

Test 3 – Solutions

April Camp 2017

1. Given 5 positive real numbers, show that there exist two of them, a and b , for which

$$0 \leq \frac{a}{1+a^2} - \frac{b}{1+b^2} \leq \frac{1}{8}.$$

2. Nonzero real numbers a, b, c, d satisfy the equations

$$a + b + c + d = 0, \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{abcd} = 0.$$

Find all possible values of the product $(ab - cd)(c + d)$.

3. Find all primes p such that $5^p + 4p^4$ is the square of an integer.
4. $ABCD$ is a cyclic quadrilateral. Let the circle Γ_1 pass through A and B and touch CD at E ; let the circle Γ_2 pass through B and C and touch DA at F ; let the circle Γ_3 pass through C and D and touch AB at G ; and let the circle Γ_4 pass through D and A and touch BC at H . Prove that $EG \perp FH$.
5. Given a polynomial P with positive real coefficients, show that $P(1)P(xy) \geq P(x)P(y)$ for all $x, y \geq 1$.