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(1)

For type 1 : Saddle Point

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$\det(A) = -4 < 0$, oburously the type of A is Saddle Point.

Assuming that $\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}$,

$$\bar{A} = A + \tilde{A} = \begin{bmatrix} 2+\tilde{\alpha} & 0 \\ 0 & -2+\tilde{\gamma} \end{bmatrix}$$

To make $\bar{A}$'s type Saddle Point,

$\det(\bar{A}) < 0 \Rightarrow (2+\tilde{\alpha})(-2+\tilde{\gamma}) < 0$

Let $\tilde{\alpha} = 1$, $\tilde{\gamma} = 1$, the above inequality is well met

Thus we have $\tilde{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ whose type is also Saddle Point like A.

(2)

For type 2 : Center

$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, $tr(A) = 0$, $\det(A) = 4 > 0$.

oburously the type of A is Center.

Assuming that $\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}$

then $\bar{A} = A + \tilde{A} = \begin{bmatrix} \tilde{\alpha} & 2 \\ -2 & \tilde{\gamma} \end{bmatrix}$

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To make $\bar{A}$'s type Center, conditions $tr(\bar{A}) = 0$ and $\det(\bar{A}) > 0$ should be met.
That is, $tr(\bar{A}) = \tilde{\alpha} + \tilde{\gamma} = 0$

$\det(\bar{A}) = \tilde{\alpha} \cdot \tilde{\gamma} + 4 > 0$

Let $\tilde{\alpha} = 1$, $\tilde{\gamma} = -1$, the above conditions are well met

Thus we have $\tilde{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$ whose type is also Center like A

(3)

For type 3 : Stable Node :

$A = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$, $tr(A) = \cancel{-4}\,-6 < 0$, $\det(A) = \cancel{4}\,8 > 0$

$\Delta = tr(A)^2 - 4\det(A) = \cancel{0}\ 4 > 0$

obviously the type of A is Saddle Node

Assuming that $\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}$, $\bar{A} = A + \tilde{A} = \begin{bmatrix} \tilde{\alpha}-2 & 0 \\ 0 & \tilde{\gamma}-4 \end{bmatrix}$

To make $\bar{A}$'s type Saddle Node

$tr(\bar{A}) = \tilde{\alpha} + \tilde{\gamma} - \cancel{4}\,6 < 0$, $\det(\bar{A}) = (\tilde{\alpha}-2)(\tilde{\gamma}-4) > 0$

$\Delta = tr(\tilde{A})^2 - 4\det(\tilde{A}) = (\tilde{\alpha} + \tilde{\gamma} - \cancel{4}\,6)^2 - 4(\tilde{\alpha}-2)(\tilde{\gamma}-4) > 0$

Let $\tilde{\alpha} = -1$, $\tilde{\gamma} = -1$, the above inequalities are well met

Thus we have $\tilde{A} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A = \begin{bmatrix} -3 & 0 \\ 0 & -5 \end{bmatrix}$ whose type is also Saddle Node like A

(4)

For type 4 : Unstable Node

$A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$, $tr(A) = 6 > 0$; $det(A) = 8 > 0$

$\Delta = tr(A)^2 - 4\,det(A) = 4 > 0$

obviously the type of A is Unstable Node.

Assuming that $\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}$

then $\bar{A} = A + \tilde{A} = \begin{bmatrix} 2+\tilde{\alpha} & 0 \\ 0 & 4+\tilde{\gamma} \end{bmatrix}$.

To make $\bar{A}$'s type Unstable Node.

$tr(\bar{A}) = 6 + \tilde{\alpha} + \tilde{\gamma} > 0$

$det(\bar{A}) = (2+\tilde{\alpha})(4+\tilde{\gamma}) > 0$

$\Delta = tr(A)^2 - 4\,det(\bar{A}) = (6+\tilde{\alpha}+\tilde{\gamma})^2 - 4(2+\tilde{\alpha})(4+\tilde{\gamma}) > 0$

Let $\tilde{\alpha} = 1$, $\tilde{\gamma} = 1$, the above inequalities are well satisfied.

