Advanced Test 1

January Camp 2021

Time: $2\frac{1}{2}$ hours

1. Find all possible real numbers k such that the values of x satisfying

$$k(2-k)x^2 - (k+4)x + 6 = 0$$

are positive integers.

2. Let O be the circumcentre of $\triangle ABC$. Let X, Y and Z be the reflections of O over BC, CA and AB respectively. Prove that $\triangle XYZ$ is congruent to $\triangle ABC$ and the corresponding sides are parallel.

3. Find the smallest non-negative integer which can not be written in the from

$$\frac{2^a - 2^b}{2^c - 2^d}$$

for some positive integers a, b, c, and d.

4. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$f(x+y) = f(x) + f(y) + \frac{1}{2021}$$

Where $\mathbb{R}^+$ is the set of positive real numbers.

5. Jon has a collection of weights with different positive integer values. Is it possible that there are exactly 2020 ways to choose some of these distinct weights such that their total weight is 2020?

- Submit your solutions at `https://forms.gle/M1L9KgbwzDxCKEjD9`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.