DS 4400

Machine Learning and Data Mining I
Spring 2024

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March 15 2024

Outline

- Ensemble learning
  - Boosting
    - AdaBoost
    - Properties of boosting
    - Bagging vs Boosting
- Deep learning demo
- Overview to deep learning
  - History of deep learning
  - Perceptron and limitations

Ensemble Learning

Consider a set of classifiers $h_1, \ldots, \mathrm{h}_\mathrm{L}$

Idea: construct a classifier $H(\mathbf{x})$ that combines the individual decisions of $h_1, \ldots, \mathrm{h}_\mathrm{L}$

- e.g., could have the member classifiers vote, or
- e.g., could use different members for different regions of the instance space

Successful ensembles require **diversity**

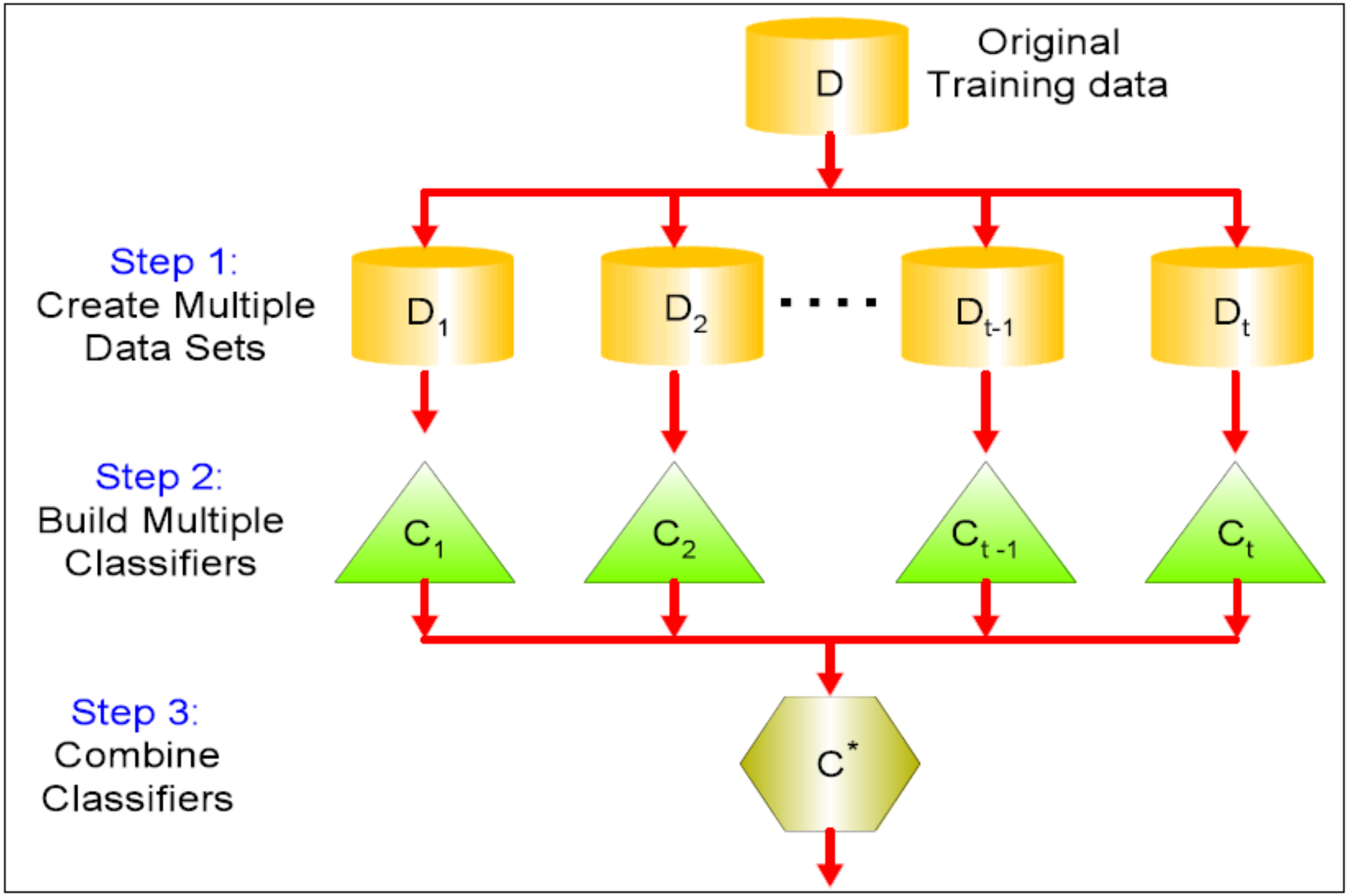
- Classifiers should make different mistakes
- Can have different types of base learners

How to Achieve Diversity

- Avoid overfitting
  - Randomize the training data
- Features are noisy
  - Randomize the set of features

Two main ensemble learning methods

Bagging



Majority Votes

Random Forests

- Ensemble method specifically designed for decision tree classifiers
- Introduce two sources of randomness: "Bagging" and "Random input vectors"
  - Bagging method: each tree is grown using a bootstrap sample of training data
  - Random vector method: At each node, best split is chosen from a random sample of *m* attributes instead of all attributes

AdaBoost

- A meta-learning algorithm with great theoretical and empirical performance
- Turns a base learner (i.e., a "weak hypothesis") into a high performance classifier
- Creates an ensemble of weak hypotheses by repeatedly emphasizing mispredicted instances

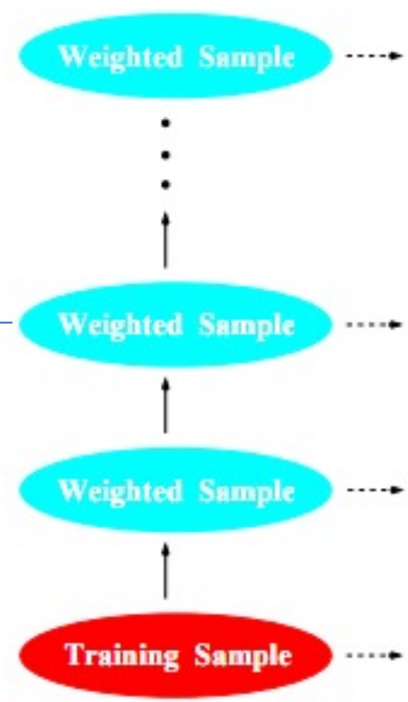
Adaptive Boosting
Freund and Schapire 1997

Overview of AdaBoost

Sequential training process

$$G(x) = sign(\sum_{t=1}^{T} \beta_t h_t(x))$$

Better classifiers will get higher weights



$h_T(x)$

$h_3(x)$

$h_2(x)$

$h_1(x)$

- Mis-classified examples get higher weights
- Correct examples get lower weights

Uniform weights

FIGURE 10.1. *Schematic of AdaBoost. Classifiers are trained on weighted versions of the dataset, and then combined to produce a final prediction.*

