))

ggplot(data = d, aes(x = t, y = abund, color = lambda, group = lambda)) +
  geom_hline(yintercept = n0) +
  geom_point() +
  geom_line()
```

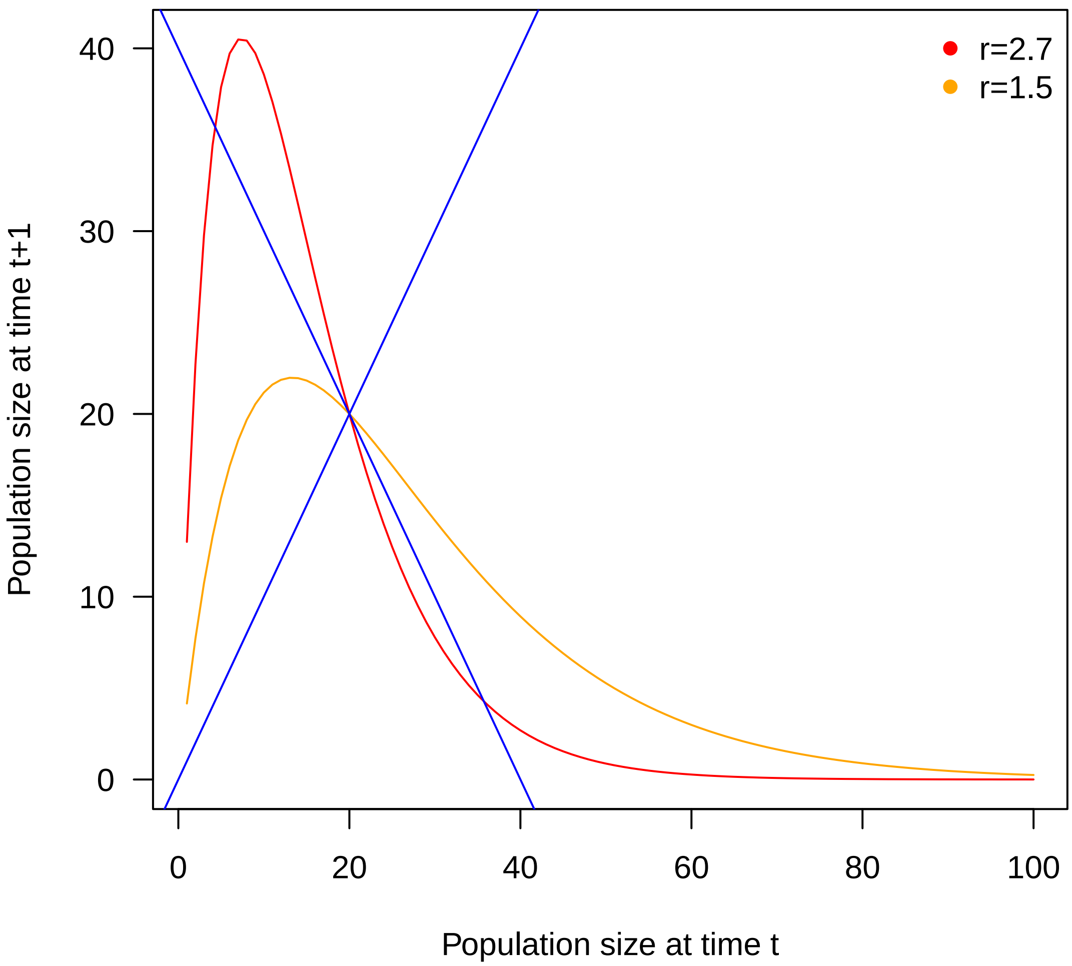


Logistic growth

```r
n0 <- 1:100
k <- 20
r <- 0.5
```

```r
logisticGrowth <- function(n, r, k) {
  n * exp(r * (1 - (n / k)))
}

colz <- c(grey(0.1, 0.9), 'dodgerblue', 'firebrick', 'forestgreen')
#effect of growth rate
plot(n0, logisticGrowth(n = n0, r = 2.7, k = 20),
  type='l', las=1,
  xlab='Population size at time t',
  ylab='Population size at time t+1',
  col="red")
lines(n0, logisticGrowth(n = n0, r = 1.5, k = 20), col = "orange")
abline(a = 0, b = 1, col = "blue")
abline(a = 40, b = -1, col = "blue")
legend('topright', bty='n', c('r=2.7', 'r=1.5'),
  pch = 16, col = c("red", "orange"))
```



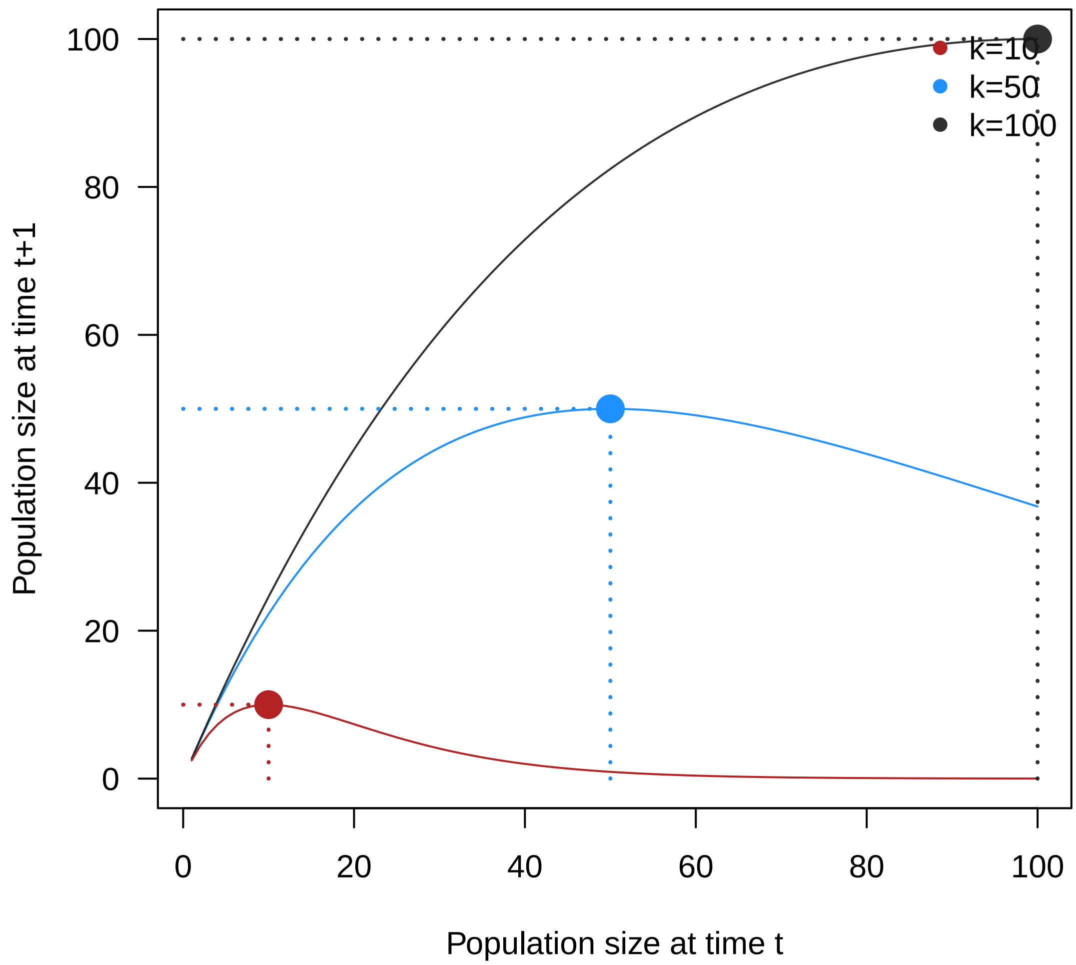
```r
#effect of carrying capacity
plot(n0, logisticGrowth(n0,1, 50),
  type='l', las=1, ylim=c(0,100),
  xlab='Population size at time t',
  ylab='Population size at time t+1',
  col=colz[2])
lines(n0, logisticGrowth(n0,1,100),
  col=colz[1])
lines(n0, logisticGrowth(n0,1,10),
  col=colz[3])
legend('topright', bty='n',
  c('k=10', 'k=50', 'k=100'),
  pch=16, col=colz[c(3,2,1)])



