## Exercises – Semantic Tableaux Method

**Exercise 1**

Using the semantic tableaux method decide what kind (consistent, inconsistent, valid) of formula is U1.

If U1 is consistent, find all its models.

**U1= (p ^ q) v (Ꞁp ^ Ꞁr) -> (q < - > r)**

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**

**Exercise 2**

Prove that the following formulas are tautologies using the semantic tableaux method:

1.distribution of ’->’ over ’ ^ ’:

**U1= (p-> q ^ r) <-> (p->q) ^ (p->r)**

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**

**Exercise 3**

Using the semantic tableaux method, decide whether the following logical consequence holds or not.

If a logical consequence does not hold find an anti-model of it.

1**. p-> (Ꞁ q v r ^s), p, Ꞁ s |= Ꞁq**

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**

**Exercise 4**

Write all the anti-models of the propositional formula U1 using the semantic tableaux method.

**U1= (p v q) ^ Ꞁr -> p ^q ^r**

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**

**Exercise 5**

Using the semantic tableaux method, prove the following property in predicate logic:

1.’’ is semi-distributive over ’’:

|= (ꓱx)(A(x) ^ B(x)) -> (ꓱx)A(x) ^ (ꓱx)B(x)

and

(ꓱx)A(x) ^ (ꓱx)B(x) ->(ꓱx)(A(x) ^ B(x))

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**

**Exercise 6**

Check the validity of the following first-order formulas using the semantic tableaux method:

**U1=(ꓯx) (ꓯy)P(x,y) <-> (ꓱx) (ꓯy)P(x,y)**

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**

**Exercise 7**

Using the semantic tableaux method check whether the following logical consequences hold.

1.**(ꓯx)(P(x) ->Q(x)), (ꓯx)P(x) |= (ꓯx)Q(x)**

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡ ϵ**