Vortex dipole colliding against no-slip wall with tracer analysis

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Equations

$$\frac{\partial \zeta_g}{\partial t} = -J(\psi, \zeta_g) - \beta \frac{\partial \psi}{\partial x} + \nu_{art} \left(\frac{\partial^2 \zeta_g}{\partial x^2} + \frac{\partial^2 \zeta_g}{\partial y^2} \right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \zeta_g(x, y),$$

- Same as MT06
- QG barotropic vorticity equations
- f-plane approximation
- Used Arakawa Jacobian

No-slip boundary condition

- Forces the velocity tangent to the surface to be zero.
- Slightly tricky to deal with in vorticity-streamfunction formulation since we don't deal with the velocities directly.
- The condition creates vorticity near the wall because the particles being moved by the vortices are sliding past the particles stuck at the wall.
- Initially estimated the vorticity at the wall:

$$\zeta_w = \frac{2(\psi_{w+1} - \psi_w)}{\Delta n^2}$$

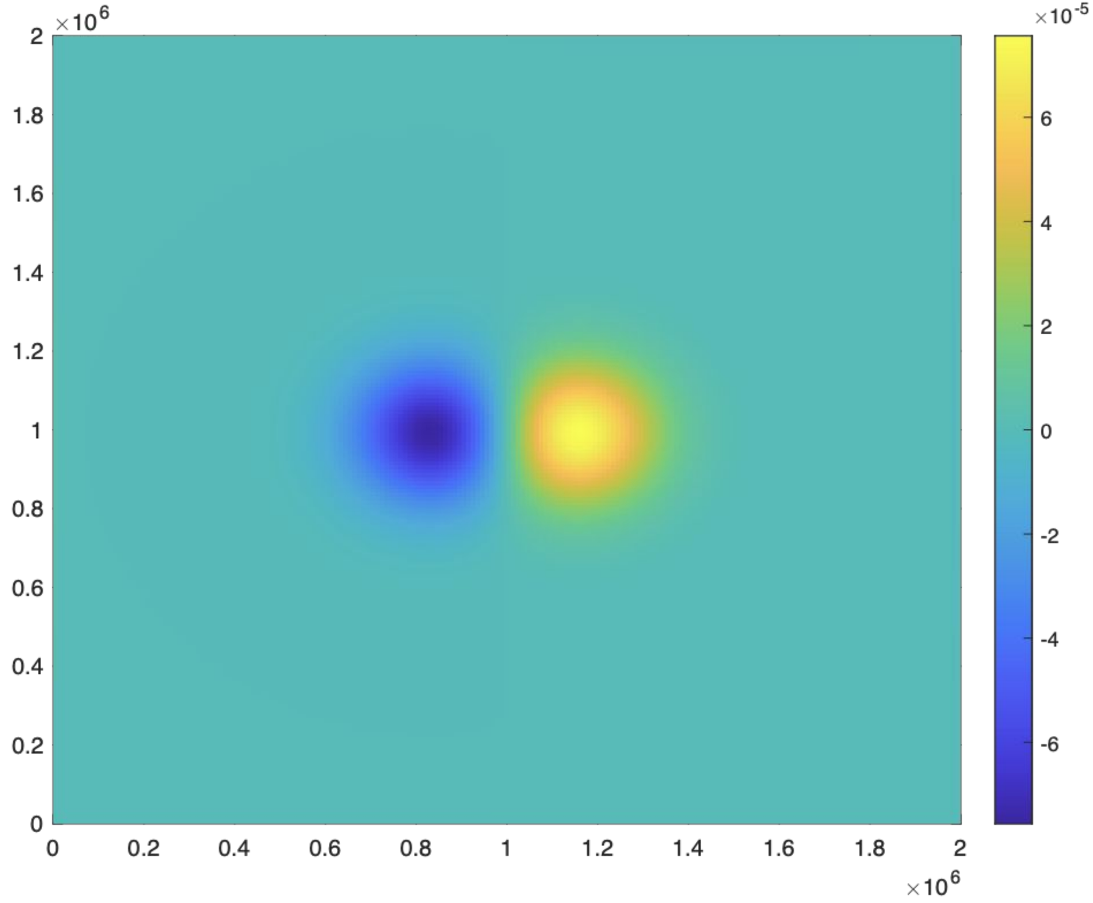
- This blew up, but a paper Professor Chamecki provided instead suggested:

$$\zeta_w = \frac{6\psi_{w+1} - \frac{3}{2}\psi_{w+2} + \frac{2}{9}\psi_{w+3}}{\Delta n^2}$$