#Look at the peaks of the growth (where is the maximum population size here?)

plotSegs <- function(kx,ky, color){
  segments(x0=0,x1=kx, y0=ky,y1=ky, col=color, lwd=2, lty=3)
  segments(x0=kx,x1=kx, y0=0,y1=ky, col=color, lwd=2, lty=3)
  points(kx,ky, pch=16, cex=2, col=color)
}

plotSegs(10,10, color=colz[3])
plotSegs(50,50, color=colz[2])
plotSegs(100,100, color=colz[1])
```



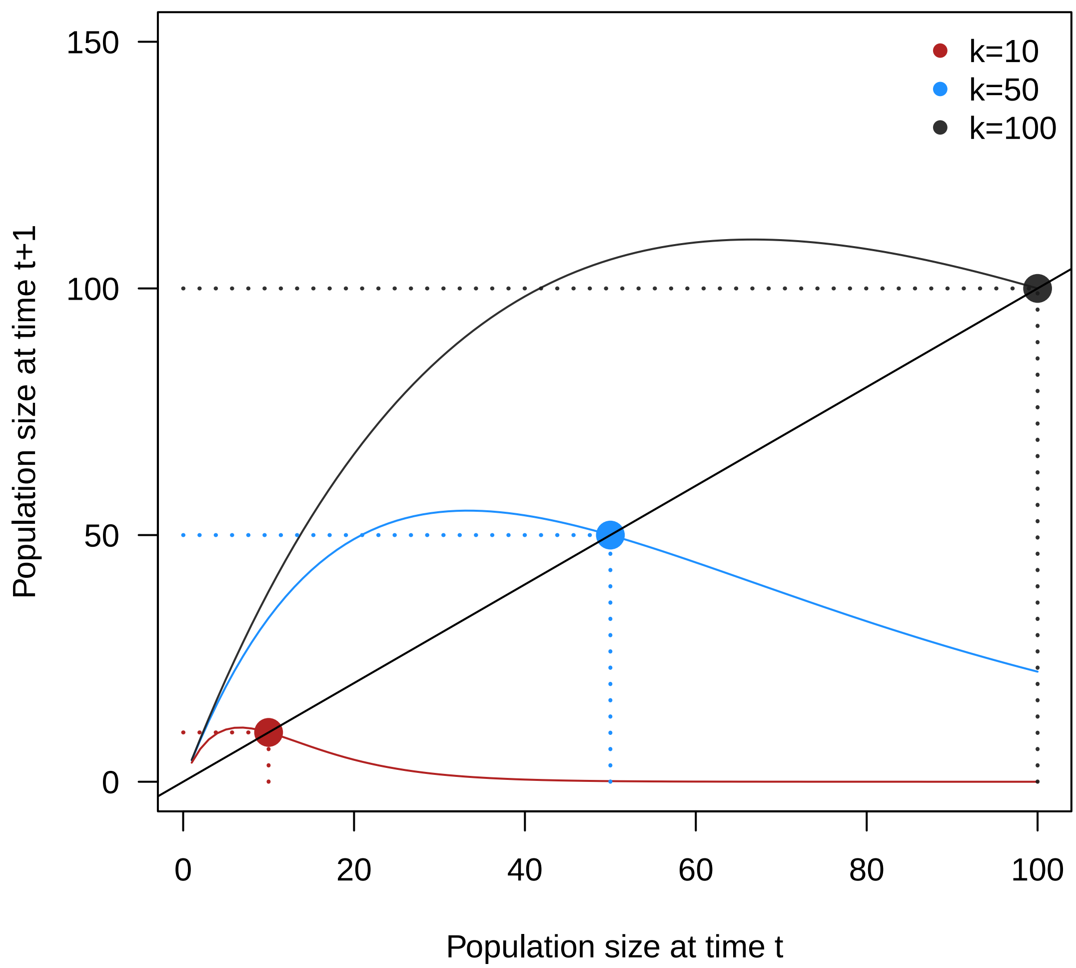

```
# But why is this? What situations would cause this to not be the case?

plot(n0, logisticGrowth(n0,1.5, 50),
  type='l', las=1, ylim=c(0,150),
  xlab='Population size at time t',
  ylab='Population size at time t+1',
  col=colz[2])
lines(n0, logisticGrowth(n0,1.5,100),
  col=colz[1])
lines(n0, logisticGrowth(n0,1.5,10),
  col=colz[3])
legend('topright', bty='n',
  c('k=10', 'k=50', 'k=100'),
  pch=16, col=colz[c(3,2,1)])


#Look at the peaks of the growth (where is the maximum population size here?)
```

```
plotSegs(10,10, color=colz[3])
plotSegs(50,50, color=colz[2])
plotSegs(100,100, color=colz[1])

# this line intersects points where population change from t to t+1 is 0. These are equilibrium.
abline(a=0,b=1)
```



Alright. So now we can look at the actual dynamics across many generations.

