# SPRING 2025 MATH 540: QUIZ 10

**Name:**

1. Give an example showing that the quotient and remainder found in the division algorithm over the Gaussian integers need not be unique. (5 points)

Solution. The point is to exploit the lack of uniqueness in the proof og the algorithm, when one approximates the real and imaginary parts of $\frac{u}{v}$, for $u, v \in G$.

Take $u = 2 + 2i$ and $v = 1 + 2i$. Then $\frac{u}{v} = 1.5 - .4i$. SO we make take $q = qorq = 2$.Then:

$2 + 2i = 1 \cdot (1 + 2i) + 1$, with $N(1) = 1 < 4 = N(1 + 2i)$

$2 + 2i = 2 \cdot (1 + 2i) + -2i$, with $N(2i) = 4 < 5 = N(1 + 2i)$.

2. Find a prime factorization in the Gaussian integers of $10 + 15i$. Be sure to justify your answer. You may use results from class. (5 points)

Solution. $10 + 15i = 5(2 + 3i) = (1 - 2i)(1 + 2i)(2 + 3i)$. Each of the three factors is a Gaussian prime since $N(1 \pm 2i) = 5$ and $N(2 + 3i) = 13$ are prime.