REPORT FOR THE ELE2024 COURSEWORK

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1. Part A: Control Theory

1.1. **Given Equations.** The following equations were given by Dr P. Soposakis as part of the coursework brief:

$$L = L_0 + L_1 exp(-\alpha y) \tag{1}$$

$$F_{mag} = c\frac{I^2}{y^2} \tag{2}$$

1.2. **Problem A1.** Use first principles from physics, as well as Equations (1) and (2), to derive a system of ordinary differential equations that describes how the input voltage, V , affects the position, x, of the ball on the inclined plane. Note: introduce an inertial frame of reference where counterclockwise rotations are positive.

$$F_{spring} = k(x - d) \tag{3}$$
$$F_{damper} = b\dot{x} \tag{4}$$
$$F_{mag} = c\frac{I^2}{y^2}, \text{ where } y = d - x \tag{5}$$

$$-Tr = I\ddot{\theta} \tag{6}$$
$$a = \ddot{x} = \ddot{\theta}r \tag{7}$$
$$\therefore T = \frac{I\ddot{\theta}}{r} \tag{8}$$
$$\ddot{\theta} = \frac{\ddot{x}}{r} \tag{9}$$
$$I = \frac{2}{5}mr^2 \tag{10}$$

$$T = -\frac{2mr^2\ddot{x}}{5r^2}$$
$$T = -\frac{2m\ddot{x}}{5} \tag{11}$$

$$F = m\ddot{x}$$
$$F_{mag} + mg\sin\phi - T - F_{spring} - F_{damper} = m\ddot{x}$$
$$\frac{cI^2}{y^2} + mg\sin\phi + \frac{2m\ddot{x}}{5} - k(x - d) - b\dot{x} = m\ddot{x} \tag{12}$$

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Some note goes here. Version 0.0.1. Last updated: November 24, 2020.

$$V_R = IR \tag{13}$$
$$V_L = L\dot{I} \tag{14}$$
$$L = L_0 + L1\exp(-\alpha y), \text{ where } y = d - x \tag{15}$$

$$\begin{aligned} V &= V_R + V_L \\ V &= IR + L\dot{I} \\ V &= IR + (L_0 + L_1\exp(-\alpha y))\dot{I} \end{aligned}$$

$$V = IR + (L_0 + L_1\exp(-\alpha(d - x))\dot{I} \tag{16}$$

$$\begin{aligned} \frac{cI^2}{y^2} + mg\sin\phi + \frac{2m\ddot{x}}{5} - k(x-d) - b\dot{x} &= m\ddot{x} \\ \frac{cI^2}{y^2} + mg\sin\phi - k(x-d) - b\dot{x} &= m\ddot{x} - \frac{2m\ddot{x}}{5} \\ \frac{cI^2}{y^2} + mg\sin\phi - k(x-d) - b\dot{x} &= \frac{3m}{5}\ddot{x} \end{aligned}$$

$$\frac{cI^2}{(d-x)^2} + mg\sin\phi - k(x-d) - b\dot{x} = \frac{3m}{5}\ddot{x} \tag{17}$$

1.3. **Problem A2.** Refer to other sections as Section 1.2. An example of a numbered list

(1) first item,
(2) second item.

Links are like this. We also have **boldface**, *italics*, *emphasised*, `truetype`, SMALL CAPS and so on.

1.4. **Problem A3.** Denote the real numbers as $\mathbb{R}$ and the complex numbers as $\mathbb{C}$. Example of a limit:

$$z = \lim_{s\to 0^+} \frac{s+1}{s^3+s^2-5s+9}. \tag{18}$$

Another example

$$\lim_{s\to\infty} \frac{s+1}{s^3+s^2-5s+9}. \tag{19}$$

Example of an integral

$$\int_0^\infty e^{-s\tau} f(\tau)\mathrm{d}\tau. \tag{20}$$

Three aligned equations

$$a =1, \tag{21}$$
$$b =2, \tag{22}$$
$$c =3. \tag{23}$$

Two aligned equations without equation numbers

$$\begin{aligned} a &=1, \\ b &=2. \end{aligned}$$

Mathematical derivations aligned at the "=" sign:

$$\begin{aligned}\frac{1}{2+3j} &= \frac{2-3j}{(2+3j)(2-3j)}\\ &= \frac{2-3j}{2^2+3^2}\\ &= \frac{2-3j}{13}\\ &= \frac{2}{13} - j\frac{3}{13}.\end{aligned} \tag{24}$$