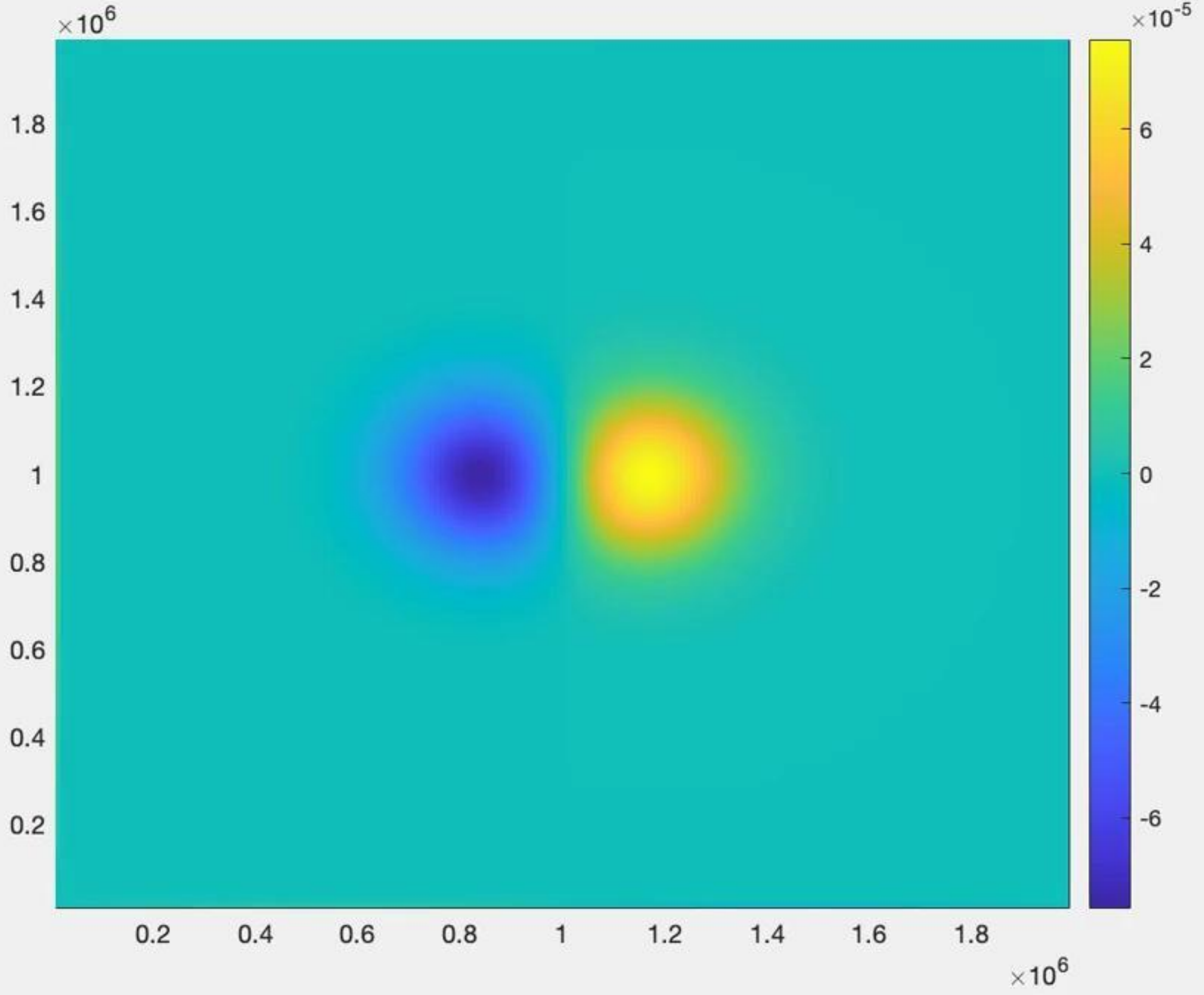
Initial Condition

- Used almost the same parameters as in Modeling Task 6
- Changed the magnitude of one of the vortices so that they form a dipole and ran the simulation for longer

$\Gamma/(\pi a^2) = 8 \times 10^{-5}\ s^{-1}$
a = 180 km
b = 300 km
$L_x = L_y = 2000$ km
$T_{total} = 30$ days
dt = 12 minutes
dx = dy = 10 km
$\beta = 0$
$\nu = 0.01$
tol = 10^{-10}

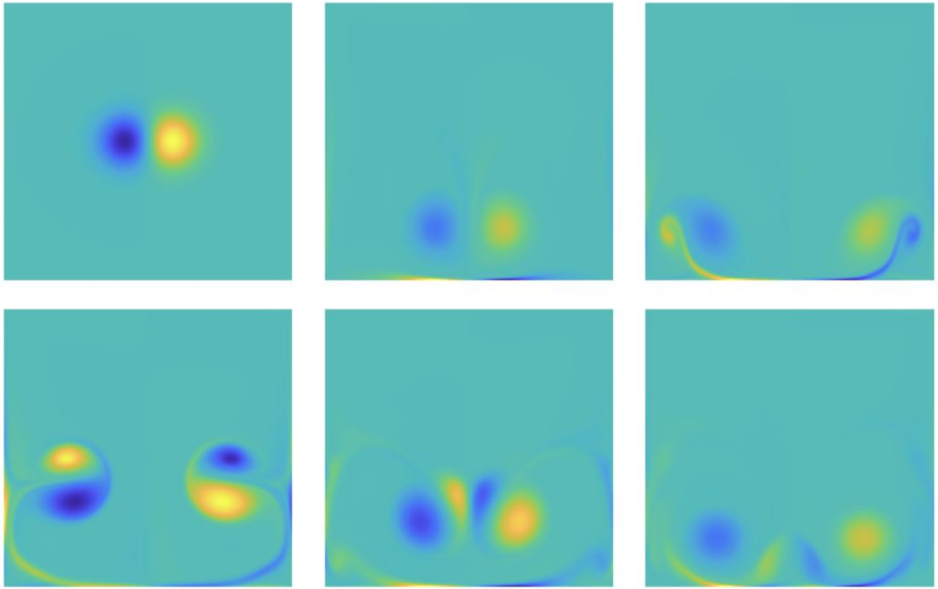


Vorticity Animation

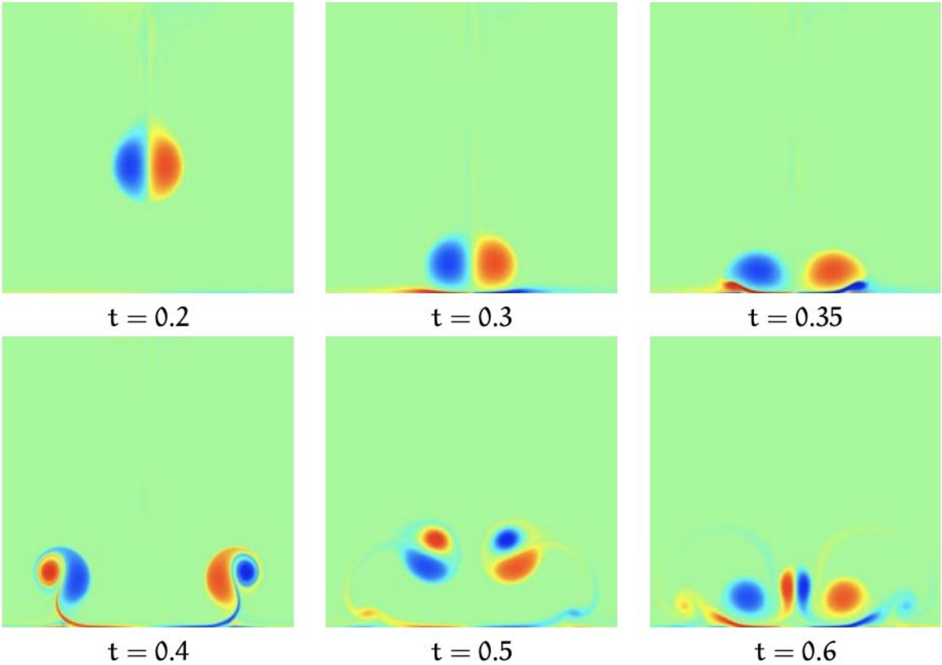


Comparison

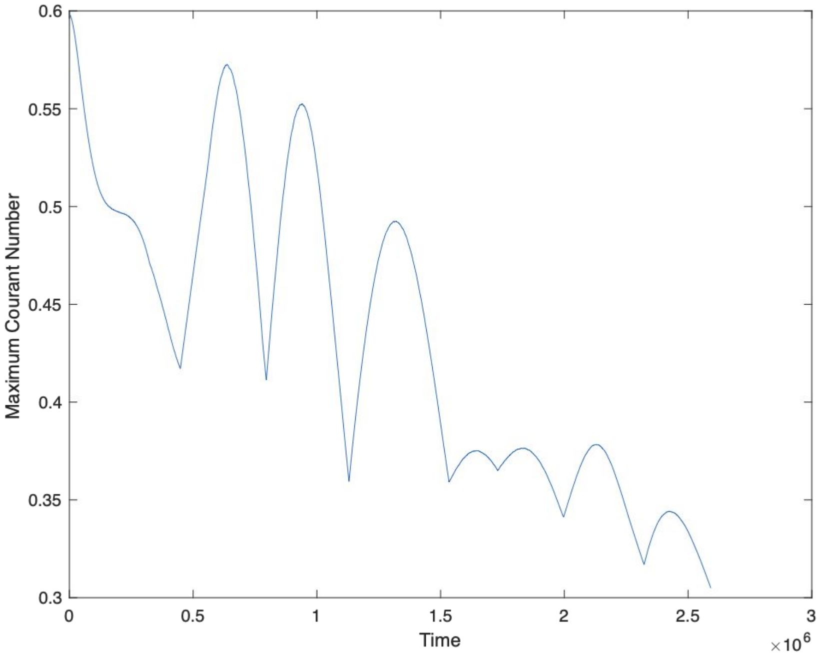
My result

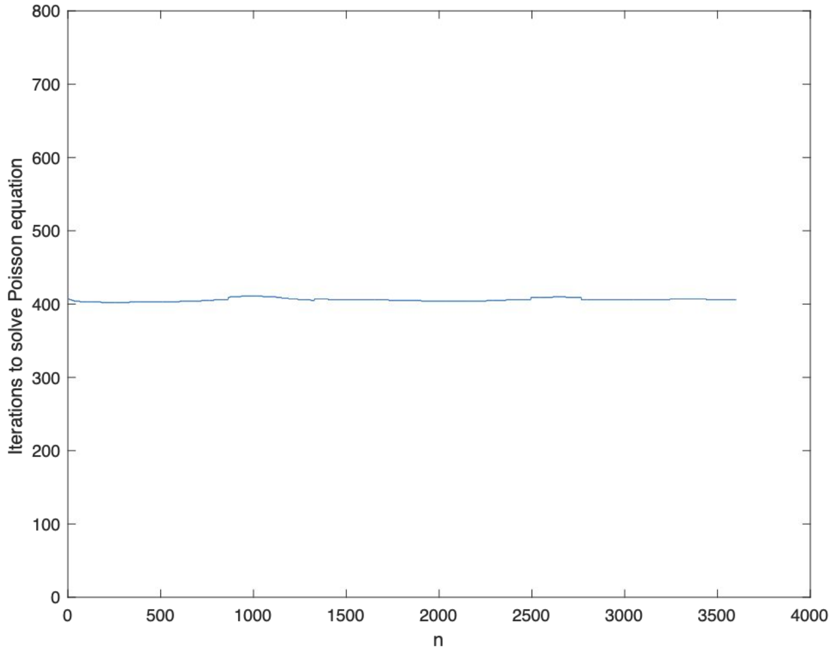


Result from Kramer (2007)