AdaBoost

```
Initialize a vector of n uniform weights w_1
for t = 1, ..., T
    Train model h_t on X, y with weights w_t
    Compute the weighted training error of h_t
    Choose β_t = 1/2 ln((1 − ε_t)/ε_t)
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        w_{t+1,i} = w_{t,i} exp(−β_t y_i h_t(x_i))
    Normalize w_{t+1} to be a distribution
end for
Return the hypothesis
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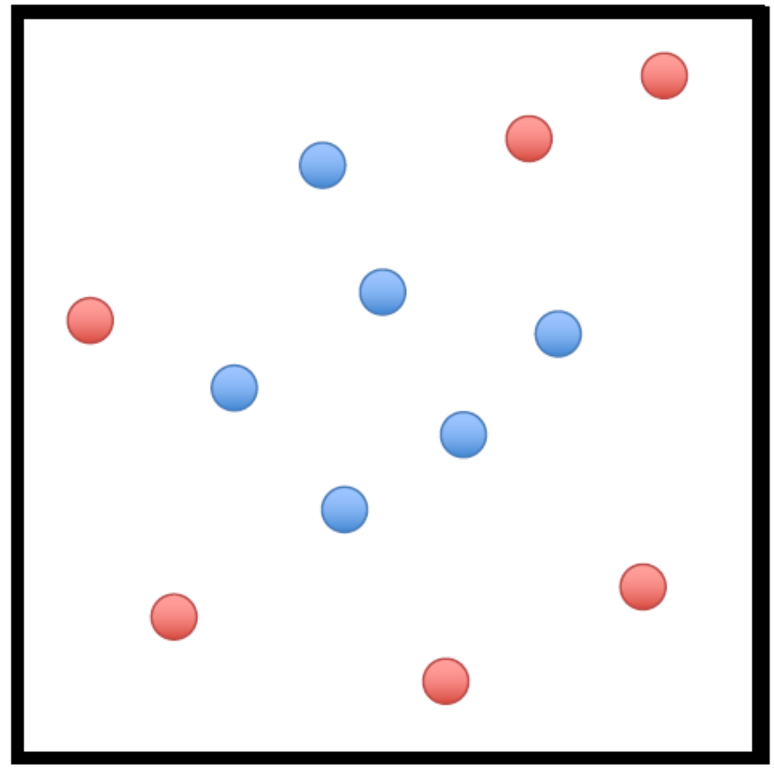
Normalize $\mathbf{w}_{t+1}$ to be a distribution

end for

Return the hypothesis

$$H(\mathbf{x}) = \operatorname{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x}) \right)$$



- Size of point represents the instance's weight

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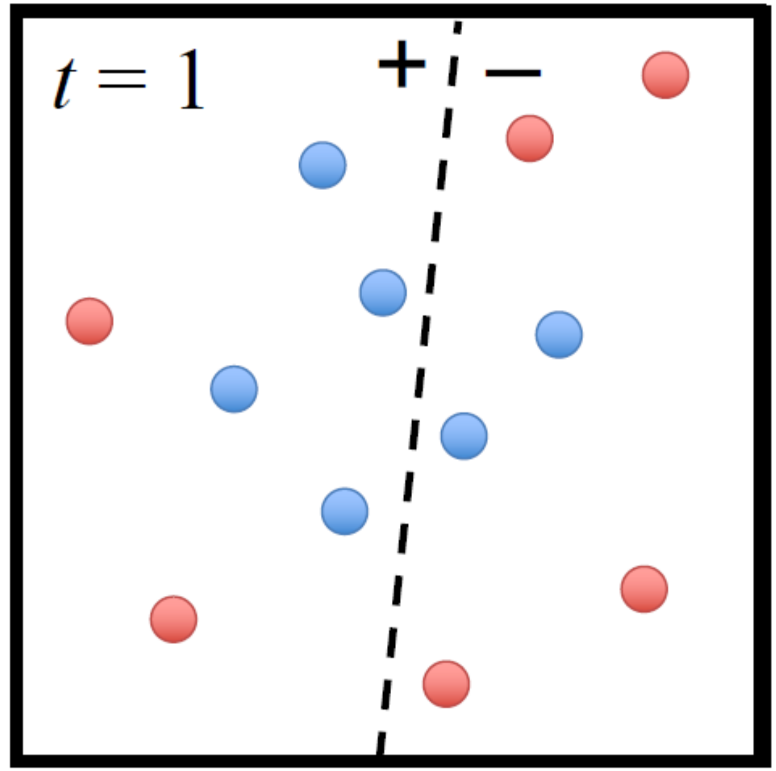

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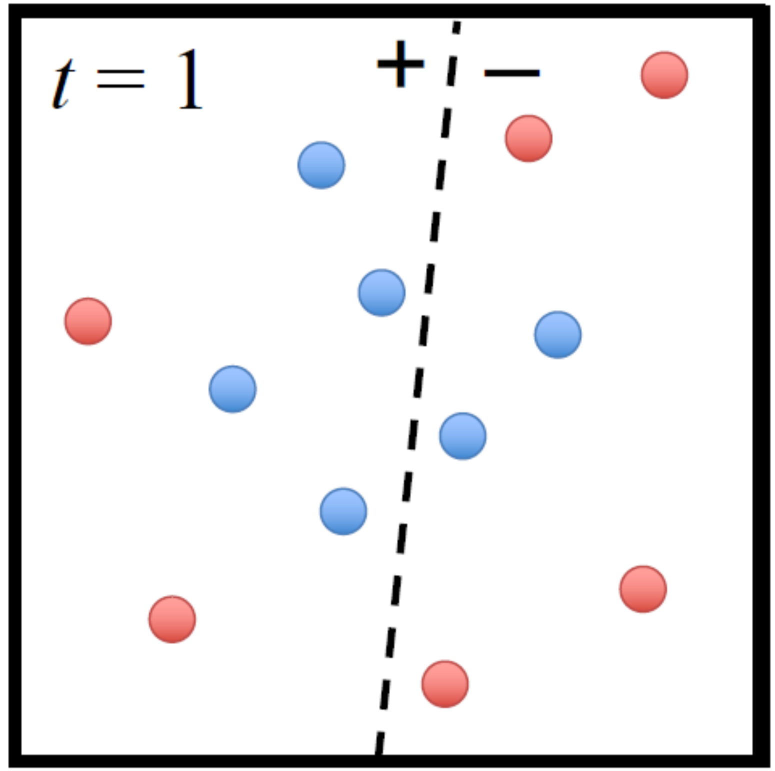

- β_t measures the importance of h_t
- If $\epsilon_t \leq 0.5$, then $\beta_t \geq 0$ (can trivially guarantee)

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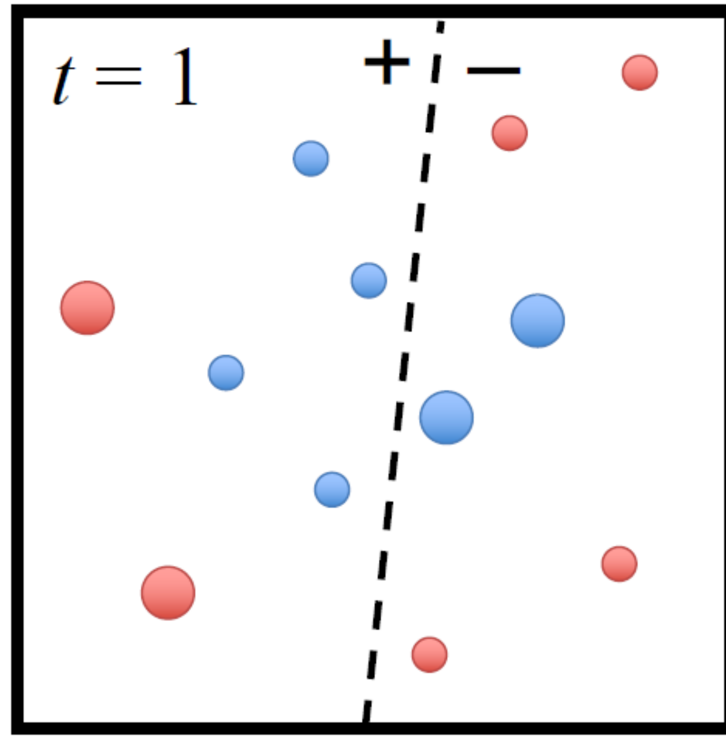

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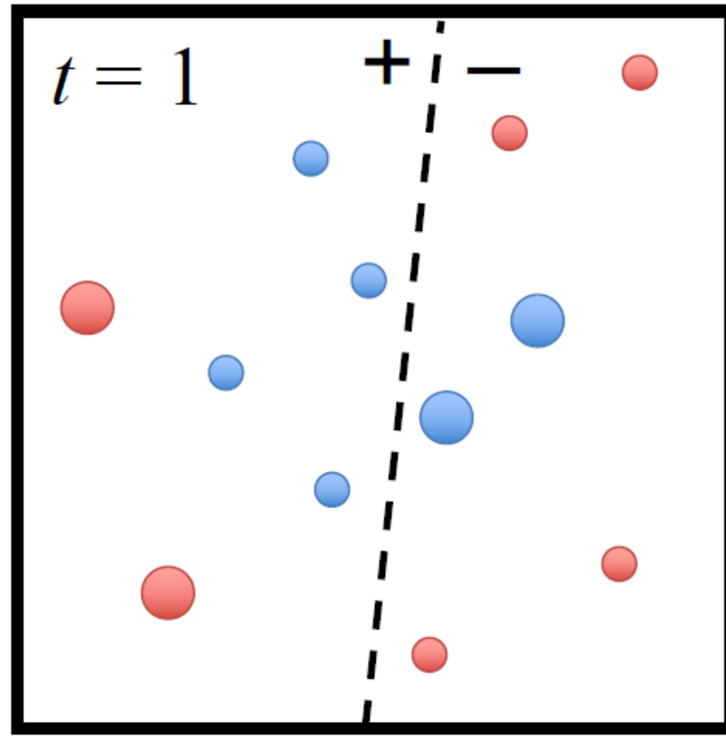

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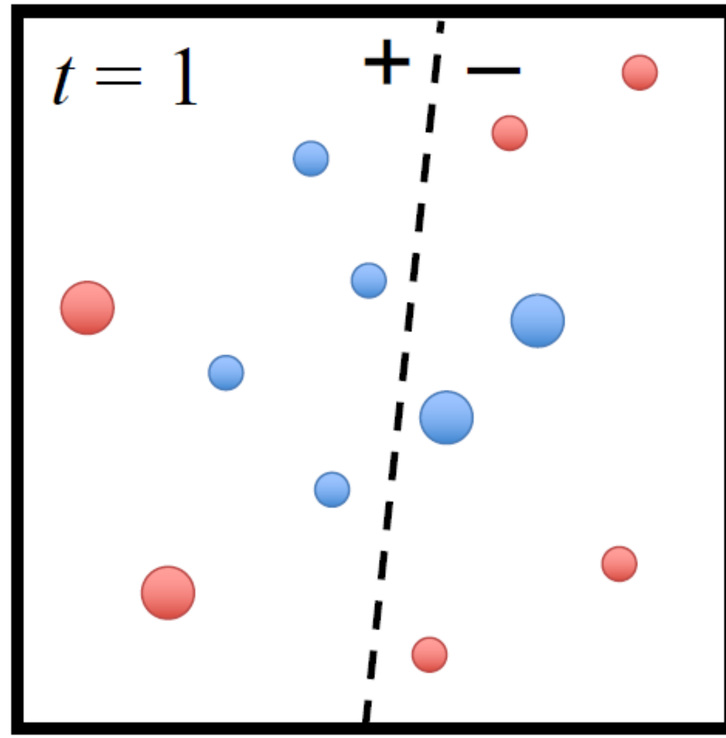

- Weights of correct predictions are multiplied by $e^{-\beta_t} \leq 1$
- Weights of incorrect predictions are multiplied by $e^{\beta_t} \geq 1$

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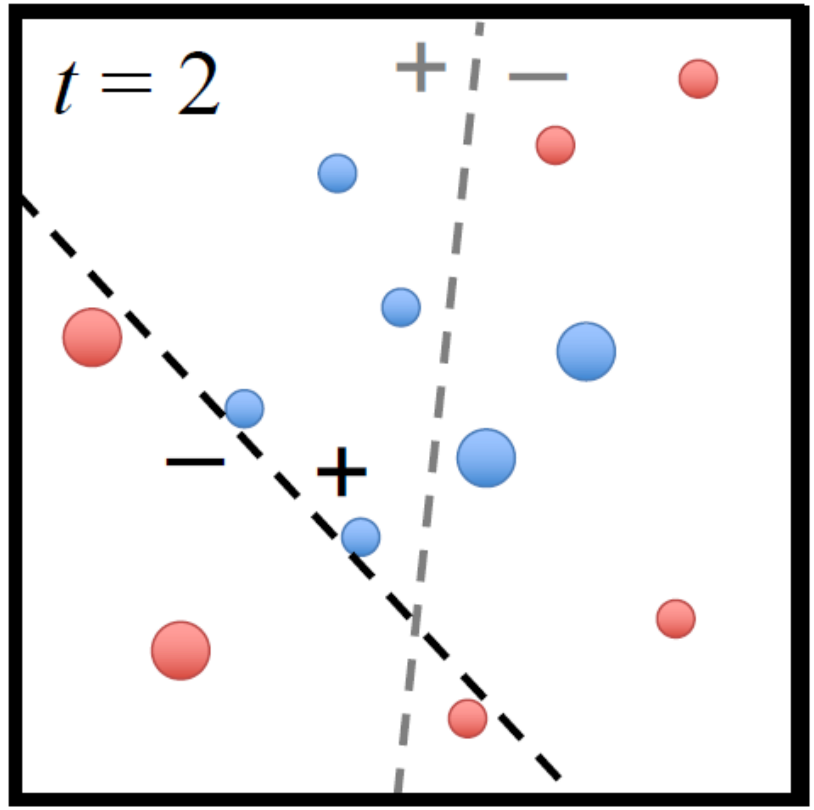

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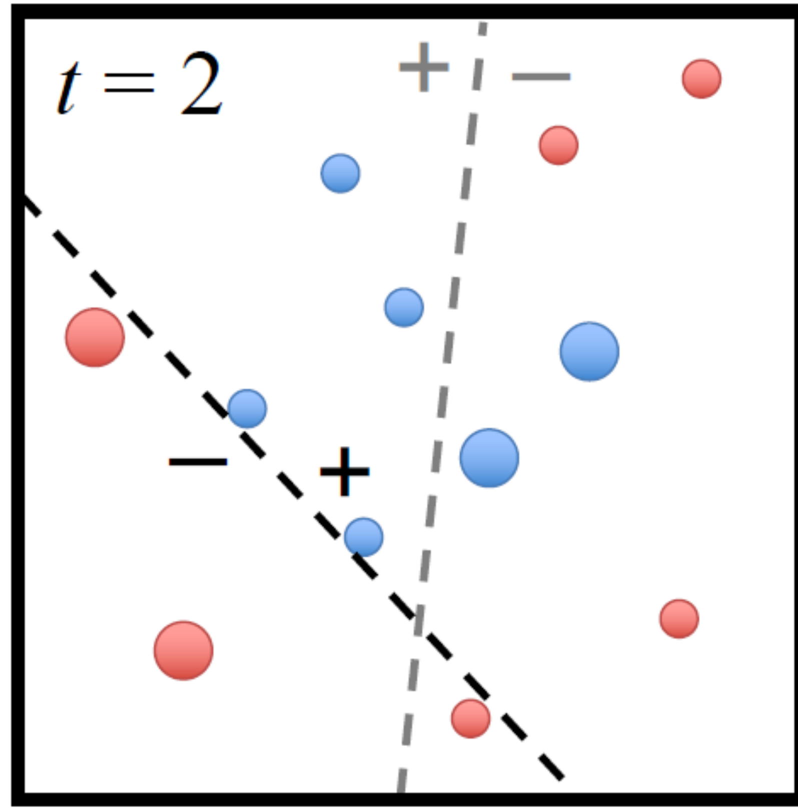

- Compute importance of hypothesis β_t
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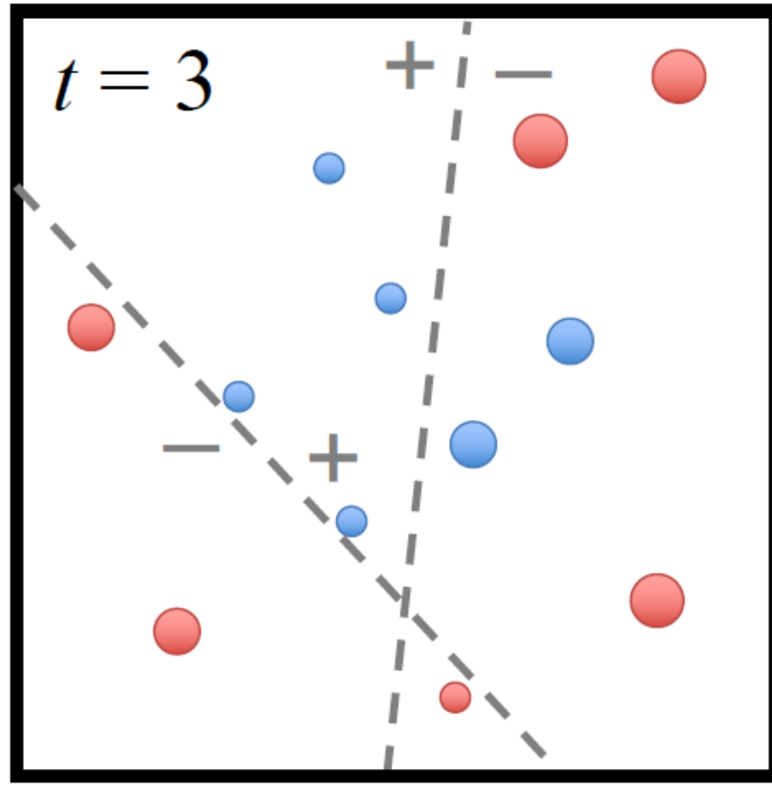

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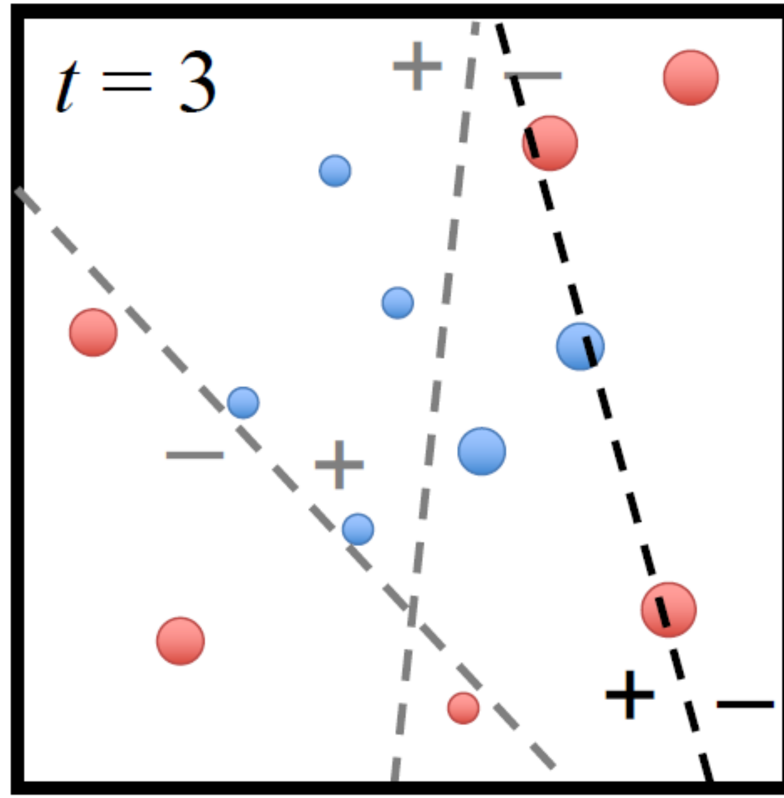

