Intermediate April Monthly Assignment

Due Date: 30 April 2021

1. Find all natural numbers k which can be represented as the sum of two relatively prime numbers not equal to 1.

2. $ABCD$ is a convex quadrilateral having perpendicular diagonals and it can also be inscribed in a circle with centre O. Prove that the broken line AOC divides the quadrilateral into two parts of equal area.

3. There are 68 coins, each coin having a different weight than that of the others. Show how to find the heaviest and lightest coin in 100 weighings on a balance beam.

4. Inside square $ABCD$ consider a point M. Prove that the points of intersection of the medians of triangles ABM, BCM, CDM and DAM form a square.

5. For $x, y, z > 0$, prove that
$$\frac{x^3}{x+y} + \frac{y^3}{y+z} + \frac{z^3}{z+x} \geq \frac{xy+yz+zx}{2}.$$

6. Find all functions $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ such that for all $x \in \mathbb{R}$, $x \neq 0, 1$ we have
$$f(x) + f\left(\frac{1}{1-x}\right) = x.$$

- Submit your solutions at `https://forms.gle/Kx1QDxDT5xP3Ez547`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.