

# Tutorial 1

Janine Egert, Daniel Lill, Daniel Kaschek, Mirjam Kaschek

May 8th, 2019

If you use git, update the exercises-folder by running: `cd DynSysBio2019; git pull`

## 1 Exercise 5: Lotka-Volterra

The ODEs of the Lotka-Volterra model  $\dot{\vec{x}} = f(\vec{x})$  are given by:

$$\dot{x}(t) = a \cdot x(t) - b \cdot x(t) \cdot y(t), \quad (1)$$

$$\dot{y}(t) = c \cdot x(t) \cdot y(t) - d \cdot y(t), \quad (2)$$

where all parameters  $(a, b, c, d) \in \mathbb{R}_+^4$  are positive.

- Write a 'function `euler(ini, pars, dt, tmax)`' integrating the LV system via the Euler method for a set of initial values *ini*, parameters *pars* and stepsize *dt* for  $t \in [0, tmax]$ . The function should return the variables *time* and  $\vec{x}$ .

Reminder Euler method:  $\vec{x}_{n+1} = \vec{x}_n + f(\vec{x}_n) \cdot dt$ .

- Plot solutions of the LV system in configuration  $(t, \vec{x})$  and phase space  $(x, y)$  for  $a = 2/3$ ,  $b = 4/30$ ,  $c = 1/10$ ,  $d = 1$  and a set of initial values  $(x(t=0), y(t=0)) = \{(10, 15), (15, 10)\}$ .

- Add a noise term to the euler integration:  $\vec{x}_{n+1} = \vec{x}_n + f(\vec{x}_n) \cdot dt + \epsilon \cdot \sqrt{dt}$  with  $\epsilon \propto \mathbb{N}(0, sd)$ . Use  $sd = [sdx, sdy]$  as additional argument of your euler function and avoid  $x_n < 0$ . Compare the solutions obtained with  $sd = [.3, .3]$  to the standard solutions without noise by plotting.

## 2 Exercise 6: Lotka-Volterra extended

The LV system can be extended to:

$$\dot{x}(t) = a \cdot x(t) \cdot \left(1 - \frac{x(t)}{K}\right) - b \cdot \frac{x(t)}{x(t) + S} \cdot y(t), \quad (3)$$

$$\dot{y}(t) = c \cdot \frac{x(t)}{x(t) + S} \cdot y(t) - d \cdot y(t), \quad (4)$$

where also  $K$  and  $S$  are positive parameters.

- Repeat Exercise 5 for the extended equations using  $a = b = c = 1; d = \frac{1}{3}; K = 30, S = 10$  and  $(x(0), y(0)) = \{(1, 1), (10, 10)\}$ .

- Interpret the extended system:
  - What is the meaning of the parameters?
  - What are the qualitative differences of both systems?
  - Compare how the noise affects both systems.

## 3 Cathedral exercise

The choir of the cathedral is slightly inclined relative to the main nave. Why?