# Dark Energy and the Ultimate Fate of the Universe

David Llewellyn Smith

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## 1 Scientific Background

Albert Einstein's field equation for general relativity enabled cosmologists to study the evolution of the universe.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{1}$$

He originally added the cosmological constant to balance the effects of gravity and have a static universe, which he later called his greatest regret. However, it was shown that Einstein's static universe is unstable to any fluctuation in size, and when Hubble showed that the universe is expanding it was rendered moot anyways. Soon after Hubble's discovery, Georges Lemaître suggested what would become known as the Big Bang. With a working theory for the beginning, cosmologists began to ponder what would happen at the end. For a long time, it was thought that the curvature of the universe would determine its fate, until the discovery of dark energy. [1]

## 2 Dark Energy

### 2.1 Overview

In 1998, the observation of supernovae in distant galaxies showed that the acceleration of the universe is expanding. A negative pressure of the vacuum was suggested as an explanation for said acceleration, a phenomenon that would be dubbed dark energy by current UCLA professor Michael Turner. [2] Many different models for what dark energy is have been proposed. A very important distinction is how they treat the equation of state that relates pressure to energy density as a dimensionless number:

$$w = \frac{p}{\rho} \tag{2}$$

Using the Friedmann acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i) \tag{3}$$

we can see that the evolution of the scale factor and therefore the size of the universe is dependent on $w$. [1] $w$ has to be less than $-\frac{1}{3}$ in order for $\ddot{a}$ to be positive and therefore for expansion to be accelerating. [3]

## 2.2 Λ-CDM and the Cosmological Constant

Einstein's regretted cosmological constant turns out to be the simplest model for dark energy. It's an important part of the Λ-CDM model that is widely accepted by modern cosmologists. In this model,

$$p = -\rho \tag{4}$$

$$w = \frac{p}{\rho} = -1. \tag{5}$$

This represents an intrinsic energy for the vacuum, one that we know is there from experiments like the Casimir Effect. However, attempting to find that energy by summing over energy states of quantum harmonic oscillators

$$\frac{\Lambda}{8\pi G} \propto \int_0^\infty \sqrt{k^2 + m^2} k^2 dk \tag{6}$$

leads to divergence, and even using a cutoff at Planck scale wavelengths still leads to an absurdly large value on the order of $10^76GeV^4$, when the observed value is more like $10^-46$. [4] This problem probably can't be solved until we have a unified quantum gravity theory.

## 2.3 Quintessence

A popular type of dark energy model other than Λ-CDM is the Quintessence, in which dark energy is a time-dependent scalar field. A field like that is subject to

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{7}$$

in which $\phi$ is the field and V is a potential energy. Then, the equation of state has the form

$$w_q = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{8}$$

Many different models for the quintessence have been suggested, using different functions for the potential. [5]

## 2.4 Other Models

### 2.4.1 $w$CDM

$w$CDM refers to models in which $w$ is a constant but not equal to -1 as in Λ-CDM. [6]

### 2.4.2 Parameterizations

These are models in which $w$ is a function of the redshift $z$. [6] A popular example is the Chevallier-Polarski-Linder model, in which

$$w(z) = w_0 + w_1 \frac{z}{1+z} \tag{9}$$

### 2.4.3 Braneworld

The main idea of braneworld theories is that our 3-dimensional universe is part of a 4-dimensional brane inside a 5-dimensional space known as a bulk. [6]

### 2.4.4 Chaplygin Gas

This is part of a type of model called dark fluids that treat dark energy and dark matter as different manifestations of the same phenomenon. The equation of state in Chaplygin Gas is

$$p = -\frac{A}{\rho}. \tag{10}$$

Chaplygin Gas reproduces the acceleration of the universe nicely, it has trouble describing anisotropies of the Cosmic Microwave Background. [6]

### 2.4.5 Phantom Energy

Phantom energy is any theory in which the parameter $w < -1$. This has some interesting consequences covered in section 3.2. [5]

### 2.4.6 More

There exist many more models in the literature beyond those described above.

# 3 Theories of the Ultimate Fate

## 3.1 Big Crunch

The Big Crunch refers to a model in which the expansion of the universe eventually reverses, and the universe recollapses into the singularity it started as. This could lead to an oscillating universe, expanding and then collapsing forever. This was a more accepted theory before the discovery of the acceleration of expansion. There is some current literature that suggests it could be possible with the right form of dark energy. [7] suggests a nonlinear form using two cosmological constants.

### 3.2 Big Rip

The Big Rip is a consequence of the previously-mentioned phantom energy, any model in which $w < -1$. Given the relationship between the energy density of dark energy and the scale factor,

$$\frac{\rho}{\rho_0} = a^{-3(1+w)}, \tag{11}$$

$w < -1$ implies the energy density increases as the universe expands, which means the expansion rate of the universe becomes infinite in finite time. Everything will be ripped apart, distances will become arbitrarily big, and all interactions will stop. [8] Based on the current observed value of $w$, this can't happen for at least another 152 billion years. [9]

### 3.3 False Vacuum Decay

In this scenario, the vacuum of the universe is not in its lowest possible energy state. Then, it could tunnel into said lowest state at any time with no foreknowledge. This could potentially destroy everything or drastically alter life as we know it, or it could do nothing. [10]

### 3.4 Heat Death

Finally, the currently accepted theory is what's known as heat death. In this scenario, the universe expands forever and approaches thermodynamic equilibrium and maximum entropy. By the end, all that's left would be a very diffuse gas composed of fundamental particles.
Current evidence suggests this is the case. It is the situation predicted by Λ-CDM, which is currently widely accepted. The results from the Planck spacecraft are that $w = -1.028 \pm 0.032$, which is about -1. [11]

## 4 Timeline of the Far Future

- 5 Billion Years: Andromeda and the Milky Way merge into Milkomeda
- 100 Billion Years: Local Group fully merged into Milkomeda
- 150 Billion Years: Galaxies outside local group leave cosmological event horizon
- 2 Trillion Years: Galaxies outside local group are so redshifted photons they emit have wavelengths longer than observable universe
- 100 Trillion Years: Star formation ceases: end of Stelliferous Era and beginning of Degenerate Era
- $10^{43}$: If protons decay, when black holes are the last remaining celestial object

- $10^{106}$: When all black holes will have evaporated via Hawking Radiation and universe enters the Dark Era
- $10^{1500}$: If protons don't decay, time until all remaining matter becomes iron stars
- $10^{10^{26}}$: Time for those iron stars to quantum tunnel into black holes and evaporate - universe enters the Dark Era
- Beyond: nothing remaining but subatomic particles in an almost pure vacuum
- A new big bang?: Quantum tunneling could cause spontaneous inflation and create a new universe (on time scales on the order of $10^{10^{10^{56}}}$ [12]

# 5 References


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