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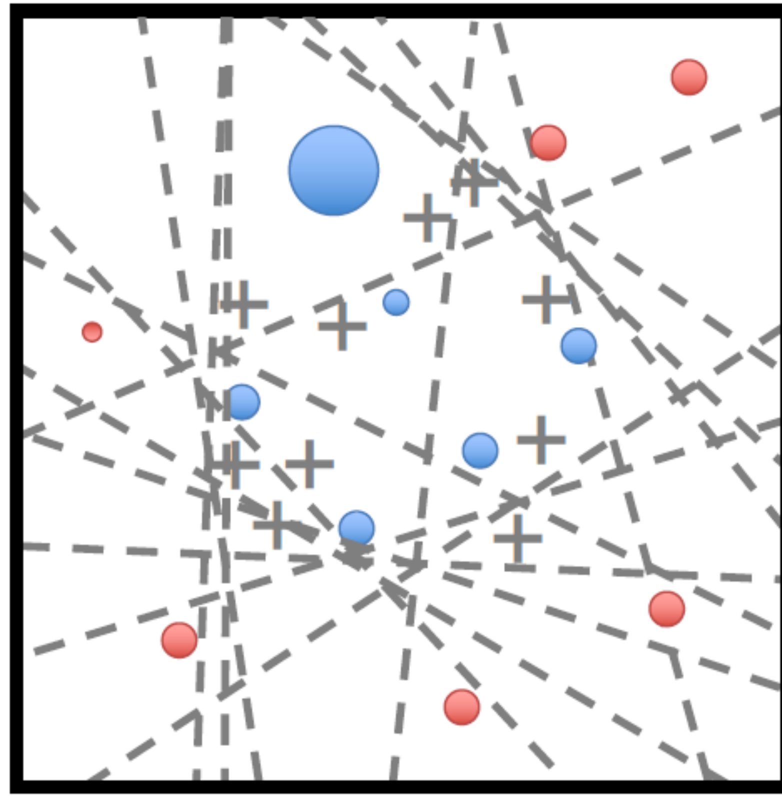
$t = \mathrm{T}$



- Final model is a weighted combination of members
  - Each member weighted by its importance

AdaBoost

INPUT: training data $X, y = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n}$,
the number of iterations T

1: Initialize a vector of n uniform weights $\mathbf{w}_1 = \left[\frac{1}{n}, \ldots, \frac{1}{n}\right]$
2: **for** $t = 1, \ldots, T$
3: Train model h_t on X, y with instance weights $\mathbf{w}_t$
4: Compute the weighted training error rate of h_t:

$$\epsilon_t = \sum_{i:y_i \neq h_t(\mathbf{x}_i)} w_{t,i}$$

5: Choose $\beta_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$
6: Update all instance weights:

$$w_{t+1,i} = w_{t,i} \exp\left(-\beta_t y_i h_t(\mathbf{x}_i)\right) \quad \forall i = 1, \ldots, n$$

7: Normalize $\mathbf{w}_{t+1}$ to be a distribution:

$$w_{t+1,i} = \frac{w_{t+1,i}}{\sum_{j=1}^{n} w_{t+1,j}} \quad \forall i = 1, \ldots, n$$

8: **end for**
9: **Return** the hypothesis

$$H(\mathbf{x}) = \mathrm{sign}\left(\sum_{t=1}^{T} \beta_t h_t(\mathbf{x})\right)$$

Train with Weighted Instances

Train with Weighted Instances

- For algorithms like logistic regression, can simply incorporate weights w into the cost function
  - Essentially, weigh the cost of misclassification differently for each instance

$$J_{\text{reg}}(\boldsymbol{\theta}) = -\sum_{i=1}^{n} w_i \left[y_i \log h_{\boldsymbol{\theta}}(\mathbf{x}_i) + (1 - y_i) \log \left(1 - h_{\boldsymbol{\theta}}(\mathbf{x}_i)\right)\right] + \lambda \|\boldsymbol{\theta}_{[1:d]}\|_2^2$$

- For algorithms that don't directly support instance weights (e.g., ID3 decision trees, etc.), use weighted bootstrap sampling
  - Form training set by resampling instances with replacement according to w

Properties

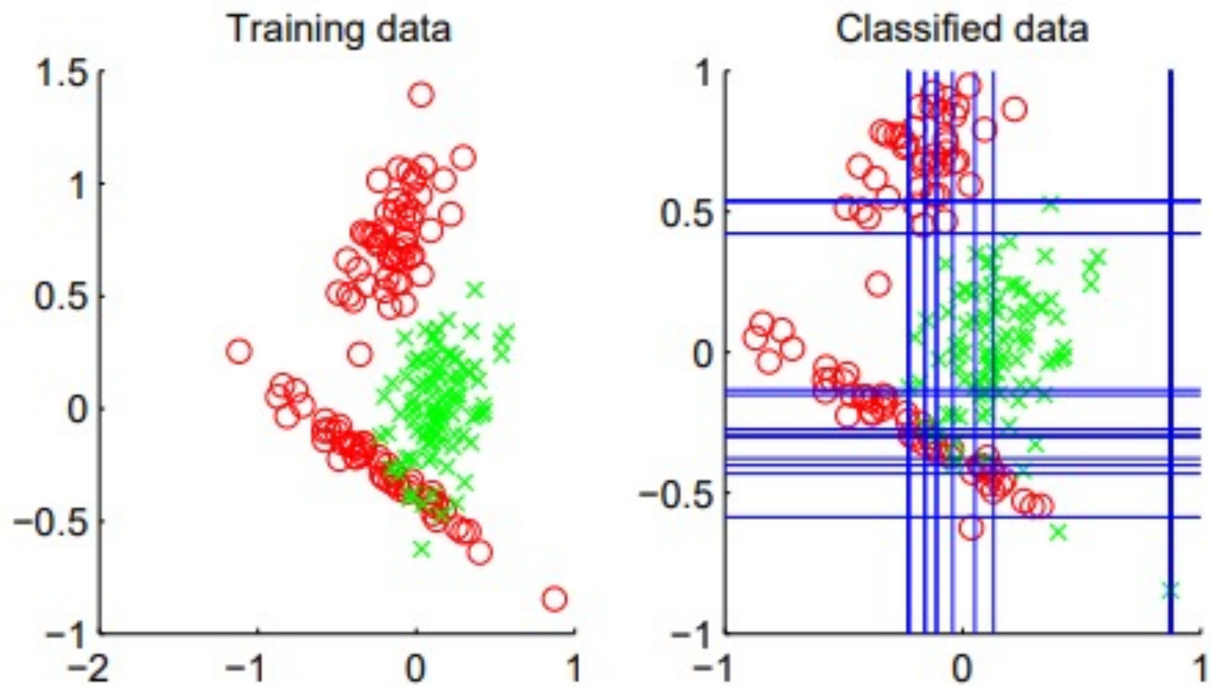
