Intermediate March Monthly Assignment

Due date: 31 March 2021

1. Two players play a game on a $4 \times n$ checkerboard. They take turns to place checkers according to the following rule: no checker can be on a square that is adjacent (by either edge or vertex) to a square that already contains a checker. The last player to place a checker wins. Show that Player 2 can always win such a game.

2. We are given ΔABC. Lines l_B and l_C are the external angle bisectors at B and C respectively. p_1 is the perpendicular line from B to l_C and p_2 from C to l_B. A' is the intersection of lines p_1 and p_2. B' and C' are defined analogously (as the intersection of *the perpendicular from one vertex of the triangle to the external angle bisector of another vertex* with *the perpendicular from the second vertex to the external angle bisector of the original vertex*). Show that the area of hexagon $AB'CA'BC'$ is twice the area of ΔABC.

3. Solve for positive integers a, b, and c:

$$a^2 + bc = 181$$
$$ab + ac = 820.$$

4. Show that for all $x, y > 0$, we have:

$$x^4 + y^4 \geq \frac{1}{2}xy(x + y)^2$$

5. There are n towns in a country. Every road in this country goes from one of these towns to another (different) town. These are not one-way roads. Two independent routes from town A to town B are routes that don't have any intermediate towns in common (i.e. there are no towns that lie between A and B on both routes). If we are told that there are at least two independent routes from any town A to any other town B, what is the minimum number of roads that could be in this country?

6. Let ABC be a triangle with centroid G and $AB \neq BC$. Γ is the circle with diameter BG and H is its centre. Let CG extended intersect AB at F and Γ at I different to G. Let AG extended intersect BC at D and Γ at J different to G. Show that IJ is parallel to FD if and only if I, H, and J are collinear.

- Submit your solutions at `https://forms.gle/Kx1QDxDT5xP3Ez547`.

- Submit each question in a single separate PDF file (with multiple pages if necessary).

- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.

- If you have multiple PDF files for a question, combine them using software such as PDFsam.