Week 1

Note Title 3/30/2015

Agenda :

- Basic financial instruments
- Arbitrage pricing
- Put-call parity (in practice)

HW 1

- probability
- recursion / mathematical induction
- compound interest

Bond basics

- like an I.O.U. given by a
  Borrower (typically a corporate or government)
  to a
  Lender (the investor)

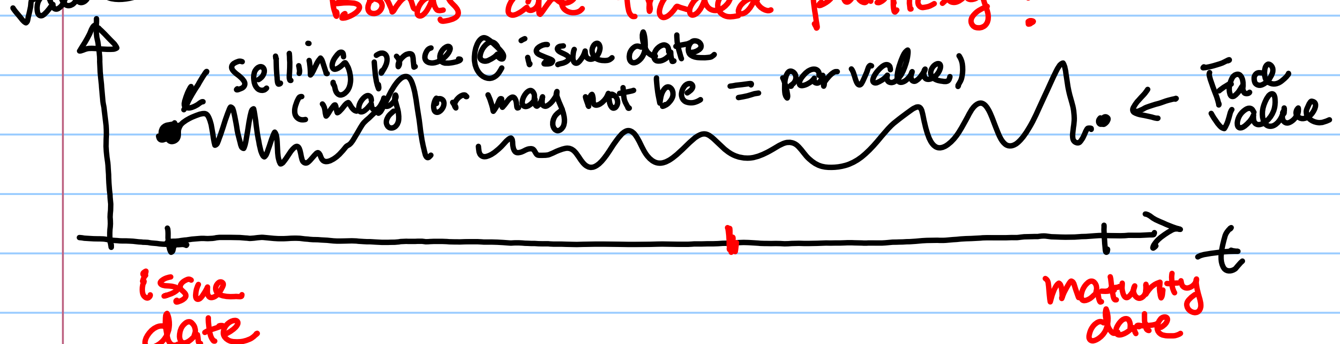
- Bonds are debt, stocks are equity

  You may want to understand the defference between owning the bonds of a company and the stocks of the same company.

Face Value / Par Value

- the amount of money a holder will get back once a bond matures

Bonds are traded publicly!



Coupon (the interest rate)

- the amount the bondholder will receive as interest payment

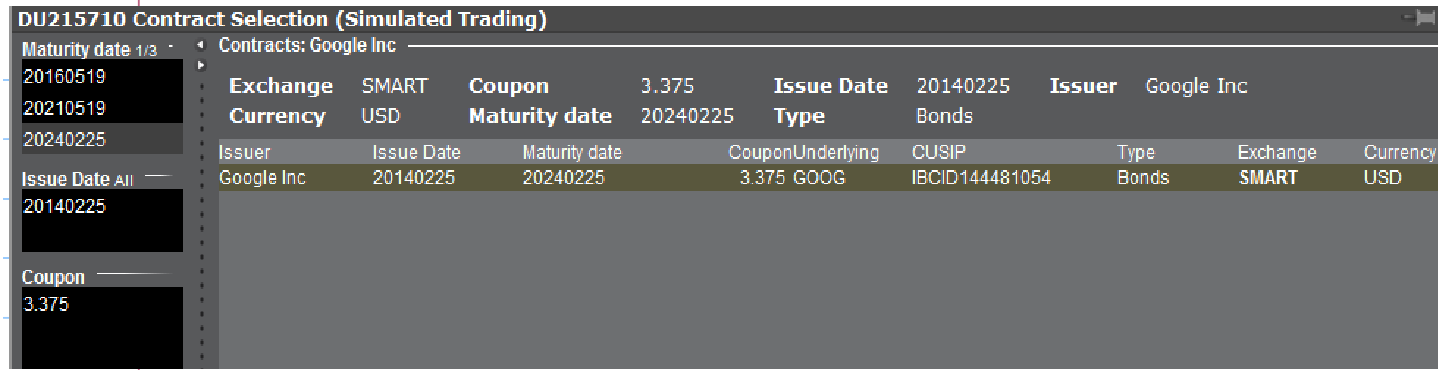
e.g. 3.375% (per year, but pay every six months)

- Bonds are traded in the open market
- NOT every investor would hold the bonds to maturity.