```
logisticDynamics <- function(n,r,k, steps=100){
  ret <- c()
  ret[1] <- n
  if(length(r) == 1){
    r <- rep(r, steps)
  }
  for(i in 1:(steps-1)){
    ret[i+1] <- logisticGrowth(ret[i], r[i], k)
```

```r
  }
  return(ret)
}
```

```r
stps <- 20

plot(1:stps,
 logisticDynamics(n=30, r=1, k=30, steps=stps),
  type='l', las=1, ylim=c(0,50),
  xlab='Time',
  ylab='Population size',
  col=1)

#sapply(seq(1,25,by=1), function(x){
# lines(logisticDynamics(n=x, r=1, k=30, steps=stps), col='firebrick')
#})


sapply(seq(5,30,by=1), function(x){
 lines(logisticDynamics(n=x, r=1, k=30, steps=stps), col='dodgerblue')
})
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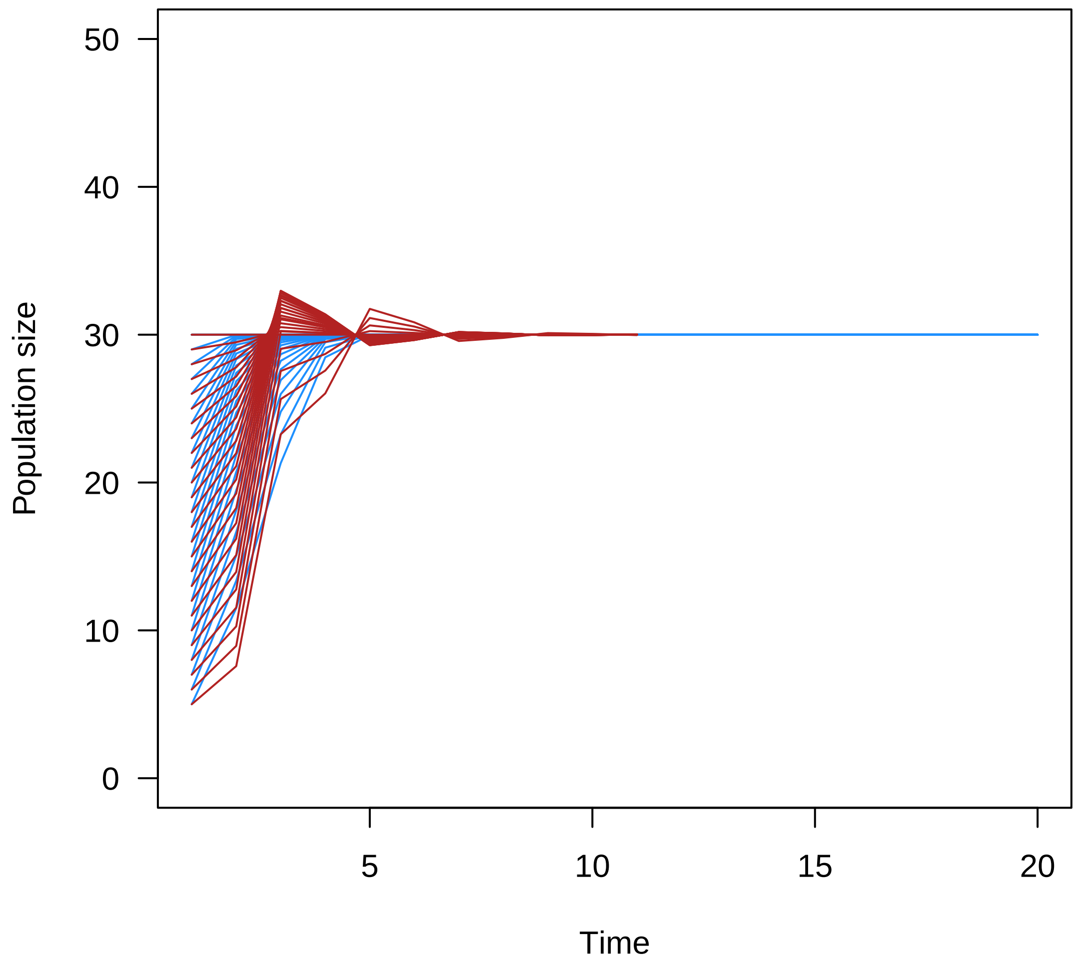
```r
sapply(seq(5,30,by=1), function(x){
 lines(logisticDynamics(n=x, r=rep(c(0.5,1.5),5), k=30, steps=stps), col='firebrick')
})
```

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What if growth rate is not 1?

