A. Teaching notes for the guided questions of Simulation III

Simulation III employs the parameters in Table I.

Center of Mass	Male Ref. %	Female Ref. %	Sim. %
cgf/f	43.0	43.4	43.2
cgu/u	43.6	45.8	44.7

Body Parameters	Male Ref. %	Female Ref. %	Sim. %
W_{FA}/W_P	2.5	2.1	2.3
W_{UA}/W_P	3.3	2.9	3.1
f/H_P	21.5	21.8	21.6
u/H_P	17.2	17.3	17.3
b/f	–	–	11.0
$d/(f+u)$	–	–	25.0

TABLE I. Parameters of the human segments in reference and in our Simulation III (the average of male and female).[1,2] The first table is for the positions of center of mass. The bottom table shows the weight and length ratios of body segments as well as the muscle insertions.

- "How do the forces on biceps and elbow joint change with different angles of their elbow bending when $\theta_{arm} = 0°$? Does the change become different for people with different BMI?"

  When $\theta_{arm} = 0°$, the calculation of the forces on biceps and elbow joint is suggested with a given value of $\theta_u (> 90°)$ before this guided question. The difficulty level of the calculation is moderate when the lever arm value of F_{bic} is given. This calculation practice can help students have deep understanding when they move to the guided question and give reasonable explanation of what they discovered from the simulation.

  Both forces on biceps and elbow joint are at a minimum when $\theta_u = 90°$ and the forces increase symmetrically when θ_u departs from 90°. Similarly to one of the guided questions in Simulation II, people with larger BMI always experience larger forces, although the difference is less pronounced for the biceps and elbow.

- “Most forces except F_{del} have the lowest values when the forearm remains vertical while the shoulder extends and flexes. What is the reason behind it?”

- “What are the optimal and most challenging poses for lifting objects? What is the range of each force experienced for different poses of their arm?” Hint: control variables. Fix one of the two variables θ_{arm} (90°, 90°) and θ_u (30°, 180°), then explore the force change with the other. The range of the second variable is also restricted by the condition $\theta_u - \theta_{arm} > 90°$,where bicep force plays the major role of maintain balance.

The sequence of guided questions is designed to encourage students to adapt their reasoning from one question to the next. Starting with a simple case where the upper arm is horizontal in the first guided question, it is straightforward for the students to geometrically connect that the zero lever arms of the weight of the ball and the weight of the forearm are the reason for the most comfortable position when the forearm is upright and to realize the role of lever arm in equilibrium. Moving to the second question, it leads the students to conclude the same reason that the easiest positions always have the forearm upright. In teaching practice, the last question can be assigned for assignments for extra credits.

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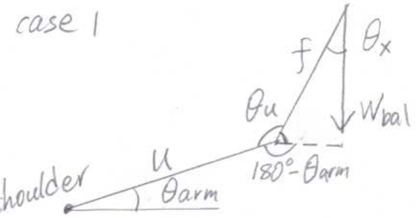
[1] Stanley Plagenhoef, F. Gaynor Evans and Thomas Abdelnour,“Anatomical data for analyzing human motion,” Res. Q. Exerc. Sport,” **54**, 169–178 (1983).

[2] Howard D. Goldick, *Mechanics, Heat and the Human Body: An Introduction to Physics*, (Pearson, New Jersey, 2001), p. 99.

Derivations of the equations in the manuscript "Interactive simulations of upper limbs"
The three cases represent different configurations of upper limbs, the upper arm and forearm above and below the horizon.
"u" and "f" are the lengths of upper arm and forearm. Weight of a ball, W_bal acts on the end of the forearm.

case 1



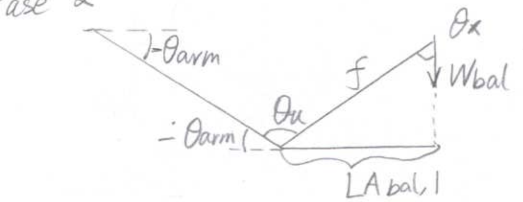
$$LA_{bal,1} = f \cdot \sin\theta_x$$

Eq(5)

Same metrical analysis for Eq(4)

