

Advanced April Monthly Assignment

Due Date: 30 April 2021

1. For $x, y, z > 0$, prove that

$$\frac{x^3}{x+y} + \frac{y^3}{y+z} + \frac{z^3}{z+x} \geq \frac{xy + yz + zx}{2}.$$

2. Find all functions $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ such that for all $x \in \mathbb{R}$, $x \neq 0, 1$ we have

$$f(x) + f\left(\frac{1}{1-x}\right) = x.$$

3. You are given a convex quadrilateral $ABCD$ such that $OA = \frac{OB \cdot OD}{OC + CD}$, where O is the intersection point of the diagonals of $ABCD$. The circumcircle of $\triangle ABC$, intersects the line BD in point Q . Prove that CQ bisects $\angle DCA$.

4. Find all triples of natural numbers x, y, z such that

$$7^x + 13^y = 2^z.$$

5. Equilateral triangle ABC has an area of 7. M and N are points on the sides AB and AC respectively, such that $AN = BM$. Let O be the intersection point of BN and CM . The area of triangle BOC is 2.

(a) Prove that $MB : AB = 1 : 3$ or $MB : AB = 2 : 3$.

(b) Find the size of $\angle AOB$.

6. n points are given in the plane ($n > 4$), such that no three of them are collinear. The points are used as vertices to form more than n triangles. Show that there exist two triangles which have exactly one vertex in common.

- Submit your solutions at <https://forms.gle/yoD4U3zZxs2yuP4d9>.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.