Intermediate Test 3

January Camp 2021

Time: $2\frac{1}{2}$ hours

1. Find the smallest positive integer which only has 1 and 2 as digits and which is divisible by 99.

2. Γ is a circle and AB and CD are $\perp$ chords that intersect inside Γ. P is on Γ, with Q diametrically opposite to P. Let the feet of the perpendiculars from P to AB and CD be P' and P'' respectively, and the feet of the perpendiculars from Q to AB and CD be Q' and Q'' respectively. Show that:

$$(P'P)^2 + (P''P)^2 + (Q'Q)^2 + (Q''Q)^2 < d^2$$

   where d is the diameter of Γ.

3. Let $q(x) = x^3 - x^2 - 2x + 1$ with roots a, b, and c. Find $a^2 + b^2 + c^2$.

4. There is a book with n chapters where chapter i has i pages. The probability of opening the book in the same chapter twice in a row is p. Is it possible for p to be $1/k$ for some integer k?

5. Points D, E, and F lie respectively on sides BC, CA, and AB of triangle ABC such that $BDEF$ is a parallelogram. Prove that the area of $BDEF$ is maximal when D, E, and F are the midpoints of the sides.

- Submit your solutions at `https://forms.gle/QiVwLteHxnQSUF9y7`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.