

April Camp 2017 Test 1

Time: 4 Hours

1. Determine all positive integers k and n satisfying the equation

$$k^2 - 2016 = 3^n.$$

2. Let ABC be an acute angled triangle. Let H be the foot of the altitude from C onto AB . Suppose that $|AH| = 3|BH|$. Let M and N be the midpoints of the segments AB and AC respectively. Let P be a point such that $|NP| = |NC|$ and $|CP| = |CB|$ and such that B and P lie on opposite sides of the line AC . Show that $\angle APM = \angle PBA$.

3. Consider a 4×4 grid of unit squares. How many ways are there to write a 0 or 1 in each 1×1 square so that the product of the two numbers written on every neighbouring pair of squares (sharing a common edge) is always 0?

4. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfying

$$f(xy - 1) + f(x)f(y) = 2xy - 1$$

for all $x, y \in \mathbb{R}$.

5. Find all infinite sequences $a_1, a_2, a_3 \dots$ of positive integers such that

- (a) $a_{mn} = a_m a_n$ for all positive integers m and n , and
- (b) there are infinitely many positive integers n such that

$$\{1, 2, \dots, n\} = \{a_1, a_2, \dots, a_n\}.$$

Each problem is worth 7 points.