Bond	PRICE/CHG	YIELD/CHG
*US 3-MO U.S. 3 Month Treasury	0.015 0.005	0.01 +0.005
*US 2-YR U.S. 2 Year Treasury	99.7422 0.0469	0.68 -0.024
*US 5-YR U.S. 5 Year Treasury	98.8438 0.1016	1.56 -0.022
*US 10-YR U.S. 10 Year Treasury	100.125 -0.00	2.05 0.00
*US 30-YR U.S. 30 Year Treasury	108.8125 -0.2188	2.63 +0.01

e.g.

GOOGLE CORPORATE BOND:



DU215710 Contract Selection (Simulated Trading)

Maturity date 1/3: 20160519, 20210519, 20240225

Issue Date All: 20140225

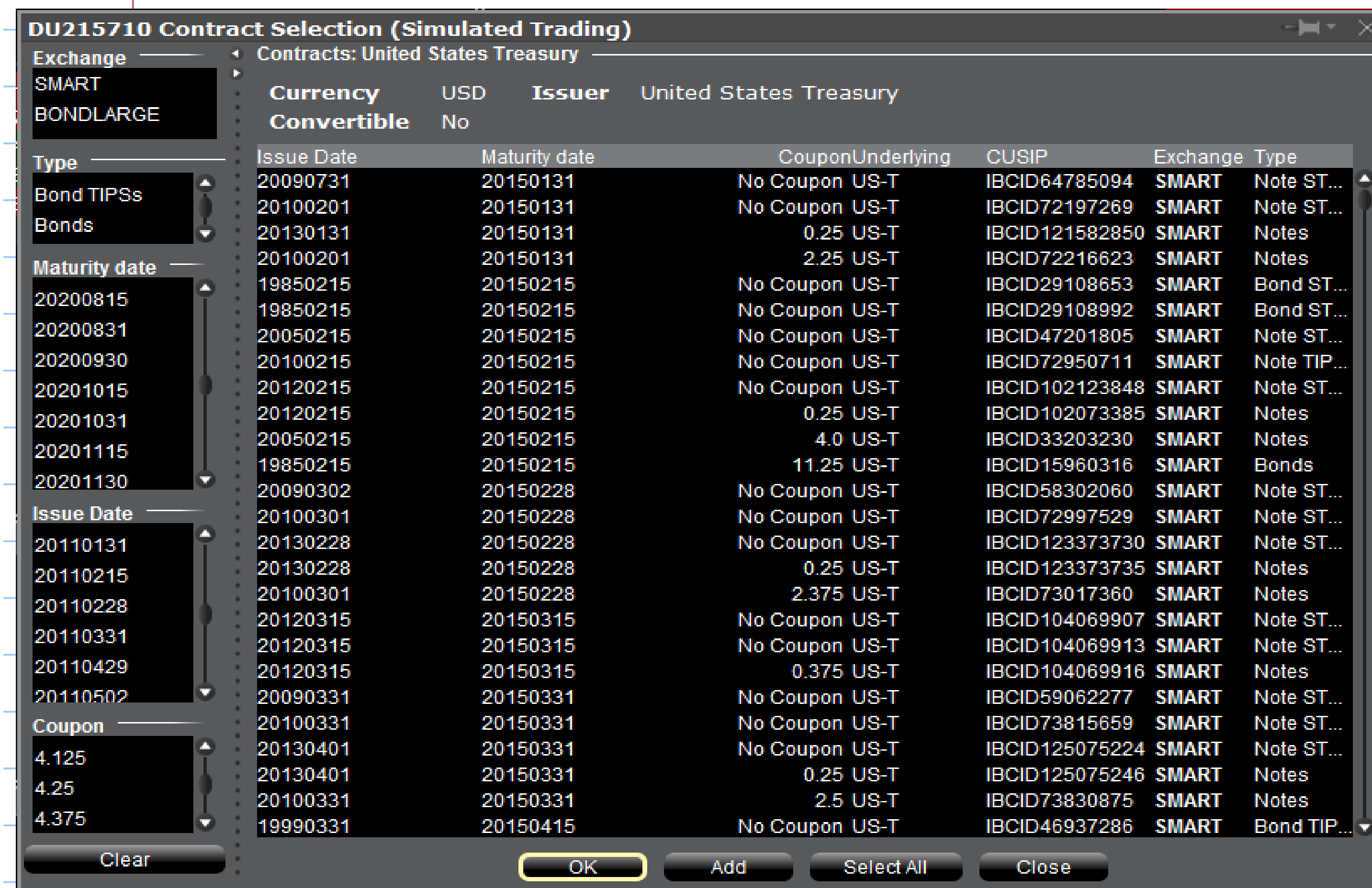
Coupon: 3.375

Contracts: Google Inc

Exchange SMART · Coupon 3.375 · Issue Date 20140225 · Issuer Google Inc
Currency USD · Maturity date 20240225 · Type Bonds

Issuer	Issue Date	Maturity date	Coupon	Underlying	CUSIP	Type	Exchange	Currency
Google Inc	20140225	20240225	3.375	GOOG	IBCID144481054	Bonds	SMART	USD

US Treasury Bonds:



DU215710 Contract Selection (Simulated Trading)

Exchange: SMART, BONDLARGE

Type: Bond TIPSs, Bonds

Maturity date: 20200815, 20200831, 20200930, 20201015, 20201031, 20201115, 20201130

Issue Date: 20110131, 20110215, 20110228, 20110331, 20110429, 20110502

Coupon: 4.125, 4.25, 4.375

Clear

Contracts: United States Treasury

Currency USD · Issuer United States Treasury
Convertible No

Issue Date	Maturity date	Coupon	Underlying	CUSIP	Exchange	Type
20090731	20150131	No Coupon	US-T	IBCID64785094	SMART	Note ST...
20100201	20150131	No Coupon	US-T	IBCID72197269	SMART	Note ST...
20130131	20150131	0.25	US-T	IBCID121582850	SMART	Notes
20100201	20150131	2.25	US-T	IBCID72216623	SMART	Notes
19850215	20150215	No Coupon	US-T	IBCID29108653	SMART	Bond ST...
