Intermediate Test 4

January Camp 2021

Time: $2\frac{1}{2}$ hours

1. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$ we have that
$$xf(y) = yf(x).$$

2. Find all positive integers m such that $2^{m^2} - 4$ is divisible by 7.

3. Consider a triangle ABC with circumcentre O. The angle bisector of $\angle BAC$ meets the opposite side BC at D, and the altitude from B onto AD intersects line AO at E. Show that A, B, D, and E are concyclic.

4. Consider a $3 \times 3 \times 3$ 3-dimensional chess cube with some hyperrooks. Hyperrooks can move along any direction parallel to an edge of the cube (like a normal rook, but also up and down). What is the maximum number of hyperrooks you can place in the chess cube without any of them attacking each other?

5. Find all positive integers a, b and c satisfying
$$a + b - c = 14$$
$$a^2 + b^2 - c^2 = 14.$$

- Submit your solutions at `https://forms.gle/QiVwLteHxnQSUF9y7`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.