r=0

```
stps <- 20

plot(1:stps,
 logisticDynamics(n=30, r=0, k=30, steps=stps),
  type='l', las=1,ylim=c(0,100),
  xlab='Time',
  ylab='Population size',
  col=1)

sapply(seq(1,25,by=1), function(x){
 lines(logisticDynamics(n=x, r=0, k=30, steps=stps), col='firebrick')
})
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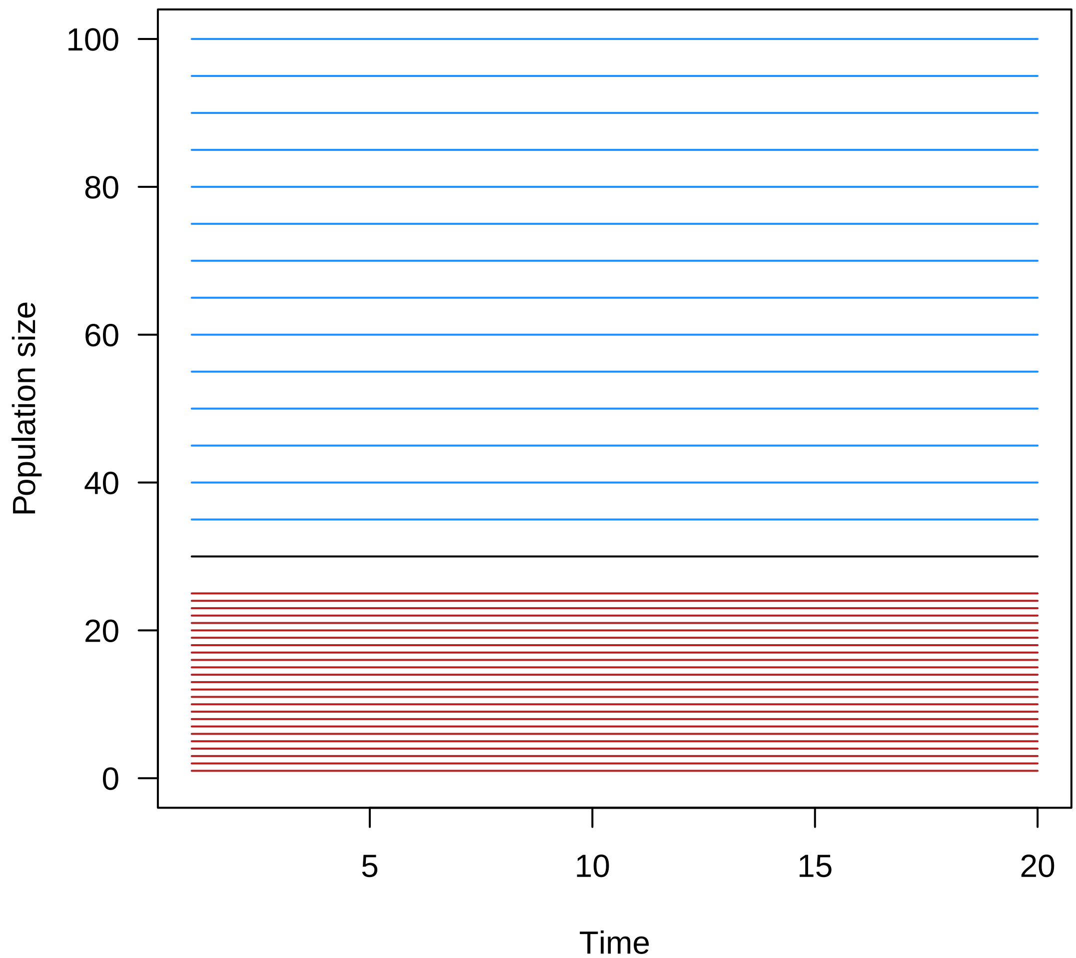
```
sapply(seq(35,100,by=5), function(x){
 lines(logisticDynamics(n=x, r=0, k=30, steps=stps), col='dodgerblue')
})
```

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r=1.5

```
stps <- 20

plot(1:stps,
 logisticDynamics(n=29, r=1.5, k=30, steps=stps),
  type='l', las=1,ylim=c(0,100),
  xlab='Time',
  ylab='Population size',
  col=1)

