Math T680 Topics in Geometry

$\boxed{\text{HW \#1}}$

Due: Monday, April 17, 2017

1. Prove that $H^2(\mathbb{R}^3 - \{x_0\}) \neq 0$. You can assume Stoke's theorem from vector calculus.
2. Study Example 1.7 in the textbook. Fill in the missing details.

   Trim down the domain from U to $W := \{(x_1, x_2, x_3) \in \mathbb{R}^3 :\ \text{either } x_3 \neq 0 \text{ or } x_1^2 + x_2^2 < 1\}$.

   There must be a function $F \in C^\infty(W, \mathbb{R})$ such that $\mathrm{grad}(F)$ is the vector field in Example 1.7. Why?

   Find a simple expression for F valid when $x_1^2 + x_2^2 < 1$.
3. Explain why the $1/p!q!$ factor has nothing to do with the alternating property of $\omega_1 \wedge \omega_1$.
4. Dissect the proof of the fact that $\wedge$ is associative; explain why the $1/p!q!$ factor is crucial. Use your best writing.