Research summary in Spring 2020

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Preface

- Topics we covered
  - Decentralized algorithms.
  - Large-scale distributed optimization.
- Tools
  - MPI, Python
- Results
  - Documents and corresponding codes.
- UIC HPC Platform – ACER
  - ACER issued free accounts to students taking Parallel Computing course, each program can be allocated the maximal 7x16 processors.
  - ACER had its own MPI scheduler and compilers.
- Great thanks to Shuo Han!

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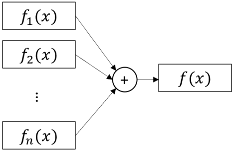
Objective Scenario

Many ML problems could be decomposed as following form:

$$\min \quad f(x) = \frac{1}{n}\sum_{i=1}^{n} f_i(x; A_i) = \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

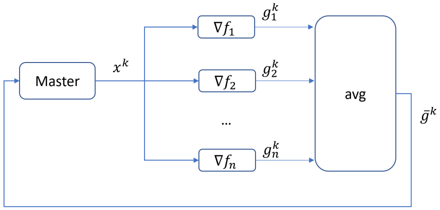


A_i is data used in f_i, f_i is convex, f has a optimal solution x^*.
In this case, we could assign each f_i into a node allocated in a cluster network.

Optimization

We could use Gradient Descent to minimize f:

$$x^{k+1} = x^k - \frac{\eta}{n}\sum_{i=1}^{n} \nabla f_i(x^k)$$
$$= x^k - \frac{\eta}{n}\sum_{i=1}^{n} g_i^k$$

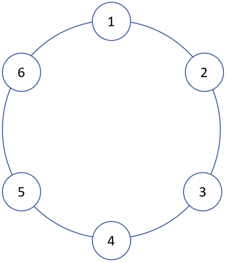


In centralized network, we could ask master node collect gradients from workers, then broadcast updated x.
What if a decentralized network, in other word, how to achieve a average mechanism?

Gossip Matrix

Let's think about d2d networks, each node i has a value to share with its neighbors. Recall the Markov chain, we use a doubly stochastic matrix W representing this "transition", for example a ring.

$$W = \begin{bmatrix} 0.5 & 0.25 & 0 & 0 & 0 & 0.25 \\ 0.25 & 0.5 & 0.25 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.25 & 0 & 0 & 0 & 0.25 & 0.5 \end{bmatrix}$$



$\forall x^0 \in \mathbb{R}^N, x^{k+1} = Wx^t,\ J = \frac{1}{n}\mathbf{1}\mathbf{1}^T$. As $k \to \infty$, $x^k = Jx^0$

Gossip Matrix

Theorem 1

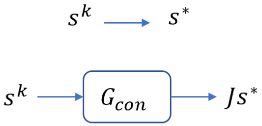
Given a W, if W satisfy following properties,

- $W_{ij} \in [0, 1]$, $W_{ii} > 0$.
- W is symmetry, i.e. $W = W^T$.
- $W \cdot \mathbf{1} = \mathbf{1}$, i.e. $\mathbf{1}$ is an eigenvector of W.
- $\lambda(W) \leq 1$.

Then, W is a gossip matrix, i.e. $\lim_{k\to\infty} x^k = Jx^0$.

Average Consensus Tracking

Suppose that we have a dynamical system G_{con}, G_{con} achieves **average consensus tracking** if we have a sequence $s = \{s^k \in \mathbb{R}^N\}$ converging to s^*, the $\hat{s} = G_{con}(s)$ converge to Js^*.