- If a point is repeatedly misclassified
  – Its weight is increased every time
  – Eventually it will generate a hypothesis that correctly predicts it
- In practice AdaBoost does not typically overfit
- Does not use explicitly regularization

Base Learner Requirements

- AdaBoost works best with "weak" learners
  - Should not be complex
  - Typically high bias classifiers
  - Works even when weak learner has an error rate just slightly under 0.5 (i.e., just slightly better than random)
    - Can prove training error goes to 0 in $O(\log n)$ iterations

- Examples:
  - Decision stumps (1 level decision trees)
  - Depth-limited decision trees
  - Linear classifiers

AdaBoost with Decision Stumps



AdaBoost in Practice

Strengths:

- Fast and simple to program
- No parameters to tune (besides T) Learn with Cross-Validation
- No assumptions on weak learner Error less than ½

When boosting can fail:

- Given insufficient data
- Overly complex weak hypotheses
- Can be susceptible to noise
- When there are a large number of outliers

Bagging vs Boosting

Bagging vs Boosting

Bagging vs.	**Boosting**
Resamples data points	Reweights data points (modifies their distribution)
Weight of each classifier is the same	Weight is dependent on classifier's accuracy
Only variance reduction	Both bias and variance reduced – learning rule becomes more complex with iterations

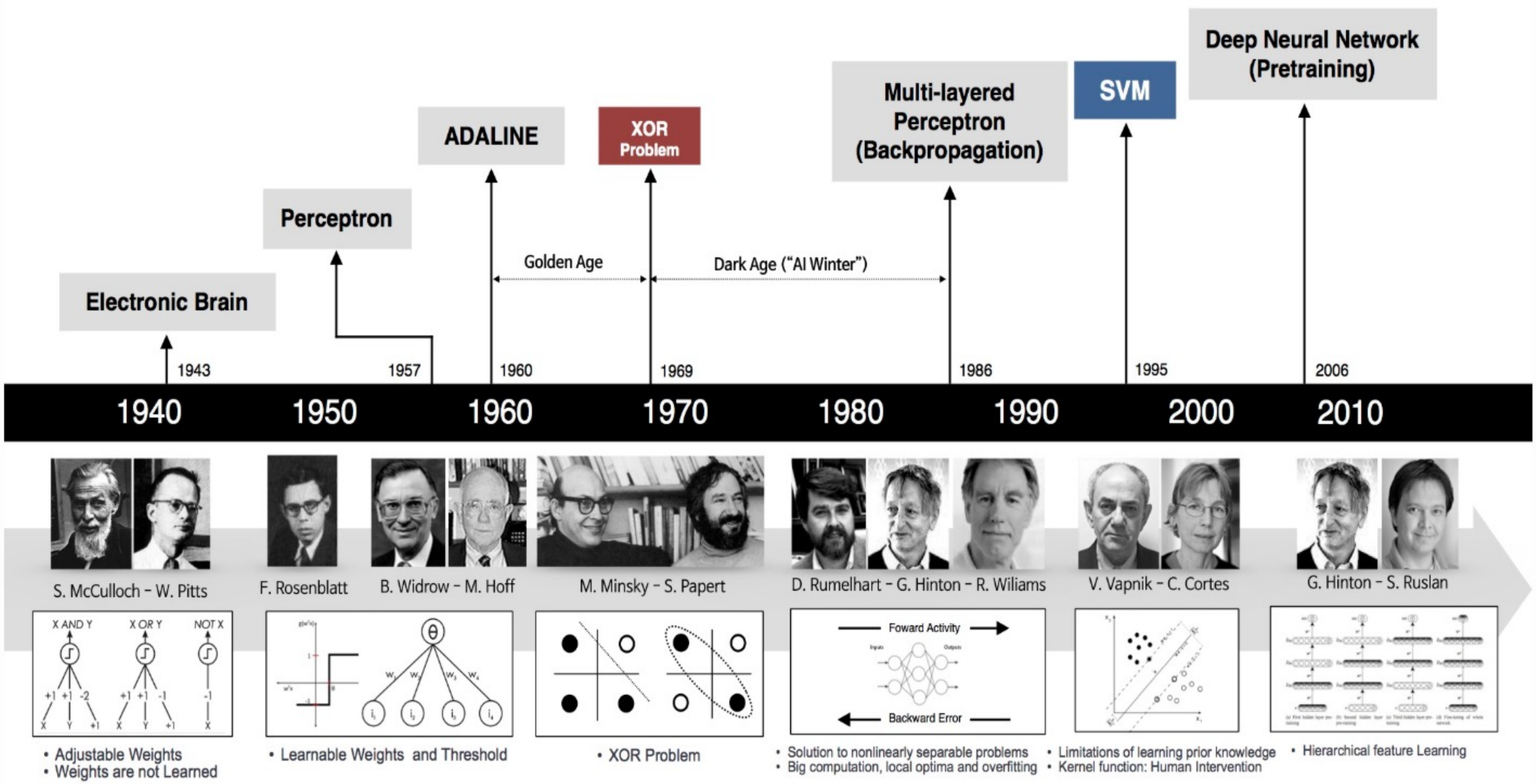
Applicable to complex models with low bias, high variance

Applicable to weak models with high bias, low variance

Review

- Ensemble learning are powerful learning methods
  - Better accuracy than standard classifiers
- Bagging uses bootstrapping (with replacement), trains T models, and averages their prediction
  - Random forests vary training data and feature set at each split
- Boosting is an ensemble of T weak learners that emphasizes mis-predicted examples
