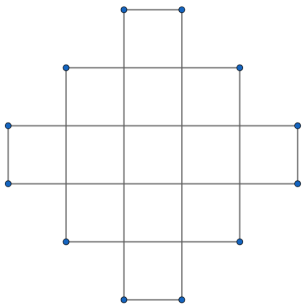
Intermediate Test 1

January Camp 2021

Time: $2\frac{1}{2}$ hours

1. You are given the following shape:



You need to tile this with L-shapes made of 3 blocks. Which single blocks could you shade out of the original diagram to make this possible?

2. Let $ABCD$ be a trapezoid with $AD \parallel BC$. The angle bisector of $\angle DAB$ intersects the angle bisectors of $\angle ABC$ and $\angle CDA$ at points P and S respectively, and the angle bisector of $\angle BCD$ intersects the angle bisectors of $\angle ABC$ and $\angle CDA$ at points Q and R respectively. Furthermore, $PS \parallel RQ$. Prove that $AB = CD$.

3. Find all natural numbers x, y and z satisfying

$$x + \frac{1}{y + \frac{1}{z}} = \frac{850862}{421}$$

4. Find all possible real numbers k such that the values of x satisfying

$$k(2-k)x^2 - (k+4)x + 6 = 0$$

are positive integers.

5. Let O be the circumcentre of $\triangle ABC$. Let X, Y and Z be the reflections of O over AB, BC and CA respectively. Prove that $\triangle XYZ$ is congruent to $\triangle ABC$ and the corresponding sides are parallel.

- Submit your solutions at `https://forms.gle/QiVwLteHxnQSUF9y7`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.

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