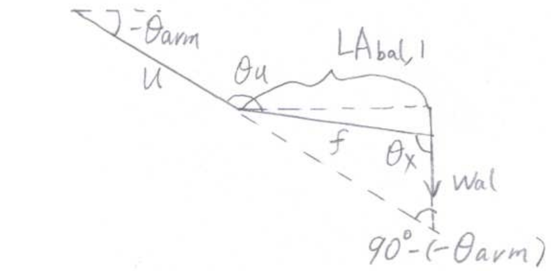
$$90° - \theta_x = 360° - \theta_u - (180° - \theta_{arm})$$
$$\theta_x = \theta_u - \theta_{arm} - 90°$$

case 2

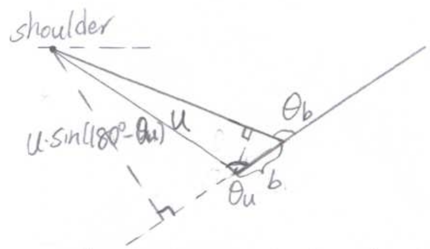


$$\theta_u - \theta_{arm} + (90° - \theta_x) = 180°$$
$$\theta_x = \theta_u - \theta_{arm} - 90°$$

case 3



$$\theta_u = \theta_x + (90° + \theta_{arm})$$
$$\theta_x = \theta_u - \theta_{arm} - 90°$$



Method 1.

$$(u^2 + b^2 - 2ub\cos\theta_u) + b^2 - 2\sqrt{u^2 + b^2 - 2ub\cos\theta_u} \cdot b \cdot \cos\theta_b = u^2$$

$$\cos\theta_b = \frac{2b^2 - 2ub\cos\theta_u}{2\sqrt{u^2+b^2-2ub\cos\theta_u} \cdot b} = \frac{b - u\cos\theta_u}{\sqrt{u^2+b^2-2ub\cdot\cos\theta_u}}$$

Method 2. check the triangle area

$$b \cdot u \sin(180° - \theta_u) = \sqrt{u^2+b^2-2ub\cdot\cos\theta_u} \cdot b \cdot \sin\theta_b$$

$$\sin\theta_b = \frac{u\sin\theta_u}{\sqrt{u^2+b^2-2ub\cos\theta_u}}$$

Eq(7) for θ_b

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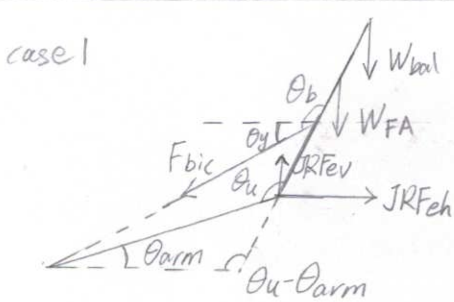
case 1



$$JRF_{eh} = F_{bic} \cdot \cos\theta_y$$

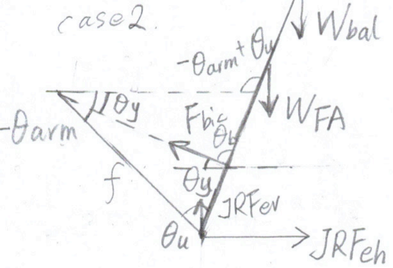
$$(\theta_u - \theta_{arm}) + \theta_y = \theta_b \quad \therefore \theta_y = \theta_b - \theta_u + \theta_{arm}$$

$$JRF_{eh} = F_{bic}\cos\theta_y = F_{bic} \cdot \cos(-\theta_y)$$
$$= F_{bic}\cos(-\theta_{arm} + \theta_u - \theta_b)$$

Eq(8)

$$JRF_{ev} = F_{bic}\sin\theta_y + W_{FA} + W_{bal}$$
$$= F_{bic}\sin(\theta_b - \theta_u + \theta_{arm}) + W_{FA} + W_{bal}$$
$$= -F_{bic}\sin(-\theta_{arm} + \theta_u - \theta_b) + W_{FA} + W_{bal}$$

case2.



$$-\theta_{arm}+\theta_u = \theta_b+\theta_y \quad \therefore \theta_y = -\theta_{arm}+\theta_u-\theta_b$$

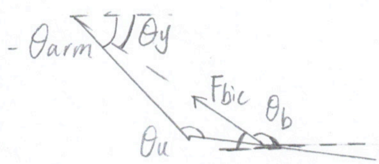
$$F_{bic}\cdot \sin\theta_y + JRF_{ev} = W_{FA}+W_{bal}$$

$$JRF_{ev} = -F_{bic}\cdot\sin\theta_y + W_{FA}+W_{bal}$$

$$= -F_{bic}\sin(-\theta_{arm}+\theta_u-\theta_b)+W_F+W_{bal}$$

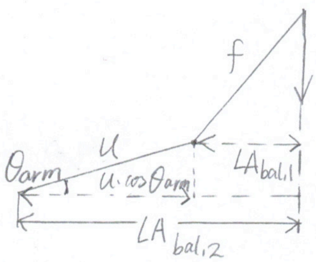
$$JRF_{eh} = F_{bic}\cos\theta_y = F_{bic}\cos(-\theta_{arm}+\theta_u-\theta_b) \qquad Eq(8)$$

case 3

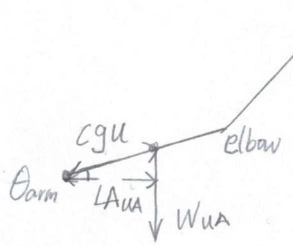


$$\theta_b = \theta_u + (-\theta_{arm}-\theta_y)$$

$$\therefore \theta_y = -\theta_{arm}+\theta_u-\theta_b$$



Eq(12)



$$LA_{uA} = cgu\cdot\cos\theta_{arm}$$

Eq(13)