19850215	20150215	No Coupon	US-T	IBCID29108992	SMART	Bond ST...
20050215	20150215	No Coupon	US-T	IBCID47201805	SMART	Note ST...
20100215	20150215	No Coupon	US-T	IBCID72950711	SMART	Note TIP...
20120215	20150215	No Coupon	US-T	IBCID102123848	SMART	Note ST...
20120215	20150215	0.25	US-T	IBCID102073385	SMART	Notes
20050215	20150215	4.0	US-T	IBCID33203230	SMART	Notes
19850215	20150215	11.25	US-T	IBCID15960316	SMART	Bonds
20090302	20150228	No Coupon	US-T	IBCID58302060	SMART	Note ST...
20100301	20150228	No Coupon	US-T	IBCID72997529	SMART	Note ST...
20130228	20150228	No Coupon	US-T	IBCID123373730	SMART	Note ST...
20130228	20150228	0.25	US-T	IBCID123373735	SMART	Notes
20100301	20150228	2.375	US-T	IBCID73017360	SMART	Notes
20120315	20150315	No Coupon	US-T	IBCID104069907	SMART	Note ST...
20120315	20150315	No Coupon	US-T	IBCID104069913	SMART	Note ST...
20120315	20150315	0.375	US-T	IBCID104069916	SMART	Notes
20090331	20150331	No Coupon	US-T	IBCID59062277	SMART	Note ST...
20100331	20150331	No Coupon	US-T	IBCID73815659	SMART	Note ST...
20130401	20150331	No Coupon	US-T	IBCID125075224	SMART	Note ST...
20130401	20150331	0.25	US-T	IBCID125075246	SMART	Notes
20100331	20150331	2.5	US-T	IBCID73830875	SMART	Notes
19990331	20150415	No Coupon	US-T	IBCID46937286	SMART	Bond TIP...

OK · Add · Select All · Close

- Default risk (what happens to the coorporate bonds you own if the company go bankrupt?)
- Bond rating
- US government securities are considered 'risk-free assets'

Forward basic

A forward contract on a commodity is a contract to purchase or sell a specific amount of the commodity at a specific price and at a specific time in the future.

