Bayesian Estimation of Two value Distribution Patameter under a Weighted Balanced Loss Function

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Abstract—**In the paper through comparing all sorts of distributed parameter estimation In different papers. This paper considers a new loss function from goodness of fit.This paper mainly discussed the Bayesian estimation of Pareto distribution parameter and Pascal distribution parameter, and had proven that this estimation was admissible with given $q(\theta)$.**

Keywords-Weighted balanced loss function; Pareto distribution; Pascal distribution; Bayes estimation; admissibe

I. INTRODUCTION

With the development of modern statistical estimate. The demand for estimation of model parameters are increasing, becuase industries of model dependence are increasing. About model parameter estimation study mostly under linex loss function in [1], symmetric loss function in [2], entropy loss function in [3]. These loss function mainly focused on accuracy of estimation and neglected the model of optimum fitting benign. There have been many scholars get the weighted loss function after zellner proposed square loss function

$$L_1\left(\theta,\hat{\theta}\right)=\frac{w}{n}q(\theta)\sum_{i=1}^{n}\left(X_i-\hat{\theta}\right)^2+(1-w)q(\theta)\left(\theta-\hat{\theta}\right)^2$$

$$\left(0\le w\le 1, q\left(\theta\right)>0\right)$$

Some scholars have used it to Bayes estimation of Poisson distribution parameters in [4-6].

Lemma 1 Let loss function be weighted balance loss function .Then Bayes estimation of θ is

$$\hat{\theta}=w\overline{X}+(1-w)\frac{E\left[\theta q(\theta)|X\right]}{E\left[q(\theta)|X\right]},$$

for any prior distribution $\pi(\theta)$, let $q(\theta)=\theta^2$,we consider a continuous distribution and a discrete distribution.

II. PARETO DISTRIBUTION

Pareto distribution is named by Italian economist---Pareto . It's an idempotent law distribution which is found from the phenomenon of the real world .It's also called Bradford distribution other than economics. In recent years ,because of its good property ,it has been receive more and more attention from researchers. Many scholars discussed the estimate and property of Pareto distribution. Li Yan Ying discussed Bayes estimation of Pareto distribution parameters in [7].Li Fan Qun discussed Bayes estimation of Pareto distribution parameters under entropy loss in [8].

Let θ and a be positive numbers, the distribution function of the Pareto distribution with shape parameter θ and scale parameter a is given by $F(x)=0$ for $x\le a$ and

$F(x)=1-a^{\theta}x^{-\theta}, x>a$

Theorem 1 Let loss function be weighted balance loss function and let $\pi(\theta)$ have the gamma distribution with shape parameter α and scale parameter λ .Then Bayes estimation of θ is $\hat{\theta}=w\overline{X}+(1-w)\cdot\frac{n+\alpha+2}{T+\lambda}$ for Pareto distribution ,where

$$T=\sum_{i=1}^{n}\left(\ln X_i-\ln\alpha\right)$$

Proof Since $\pi(\theta)$ have the gamma distribution.

Thus, $\pi(\theta)=\frac{\lambda^{\alpha}\theta^{\alpha-1}}{\Gamma(\alpha)}$, and hence Posterior density of θ is

$\pi\left(\theta|X\right)=\frac{(T+\lambda)^{n+\alpha}}{\Gamma(n+\alpha)}\theta^{n+\alpha-1}e^{-(T+\lambda)\theta}$,which have the $\Gamma-distribution$ with shape parameter $n+\alpha$ and scale parameter $T+\lambda$,where $T=\sum_{i=1}^{n}\left(\ln X_i-\ln\alpha\right)$.Observe that

$$E\left(\theta^2|X\right)=\frac{\Gamma(n+\alpha+2)}{\Gamma(n+\alpha)(T+\lambda)^2},$$

$$E\left(\theta^3|X\right)=\frac{\Gamma(n+\alpha+3)}{\Gamma(n+\alpha)(T+\lambda)^3}$$

Thus by lemma 1 , the Bayes estimation of θ is given by

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$$\hat{\theta} = w\overline{X} + (1-w)\frac{E\left[\theta q(\theta)|X\right]}{E\left[q(\theta)|X\right]}$$

$$= w\overline{X} + (1-w)\frac{E\left[\theta^2|X\right]}{E\left[\theta^3|X\right]}$$

$$= w\overline{X} + (1-w)\cdot\frac{n+\alpha+2}{T+\lambda}$$

Lemma 2 Let loss function $L\left(\theta,\hat{\theta}\right)$ be a strictly convex function of $\hat{\theta}$.Then Bayesian Estimation will unique.

Lemma 3 In a given Bayesian decision, suppose the prior distribution is proper and $\delta_B(X)$ which is the estimation of θ is unique .Then $\delta_B(X)$ is admissible estimates.

Let loss function be weighted balance loss function and let $q(\theta)=\frac{1}{\theta^2}$ .According to Theorem 1, Lemma 2 and Lemma 3,we can get Bayesian Estimation will unique and admissible estimates .

III. PASCAL DISTRIBUTION

In the modern life, many products demand a higher reliability index .Therefore products must have a reliability evaluation .Its method is pass-fail model .Its life is measured by pass-fail number ,but most experiments are very expensive such as Satellite launch .Multiple launch is impossible, hence it often specified number of failure .Trial is stopped as soon as reach the specified number of failure .In the family of Bernoulli trials , there is a famous distribution--- Pascal distribution .The probability of events A which is occurred is θ,the distribution law is $P(X=x)=C_{x-1}^{r-1}\theta^{x-r}(1-\theta)^r \overset{\Delta}{=} P(X|\theta)$, where r denote experiment times and x denote event A is occurred in the x th times.

