ELE2024 — CONTROL COURSEWORK (2020-21)

LECTURER: PANTELIS SOPASAKIS

Deadline: ~~14 December 2020, 23:59~~ 11 January 2021, 23:59 (UK Time).

Instructions: Please, take into account the following instructions

(1) Answer all questions in Parts A, B and D. In Part C you will find three (optional) bonus questions.
(2) Prepare you report using the LaTeX template provided on Canvas (You can use your own LaTeX style if you prefer). You may use MS Word, LibreOffice Writer or other similar word processing software, but it will be significantly more difficult and time consuming to write equations and the final result will, most likely, not be of the best possible quality.
(3) Compile your answers into a single PDF file (you will not be able to upload `doc(x)` or other files)
(4) The maximum length of your answers should be 8 A4 pages using the above template, plus 1 page for Part D, plus another page for Part C. In order not to exceed the page limit use the LaTeX template provided (a figure about the size of Figure 1, or even smaller, is perfectly legible).
(5) Submit your answers on Canvas before the above deadline
(6) It is advisable that you read Part D first; you can have a first meeting to decide how you will work as a team, how the various tasks will be distributed, how you will share your code and how you will be able to contribute to the report you need to prepare.

Grading criteria: *Part A* (42 marks) is theoretical and the grading criteria are:

(1) Technical correctness (80% $\equiv$ 33.6 marks) with main focus on the correctness of the procedure you will follow
(2) Quality of presentation (20% $\equiv$ 8.4 marks): clarity of presentation of your solutions, quality of typesetting of your equations

Part B (53 marks) is a glimpse into a typical day of a control engineer at work. The idea is that you need to design a control system for a client and you need to prepare a technical report to convince them that your control system is well designed (it is stable, does not have offset, and so on). In particular, you need to convince them that it will work well in practice.

Problem B6 is a design question: you will need to combine the results from the previous steps to design a control system. Your solution may involve theoretical results as well as computations, simulations and plots. There is no single correct answer to this problem: all justified design choices are acceptable.

In Part B the grading criteria will be:

(1) Technical correctness (50% $\equiv$ 26.5 marks): correct application of control theory, articulation of assumptions, and that conditions of theorems are carefully checked.
(2) Discussion of results (25%$\equiv$ 13.25 marks): avoid mathematical derivations with no discussion; likewise, if you decide to include simulation data (such as plots), provide a discussion of your results.
(3) Quality of presentation (25% $\equiv$ 13.25 marks): clarity of presentation and whether your report looks professional (typesetting of your equations, whether your plots are legible

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and have axis labels, etc). For best results, use the LaTeX template provided and read the instructions therein.

Part C (20 bonus marks) three theoretical questions for an extra 20 marks. If you get more than 100 marks in this assignment, you will be able to use the extra marks to increase your overall score in the module (up to a maximum of 100%).

Part D (Collaboration Questionnaire, 5 marks). In this part you need to provide *factual* data regarding your collaboration.

Additional information. You may find additional information on Canvas. Have a look at the following posts before you start working on the coursework:

- Getting started with git
- What are issue trackers and why you need one
- Getting started with LaTeX

1. Problem Description

Consider the system shown in Figure 1.