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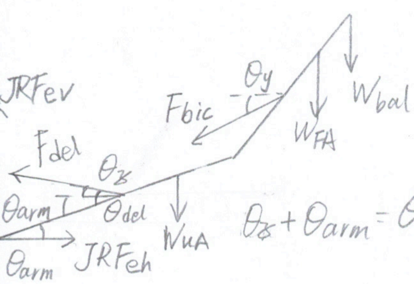
case 1



$$JRF_{sh} = F_{bic}\cdot\cos\theta_y + F_{del}\cdot\cos\theta_z$$

$$JRF_{sh} = F_{del}\cdot\cos\theta_z + \underbrace{F_{bic}\cdot\cos\theta_y}_{\text{from } Eq(8)} \qquad Eq(14)$$

$$= F_{del}\cos(\theta_{del}-\theta_{arm}) + F_{bic}\cos(-\theta_{arm}+\theta_u-\theta_b)$$

$$\theta_z+\theta_{arm} = \theta_{del} \Rightarrow \theta_z = \theta_{del}-\theta_{arm}$$

$$JRF_{sv} + F_{del}\cdot\sin\theta_z = F_{bic}\cdot\sin\theta_y + W_{FA}+W_{uA}+W_{bal}$$

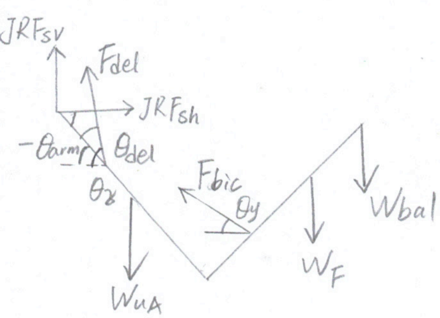
$$JRF_{sv} = \underset{\Downarrow}{-F_{del}\cdot\sin\theta_z} + \underbrace{F_{bic}\sin\theta_y}_{Eq(8).\ case1} + W_{FA}+W_{uA}+W_{bal}$$

$$F_{del}\cdot\sin(-\theta_z)$$

$$\Downarrow$$

$$\therefore JRF_{sv} = F_{del}\cdot\sin(\theta_{arm}-\theta_{del}) - F_{bic}(-\theta_{arm}+\theta_u-\theta_b)+W_{FA}+W_{uA}+W_{bal}$$

case2.



$$\theta_z = -\theta_{arm}+\theta_{del}$$

$$JRF_{sh} = F_{del}\cdot\cos\theta_z + F_{del}\cdot\cos\theta_y$$

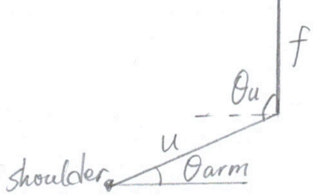
$$= F_{del}\cdot\cos(\theta_{del}-\theta_{arm}) + F_{bic}(-\theta_{arm}+\theta_u-\theta_b$$

$$JRF_{ev} + F_{del}\cdot\sin\theta_z + F_{bic}\cdot\sin\theta_y = W_{FA}+W_{uA}+W_{bal}$$

$$JRF_{ev} = \underset{\Downarrow}{-F_{del}\cdot\sin\theta_z} - \underbrace{F_{bic}\sin\theta_y}_{Eq(8).\ case\ 2} + W_{FA}+W_{uA}+W_{bal}$$

$$F_{del}\cdot\sin(\theta_{arm}-\theta_{del})$$

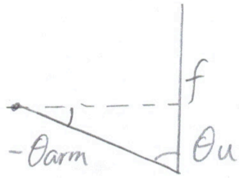
condition of bicep being the major muscle : $\theta_u - \theta_{arm}$ 9



$\theta_u = 90^\circ + \theta_{arm}$

$\theta_u - \theta_{arm} = 90^\circ$

when the forearm goes counter clock wise, even with a small angle bicep force is not the reason of keep equlibrium.



same for this case $\theta_u + (-\theta_{arm}) = 90^\circ$