  - AdaBoost has great theoretical and experimental performance
  - Can be used with linear models or simple decision trees (stumps, fixed-depth decision trees)

History of Deep Learning

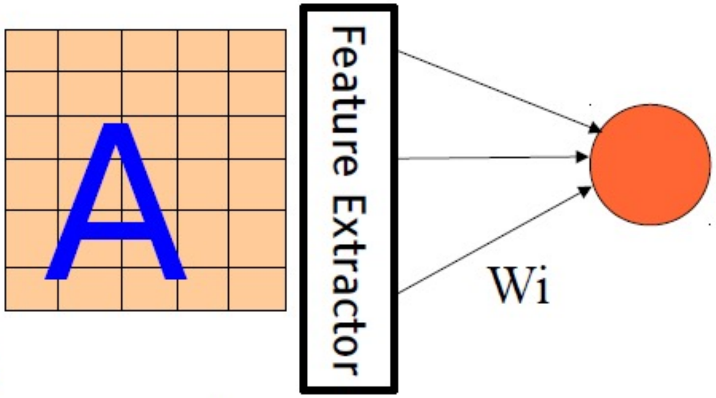


References

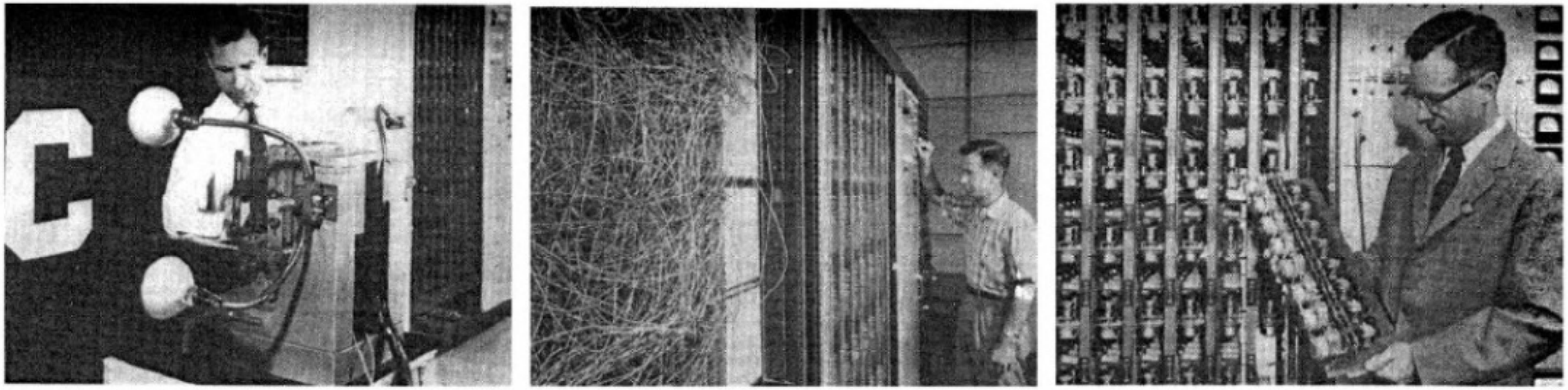
- Deep Learning books
  - https://d2l.ai/ (D2L)
  - https://www.deeplearningbook.org/ (advanced)
- Stanford notes on deep learning
  - http://cs229.stanford.edu/summer2020/cs229-notes-deep_learning.pdf

Early Models

- The first learning machine: the Perceptron
  - Built at Cornell in 1960
- The Perceptron was a linear classifier on top of a simple feature extractor
- The vast majority of practical applications of ML today use glorified linear classifiers or glorified template matching.
- Designing a feature extractor requires considerable efforts by experts.

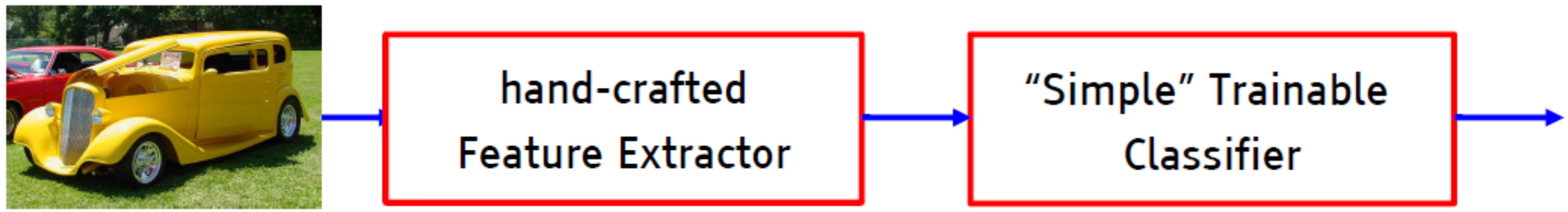


$$y = sign\left(\sum_{i=1}^{N} W_i F_i(X) + b\right)$$

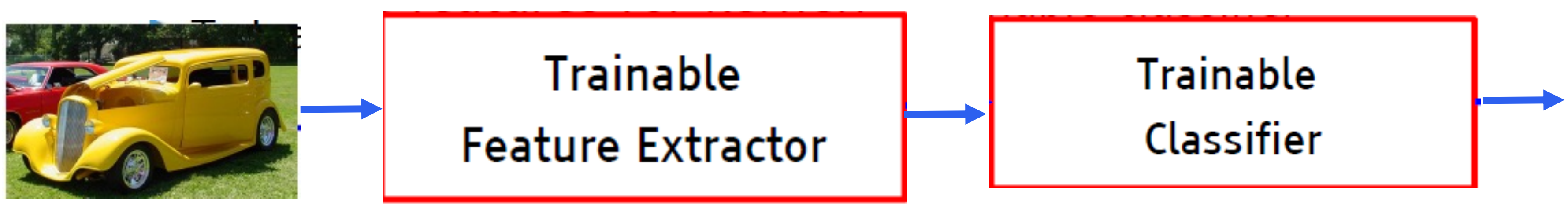


Deep Learning

- **The traditional model of pattern recognition (since the late 50's)**
  - Fixed/engineered features (or fixed kernel) + trainable classifier

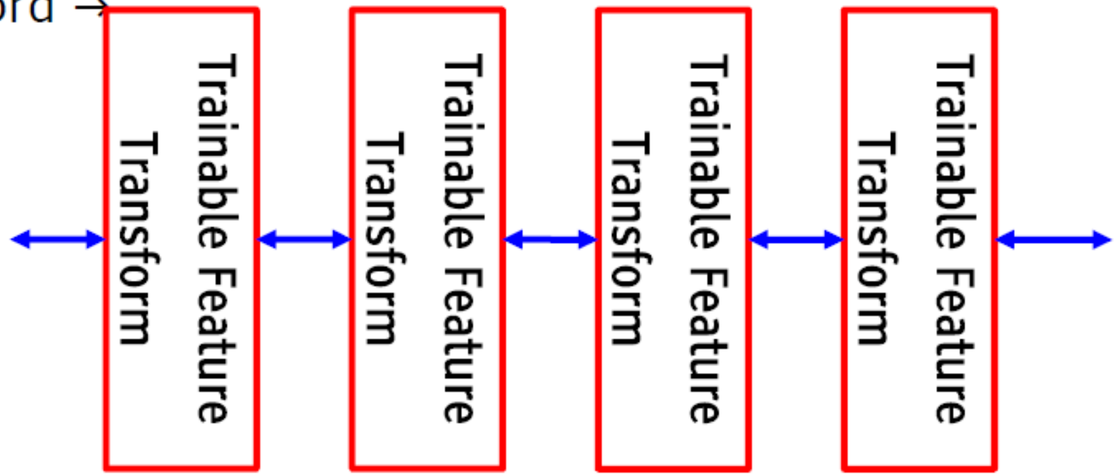


- End-to-end learning / Feature learning / Deep learning

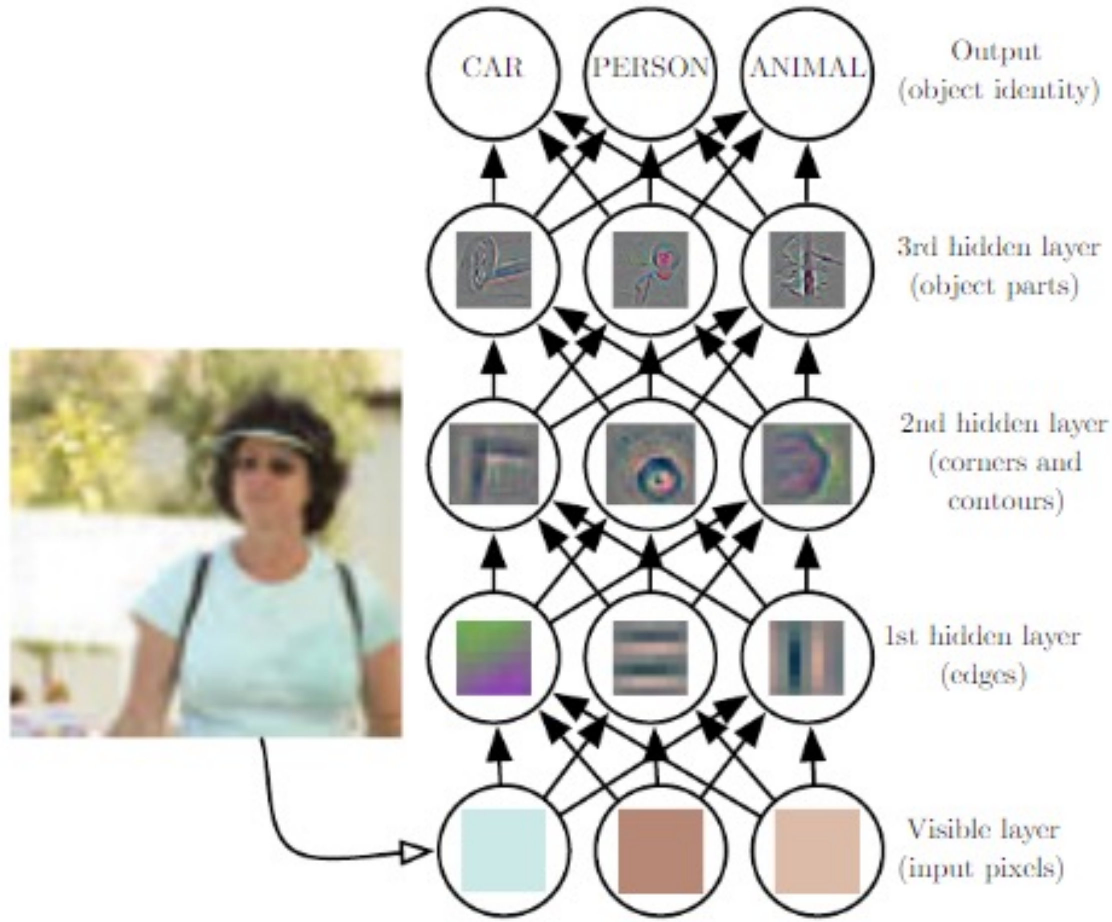


Trainable Feature Hierarchy

- Hierarchy of representations with increasing level of abstraction
- Each stage is a kind of trainable feature transform
- Image recognition
  - Pixel → edge → texton → motif → part → object
- Text
  - Character → word → word group → clause → sentence → story
- Speech
  - Sample → spectral band → sound → ... → phone → phoneme → word →



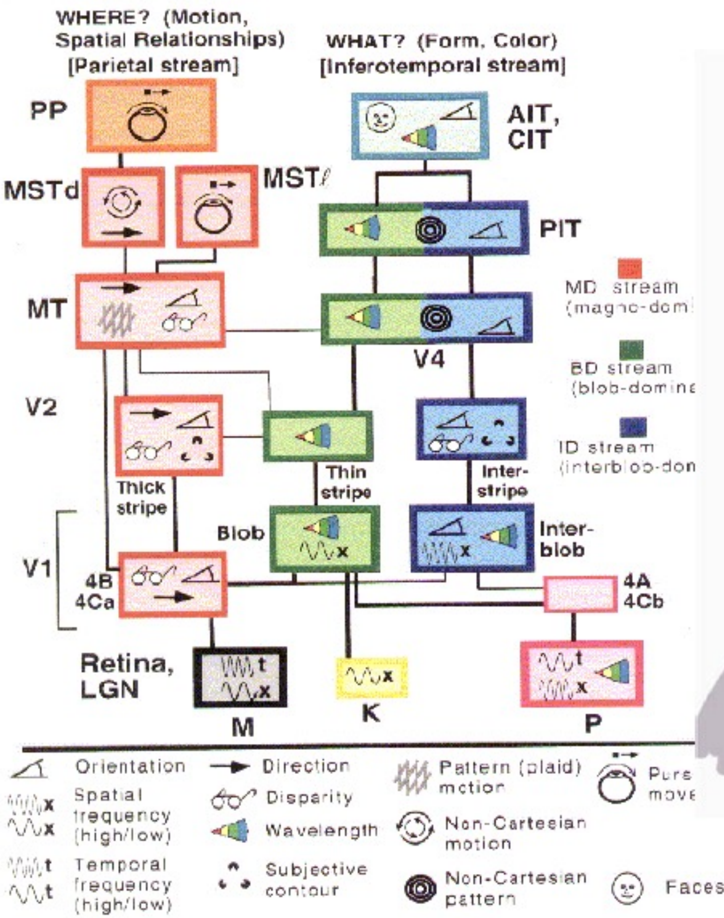
Learning Representations

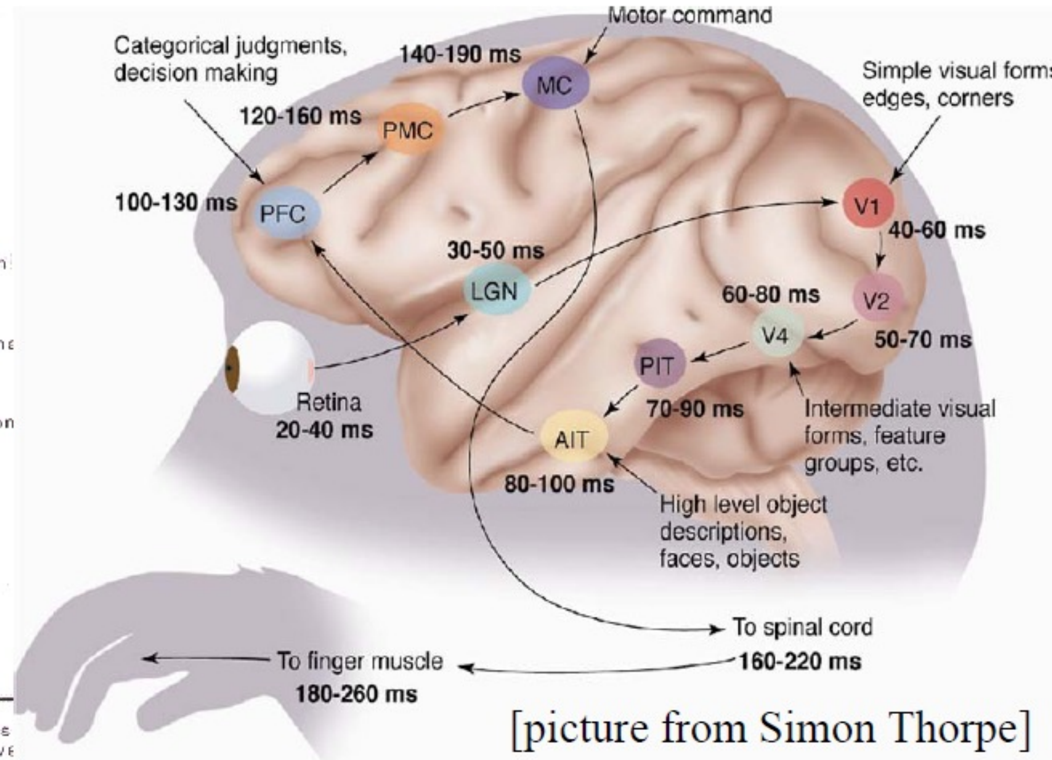


Deep Learning addresses the problem of learning hierarchical representations

The Visual Cortex is Hierarchical

