Intermediate Test 2

January Camp 2021

Time: $2\frac{1}{2}$ hours

1. Let $\triangle ABC$ be an isosceles triangle with $AB = BC$. Let H be the orthocentre of $\triangle ABC$ and let K be the reflection of H across BC. Prove that $HBKC$ is a parallelogram.

2. Let a and r be real numbers such that

$$a + ar + ar^2 + ar^3 + \cdots + ar^{2020} = 200 \qquad \text{and} \qquad a + ar + ar^2 + ar^3 + \cdots + ar^{4041} = 380.$$

   Find the value of

$$a + ar + ar^2 + ar^3 + \cdots + ar^{6062}.$$

3. Isaac is planning a nine-day holiday. Every day he will go surfing, or water skiing, or he will rest. On any given day he does just one of these three things. He never does different water-sports on consecutive days. How many schedules are possible for the holiday?

4. A positive integer N has exactly 2021 positive divisors (including 1 and N itself), and it is divisible by 2021. Prove that N is not divisible by 2021^{43}.

5. Let a, b, c, x, y and z be positive real numbers with $a + b + c = x + y + z$. Prove that

$$\frac{a}{x+y} + \frac{b}{y+z} + \frac{c}{z+x} + \frac{x}{a+b} + \frac{y}{b+c} + \frac{z}{c+a} > 2.$$

- Submit your solutions at `https://forms.gle/QiVwLteHxnQSUF9y7`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.