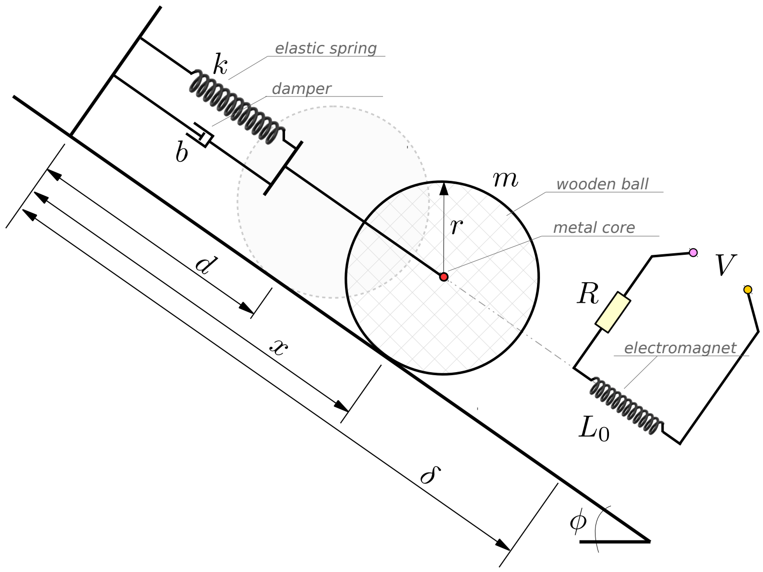

Figure 1. System of a wooden ball on an inclined plane. The ball can be attracted downwards by an electromagnet, which is controlled by a voltage V.

A wooden ball of total mass m and radius r is placed on an inclined plane, which is at an angle ϕ with respect to the horizontal plane as shown in Figure 1. The ball can roll on the inclined plane without sliding. The ball is connected to an elastic spring of stiffness k and a linear damper with viscous damping coefficient b. Let x denote the distance of the centre of the ball from the wall. When $x = d$, the spring is at its natural length and no restoring force is applied. The ball is considered to be approximately isotropic.

At the centre of the ball there is a metal core of small radius which can be attracted by an electromagnet (which is nothing but an inductor). The centre of the electromagnet is positioned at $x = \delta > d$. This inductor is connected in series with an Ohmic resistor of resistance R and a voltage V is applied to the circuit as shown in Figure 1. The nominal inductance of the inductor is L_0; however, as the ball approaches at a distance y from the centre of the inductor, its inductance increases and is given by

$$L = L_0 + L_1 \exp(-\alpha y), \tag{1}$$

where L_1 and α are given positive constants.

The electromagnet can exercise an attractive force to the metal core of the ball, whose magnitude is given by

$$F_{\text{mag}} = c\frac{I^2}{y^2}, \tag{2}$$

where c is a positive constant, I is the current that runs through the circuit and y is the distance between the centre of the wooden ball and the centre of the electromagnet.

2. Part A: Control Theory

In this part of the coursework, no particular values are provided for m, g, d, δ, r, R, L_0, L_1, α, c, k, b and ϕ. You should treat these as known constants.

Problem A1 (12 marks). Use first principles from physics, as well as Equations (1) and (2), to derive a system of ordinary differential equations that describes how the input voltage, V, affects the position, x, of the ball on the inclined plane. Note: introduce an inertial frame of reference where counterclockwise rotations are positive.

Problem A2 (5 marks). Write the system you determined in Problem A1 in a state space representation. What are the states and inputs of the system?

Problem A3 (5 marks). Characterise the equilibrium points of the system using Proposition 3.4 in the textbook.

Problem A4 (12 marks). Linearise the system at an equilibrium point (use deviation variables).

Problem A5 (8 marks). Determine the transfer function of the linearised system that you determined in Problem A4. The input of the system is the voltage across the circuit and the output is the position of the ball on the inclined plane. How many poles does this transfer function have? Derive sufficient conditions on the system parameters under which the impulse response of the transfer function is oscillatory.

3. Part B: Analysis and Controller Design

In this part, you may use the following values: $m = 425\,\mathrm{g}$, $g = 9.81\,\mathrm{m/s^2}$, $d = 42\,\mathrm{cm}$, $\delta = 65\,\mathrm{cm}$, $r = 12.5\,\mathrm{cm}$, $R = 53\,\Omega$, $L_0 = 120\,\mathrm{mH}$, $L_1 = 25\,\mathrm{mH}$, $\alpha = 1.2\,\mathrm{m}^{-1}$, $c = 6815\,\frac{\mathrm{g\cdot m^3}}{\mathrm{A^2\cdot s^2}}$, $k = 1880\,\mathrm{N/m}$, $b = 10.4\,\mathrm{Ns/m}$ and $\phi = 42°$.

