

Math T680 Topics in Geometry

HW #4

Due: Wednesday, May 10, 2017

1. Prove that S^{n-1} is not contractible. (Note: at this point we haven't defined the de Rham cohomology of S^{n-1} so it is illegal to write/use $H^p(S^{n-1})$.)
2. Show that for $1 \leq k \leq n-2$,

$$H^p(\mathbb{R}^n - S^k) \cong \begin{cases} \mathbb{R} & p = 0, n-k-1, n-1 \\ 0 & \text{otherwise} \end{cases}.$$

You can (without loss of generality) identify S^k with

$$\{(x_1, \dots, x_{k+1}, \underbrace{0, \dots, 0}_{n-k-1}) \in \mathbb{R}^n : x_1^2 + \dots + x_{k+1}^2 = 1\}.$$

(Hint: what happens when $k = n-1$? The case I didn't ask you to solve is actually a key to the cases I do ask you to solve.)

3. Assume $H_p(\mathbb{R}^n - S^k) \cong H^p(\mathbb{R}^n - S^k)$, offer an intuitive interpretation of the previous result based on the meaning of homology.