- The ventral (recognition) pathway in the visual cortex has multiple stages
- Retina - LGN - V1 - V2 - V4 - PIT - AIT ....
- Lots of intermediate representations



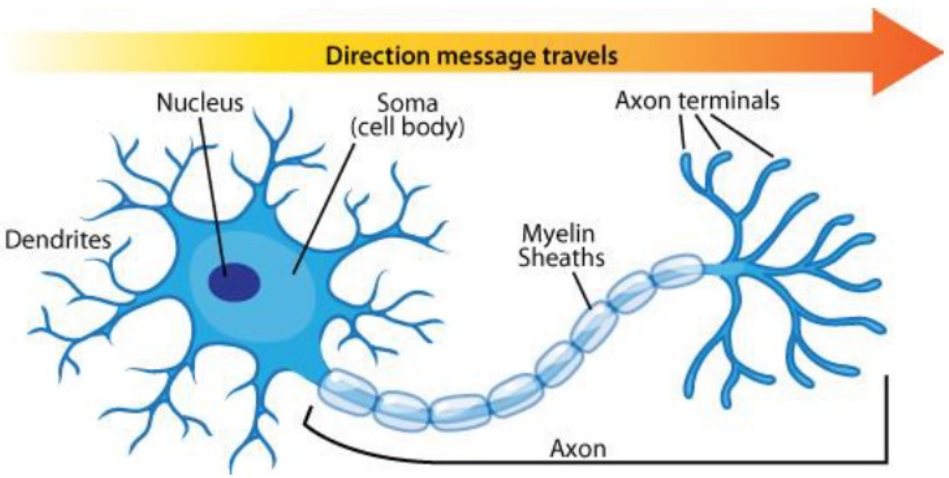


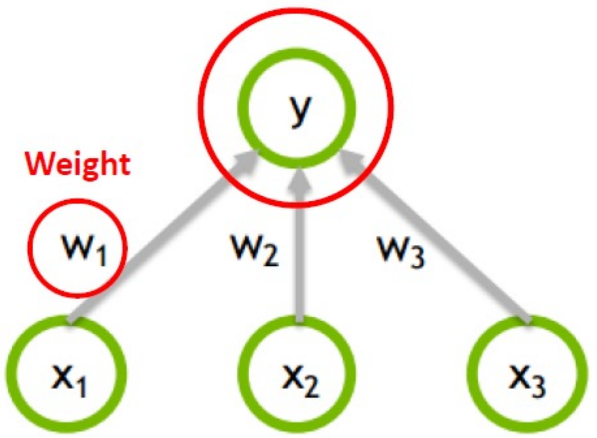
[picture from Simon Thorpe]

[Gallant & Van Essen]

Analogy to Human Brain

Human Brain





$$y=F(w_1x_1+w_2x_2+w_3x_3)$$

$$F(x)=\max(0,x)$$

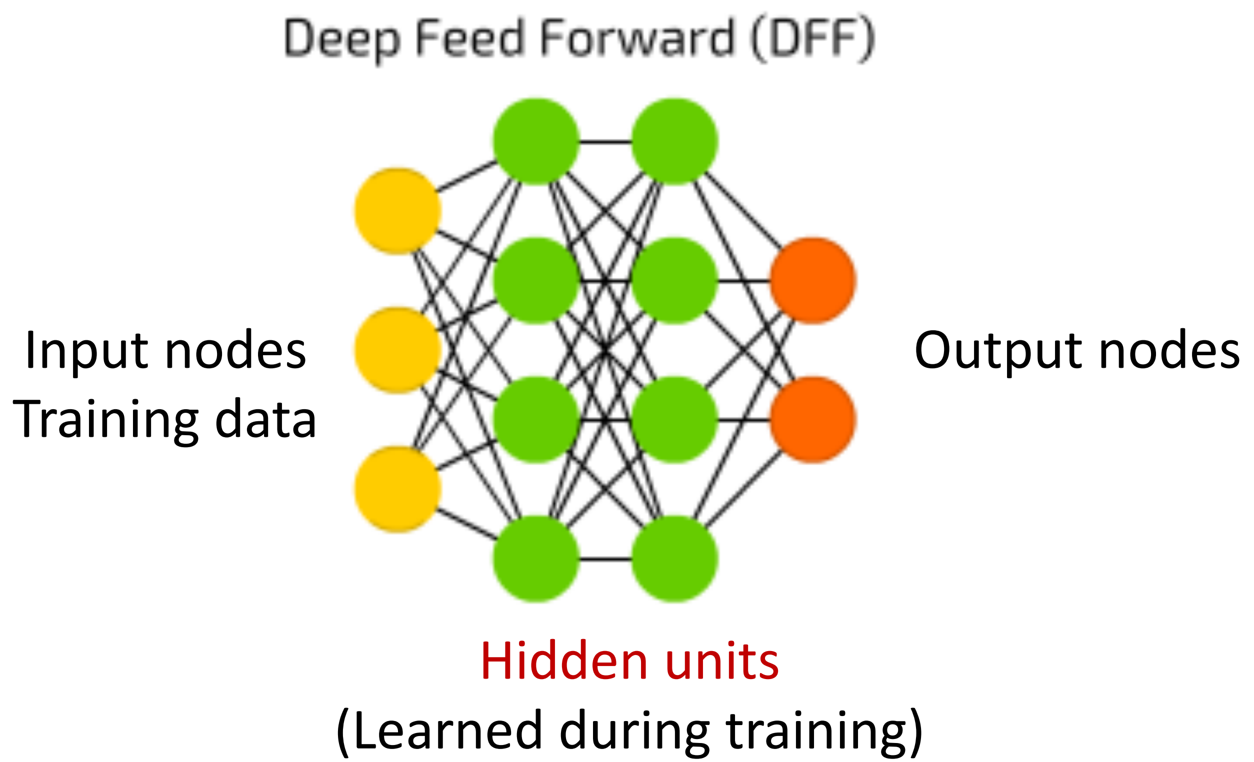
Biological Neuron

Artificial Neuron

Neural Networks

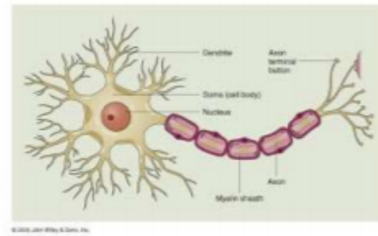
- Origins: Algorithms that try to mimic the brain.
- Very widely used in 80s and early 90s; popularity diminished in late 90s.
- Recent resurgence: State-of-the-art technique for many applications
- Artificial neural networks are not nearly as complex or intricate as the actual brain structure

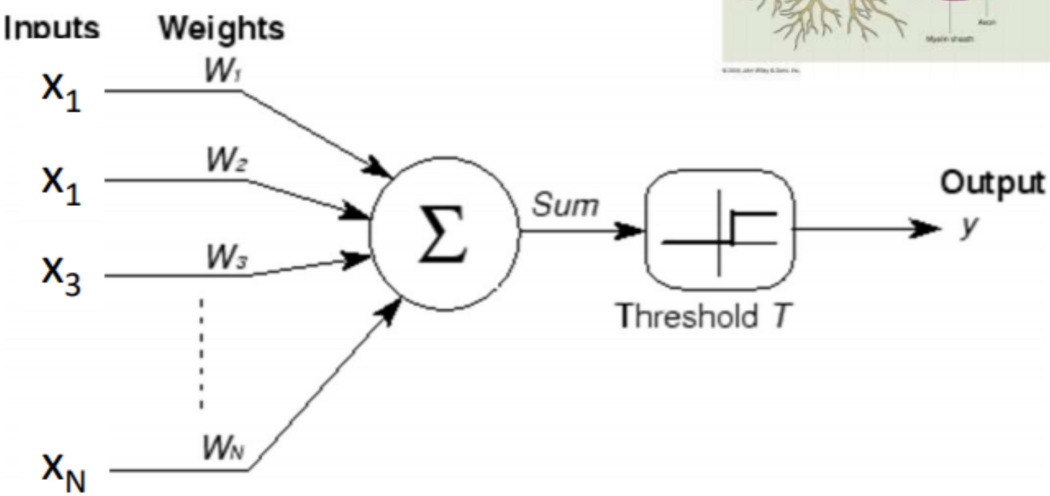
Neural Networks



- Neural networks: made up from node / units connected by links
- Each link has a weight and activation function
- Feed-forward neural networks
  - Training data is input to left and prediction are output on right

Perceptron





$$y = \begin{cases} 1 \; if \; \sum_i w_i x_i \geq T \\ 0 \; else \end{cases}$$

- A threshold unit
  - "Fires" if the weighted sum of inputs exceeds a threshold

The Perceptron

$$h(\boldsymbol{x}) = \text{sign}(\boldsymbol{\theta}^{\mathsf{T}}\boldsymbol{x}) \quad \text{where} \quad \text{sign}(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ -1 & \text{if } z < 0 \end{cases}$$

- The perceptron uses the following update rule each time it receives a new training instance (x_i, y_i)

$$\boxed{\theta_j \leftarrow \theta_j - \frac{1}{2}\left(h_\theta(x_i) - y_i\right)x_{ij}}$$

  - If the prediction matches the label, make no change
  - Otherwise, adjust $\boldsymbol{\theta}$

The Perceptron

- The perceptron uses the following update rule each time it receives a new training instance (x_i, y_i)