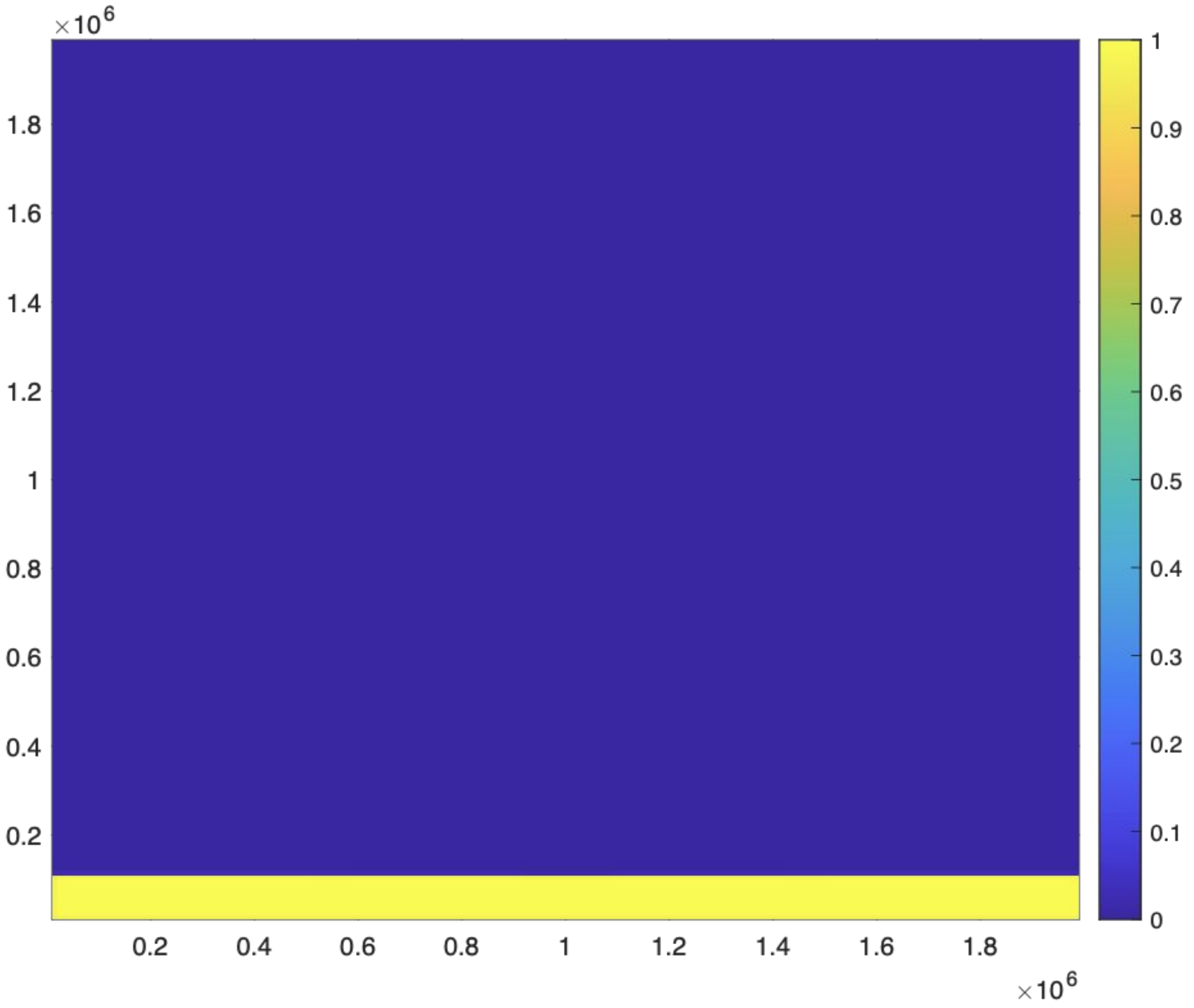
Courant number and iterations



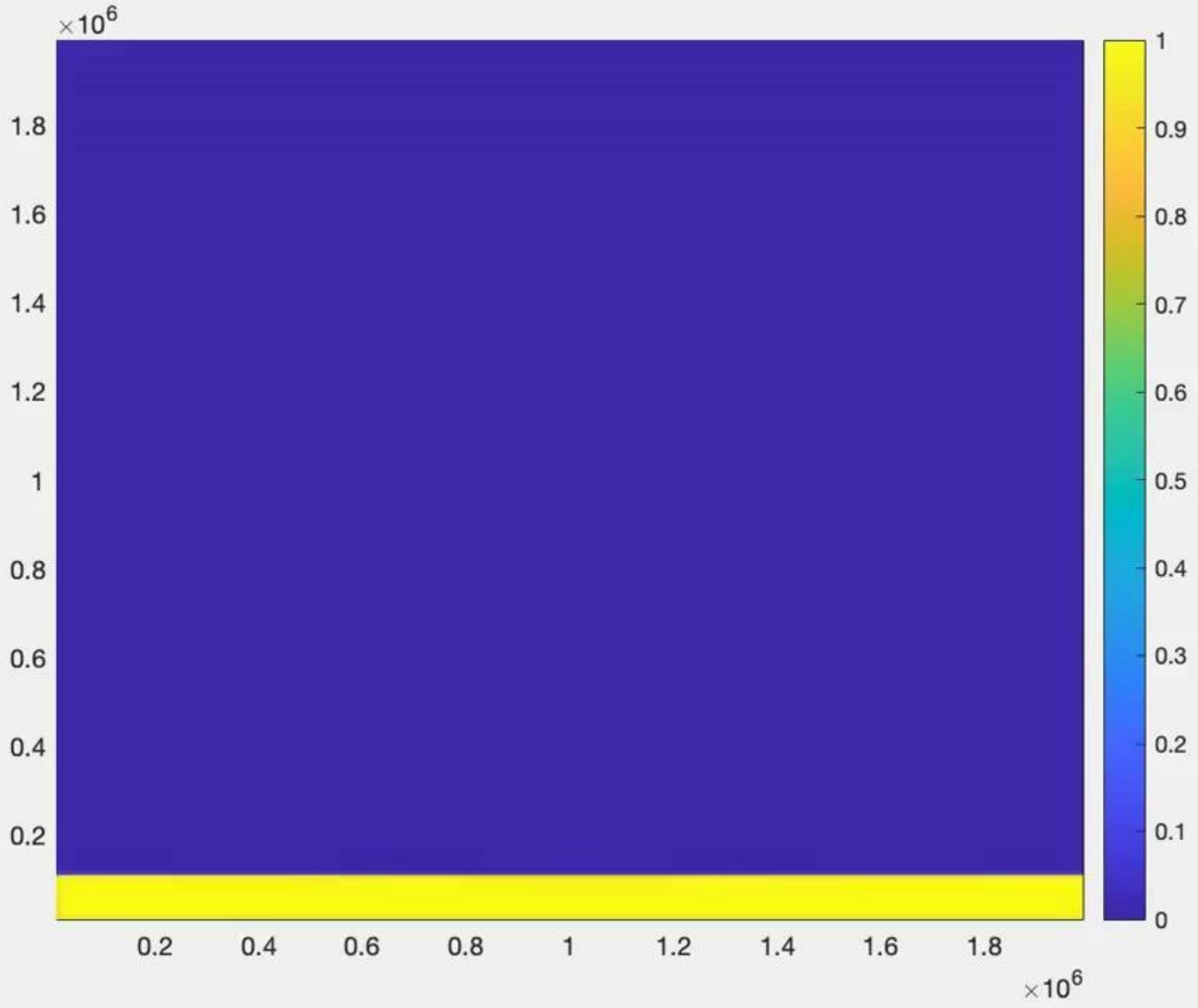