sapply(seq(1,25,by=1), function(x){
 lines(logisticDynamics(n=x, r=1.5, k=30, steps=stps), col='firebrick')
})
```

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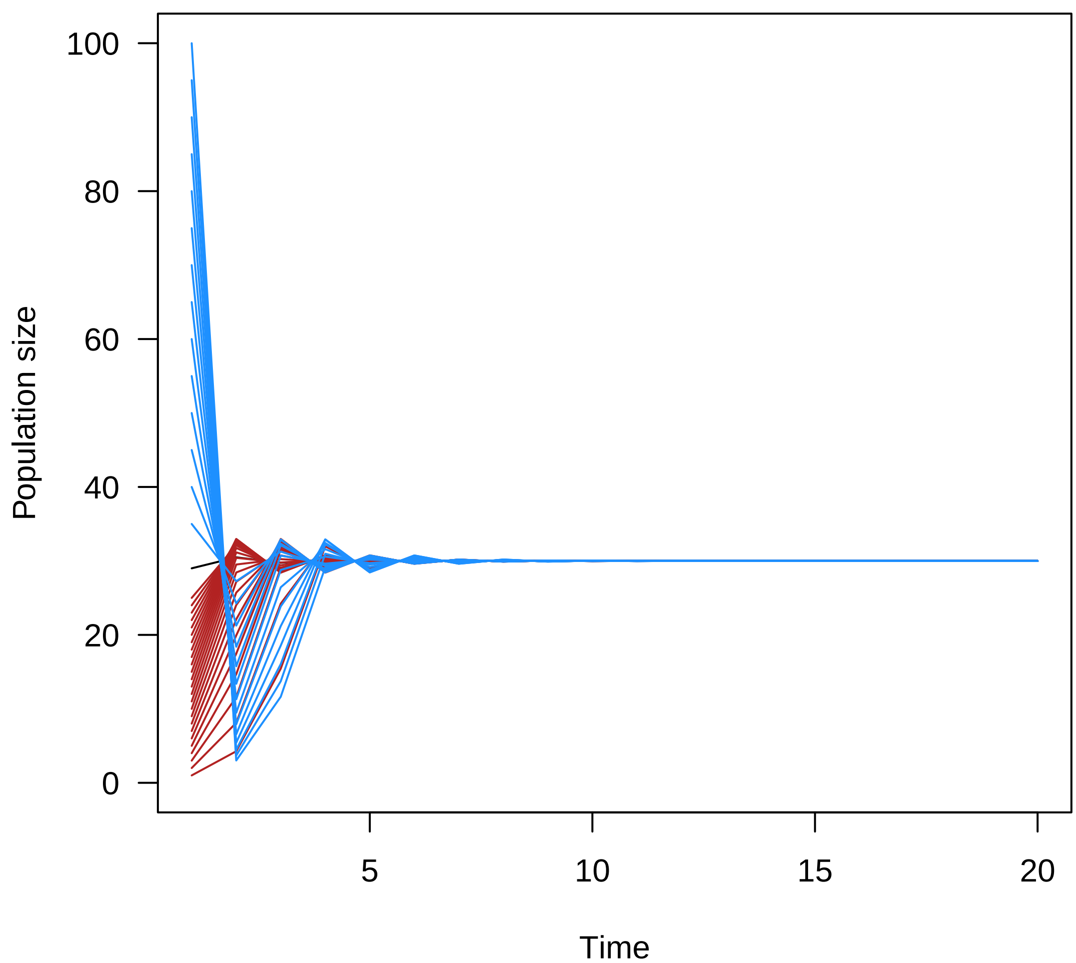
```
sapply(seq(35,100,by=5), function(x){
 lines(logisticDynamics(n=x, r=1.5, k=30, steps=stps), col='dodgerblue')
})
```

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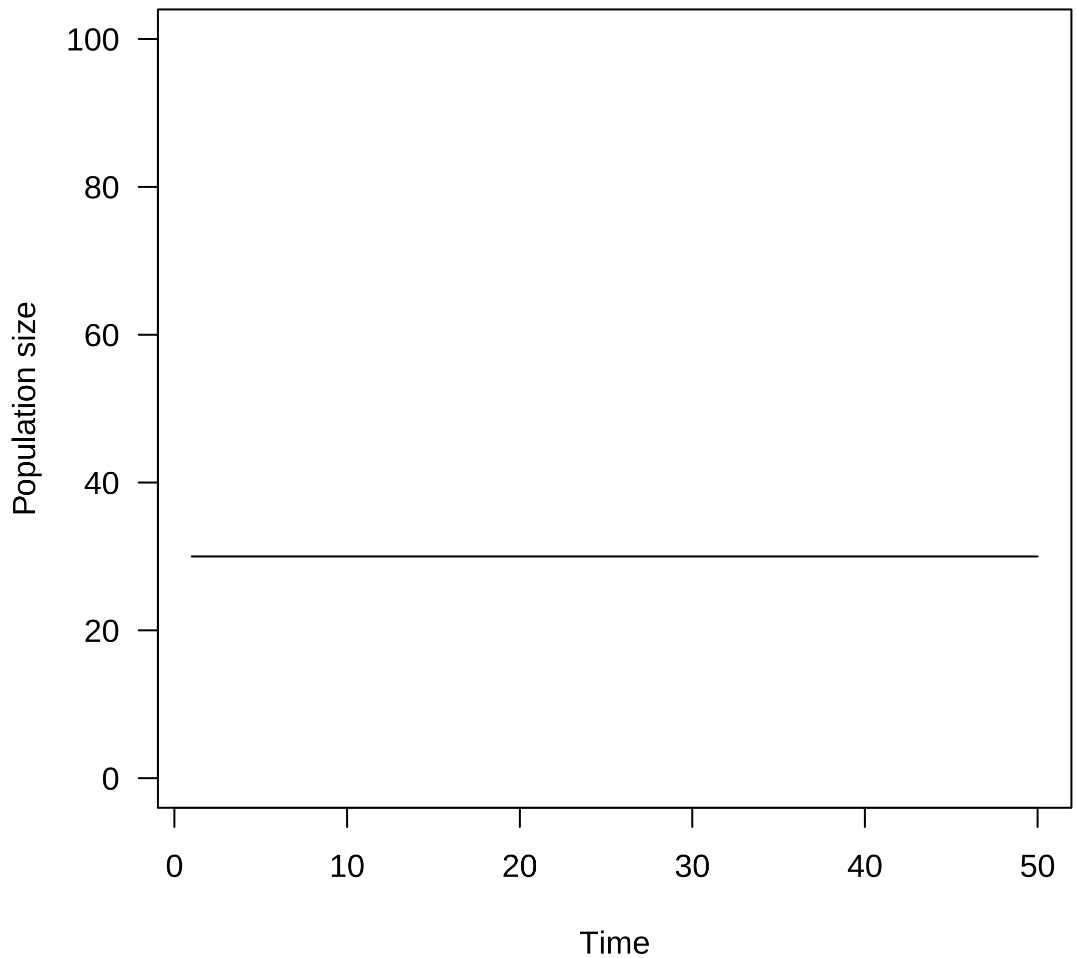
r=2

```
stps <- 50

plot(1:stps,
 logisticDynamics(n=30, r=2, k=30, steps=stps),
  type='l', las=1,ylim=c(0,100),
  xlab='Time',
  ylab='Population size',
  col=1)
```




```
plot(1:stps,
 logisticDynamics(n=29, r=2, k=30, steps=stps),
  type='l', las=1,ylim=c(0,100),
  xlab='Time',
  ylab='Population size',
  col=1)

