Image Quality Comparison of Reconstruction Using Total Variation-Based Regularizers

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***Abstract*–Regularization methods are commonly used for noise reduction in SPECT image reconstruction. Total variation (TV) is a well-accepted method for suppressing noise while preserving edges. However, using TV as regularizer implies that the image is piecewise constant. Usually this is untrue in clinical settings and leads to staircasing artifacts. High-order TV (HOTV) and infimal-convolution TV (ICTV) are proposed here to avoid such artifacts. Previously, we proposed the preconditioned alternating projection algorithm (PAPA) to address the nondifferentiability of the TV regularizer. Here we generalize PAPA to solve for these regularizers, and apply them to reconstruction of Monte Carlo-simulated SPECT data. We designed two numerical phantoms with lumpy background, with and without targets, to compare the performance of these algorithms. We analyzed the reconstructed images using the channelized Hotelling observer (CHO). The signal-to-noise ratio of CHO was used as a figure of merit. The hyperparameters of the regularization methods were selected using the point-spread function of point sources inside the phantom. Images reconstructed using PAPA-TV, PAPA-HOTV, and PAPA-ICTV, along with traditional EM-TV were studied. All such images have similar local spatial resolution. Images reconstructed using EM-TV and PAPA-TV show significant staircasing artifacts. While both PAPA-HOTV and PAPA-ICTV reduce these artifacts very well, PAPA-HOTV cannot suppress noise as well as PAPA-ICTV does, without compromising spatial resolution. According to CHO analysis, PAPA-ICTV provides higher lesion detectability, compared to PAPA-HOTV. We conclude that PAPA-ICTV is a superior image-reconstruction method and has high potential for use in clinical settings due to more efficient noise suppression and significantly improved image quality.**

I. Introduction

SPECT reconstruction generally requires noise reduction. The expectation maximization (EM) algorithm, without regularization or filtering, tends to increase the noise level as iteration number increases. The penalized likelihood (or equivalently maximum *a posteriori*, MAP) reconstruction addresses this problem. Total variation (TV) [1] is widely used as regularizer in SPECT reconstruction, for its ability to preserve edges. We have evaluated noise level and lesion detectability using two regularizers—high-order total variation (HOTV) [2] and infimal-convolution total variation (ICTV) [3]—in comparison with conventional TV.

II. Methods and Materials

A. Penalized likelihood reconstructions with different regularizers

The SPECT objective function solver can be written as:

$$f_* = \arg\min_{f\geq 0}\left\{\langle Af,1\rangle - \langle \ln(Af+\gamma), g\rangle + \lambda U(f)\right\} \quad (1)$$

where f is the reconstructed image vector, g is the observed data, and A is the system-response matrix. The following regularizers $U(f)$ have been used in this study:

a. total variation (TV):

$$\int_\Omega \lambda |\nabla f| dx \quad (2)$$

b. high-order total variation (HOTV)

$$\int_\Omega \lambda_1 |\nabla f| + \lambda_2 |\nabla\cdot(\nabla f)| dx \quad (3)$$

c. infimal-convolution total variation (ICTV):

$$\min_{f=f_1+f_2}\int_\Omega \lambda_1 |\nabla f_1| + \lambda_2 |\nabla\cdot(\nabla f_2)| dx \quad (4)$$

B. Preconditioning alternating projection algorithm (PAPA)

PAPA was proposed [4] to solve the following problem:

$$f_* = \arg\min_{f\geq 0}\left\{\langle Af,1\rangle - \langle \ln(Af+\gamma), g\rangle + \lambda\varphi(Bf)\right\}, \quad (5)$$

where B is a matrix, and φ is a convex non-negative function.

Using proximity operators, we formulated the optimization problem (1) as a system of fixed-point equations. Such fixed-point characterization naturally leads to an alternating projection algorithm. PAPA does not require an *ad hoc* parameter as is needed in EM-TV [5] to deal with non-differentiability of TV. Moreover, PAPA can solve a wide range of optimization problems, provided that the regularizer can be formulated in the form of $\varphi(Bf)$. PAPA was successfully applied to TV [4]. Since HOTV and ICTV can be rewritten as $\varphi(Bf)$ (Table 1), PAPA can also solve these regularization problems efficiently.