Someone studied Bayes estimation of Pascal distribution under symmetrical loss in [9-10].

Theorem 2 Let loss function be weighted balance loss function and let $\pi(\theta)$ have the Beta distribution with parameter a and b .Then the one and only Bayesian estimation of θ is $\hat{\theta} = w\overline{X} + (1-w)\frac{a+b+x+2}{a+x-r+2}$ for Pascal distribution

Proof Since $\pi(\theta)\sim Be(a,b)$, thus $\pi(\theta)=\frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)}$ .Hence posterior density of θ is $\pi(\theta)\propto\theta^{a+x-r-1}(1-\theta)^{r+b-1}$, which have the Beta with parameters $a+x-r$ and $r+b$,so we get

$$E\left(\theta^2|X\right)=\int_0^1\theta^{-1}\frac{\theta^{a+x-r-1}(1-\theta)^{r+b-1}}{B(a+x-r,r+b)}d\theta$$

$$=\frac{(a+x-r)(a+x-r+1)}{(a+b+x+1)(a+b+x)}$$

$$E\left(\theta^3|X\right)=\int_0^1\theta^{-2}\frac{\theta^{a+x-r-1}(1-\theta)^{r+b-1}}{B(a+x-r,r+b)}d\theta$$

$$=\frac{(a+x-r+1)(a+x-r+2)(a+x-r)}{(a+x+b+2)(a+b+x+1)(a+b+x)}$$

It follows from lemma 1 that Bayesian Estimation of θ is

$$\hat{\theta} = w\overline{X} + (1-w)\frac{E\left(\theta^2|X\right)}{E\left(\theta^3|X\right)}$$

$$= w\overline{X} + (1-w)\frac{a+b+x+2}{a+x-r+2}$$

Theorem 2 Let loss function be weighted balance loss function and let $\pi(\theta)$ have the Beta distribution with parameter a and b .Then confidence levels of θ is Bayes the confidence lower limit which confidence levels is $1-\alpha$,and $\underline{\theta}$ is $I_{\theta}(a+x-r,r+b)=\alpha$,where

$$I_x(a,b)=\frac{1}{B(a,b)}\int_0^x\theta^{a-1}\cdot(1-\theta)^{n-1}\,d\theta$$

Proof the confidence levels of θ is Bayes the confidence lower limit which confidence levels is $1-\alpha$,and $\underline{\theta}$ is

$$1-\alpha=\int_{\underline{\theta}}^1\pi(\theta|x)\,d\theta=\int_{\underline{\theta}}^1\frac{\theta^{a+x-r-1}\cdot(1-\theta)^{r+b-1}}{B(a+x-r,r+b)}d\theta$$

We get $\alpha=I_{\theta}(a+x-r,r+b)$ and we begin with a preliminary result.

IV. CONCLUSIONS

General Bayes estimation are under symmetrical loss .In the paper discussed the Bayes estimation of two different kinds of distribution---Pareto distribution parameter and Pascal distribution parameter under the weighted balance loss with given $q(\theta)$, and proved that the estimation is the admissible.

REFERENCES

[1] S. Y. Huang, T. Liang, “Empirical Bayes estimation of the truncation Parameter with linex loss,”Journal of Statistics Sinica,vol.7,pp. 755 - 769,1997.

[2] Xu Bao, “The Bayes estimation of reciprocal of Poisson mean under a symmetric loss function,” Journal of Jilin normal university journal, vol.3,pp. 53 - 54, 2006.

[3] Wang Dehui , Niu Xiaoning, “Bayes estimation of Pascal distribution parameter under entropy loss function,” Journal of Acta scientiarum naturalism universitatis jilinensis, vol.1,pp. 19-22, 2001.

[4] Sanjari Farsipour, N. Asgharzadeh, “A estimation of a normal mean relative to balanced loss function ,” Journal of Statistical Papers , vol.45,pp. 123-133, 2004.

[5] A.Asgharzadeh, F.N. Sanjari, “Estimation of the exponential mean time to failure under a weighted balanced loss function,” Journal of Statistical Papers , vol. 49,pp. 98-110, 2008.

[6] Liu Surong,Ren Haiping, “Bayes Estimation of Poisson distribution parameters under a weighted balanced loss function ,” Journal of Statistics and Decision，vol. 16,pp. 17-19 , 2010 .

[7] Li Yanying, “Bayes estimation of Pareto distribution,” Journal of Journal of Sichuan University of Science，3rd ed., vol. 23, pp. 275-277, 2010.

[8] Li Fanqun, “The Bayes estimation of Pareto distribution parameter under the entropy loss function,” Journal of Jounal of Fuyang Teachers College, 1st ed., vol. 3, pp. 275-277,2007.

[9] Wei Ling,Shi Yimin, “Bayes estimation of the pascal distribution parameter,” Journal of Pure and Applied Mathematics, vol. 2 ,pp. 13-16,1999.

[10] Zou Rongjing,Zhu Dan,Song Lixin, “Bayes estimation of Pascal distribution parameter under a symmetric loss function,” Journal of Pure and Applied Mathematics , vol. 2,pp 13-16,1999.

[11] Mao Shisong, “Advanced mathematical statistics ,” Eds BeiJing: China Higher Education Press,1998 , pp.367-372.