The "commodity" may be :
- gold
- soyabeans
- oil
- foreign currency etc.
- an index (e.g. S&P500)

Purpose : a vehicle for transferring risk from a hedger to a speculator
(e.g. airline - hedger
oil trader - speculator)

Forward price = the price that applies at delivery

(value of the contract is zero when it initiated)

Terminology :

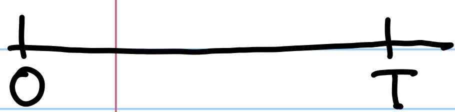
Spot market	vs	forward market
"		"
the open market for immediate delivery of the asset		the market of forward contracts for future delivery

"Theorem" Suppose an asset can be stored at zero cost and also sold short.

Suppose : current spot price ($t=0$) is S

Then : the theoretical forward price (for delivery at $t=T$)

is $$F = S \underbrace{e^{rT}}_{\substack{\text{growth factor if money is deposited in an ideal bank offering}\\ \text{continuous time compound interest with annualized rate } r \text{ for a period of}\\ \text{time } T \text{ (years)}}} \text{ ——— } \star$$



["Theorem" because I haven't defined what the theoretical forward price is supposed to mean.]

Regardless, $\star$ seems too simple to make any sense, considered that it does not account for any prediction of the <u>supply and demand</u> in $(0, T]$ of the underlying asset.

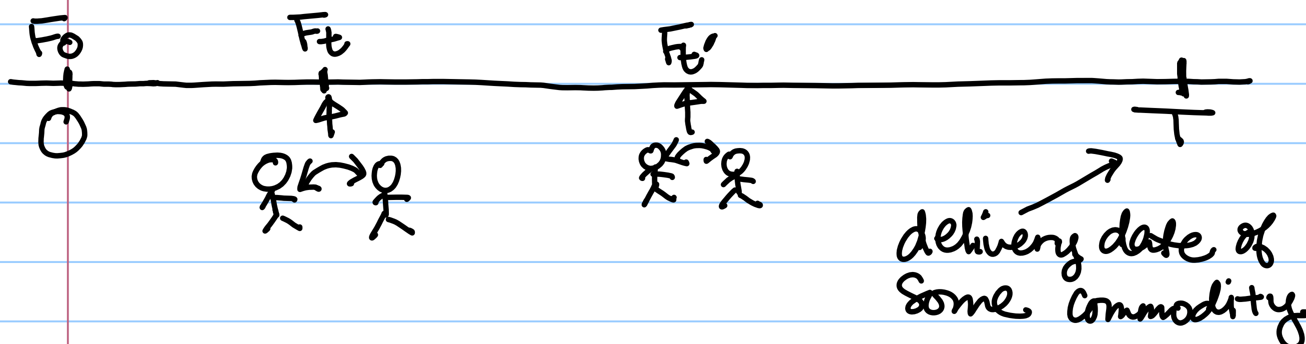
[And what about the supply and demand of such forward contracts ??]

Will get back to this .....

Futures basic

- Forward contracts are traded over the counter.
- Desirable to standardize the contracts and trade them on an organized exchange
  - convenient
  - no need to find counterparty
  - liquidity

Challenge of standardizing forward contracts:

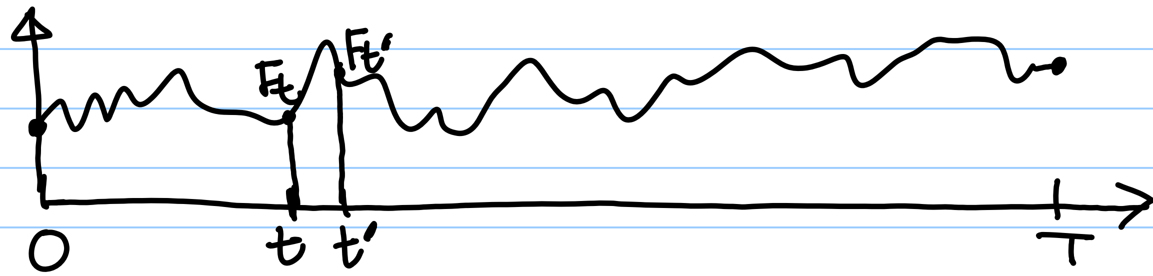


Exchange needs to keep track of the thousands of outstanding forward contracts each having a different price, even all other terms of the contract (delivery date, quantities to be delievered, quality of delivered goods etc.) are identical.