A example (not unique) of G_{con} is

$$\hat{s}^{k+1} = W\hat{s}^k + (s^{k+1} - s^k), \quad \hat{s}^0 = s^0 \tag{1}$$

Let $\hat{s}^{k+1} = s^{k+1} + \epsilon^{k+1}$, we have

$$\begin{aligned} \epsilon^{k+1} &= \hat{s}^{k+1} - s^{k+1} = W\hat{s}^k - Ws^k + Ws^k - s^k \\ &= W\epsilon^k + (W - I)s^k \end{aligned}$$

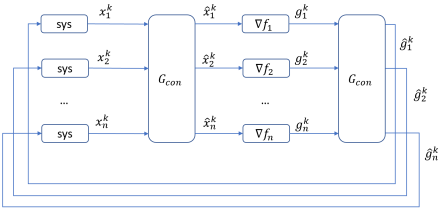
Therefore, (1) could be rewritten as,

$$\hat{s}^{k+1} = s^{k+1} + \epsilon^{k+1}, \quad \epsilon^{k+1} = W\epsilon^k + (W - I)s^k, \quad \epsilon^0 = 0 \tag{2}$$

Average Consensus Tracking

We could use (2) to track x^k and g^k, since both of them will converge to the optimal point, i.e.

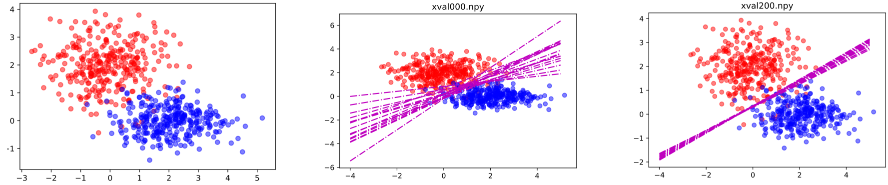
$$x \to x^*, \quad \hat{x} \to Jx^*$$
$$g \to g^*, \quad \hat{g} \to 0$$



Therefore, we succeed to generalize a decentralized gradient descent algorithm.

Average Consensus Tracking

Many optimization algorithms could be rewritten as a decentralized form by using average consensus tracking; here's an example of SVM.



The middle figure shows the optimal decision boundaries without consensus sharing, and the right one is with consensus sharing.

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Stochastic Gradient Descent

Similar to the previous scenario, we assume $\nabla^2 F \succcurlyeq L$ and $\nabla^2 f_i \preccurlyeq \mu$.

$$\min \quad F(x) = \frac{1}{n}\sum_{i=1}^{n} f_i(x; A_i) = \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

Another popular method is Stochastic Gradient Descent, it randomly pick one gradient, instead of all gradient to update x.

$$x^{k+1} = x^k - \frac{\eta}{n} \triangledown f_{i_k}(x^k) = x^k - \eta g^k$$

Comparing to GD, SGD decreases the intensity of computation, but it can't guarantee $x^k \to x^*$.

Stochastic Gradient Descent

- $g^k = \nabla f_{i_k}(x^k)$.
- $\mathbb{E}(g^k) = \nabla F(x^k)$.
- $\exists \sigma_g \geq 0,\ c_g \geq 1,\ \mathbb{E}\left[\left\|g^k\right\|_2^2\right] \leq \sigma_g^2 + c_g \left\|\nabla F(x^k)\right\|_2^2$
- $\mathbb{E}\left[F\left(x^k\right) - F\left(x^*\right)\right] \leq \frac{\eta L \sigma_g^2}{2\mu} + (1 - \eta\mu)^t \left(F\left(x^0\right) - F\left(x^*\right)\right)$

Stochastic Gradient Descent

- $g^k = \nabla f_{i_k}(x^k)$.
- $\mathbb{E}(g^k) = \nabla F(x^k)$.
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- $\mathbb{E}\left[F\left(x^k\right) - F\left(x^*\right)\right] \leq \frac{\eta L \sigma_g^2}{2\mu} + (1-\eta\mu)^t \left(F\left(x^0\right) - F\left(x^*\right)\right)$
- How to reduce the variance of g^k?
    - Find a zero-mean auxiliary variable v and it positively correlated with $\nabla f_{i_k}(x^k)$, i.e. $\left\langle v, \nabla f_{i_k}(x^k)\right\rangle > 0$.
    - Let $\hat{g}^k = \nabla f_{i_k}(x^k) - v, \mathbb{E}(\hat{g}^k) = \nabla F(x^k)$
    - $\mathbb{E}\left[\left\|\hat{g}^k\right\|_2^2\right] = \mathbb{E}\left[\left\|\nabla F(x^k)\right\|_2^2\right] + \mathbb{E}\left[\left\|v\right\|_2^2\right] - 2\mathbb{E}\left[\left\langle v, \nabla f_{i_k}(x^k)\right\rangle\right]$.
- Variance reduction is a way to reduce noise.
    - $\rightarrow$ More accurate estimate $\rightarrow \mathbb{E}\left[F(x^k) - F(x^*)\right] \rightarrow 0$.

