Tutorial 3

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Exercise 8: FitzHugh-Nagumo model

The ODEs of the FitzHugh-Nagumo model $\dot{\vec{u}} = f(\vec{u})$ are given by:

$$\dot{v}(t) = v(t) \cdot (a - v(t)) \cdot (v(t) - 1) - w(t) + I_{app}, \tag{1}$$
$$\dot{w}(t) = \epsilon \cdot (v(t) - \gamma \cdot w(t)) \tag{2}$$

where all parameters $(\alpha, \epsilon, \gamma) \in \mathbb{R}^3_+$ are positive.

- Write a function FN($\vec{du}, \vec{u}, \vec{p}, t$) that can be passed as a function argument to prob = ODEProblem(FN, $\vec{u}_0, \vec{t}_{span}$) of the Julia package DifferentialEquations. Solve the problem with sol = solve(prob).
  Hint: Check out the ODE examples on juliadiffeq.org.
- Use prob = ODEProblem(FN, $\vec{u}_0, \vec{t}_{span}$) to integrate the system for $a = 0.25, \epsilon = 0.002, \gamma = 1.1, I_{app} = 0$ and $t_{span} = (0.0, 1000)$ for different initial values $(v(0), w(0))$ varying $v(0)$ around $a = 0.25$ and keeping $w(0) = 0$. Plot the solutions in configuration space and in phase space.
- Write a function FNnc($\vec{u}$) for $\vec{u} = -0.3 : 0.1 : 1$ returning the nullclines of the system $w(v)$ for $\dot{v}(t) = 0$ and $w(v)$ for $\dot{w}(t) = 0$. Add them to your phase-space plot.
- Now test $I_{app} > 0$. Plot the solutions in config space for $I_{app} = [0, 0.02, 0.1, 0.5, 0.7]$ and use the nullclines to understand the behaviour in phase space. The initial value $(v(0), w(0))$ can be fixed to $(0, 0)$.

Cathedral exercise

The southern portal dates from the Renaissance. What is the connection between the Renaissance portal, the reformation, the counter reformation and Erasmus from Rotterdam?