sapply(seq(1,25,by=1), function(x){
 lines(logisticDynamics(n=x, r=2, k=30, steps=stps), col='firebrick')
})
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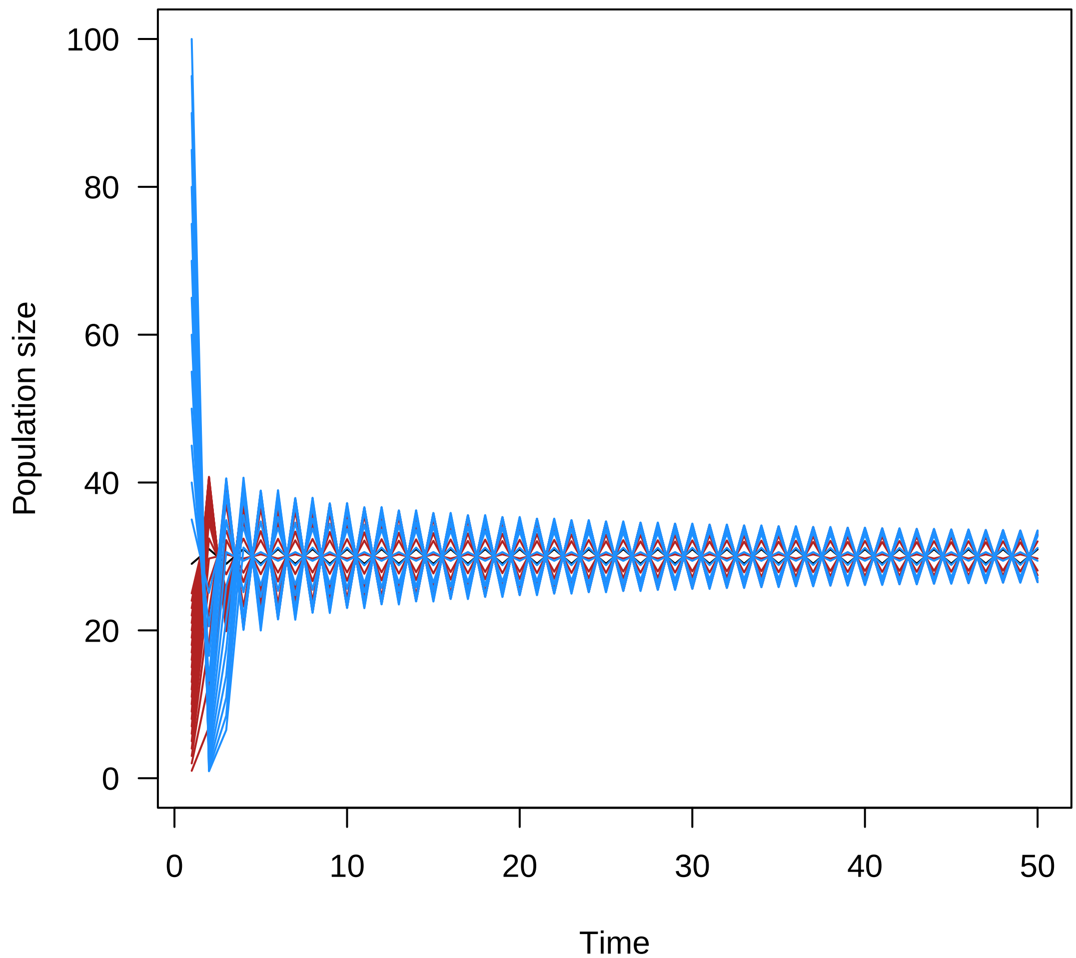
```
sapply(seq(35,100,by=5), function(x){
 lines(logisticDynamics(n=x, r=2, k=30, steps=stps), col='dodgerblue')
})
```

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```

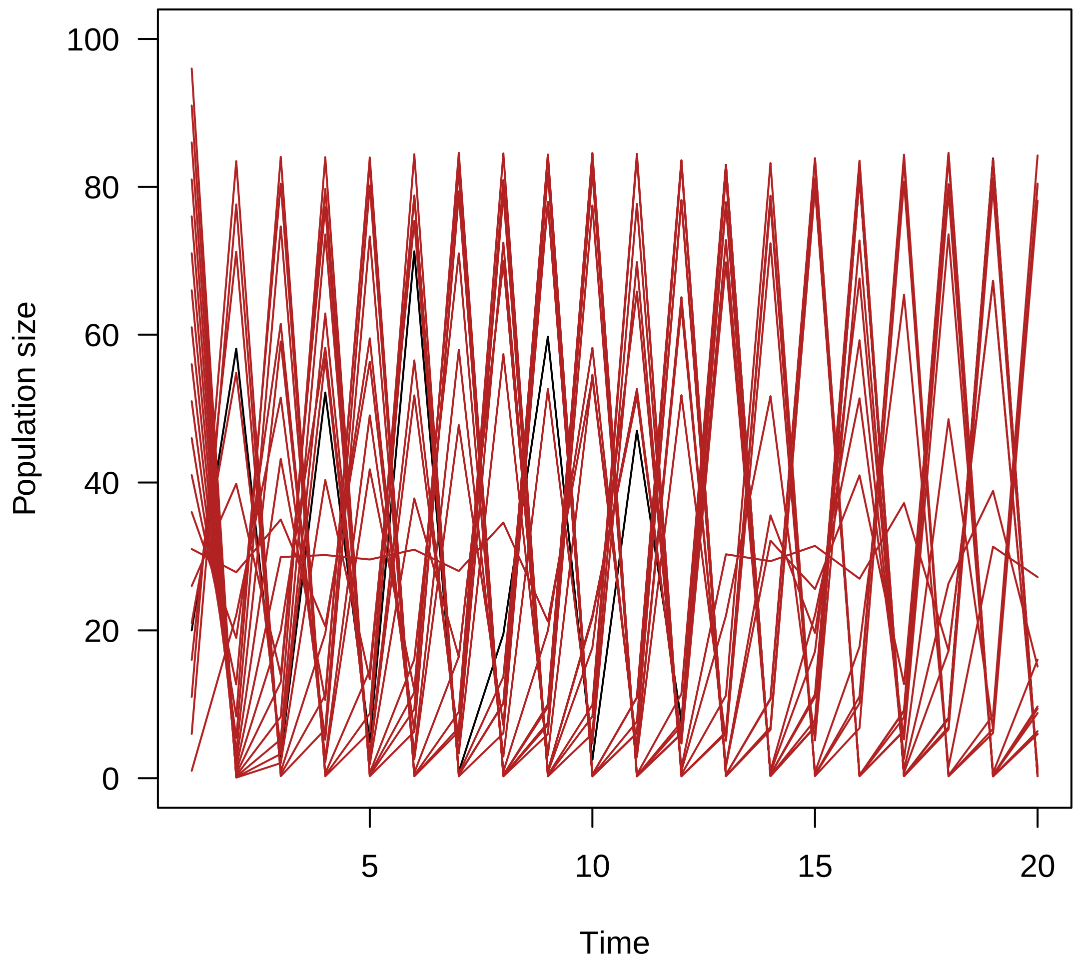
(between 3 and 3.449 – oscillates between 2 values)

r=3.2

```
stps <- 20

plot(1:stps,
 logisticDynamics(n=20, r=3.2, k=30, steps=stps),
  type='l', las=1,ylim=c(0,100),
  xlab='Time',
  ylab='Population size',
  col=1)


sapply(seq(1,100,by=5), function(x){
 lines(logisticDynamics(n=x, r=3.2, k=30, steps=stps), col='firebrick')
})
```

