Flux, Jacobian and Hessian of a three-phase subphysics

1 Definitions

Let s_w, s_o and s_g be the saturations of water, oil and gas, respectively. All three saturations are related:

$$s_\mathrm{w} + s_\mathrm{o} + s_\mathrm{g} = 1. \tag{1}$$

Let i be one of w, o or g. Then the following definitions apply: Let k_i stand for the *permeability*. Let μ_i stand for the *viscosity*. Let $\lambda_i = k_i/\mu_i$ stand for the *mobility* and let $\lambda = \lambda_\mathrm{w} + \lambda_\mathrm{o} + \lambda_\mathrm{g}$ be the *total mobility*. Let g_i stand for the *density*. Let v stand for the *velocity*.

2 Flux

The flux of a three-phase subphysics is of the form:

$$F(s_\mathrm{w}, s_\mathrm{o}) = \begin{pmatrix} f_\mathrm{w} \\ f_\mathrm{o} \end{pmatrix}. \tag{2}$$

2.1 f_w

The part of the flux that corresponds to the water is defined as:

$$f_\mathrm{w} = \frac{\lambda_\mathrm{w}}{\lambda}(v + \lambda_\mathrm{o}(g_\mathrm{w} - g_\mathrm{o}) + \lambda_\mathrm{g}(g_\mathrm{w} - g_\mathrm{g})). \tag{3}$$

2.2 f_o

The part of the flux that corresponds to the water is defined as:

$$f_\mathrm{o} = \frac{\lambda_\mathrm{o}}{\lambda}(v + \lambda_\mathrm{w}(g_\mathrm{o} - g_\mathrm{w}) + \lambda_\mathrm{g}(g_\mathrm{o} - g_\mathrm{g})). \tag{4}$$

3 Jacobian

3.1 $\partial f_\mathrm{w}/\partial s_\mathrm{w}$