A BOOKKEEPING NIGHTMARE

The brilliant invention:

FUTURES MARKET



enters a futures contract @ price F_t

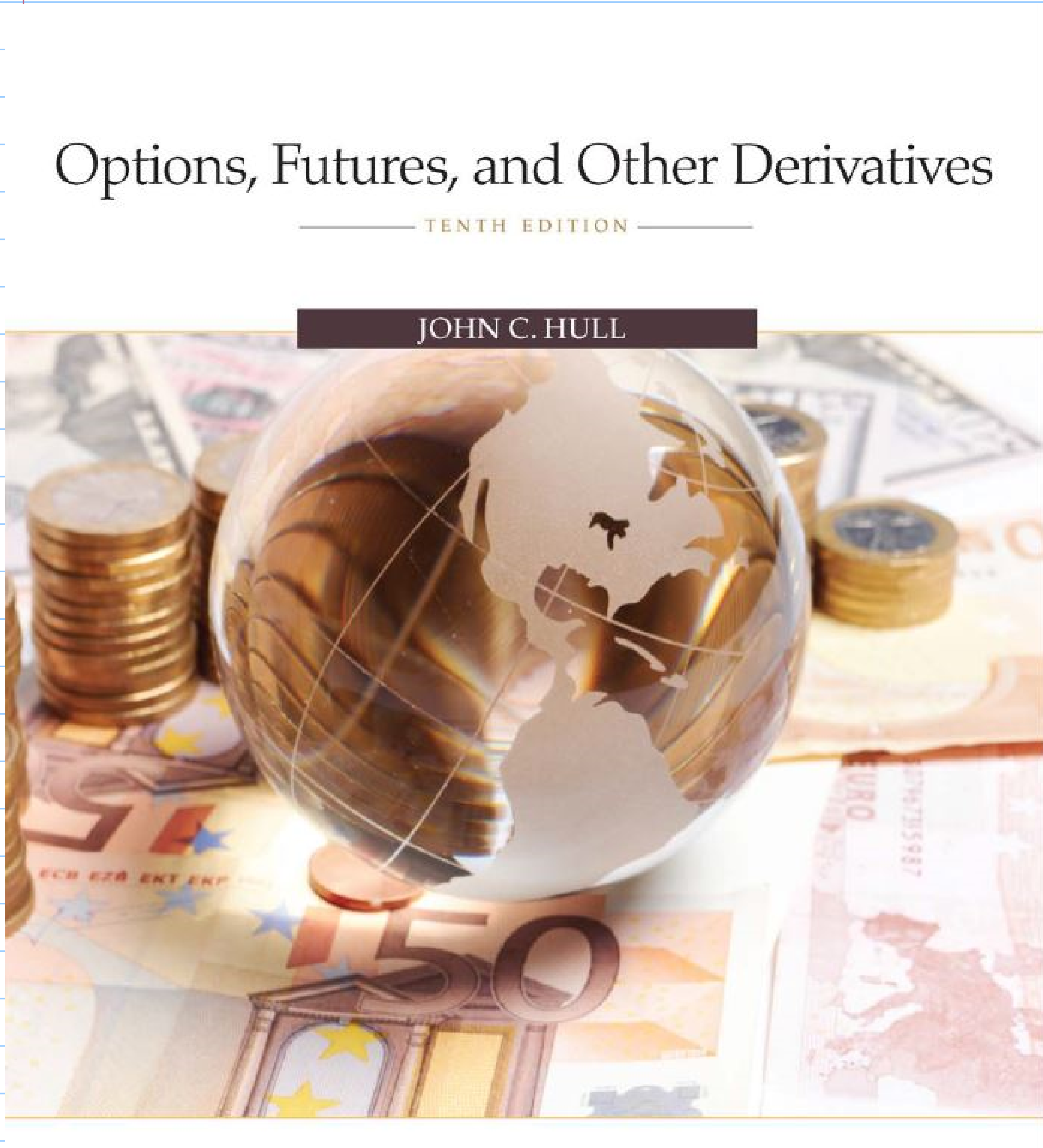
Note: the long side does <u>not</u> pay the short side $\$F_t$.
But 5-10% of the contract price is required as a deposit in each side's margin account.

