Abundances and Temperature-Density Correlations from Multi-Instrument Tomography

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1. INTRODUCTION

With the forthcoming UCoMP data, I have given some thought as to how best to make use of it. Let us assume that we have at our disposal enough data from the following instruments to perform simultaneous tomographic reconstructions for all of the following:

- Kcor, LASCO-C2
- UCoMP, 530.3 FeXIV, 637.4 FeX, 706.2 FeXV, 789.4 FeXI, 1074.7 FeXIII, 1079.8 FeXIII
- EUVI or AIA, 171, 195, 284, Å and/or other bands

It would make the most sense to use the same tomographic grid (hopefully, a sine-latitude grid) for all reconstructions. The regularization parameters would vary depending on the observation sequences and possibly the noise in the data (especially for UCoMP).

The electron density N_e is measured directly by the tomography based on white-light instruments. All of the spectral lines have collisionally excited components, and they may have radiatively excited components as well. While the collisional intensities are proportional to N_e^2, the scattered intensities are proportional to N_e. Thus, spectral line emissivities, when averaged over a volume element of the tomographic grid will not only depend on $< N_e >$, which is N_e averaged over the volume element, but on $< N_e^2 >$ as well. The scattered intensities of the optical coronal lines measured by UCoMP are insensitive to the ion temperatures and ion velocity, since they scatter radiation from the visible continuum. In contrast, any scattered portion of the intensity of the Fe lines in EUV bands will have some sensitivity to the ion temperatures (parallel and perpendicular) as well as the velocity. In the interest of tractability, we will ignore these issues and assume the velocity is negligible (at least in terms of its effect on the scattering) and $T_{i\|} = T_{i\perp} = T_e$. It should not be a bad approximation, since we will only be studying the corona at heights of less than 1.2 $R_\odot$. All of spectral line intensities are, of course, proportional to the iron abundance, which we will denote as $A \equiv [Fe]/[H]$. In addition, these lines are sensitive to the electron temperature T_e, which is not uniform within a given volume element since it is threaded by thermally isolated field lines.

2. JOINT TEMPERATURE AND DENSITY DISTRIBUTION

Given that both T_e and N_e vary within the tomographic voxel, we should admit the possibility that their variations are correlated. The easiest way to do this is assume that the probability that a tiny volume of plasma within the voxel has an electron temperature T_e and electron density N_e is given by the bivariate normal distribution:

$$\begin{aligned}\mathcal{P}(N_e, T_e | T_m, N_m, \sigma_T, \sigma_N, q) = & \frac{1}{2\pi\sigma_T\sigma_N\sqrt{1-q^2}} \\ & \times \exp\left\{ \frac{-1}{2(1-q^2)} \left[\frac{(T_e - T_m)^2}{\sigma_T^2} + \frac{(N_e - N_m)^2}{\sigma_N^2} - \frac{2q(T_e - T_m)(N_e - N_m)}{\sigma_T\sigma_N} \right] \right\} , \qquad (1)\end{aligned}$$

where N_m is the mean electron density in the voxel (which already has been estimated from white-light tomography, but it is important to not simply use that estimate here), $T_m = < T_e >$ is the mean electron temperature in the voxel, σ_T and σ_N are standard deviations, and q is the temperature-density correlation coefficient. The parameters governing this distribution $(N_m, T_m, \sigma_T, \sigma_N, q)$, as well as the iron abundance A will have to be estimated jointly from the tomographic measurements in each voxel. Note that Eq. 1 allows N_e and T_e to assume any value (even negative ones). In practice we should enforce thresholds, to ensure $N_e > N_{\min}$ and $T_e > T_{\min}$, which are physical values that coronal plasma must exceed. For example, $T_{\min} \approx 6 \times 10^6$ K, and $N_{\min} \approx 10^5$ cm^{-3} might be applicable threshold values. Drawing samples from a probability distribution that obeys these threshold values is a simple matter of drawing from Eq. 1 and throwing out values that are in violation of these constraints (note that Monte Carlo integration over the distribution will also require a re-weighting step, but that is explained later).