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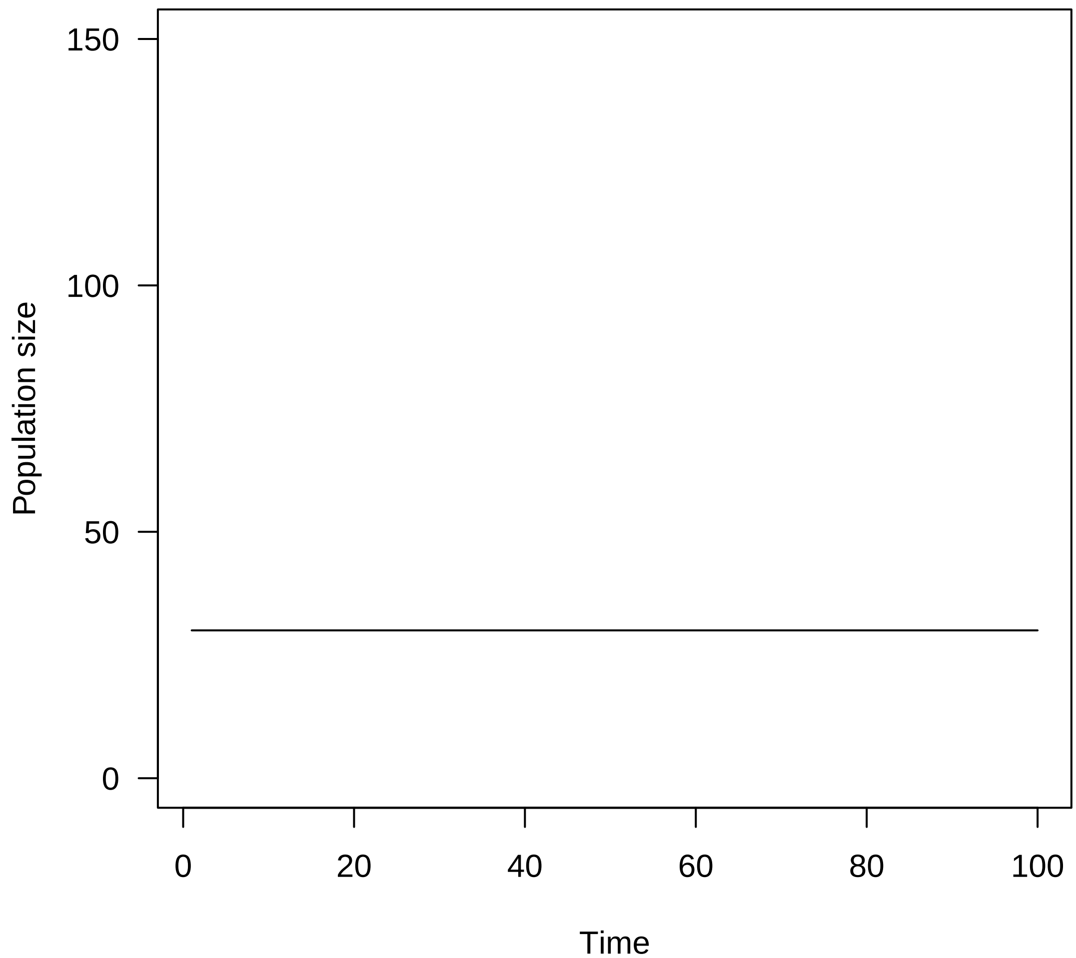
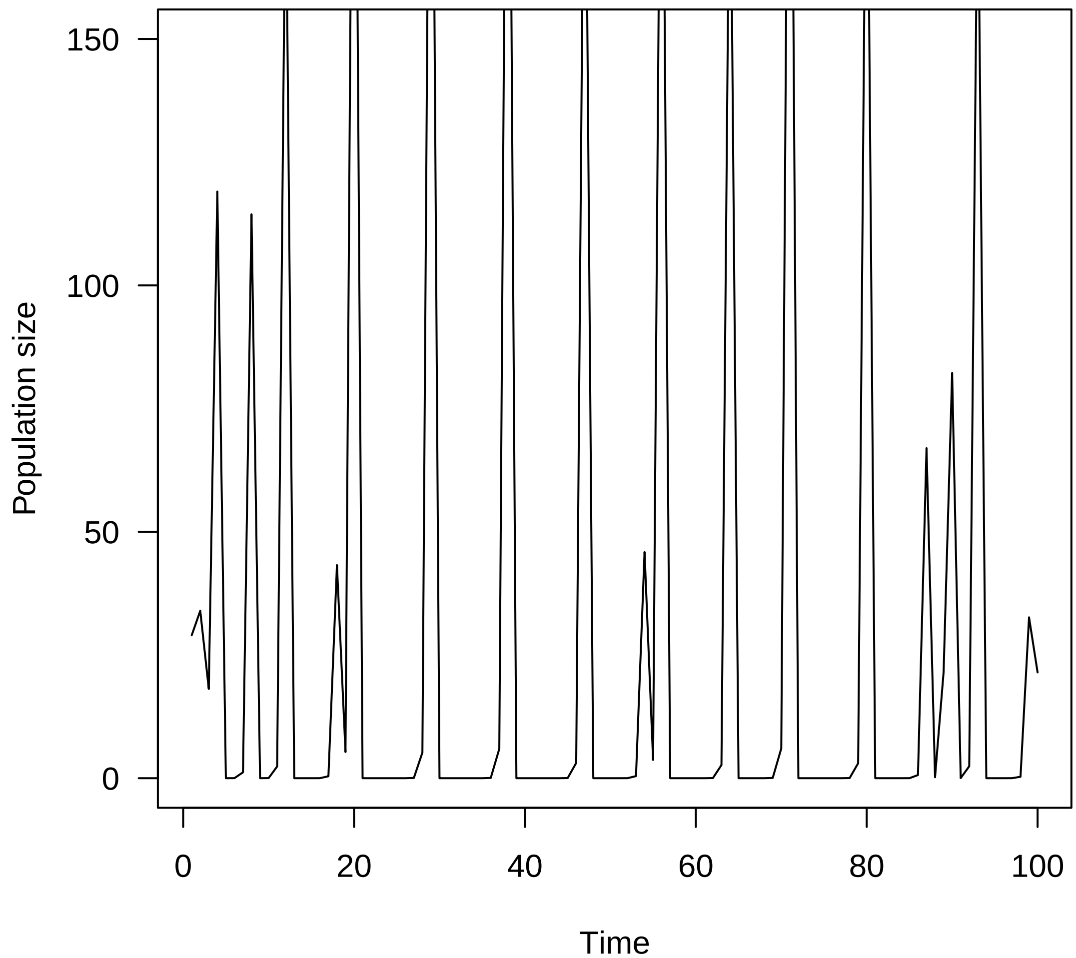
(onset of chaos) $r > 3.56$

```
stps <- 100

plot(1:stps,
 logisticDynamics(n=30, r=4.75, k=30, steps=stps),
  type='l', las=1,ylim=c(0,150),
  xlab='Time',
  ylab='Population size',
  col=1)
```




```
plot(1:stps,
 logisticDynamics(n=29, r=4.75, k=30, steps=stps),
  type='l', las=1,ylim=c(0,150),
  xlab='Time',
  ylab='Population size',
  col=1)
```



