

Test 1
April Camp 2021
Time: $4\frac{1}{2}$ hours

1. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x^3) + f(y^3) = (x+y)(f(x^2) + f(y^2) - f(xy))$$

for all real numbers x and y .

2. Let n be a positive integer. Find the number of permutations a_1, a_2, \dots, a_n of the sequence $1, 2, \dots, n$ satisfying

$$a_1 \leq 2a_2 \leq 3a_3 \leq \dots \leq na_n.$$

3. Let $ABCD$ be a convex quadrilateral with $\angle ABC > 90^\circ$, $\angle CDA > 90^\circ$, and $\angle DAB = \angle BCD$. Denote by E and F the reflections of A in lines BC and CD respectively. Suppose that the segments AE and AF meet the line BD at K and L respectively. Prove that the circumcircles of triangles BEK and DFL are tangent to each other.

- Submit your solutions at <https://forms.gle/uhMSLew7qTQ9Qbqr6>.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.

jgs