At the end of each trading day, when the contract price change from F_t to $F_{t'}$

account of long side: $+\ \$(F_{t'} - F_t)$
account of short side: $-\ \$(F_{t'} - F_t)$

This process of adjusting the contracts is called marking to the market done by the clearinghouse.

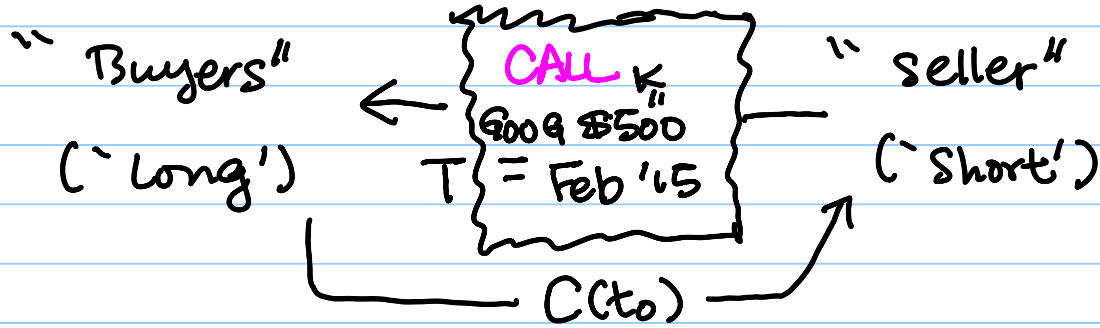
An authoritive reference:



options basic

European Call Options on an underlying asset

- Contracts between two parties



- Buyer has the right,
  <u>but not the obligation</u>,
  to buy from the seller of the option
  one unit of the asset
  (e.g. 100 shares of Google stock)
  <u>at a predetermined time T</u>
  in the future for a predetermined price K.

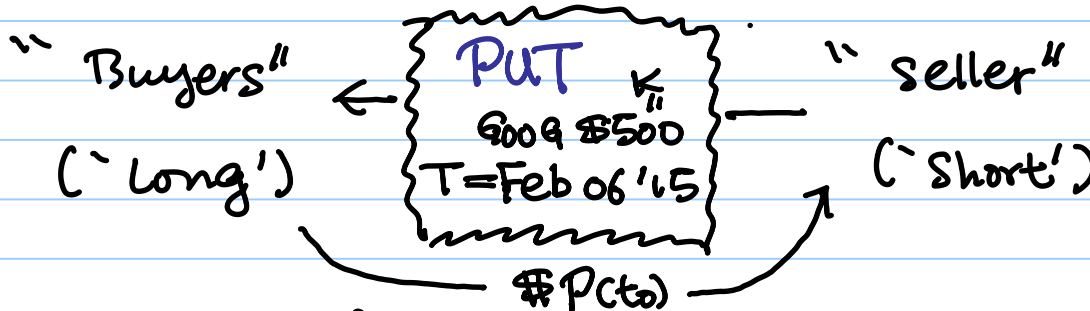
- T : maturity (or expiration) date

  K : strike price

- If "at a predetermined time T"
  is changed to "at or before time T",
  the call option is called an
  <u>American call option</u>.