Problem B1 (7 marks). Show that the system can equilibrate only at those positions x^e that satisfy

$$x_{\min} < x^e < x_{\max}, \tag{3}$$

where $x_{\min} = d + \frac{mg\sin\phi}{k}$ and $x_{\max} = \delta$. Determine the equilibrium voltage and current as a function of x^e and determine the position $x^e_\star$ where the corresponding equilibrium voltage attains its maximum value.

Note: For the following problems, linearise the system at the equilibrium point that corresponds to $x = 0.75x_{\min} + 0.25x_{\max}$.

Problem B2 (12 marks). You now need to verify the system linearisation that you determined in Problem A4. Suppose that the input voltage is equal to V^e for all $t \geq 0$ and the initial state of the system is close to its equilibrium value. Write a program in Python to simulate both the nonlinear dynamical system you determined in Problem A2 and its linearisation. Are the two responses similar?

According to the Hartman-Grobman theorem (or linearisation theorem), if the linearised dynamical system is stable, then the nonlinear system behaves like a stable system close to the equilibrium point. Does this check out in your simulations?

PROBLEM B3 (6 MARKS). Determine and plot the impulse and step responses of the transfer function of the linearised system.

PROBLEM B4 (6 MARKS). Determine the low- and high-frequency asymptotes of the magnitude (in dB) and phase lag of the transfer function of the linearised system. Include the Bode plot in your report.

PROBLEM B5 (5 MARKS). We need to design a controller for this system. Suppose we can measure the x-position of the ball on the inclined plane with a laser proximity sensor and the control objective is to manipulate the voltage V so as to steer the x-position of the ball to a set point x^{sp}. What are the desired characteristics of a controller for the system of Figure 1? If someone proposed a certain controller for this system, what evidence would you require to be confident that it is a good controller.

PROBLEM B6 (17 MARKS). Suppose that x can be measured with a sensor that can be modelled as a first-order system with time constant 30 ms. Design a control system. Evaluate how good this control system is by providing theoretical and/or simulation-based evidence as appropriate. Take into account your answer in PROBLEM B5.

4. PART C: BONUS QUESTIONS

PROBLEM C1 (5 MARKS). Determine the Laplace transform of $f(t) = \ln t^3$, $t > 0$.

PROBLEM C2 (5 MARKS). Determine the Laplace transform of $f(t) = |\cos(\omega t)|$, $t \geq 0$, $\omega > 0$.

PROBLEM C3 (10 MARKS).[1] Find a sequence of time-domain functions $(f_n)_{n\in\mathbb{N}}$ such that, (i) f_n converges pointwise to a function f for all $t \geq 0$, (ii) all f_n have a Laplace transform F_n, (iii) f has a Laplace transform F. Is is possible that the following is not true

$$\lim_{n\to\infty} (\mathscr{L} f_n)(s) = (\mathscr{L} f)(s)? \tag{4}$$

Give a counterexample. Under what additional conditions does this hold?

[1] A little comment on why this is an interesting question: how would you determine the Laplace transform of a "complicated" function (for example $f(t) = \sin(\sqrt{t})$? An idea would be to write function $f(t)$ as a limit of simpler functions $f_n(t)$ for which we can easily determine their Laplace transform. For instance, we can take the Taylor expansion of f, where f_n is the n-th order approximation of f. Since f_n is a polynomial in t, we can easily determine its Laplace transform. However we need to ask: is the limit of $F_n(s) := (\mathscr{L} f_n)(s)$ the same as $\mathscr{L} f$?

5. Part D: Planning, Organisation & Collaboration

Part D: 5 marks: provide factual answers to the following questions:

D1. How did your team collaborate on writing code? Did you use a source versioning system such as `git`? How did your team distribute tasks and responsibilities? Did you use an issue tracker? If yes, provide links.

D2. What are three (or more) ways you did well in functioning as a team?

D3. What problems/challenges did you face interacting as a team? How did you address them?