Thus $\tilde{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\bar{A} = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$

whose type is Unstable Node like A

(5)

For type 5 : Stable Spiral

$A = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix}$, $tr(A) = -4 < 0$
$det(A) = 8 > 0$

$\Delta = tr(A)^2 - 4\,det(A) = -16 < 0$

obviously the type of A is Stable Spiral

Assuming that $\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}$, $\bar{A} = \begin{bmatrix} \tilde{\alpha}-2 & 2 \\ -2 & \tilde{\gamma}-2 \end{bmatrix}$

To make $\bar{A}$'s type Stable Spiral,

$tr(\bar{A}) = \tilde{\alpha} + \tilde{\gamma} - 4 < 0$, $det(\bar{A}) = (\tilde{\alpha}-2)(\tilde{\gamma}-2)+4 > 0$, $\Delta = tr(A)^2 - 4\,det(A)$

$= (\tilde{\alpha} + \tilde{\gamma} - 4)^2 - 4[(\tilde{\alpha}-2)(\tilde{\gamma}-2)+4] < 0$

Let $\tilde{\alpha} = 1$, $\tilde{\gamma} = 1$, the above inequalities are well satisfied.

Thus $\tilde{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix}$

whose type is Stable Spiral like A

(6)

For type 6 : Unstable Spiral

$A = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$, $tr(A) = 4 > 0$
$det(A) = 8 > 0$

$\Delta = tr(A)^2 - 4\,det(A) = -16 < 0$

obviously the type of A is Unstable Spiral

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$$\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} \tilde{\alpha}+2 & 2 \\ -2 & \tilde{\gamma}+2 \end{bmatrix}$$

If $\bar{A}$'s type is Unstable Spiral,

$tr(\bar{A}) = \tilde{\alpha} + \tilde{\gamma} + 4 > 0$

$\det(\bar{A}) = (\tilde{\alpha}+2)(\tilde{\gamma}+2) + 4 > 0$

$\Delta = tr(\bar{A})^2 - 4\det(\bar{A}) = (\tilde{\alpha}+\tilde{\gamma}+4)^2$

$- 4[(\tilde{\alpha}+2)(\tilde{\gamma}+2)+4] < 0$

Let $\tilde{\alpha} = 1$, $\tilde{\gamma} = 1$, then the above inequalities are satisfied.

Thus $\tilde{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$

whose type is Unstable Spiral like A.

(7)

For type 7: Star

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $tr(A) = 4$, $\det(A) = 4 > 0$

$\Delta = tr(A)^2 - 4\det(A) = 0$

obviously A's type is Star

$$\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} 2+\tilde{\alpha} & 0 \\ 0 & 2+\tilde{\gamma} \end{bmatrix}$$

If $\bar{A}$'s type is also Star

$tr(\bar{A}) = 4 + \tilde{\alpha} + \tilde{\gamma}$, $\det(\bar{A}) = -(2+\tilde{\alpha})(2+\tilde{\gamma}) > 0$

$\Delta = tr(\bar{A})^2 - 4\det(\bar{A}) = (4+\tilde{\alpha}+\tilde{\gamma})^2$

$+ 4(2+\tilde{\alpha})(2+\tilde{\gamma}) = 0$

(8)

For type 8 : Degenerate Node

$A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$, $tr(A) = 2$ $det(A) = 0$

obviously A's type is Degenerate Node

$\tilde{A} = \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{r} \end{bmatrix}$, $\bar{A} = A + \tilde{A} = \begin{bmatrix} 2+\tilde{\alpha} & 0 \\ 0 & \tilde{\gamma} \end{bmatrix}$

If $\bar{A}$'s type is Degenerate Node

$det(\bar{A}) = (2+\tilde{\alpha})\tilde{\gamma} = 0$