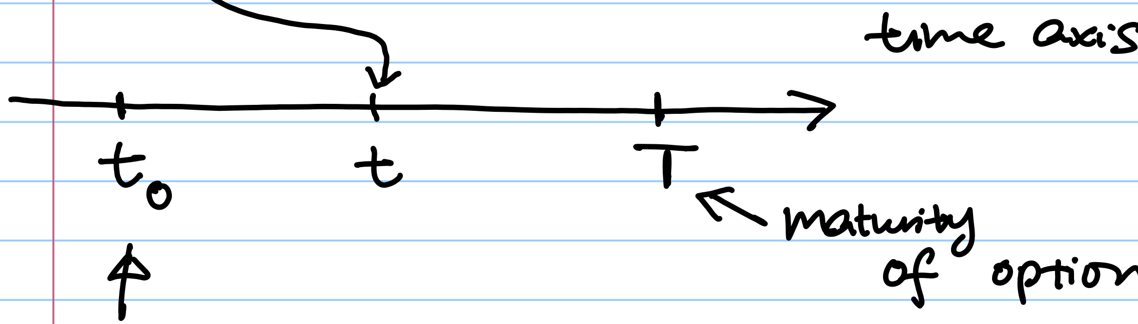
- Difference between E. and A. options has little to do with physical geography. Basically all options traded publicly on exchanges are American options.
- But we shall first develop the theory for European options.

- European Put option on an underlying asset is a contract between two parties:



- Buyer has the right,
  but not the obligation,
  to sell to the seller of the option
  one unit of the asset
  (e.g. 100 shares of Google stock)
  at a predetermined time T
  in the future for a predetermined
  price K.

Some time in $[t, T]$, presumably the buyer can sell the option to someone else at this time



maturity of option

time when the option trade happens

- Buyer of the call option pays \$ $C(t)$ at time t $(<T)$ to the seller of the call option.
- Buyer of the put option pays \$ $P(t)$ at time t $(<T)$ to the seller of the put option.

Hard Question:

$$\boxed{\text{What are the fair prices for } C(t) \ \& \ P(t)?}$$

More options terminologies:

- $S(t)$ = price of the underlying asset at time t.

In particular, $S(T)$ = price of the underlying asset at maturity

- At time t, a call option is

$$\begin{cases} \text{in the money (ITM) if } S(t) > K \\ \text{at the money (ATM) if } S(t) = K \\ \text{out of the money (OTM) if } S(t) < K \end{cases}$$

Similarly, a put option is

$$\begin{cases} \text{in the money (ITM) if } S(t) < K \\ \text{at the money (ATM) if } S(t) = K \\ \text{out of the money (OTM) if } S(t) > K \end{cases}$$

Q : You enter a long position for a call option at t_0. At maturity the call is in the money. If you exercise the option, you make a profit in this trade, is that right?

Ans: not necessarily, remember that you have to pay a premium for the option. You profit only if $S(T) - K > C(t_0)$.

Payoff of a call option at maturity is

$$C(T) = \max(S(T) - K, 0)$$

$$= \begin{cases} S(T) - K & \text{if } S(T) > K \\ 0 & \text{if } S(T) \leq K \end{cases}$$

Payoff of a put option at maturity is

$$P(T) = \max(K - S(T), 0)$$

Another teaser "theorem", called PUT-CALL PARITY:

"Theorem": Let $C(t)$ and $P(t)$ be the values at time t of a European call and put option, respectively, with maturity T and strike K, on the same non-dividend paying asset with spot price $S(t)$. Then

$$\circledast \quad - \quad P(t) + S(t) - C(t) = K e^{-r\underbrace{(T-t)}_{\text{"time to expiration"}}}$$

If not, some greedy traders will step in the market, do something smart, and quickly force $\circledast$ to be satisfied.

[April 1, Apple trading at $124]

AAPL

SIMULATED TRADING | SIMULATED TRADING

Call Last	Call Change	Call Bid	Call Ask	Description	Put Last	Put Change	Put Bid	Put Ask
				APR 02 '15				
1.50	-0.44	1.47	1.55	123	0.30	-0.21	0.28	0.31
0.80	-0.48	0.82	0.85	124	0.61	-0.24	0.59	0.64
0.40	-0.36	0.39	0.40	125	1.22	-0.09	1.12	1.19
0.18	-0.26	0.16	0.18	126	2.04	+0.03	1.90	1.98
				APR 10 '15				
2.55	-0.27	2.51	2.58	123	1.38	+0.18	1.29	1.32
1.97	-0.23	1.94	1.97	124	1.77	+0.20	1.70	1.74
1.44	-0.26	1.45	1.48	125	2.33	+0.29	2.20	2.27
1.01	-0.23	1.04	1.07	126	2.94	+0.35	2.78	2.85
				APR 17 '15				
3.50	-0.05	3.25	3.35	122.86	2.15	+0.28	1.89	1.92
3.20	-0.25			123	2.00	+0.07	1.95	1.98
2.59	-0.28			124	2.48	+0.14	2.38	2.42
2.36	-0.35	2.49	2.53	124.29	2.60	+0.13		
2.10	-0.27	2.14	2.19	125	3.00	+0.17	2.88	2.92
1.80	-0.24	1.83	1.86	125.71	3.35	+0.15	3.25	3.35
1.71	-0.18	1.71	1.74	126	3.70	+0.33	3.40	3.50

