SPRING 2025 MATH 590: QUIZ 7

Name:

1. Define what it means for $u_1, u_2, \ldots, u_n$ in $\mathbb{R}^n$ to be an orthonormal basis. (2 points)

2. For columns vectors $v, w \in \mathbb{R}^2$, define $\langle v, w \rangle := v^t \cdot \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \cdot w$. This gives a new inner product on $\mathbb{R}^2$. Show that the vectors $v = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ are orthogonal with respect to this inner product. (4 points)

3. Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the three independent vectors $v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$. (4 points)