$$\boxed{\theta_j \leftarrow \theta_j - \frac{1}{2}\left(h_\theta(x_i) - y_i\right)x_{ij}}$$

The Perceptron

- The perceptron uses the following update rule each time it receives a new training instance (x_i, y_i)

$$\boxed{\theta_j \leftarrow \theta_j - \frac{1}{2}\underbrace{(h_\theta(x_i) - y_i)}_{\text{either 2 or -2}}x_{ij}}$$

- Re-write as $\boxed{\theta_j \leftarrow \theta_j + y_i x_{ij}}$ (only upon misclassification)

Perceptron Rule: If x_i is misclassified, do

$$\theta \leftarrow \theta + y_i\, x_i$$

Online Perceptron

Let $\theta \leftarrow$[0,0,…,0]
Repeat:
    Receive training example (x_i, y_i)
    If $y_i \theta^T x_i \leq 0$ // prediction is incorrect
        $\theta \leftarrow \theta + y_i\, x_i$
Until stopping condition

Online learning – the learning mode where the model update is performed each time a single observation is received

Batch learning – the learning mode where the model update is performed after observing the entire training set

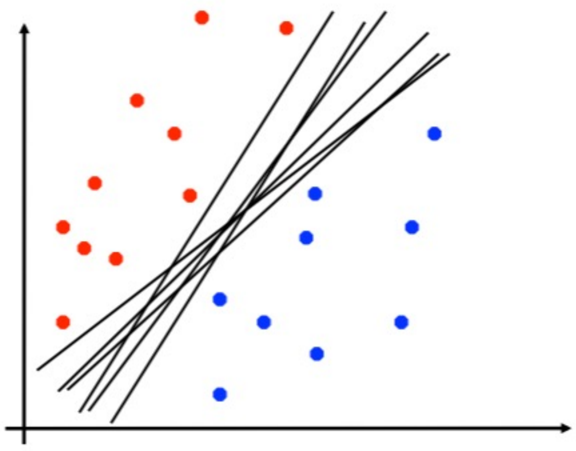
Batch Perceptron

Let $\theta \leftarrow$[0,0,…,0]
Repeat:
$\quad \Delta = [0{,}0, \ldots, 0]$
$\quad$ For $i = 1$ to N // Consider all training examples
$\quad\quad$ If $y_i \theta^T x_i \leq 0$ // Prediction is incorrect
$\quad\quad\quad \Delta \leftarrow \Delta + y_i\, x_i.$ // Accumulate all errors
$\quad \theta \leftarrow \theta + \frac{\Delta}{N}$ // Parameter update rule
Until stopping condition

- Guaranteed to find separating hyperplane if data is linearly separable

Perceptron Limitations

- Is dependent on starting point
- It could take many steps for convergence
- Perceptron can overfit
  - Move the decision boundary for every example



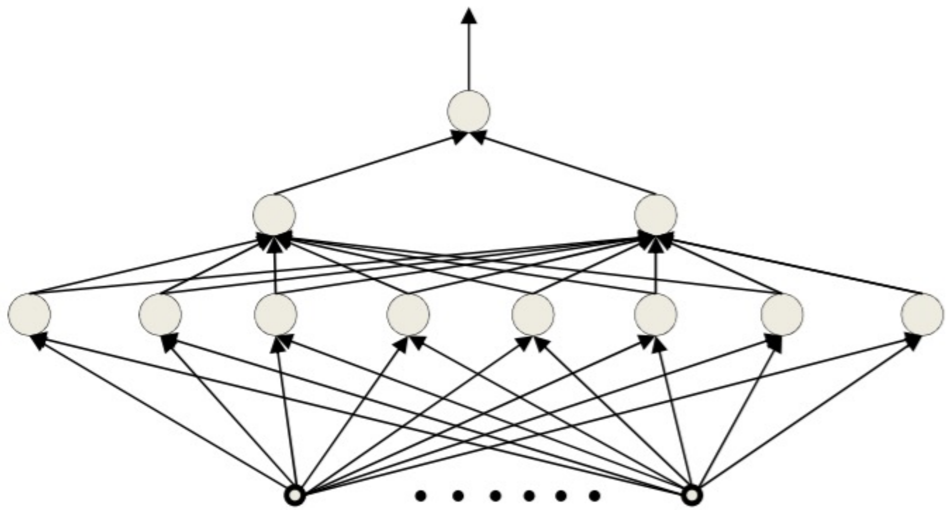
Which of this is optimal?

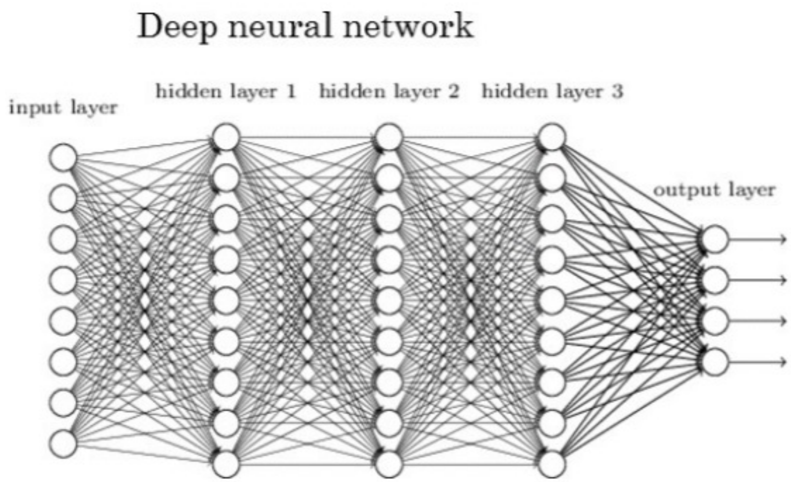
History of Perceptrons

- They were popularised by Frank Rosenblatt in the early 1960’s.