Arbitrage - free pricing

Arbitrage opportunity (arb. opp.) — an investment opportunity guaranteed to earn money without any risk.

Usually price of just about anything is decided by supply and demand. But imagine that some financial instrument is priced in such a way that creates an arb. opp., then quite likely this person (and/or other people who see the same opportunity) will try to buy/sell as many units of the instrument (and whatever else necessary for generating riskless profit) as possible so as to generate as much riskless profit as possible.

As such, the surge in supply/demand will (typically quickly) move the price to a price that eliminates the arbitrage opportunity.

Such an equilibrium price is called a no-arbitrage price of the financial instrument.

Interest

- "the time value of money"
- riskless return

Assume interest rate is
$100r\%$ per year

It means:

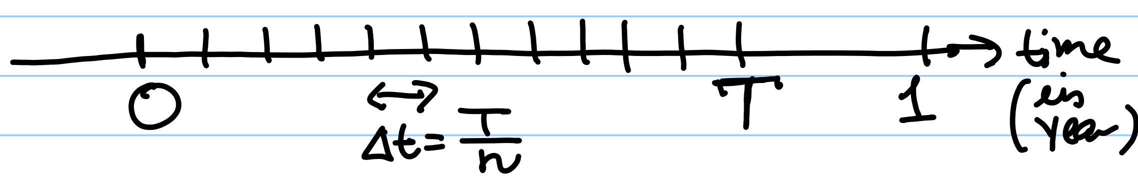
If interest is paid at the end of every year, a capital of \$C invested becomes
$\$C \cdot (1+r)$ after one year

If interest is paid quarterly, and one re-invests the interest, a capital of \$C invested becomes
$\$C\left(1+\frac{r}{4}\right)^4$ after one year

If interest is compounded daily,
\$C becomes $\$C\left(1+\frac{r}{365}\right)^{365}$ after one year

More generally, if compounded n times for a period of T (years),

$$\$C \text{ becomes } \$C(1+\Delta t\, r)^n$$
$$= \$C\left(1+\frac{T}{n}r\right)^n$$



Recall

(★) —— $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \; (\approx 2.718)$

so $\lim_{n\to\infty}\left[\left(1+\frac{Tr}{n}\right)^{\frac{n}{Tr}}\right]^{Tr} = e^{rT}$

The growth factor assuming "continuous compounding" for a period of time T (years) with an annualized interest rate r.

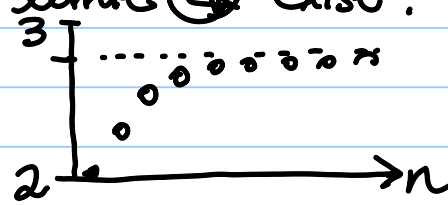
Of financial interest:

would you rather compound the interest more frequently or less frequently?

that is to ask: How does $\left(1+\frac{1}{n}\right)^n$ vary with n?

Of mathematical interest:

why does the limit (★) exist?



If we can prove:

$\left(1+\frac{1}{n}\right)^n < 3$

and $\left(1+\frac{1}{n}\right)^n \uparrow$ as $n \uparrow$ then the limit must exist.

Interestingly, proving the monotonicity of $(1+\frac{1}{n})^n$ answers both questions. (HW 1)

$\lim_{n\to\infty} (1+\frac{1}{n})^n = e$ is perhaps one of the most nontrivial limits in basic analysis. The monotonicity is a bit tricky to show. (I know of two different proofs.)