3. SPECTRAL SYNTHESIS MODEL

In order to proceed further, we will require a model that calculates the emissivities as a function of the unknown parameters. Let the spectral synthesis model of the M spectral lines and EUV bands be represented by $\{s_k(A, T_e, N_e)\}$, where $k = (1, \ldots, M)$. Then the model of the $k\underline{\text{th}}$ tomographic emissivity in the computational volume element is given by integrating s_k over the probability distribution in Eq. 1, i.e.,

$$\epsilon_k(N_m, \mathbf{c}) = \int \mathrm{d}T_e \int \mathrm{d}N_e \, s_k(A, T_e, N_e) \mathcal{P}(T_e, N_e | N_m, T_m, \sigma_T, \sigma_N, q) \,, \tag{2}$$

where ϵ_k is the modeled value of the tomographic emissivity, and $(N_m, \mathbf{c}) = (N_m, A, T_m, \sigma_T, \sigma_N, q)$ is the vector of unknown parameters we wish to determine (so, N_m is not included in the vector $\mathbf{c}$). Thus, the model values of the emissivites $\{\epsilon_k(N_m, \mathbf{c})\}$ is easily determined by combining the spectral synthesis model with an integration over the joint distribution in Eq. 1 for any value of $\mathbf{c}$. In this way, the model of the tomographic emissivity takes into account the variation of T_e and N_e within the voxel, as well as correlation between the two, into account. As stated above, the threshold constraints can be taken into account with a simple Monte Carlo integration procedure.

4. MEASUREMENT MODEL AND PARAMETER ESTIMATION

Here, I describe a statistical model of the tomographic measurements and their errors within a single voxel. The vector $\boldsymbol{y} = (y_1, \ldots, y_M)$ is the vector of tomographic measurements for the voxel, except for the white-light measurement of the density, which is y_0 (so, y_0 is not included in the vector $\boldsymbol{y}$). Let σ_{wl} be the uncertainty of the white-light measurement, and let $\boldsymbol{\Sigma}$ be the $M \times M$ uncertainty covariance matrix of the UCoMP and EUV measurements. If the UCoMP and EUV measurement errors were all uncorrelated, then $\boldsymbol{\Sigma}$ would be diagonal. However, since these measurements will all be coming from 1 or 2 telescopes (e.g., UCoMP and EUVI), there could be some correlation in the measurement errors. Even so, taking $\boldsymbol{\Sigma}$ to be diagonal might not be a bad approximation. I will assume that there is no correlation between errors of the white-light measurement and the others. This might not strictly be true, since all tomographic measurements will be corrupted by coronal dynamics, which could have similar (or at least correlated) effects in all tomographic reconstructions. Then the measurement model (likelihood) is

$$\mathcal{P}(N_m, \boldsymbol{\epsilon} | y_0, \boldsymbol{y}) = \mathcal{N}(N_m; y_0, \sigma_{\mathrm{wl}}) \mathcal{N}\big(\boldsymbol{\epsilon}(N_m, \mathbf{c}); (y_1, \ldots, y_M), \boldsymbol{\Sigma}\big) \,, \tag{3}$$

where $\boldsymbol{\epsilon}(N_m, \mathbf{c}) \equiv \big(\epsilon_1(T_m, \mathbf{c}), \ldots, \epsilon_M(N_m, \mathbf{c})\big)$, and $\mathcal{N}(x; m, C)$, is the normal distribution over x, with mean m and covariance C.

Now, the maximum likelihood estimate of the parameter vector $\mathbf{c}$ is found by maximizing the logarithm of Eq. 3, which results in

$$(\hat{N}_m, \hat{\mathbf{c}}) = \underset{N_m, \mathbf{c}}{\operatorname{argmin}} \left[\frac{(N_m - y_0)^2}{\sigma_{\mathrm{wl}}^2} + \big(\boldsymbol{\epsilon}(\mathbf{c}) - \boldsymbol{y}\big)^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \big(\boldsymbol{\epsilon}(\mathbf{c}) - \boldsymbol{y}\big) \right]. \tag{4}$$

where the superscript $^{\mathrm{T}}$ indicates matrix (or vector) transposition. Note that this will result in an estimate of the mean electron density N_m that is somewhat different than y_0, which was made from the white-light tomography alone. Joint estimation of N_m and $\mathbf{c}$, has two benefits. The first is that the EUV and UCoMP measurement carry information about N_e, which is now included. The second is that simply taking T_m to be y_0 would cause a statistical bias (most likely shrinkage) in the estimate of $\mathbf{c}$, but this procedure should remove that difficulty. The minimization in Eq. 4 should be repeated for every voxel with valid data.

5. VALIDATION

One independent validation of the results is the widths of the line profiles measured by UCoMP.