Math 538 Differential Geometry and Manifolds

HW #0

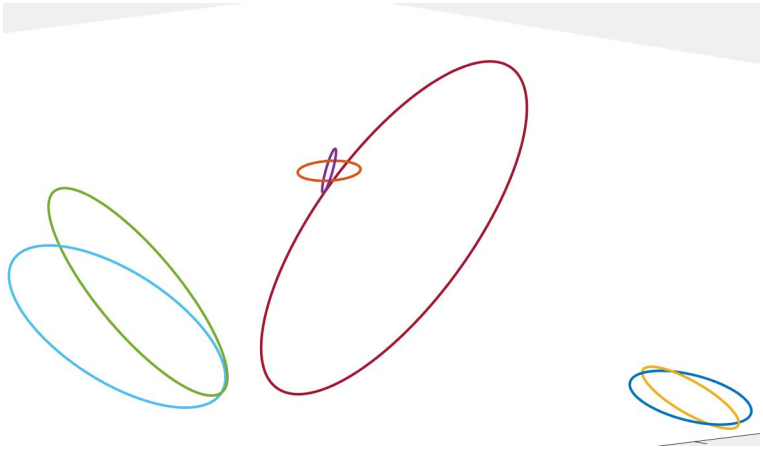
Due: Friday September 25, 2020

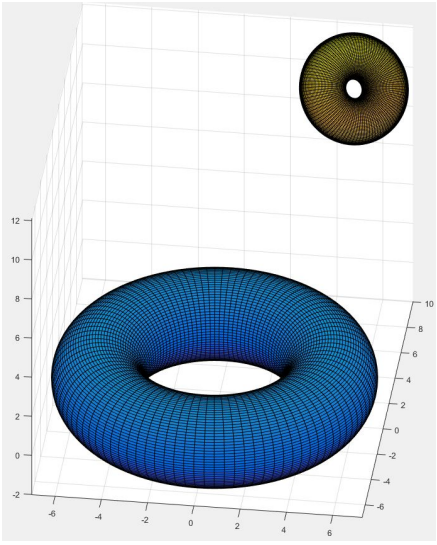
1. Explain how to represent a ellipse with a certain center, major radius, minor radius and tilt in 3-space implicity and explicitly/parametrically.

   (Suggestion: write down the equations in the most standard way on the plane – as you learned it in a highschool conics section lesson, then put in a zero to the 3rd dimension, followed by translation and rotation in $\mathbb{R}^3$ – as you learned it in linear algebra.)

2. What is the formula that gives the length of a parameterized curve in $\mathbb{R}^n$? Explain why this formula gives the arclength.

   The formula *appears* to be dependent on the way the curve is parameterized, but of course it must be independent to the way the curve is parameterize. Give a direct proof that the formula is *invariant* under reparametrization.





3. Explain how to represent a torus (with certain major and minor radii, center, and tilt) in space explicitly and implicity.

4. (Extra credit.) What is the name of the mathematical result that says "**locally** you can go back and forth explicit and implicit representations"? Explain how the concept of local linear approximation plays a central role in this theorem.