Tracer Initial Condition

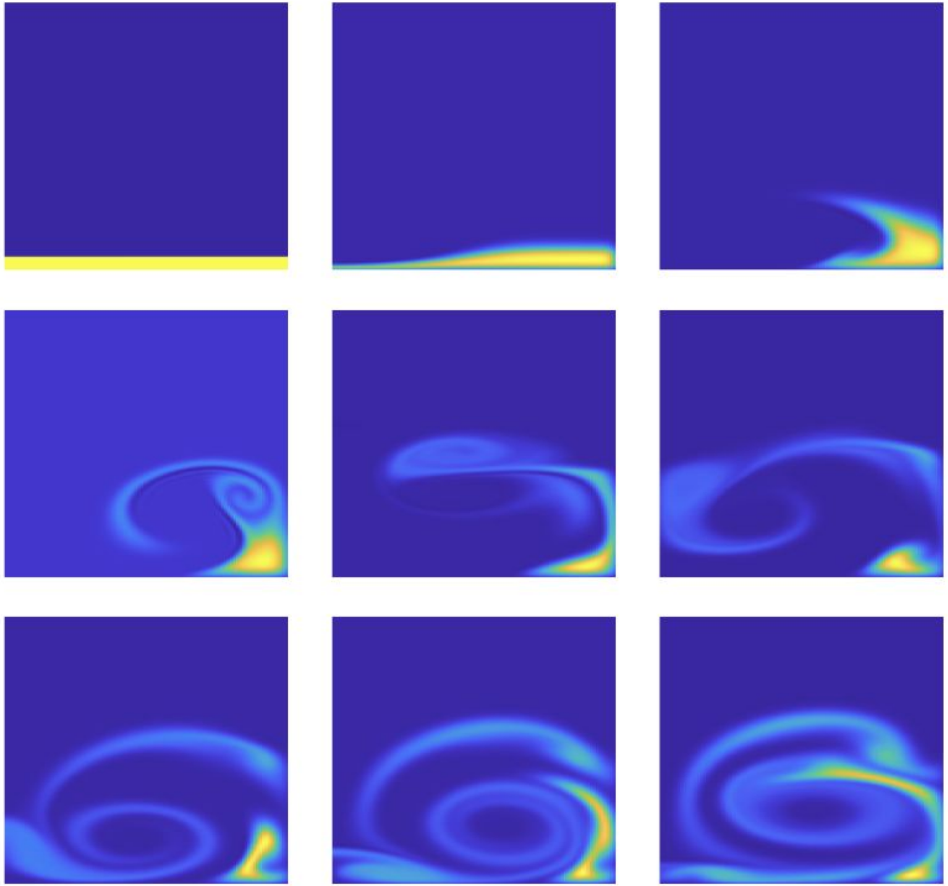
$$\frac{\partial S}{\partial t} = -J(\psi, S) + \nu \nabla^2 S$$

