Math T680 Topics in Geometry
$\boxed{\text{HW \#6}}$
Due: Wednesday, May 29, 2017

1. Prove the Mayer-Vietoris theorem (Theorem 5.2 of M&T) with U replaced by a smooth manifold M, and U_1, U_2 are two open sets in M with $M = U_1 \cup U_2$.

   Note: you only have to prove a manifold version of Theorem 5.1, and then combine it with a general algebraic result in Chapter 4.

2. Use the manifold version of Mayer-Vietoris theorem to establish

$$\sum_{i=0}^{n} (-1)^i \alpha_i = \sum_{i=0}^{n} (-1)^i \dim H^i(M^n),$$

   where α_k is the number of k-simplices in *any* triangulation of M^n.

   If M is homeomorphic to S^2, use it to prove $\alpha_0 - \alpha_1 + \alpha_2 = 2$.

3. Fill in the technical details (a)-(d) on Page 6 of the lecture notes for Chapter 10.

4. Look up the two results about $\mathbb{RP}^n$ in Chapter 9 that I did not have time to go through, one is about the orientability of $\mathbb{RP}^n$, the other about the de Rham cohomology of $\mathbb{RP}^n$. These two results can be used to exemplify a result known as "Poincaré duality" (in Chapter 13), which we also won't have time to go through in the course. Look up the key result in Chapter 13, and explain why the two results about $\mathbb{RP}^n$ exemplify it.