Intermediate February Monthly Assignment

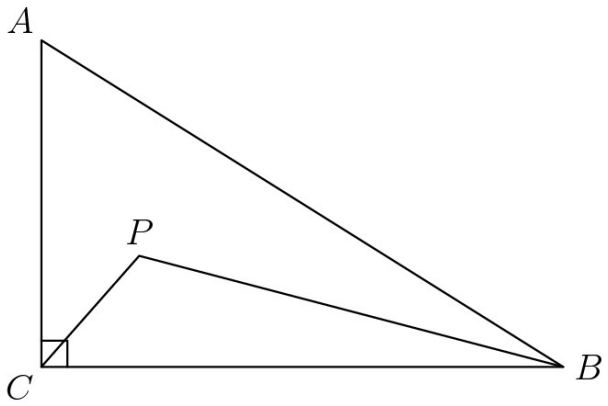
Due date: 20 February 2021

1. The natural number n can be replaced by ab if $a + b = n$, where a and b are natural numbers. Can the number 2021 be obtained from 22 after a sequence of such replacements?

2. Prove that among the first 30000 positive integers there are at least 22000 composite numbers.

3. Let a and b be positive real numbers such that $2a^2 + 2b^2 = 5ab$. If $|x|$ denotes the absolute value of x, calculate
$$\left|\frac{a+b}{a-b}\right|.$$

4. Triangle ABC is a right angled triangle with $\angle C = 90°$. P is placed randomly inside $\triangle ABC$. What is the probability that the area of $\triangle PBC$ is less than half of the area of $\triangle ABC$?



5. Let c and d be positive divisors of a natural number n such that $c > d$. Prove that
$$c > d + \frac{d^2}{n}.$$

6. Suppose $a, b, c > 0$ and $\sqrt{a-b} + \sqrt{a-c} > \sqrt{b+c}$. Prove that $a > \frac{3}{4}(b+c)$.

- Submit your solutions at `https://forms.gle/Kx1QDxDT5xP3Ez547`.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.