  - They appeared to have a very powerful learning algorithm.
  - Lots of grand claims were made for what they could learn to do.
- In 1969, Minsky and Papert published a book called “Perceptrons” that analysed what they could do and showed their limitations.
  - Many people thought these limitations applied to all neural network models.
- The perceptron learning procedure is still widely used today for tasks with enormous feature vectors that contain many millions of features.

They are the basic building blocks for Deep Neural Networks

Multi-Layer Perceptron

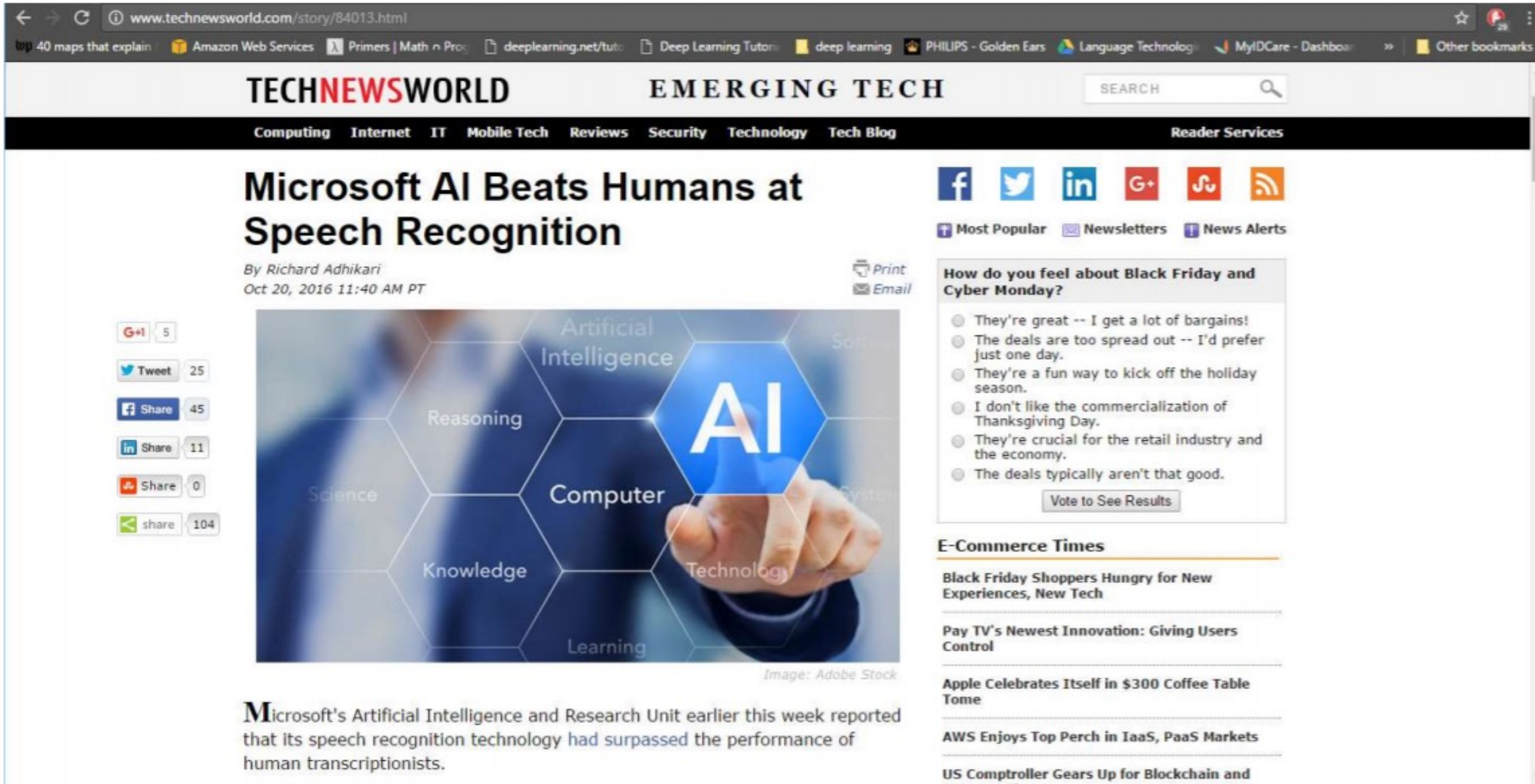




- A network of perceptrons
  - Generally "layered"

Deep Learning Applications

Success stories: Speech recognition



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Microsoft AI Beats Humans at Speech Recognition

By Richard Adhikari
Oct 20, 2016 11:40 AM PT

Image: Adobe Stock

Microsoft's Artificial Intelligence and Research Unit earlier this week reported that its speech recognition technology had surpassed the performance of human transcriptionists.

How do you feel about Black Friday and Cyber Monday?

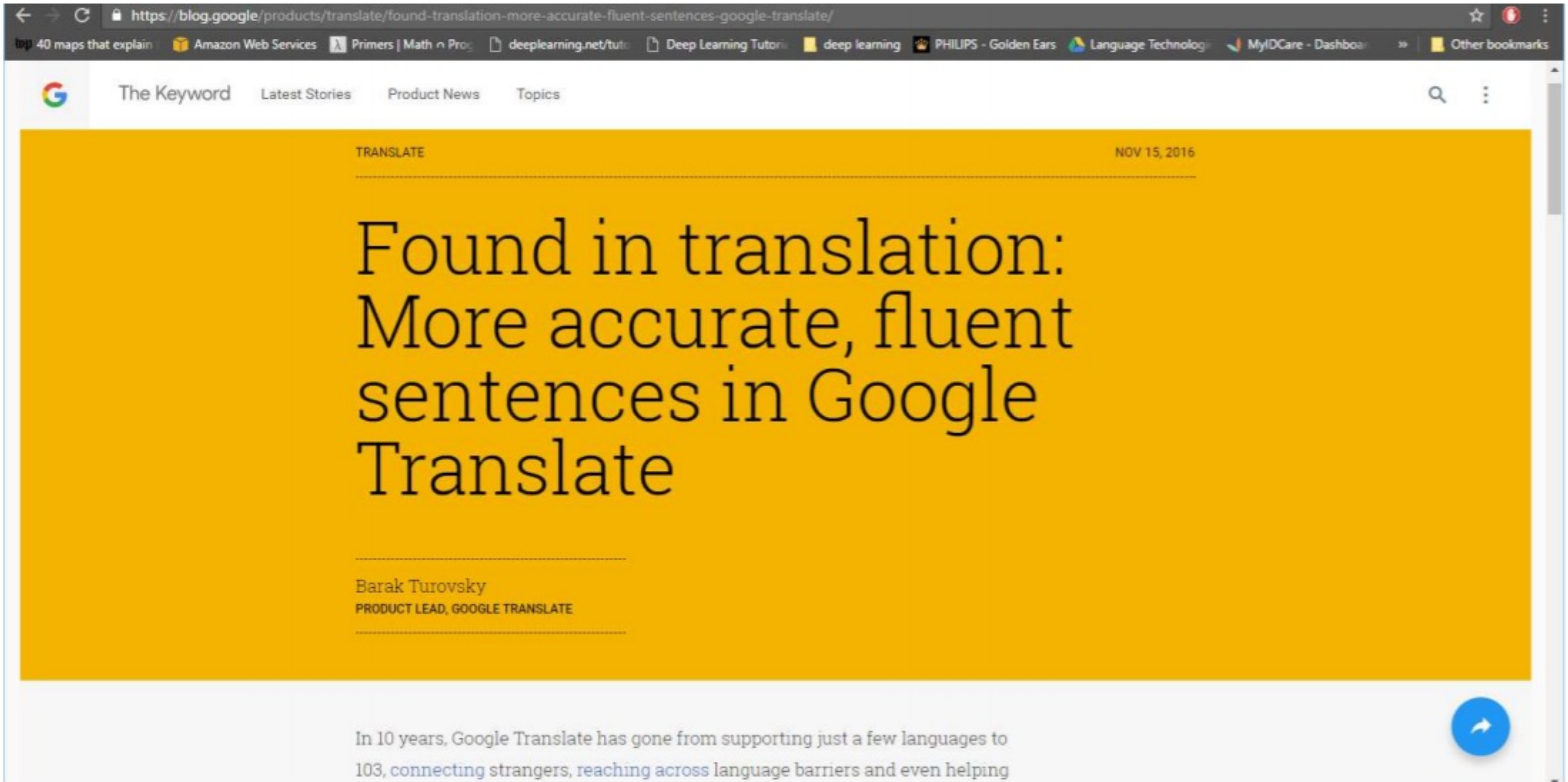
- They're great -- I get a lot of bargains!
- The deals are too spread out -- I'd prefer just one day.
- They're a fun way to kick off the holiday season.
- I don't like the commercialization of Thanksgiving Day.
- They're crucial for the retail industry and the economy.
- The deals typically aren't that good.

Vote to See Results

E-Commerce Times

- Black Friday Shoppers Hungry for New Experiences, New Tech
- Pay TV's Newest Innovation: Giving Users Control
- Apple Celebrates Itself in $300 Coffee Table Tome
- AWS Enjoys Top Perch in IaaS, PaaS Markets
- US Comptroller Gears Up for Blockchain and

Success stories: Machine Translation



Success stories: Image segmentation