1. Model Consistency of Partly Smooth Regularizers

This paper studies least-square regression penalized with partly smooth convex regularizers. This class of penalty functions is very large and versatile, and allows to promote solutions conforming to some notion of low complexity. Indeed, such penalties/regularizers force the corresponding solutions to belong to a low-dimensional manifold (the so-called model), which remains stable when the argument of the penalty function undergoes small perturbations. Such a good sensitivity property is crucial to make the underlying low-complexity (manifold) model robust to small noise. In a deterministic setting, we show that a generalized "irrepresentable condition" implies stable model selection under small noise perturbations in the observations and the design matrix, when the regularization parameter is tuned proportionally to the noise level. As an algorithmic implication, we also prove that this condition is almost necessary for stable model recovery. We then turn to the random setting, where the design matrix and the noise are random, and the number of observations grows large. We show that under our generalized "irrepresentable condition," and a proper scaling of the regularization parameter, the regularized estimator is model consistent. In plain words, with a probability tending to one as the number of measurements tends to infinity, the regularized estimator belongs to the correct low-dimensional model manifold. This paper unifies and generalizes a large body of literature, where model consistency was known to hold, for instance for the Lasso, group Lasso, total variation (fused Lasso), and nuclear/trace norm regularizers. As an algorithmic implication, we show that under the deterministic model selection conditions, the forward–backward proximal splitting algorithm used to solve the penalized least-square regression problem is guaranteed to identify the model manifold after a finite number of iterations. Finally, we detail how our results extend from the quadratic loss to an arbitrary smooth and strictly convex loss function. We illustrate the usefulness of our results on the problem of lowrank matrix recovery from random measurements using nuclear norm minimization.

2. The construction of the robust regression models with the LS–SVM method

using a nonquadratic loss function

In recent decades, there was a growing awareness of the fact that some of the most common statistical procedures are excessively sensitive to relatively small deviations from assumptions. Therefore, there were other procedures – "robust". Robustness means insensitivity to small deviations from the assumptions. The estimations are called robust, if not depending on the presence of large emissions; they do not change their values. If in the dataset missing large errors, then robust evaluations will be less effective, but the most reliable.

In this work based on LS–SVM, is provided the method of construction robust solutions using the Huber loss functions. For obtaining, such solutions are proposed the computational schemes involving the use of pseudo-observations, or a weighted variant of LS–SVM. The conducted researches have demonstrated the good possibilities with the use of adaptive Huber loss function to obtain the robust solutions with a small displacement in conditions of clogging observations. This approach can be recommended for use to obtain the robust solutions in the LS–SVM method