Matrix model

((give this as an example problem to let the students work through it))

First, build a simple projection matrix

```
projMatrix <- matrix(
  c(
    0.2,      1,     0.25,
    0.3,      0,     0,
    0,        0.3,   0.6
  )
  ,nrow=3,ncol=3,byrow=T
)
```

```r
abund0 <- matrix(c(20,20,20), ncol=1)
```

Simulate one generation into the future.

```r
abund1 <- projMatrix  %*% abund0
```

Simulate one more generation

```r
projMatrix %*% abund1
```

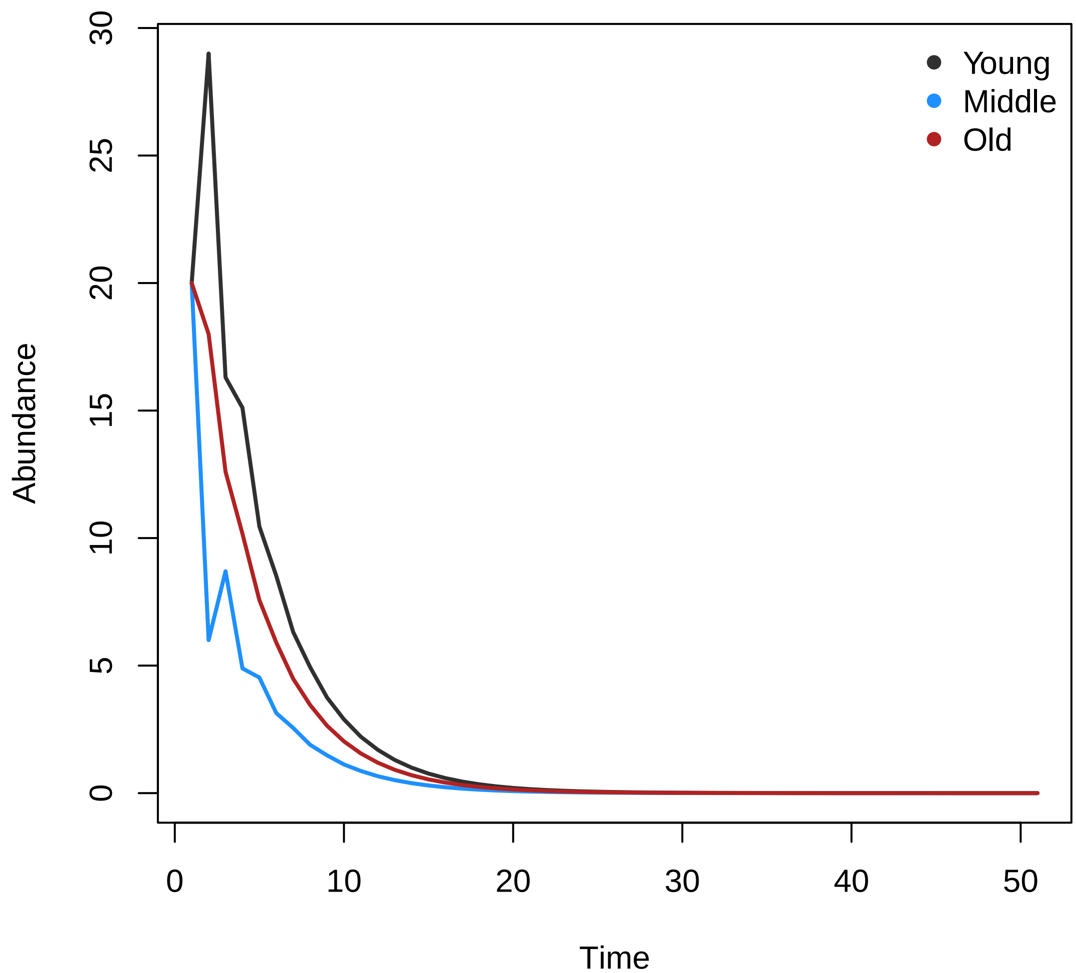
```
##      [,1]
## [1,] 16.3
## [2,]  8.7
## [3,] 12.6
```

Simulate many generations to examine dynamics

```r
getStageDynamics <- function(projMatrix, abund, steps=100){
  ret <- matrix(0, ncol=3, nrow=steps+1)
  ret[1,] <- abund
  for(i in 1:steps){
    ret[i+1, ] <- projMatrix %*% matrix(ret[i,],ncol=1)
  }
  return(ret)
}
```

```r
stageDynamics <- getStageDynamics(projMatrix, abund0, steps=50)


plot(stageDynamics[,1], type='l', lwd=2,
  col=colz[1],
  ylab='Abundance', xlab='Time')
lines(stageDynamics[,2], lwd=2, col=colz[2])
lines(stageDynamics[,3], lwd=2, col=colz[3])
legend('topright', pch=16, col=colz[1:3],
  c('Young', 'Middle', 'Old'), bty='n')
```



Explore some of the dynamics with the projection matrix and equilibrium points.