Math T680 Topics in Geometry

$\boxed{\text{HW \#3}}$

Due: Wednesday, May 3, 2017

1. Problem 4.4 M&T.

   Without knowing the background, this problem alone would not explain Euler characteristics. Enjoy the beautiful algebra for now.

2. Verify my counts of V, E and F in the following tesselation of the plane. The earth is spherical, so picture the following plane as a sphere. In particular, the 'unbounded region' (which is actually bounded when you map the plane to the sphere) should be counted as one face. (Recall that the *one point compactification* of the plane is S^2; it can be implemented, for example, by a stereographic projection $\mathbb{R}^2 \cup \{\infty\} \to S^2$.)

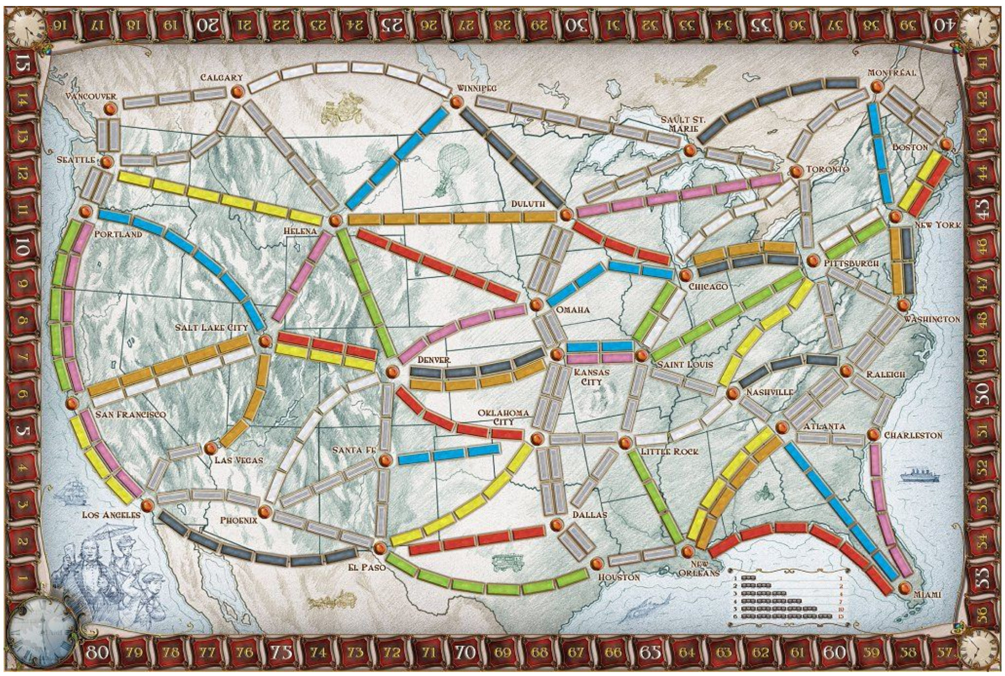

Figure 1: $V = 36$, $E = 78$, $F = 44$, $\dim H^0(S^2) = 1$, $\dim H^1(S^2) = 0$, $\dim H^2(S^2) = 1$, $V - E + F = \dim H^0(S^2) - \dim H^1(S^2) + \dim H^2(S^2) = 2$

3. Some of the cells above are 4-sided, one is even 5-sided. Add some railroads to make all the cells 3-sided. Then count V, E, F again. No matter how you do that $V - E + F$ must be 2.

I will give a proof of $V - E + F = 2$ using de Rham cohomology. Another approach, which I think is more intuitive, is to use singular homology. Either approach reduces to the algebraic structure seen in Problem 1.