Introduction to Optimization. HW#4. Due: Friday, May 16, 2025

1. Consider the LP $\min z = 2x_1 + 9x_2 + 3x_3$ s.t. $-x_1 - 6x_2 \geqslant -3$, $x_1 + 4x_2 + x_3 \geqslant 1$, $-2x_1 - 14x_2 \geqslant -5$, $x_1, x_2, x_3 \geqslant 0$. Let's call this LP the 'primal'.

   Formulate the dual of this LP, then "standardize" it, and then solve it using the usual simplex method.

   Explain how by solving the standardized version of the dual, you can find the minimizer and the minimum value of the LP above.

   For full credits, present the simplex step(s) and clearly state the **optimal value** and the **optimizer** of (i) the standardized version of the dual, (ii) the dual, (iii) the dual of the (standardized verson of the) dual, and (iv) the primal.

   Note: when you solve the primal this way, you are essentially using the so-called dual simplex method. But note that the official dual simplex method is easier to apply: it does not require us to first standardize the dual.

2. Now, apply the dual simplex method to solve the LP in Problem 1.

3. Apply the original simplex method to solve the LP in Problem 1. Which method do you prefer? Why?

4. Page 98, Exercise 4-4-1

5. Page 98, Exercise 4-4-2

6. Page 102, Exercise 4-5-2

7. Page 106, Exercise 4-6-4.

8. Solve an instance of the LP in Problem 4, HW1 using the methods we have developed so far. Reproduce the graphical outputs as in the Matlab script `LinearFit1D.m`, posted along with this assignment. (The script `LinearFit1D.m` uses `linprog()` in the optimization toolbox, your job is to solve the same optimization problem using instead the software developed by the authors. The fun thing is that for the first time you will be looking at a pretty big tableau, and you will have to do many Jordan exchanges to solve the problem!)

   Would you use the original simplex method or the dual simplex method?