More mathematical derivations:

$$\begin{aligned} & as + 4 + 2s = b + (8+a)s\\ \Leftrightarrow\ & (a+2)s + 4 = b + (8+a)s\\ \Leftrightarrow\ & (a+2)s - (8+a)s = b - 4.\end{aligned}$$

Boldface math: $\boldsymbol{x}$. Vectors:

$$\boldsymbol{x} = \begin{bmatrix} x_1\\ x_2\\ x_3\end{bmatrix}. \tag{25}$$

Another example: According to Taylor's Theorem:

$$\phi(x) \approx \phi(x_0) + \phi'(x_0)(x - x_0). \tag{26}$$

Partial derivatives:

$$\frac{\partial f(x,y)}{\partial x} = x^2\cos(xy). \tag{27}$$

Common typesetting mistakes

Important Note:

- Write $\cos x$, not $cosx$.
- Write $\sin x$, $\log x$, etc — not $sinx$, $logx$ and so on.
- Write $\lim_{s\to 0^+} sF(s)$, not $lim_{s\to 0^+} sF(s)$.
- Write $F(s)$, not $F(S)$.
- Write xy, or $x\dot{y}$, but not $x * y$ — that would be the *convolution* of x with y, not their product, so
$$2 * 3 = 3t^2. \tag{28}$$
- To denote a variable with a subscript, write x_1, not $x1$.
- For superscripts, write x^2, not $x2$.
- Denote a variable by x, not x.
- Double quotes are "like this", not ”like this”.
- Reference equations using: Equation (27); not Equation 27 and not Equation (12).

1.5. **Problem A4.**

1.6. **Problem A5.**

2. Part B: Analysis and Controller Design

2.1. **Problem B1.**

2.2. **Problem B2.**

2.3. **Problem B3.**

2.4. **Problem B4.**

2.5. **Problem B5.**

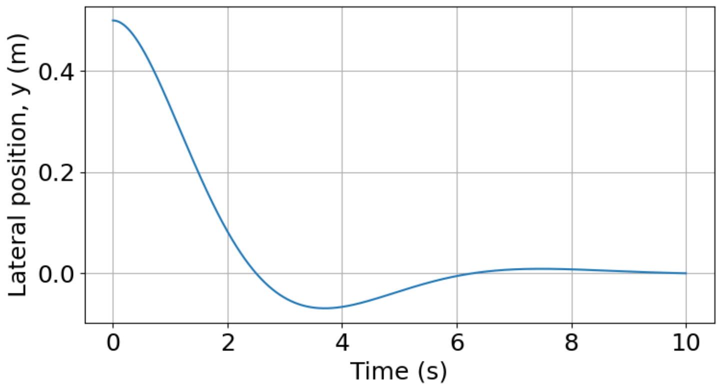

FIGURE 1. You may of course include figures in your document. Make sure your figures have legible axis labels. Figures of about this size are perfectly legible.

2.6. **Problem B6.**

3. Part C: Bonus Questions

3.1. **Problem C1.** Determine the Laplace transform of f(t) = $\log\left(t^3\right)$, $t > 0$.

The solution is:

$$\left(\frac{\sqrt{3}\left(-G_{5,2}^{2,3}\left(\begin{matrix} \frac{2}{3}, \frac{1}{3}, 0 & 1, 1 \\ 0, 0 & \end{matrix} \middle| \frac{27}{s^3}\right) + G_{5,2}^{0,5}\left(\begin{matrix} \frac{2}{3}, \frac{1}{3}, 0, 1, 1 & \\ & 0, 0 \end{matrix} \middle| \frac{27}{s^3}\right)\right)}{2\pi s},\ 0,\ \text{True} \right) \tag{29}$$

3.2. **Problem C2.** Determine the Laplace transform of f(t) = $|\cos(\omega)|$), $t \geq 0$, $\omega > 0$.

The solution is:

$$\left(\frac{|\cos(\omega)|}{\omega},\ 0,\ \text{True} \right) \tag{30}$$

3.3. **Problem C3.**

4. Part D: Planning, Organisation & Collaboration

4.1. **D1.** We used a Github repository to collaborate on writing code. We used the git issue tracker to keep track of the tasks we were assigned and how those tasks were progressing. Here is the link to the issue tracker here.

4.2. **D2.**

4.2.1. *Communication.* We communicated clearly

4.2.2. *Organisation.* We began early and kept on top of our work.

4.2.3. *Education.* We all taught each other new things. etc

4.3. **D3.** The restrictions that exist due to Covid-19 presented was the main challenge that was presented to us. The fact that we were apart meant communication was limited to calls, texts and emails. Because these forms of communication are inherently more limited than face-to-face conversing it was important that we we communicate, we clearly define the goals of it.