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TABLE 1. Regularizers TV, HOTV, and ICTV. Here ∇ represents first-order difference matrix.

	φ	B	f
TV	L1-norm	∇	f
HOTV	L1-norm	$\begin{pmatrix} \nabla \\ -\nabla^T \nabla \end{pmatrix}$	f
ICTV	L1-norm	$\begin{pmatrix} \nabla & \\ & -\nabla^T \nabla \end{pmatrix}$	$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

C. Phantom Simulation

Two numerical phantoms were created: a cylinder with lumpy background (Fig. 1a) and target absent; and an identical cylinder with target present, i.e. containing one set of hot spheres and one set of point sources, respectively (Fig. 1b,c).

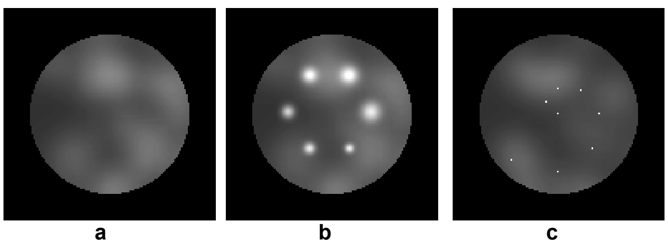

Fig. 1. Phantom cross sections. **a.** Cylinder with lumpy background activity (randomly positioned, overlapping, Gaussian-blurred spheres) with target absent. **b.** Cylinder with lumpy background activity with targets present: with six hot spheres; and **c.** with seven point sources.

D. Parameter selection

We required that all the reconstruction methods studied had the same spatial resolution. We first selected regularization parameter λ for PAPA-TV to balance the noise-resolution tradeoff. Then, we selected parameters for the other three methods to match the spatial resolution of PAPA-TV. λ is selected to be 0.018 for both OSL-TV and PAPA-TV. PAPA-ICTV parameters λ_1 and λ_2 were selected as 0.016 and 0.010, respectively. PAPA-HOTV parameters λ_1 and λ_2 were selected to be 0.007 and 0.0035.

III. Results and Discussion

A. Reconstructed images

Figure 2 shows representative reconstructed images. Staircasing artifacts are evident in both EMTV (2a) and PAPA-TV (2b) reconstructions, but they are significantly reduced in PAPA-HOTV (2c) and PAPA-ICTV (2d).

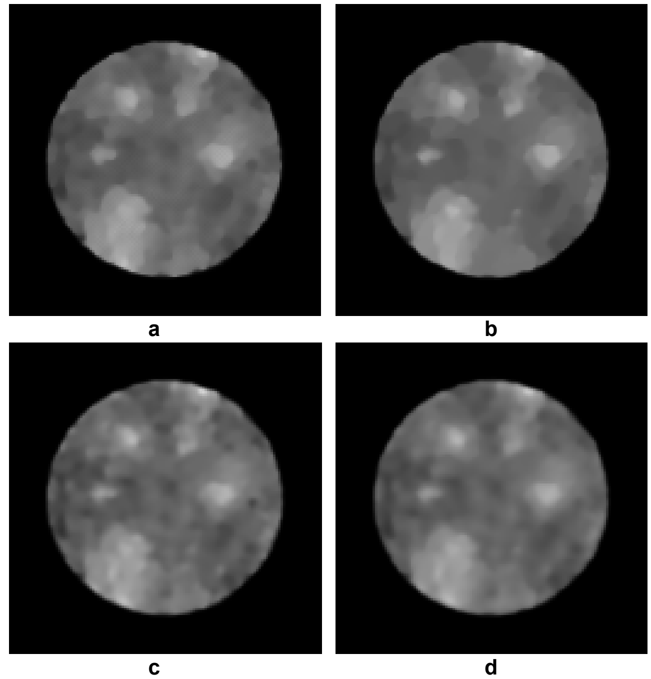

Fig. 2. Reconstructed images studied by CHO. (a) EM-TV (b) PAPA-TV (c) PAPA-HOTV (d) PAPA-ICTV.

