Test 5

April Camp 2021

Time: $4\frac{1}{2}$ hours

1. Let a and b be integers, and suppose that $a^n + n \mid b^n + n$ for all positive integers n. Show that $a = b$.

2. Suppose that a, b, c, and d are positive real numbers satisfying $(a + c)(b + d) = ac + bd$. Find the smallest possible value of
$$\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}.$$

3. Let $ABCD$ be a cyclic quadrilateral with no two sides parallel. Let K, L, M, and N be points lying on segments AB, BC, CD, and DA respectively such that $KLMN$ is a rhombus with $KL \parallel AC$ and $LM \parallel BD$. Let ω_1, ω_2, ω_3, and ω_4 be the incircles of triangles ANK, BKL, CLM, and DMN respectively. Prove that the internal common tangents to ω_1 and ω_3 and the internal common tangents to ω_2 and ω_4 are concurrent.

- Submit your solutions at `https://forms.gle/uhMSLew7qTQ9Qbqr6`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.