Stochastic Variance Reduced Gradient - SVRG

Store the history x^{old} and substitute it for v, i.e.

$$\hat{g}_s^k = \underbrace{\nabla f_{i_k}\left(\boldsymbol{x}_s^k\right) - \nabla f_{i_k}\left(\boldsymbol{x}_s^{\text{old}}\right)}_{\rightarrow \mathbf{0} \text{ if } \boldsymbol{x}_s^k \approx \boldsymbol{x}_s^{\text{old}}} + \underbrace{\nabla F\left(\boldsymbol{x}_s^{\text{old}}\right)}_{\rightarrow \mathbf{0} \text{ if } \boldsymbol{x}_s^{\text{old}} \approx \boldsymbol{x}^*}$$

In s^{th} epoch,

- take snapshots for every $\nabla f_i(x_s^0)$ as $\nabla f_i(x^{\text{old}})$.
- inner loop: for $k = [1, 2, \cdots, m]$
  - $x_s^{k+1} = x_s^k - \eta \hat{g}_s^k$
- Update x_{s+1}^0 for next epoch.

Stochastic Variance Reduced Gradient - SVRG

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- $\mathbb{E}\left[\left\|\hat{g}^k\right\|_2^2\right] \leq 4L\left[F(x_s^k) - F(x^*) + F(x_s^{\text{old}}) - F(x^*))\right]$
- $\mathbb{E}\left[F(x_s^{\text{old}}) - F(x^*)\right] \leq \rho^s\left[F(x^0) - F(x^*)\right]$
- $\rho = \frac{1}{\mu\eta(1-2L\eta)m} + \frac{2L\eta}{1-2L\eta} < 1$

Stochastic Variance Reduced Gradient - SVRG

- Built & tested the SVRG Python program with MPI.
- UIC high performance cluster ACER supports MPI.
- We also succeed built other variance reduction algorithms on ACER, including SAGA.

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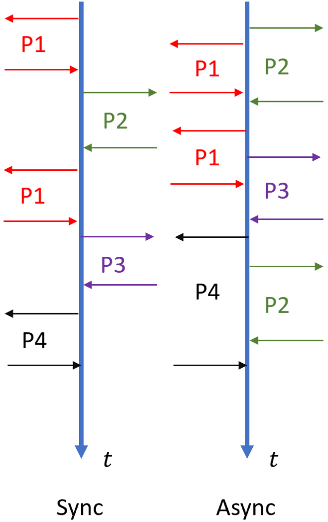
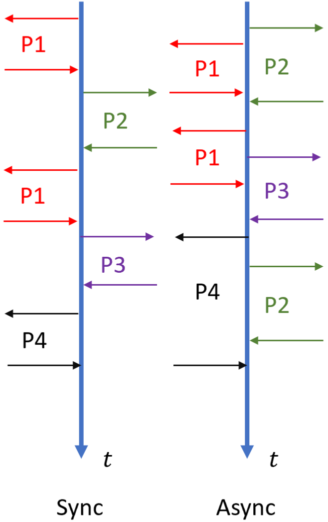
Asynchronous Optimization

- Sync: R/W in interleaved style.
  - Common in analysis.
  - Damage performance a lot.
- Async: R/W in any order.
  - Remove sync lock.
  - Hard to analyze.
  - Achieve linear speedup for parallel scenario using shared memory.



Asynchronous Optimization

- Problems of async:
    - Multiple agents will read the same x.
- Problems of MPI.
    - MPI is built on socket, not shared memory, it already has locks.
    - Only verify if analysis fits practical.
- Still investigating async algorithms, including variance reduction ones.



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