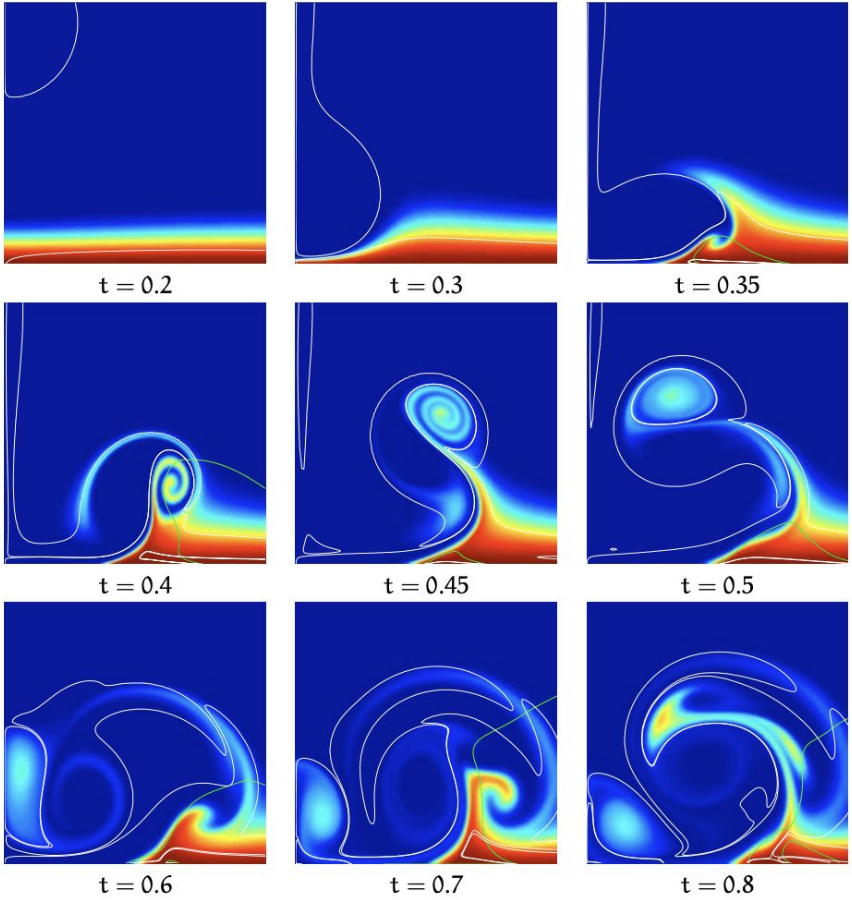


Tracer Animation



Comparison





References

- Kramer, W. (2007). Dispersion of tracers in two-dimensional bounded turbulence. [Phd Thesis 1 (Research TU/e/ Graduation TU/e), Applied Physics and Science Education]. Technische Universiteit Eindhoven.
- Foreman, M.G.G. and Bennett, A.F., 1988. On no-slip boundary conditions for the incompressible Navier-Stokes equations, Dyn. Atmos. Oceans, 12: 47-70.