B. Localized noise power spectrum (NPS)

For each reconstruction method, the noise power spectrum was calculated for a 6 cm × 6 cm region in the center of the slice in Fig. 1a, using [6; Eq. 5.1] to account for image structures. Figure 3 shows that all four algorithms suppress high frequency noise very well. PAPA-TV and PAPA-ICTV provide significantly less (~2-fold) noisy images than EM-TV and PAPA-HOTV, as is evident in the reconstructed images in Fig. 2.

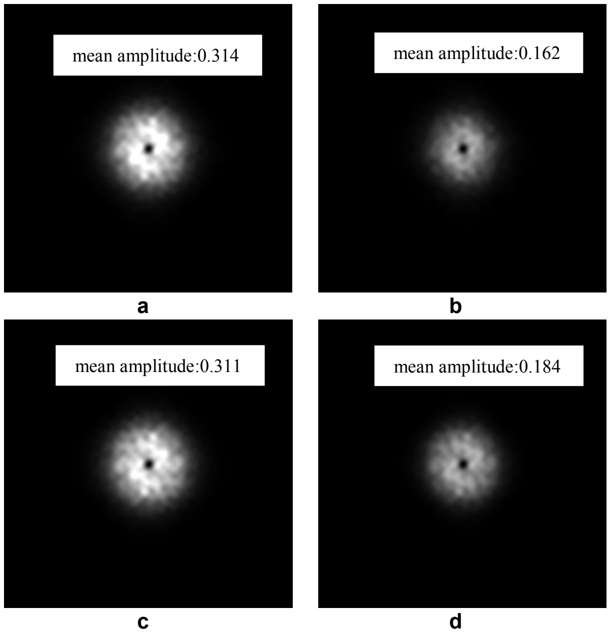

Fig. 3. Local noise power spectra for **a.** EM-TV **b.** PAPA-TV **c.** PAPA-HOTV **d.** PAPA-ICTV. Images shown here have same scale of brightness and contrast.

C. *Channelized Hotelling observer*

We applied Channelized Hotelling observer (CHO) [7] to our reconstructed images to analyze the detectability of the six spheres shown in Fig. 1b. Five channels were selected as follows: 1/64–1/32, 1/32–1/16, 1/16–1/8, 1/8–1/4, 1/4–1/2 cycles/pixel. For each algorithm, 100 noise realizations were used. The results are compiled in Table 2. We observe that SNR is higher for the PAPA-TV and PAPA-ICTV reconstructed images compared with PAPA-HOTV, and EMTV gives the lowest SNR.

TABLE 2. CHO SNRs of EM-TV PAPA-TV, PAPA-HOTV, and PAPA-ICTV with standard error.

sphere FWHM (mm)	EMTV	PAPA-TV	PAPA-HOTV	PAPA- ICTV
9.4	2.67 ± 0.019	3.22 ± 0.021	2.71 ± 0.019	2.76 ± 0.020
11.75	3.74 ± 0.023	3.86 ± 0.024	3.85 ± 0.023	3.69 ± 0.023
14.1	5.23 ± 0.030	5.69 ± 0.032	5.17 ± 0.031	5.41 ± 0.033
16.45	7.05 ± 0.041	7.21 ± 0.039	7.60 ± 0.041	8.09 ± 0.043
18.8	7.62 ± 0.042	9.71 ± 0.052	8.08 ± 0.044	8.67 ± 0.047
21.15	8.96 ± 0.047	12.83 ± 0.067	9.71 ± 0.050	10.30 ± 0.053

IV. Conclusions

Our results demonstrate that both PAPA-HOTV and PAPA-ICTV can preserve edges without creating staircasing artifacts. PAPA-ICTV suppresses noise better than PAPA-HOTV, thus providing better images. Therefore, PAPA-ICTV has superior potential for use in clinical practice.

References

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