Test 3

April Camp 2017

Time: 4 hours

1. Find the least positive integer m such that for every prime number $p > 3$ we have $105 \mid 9^{p^2} - 29^p + m$.

2. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$,
$$f(xf(x) + f(xy)) = f(x^2) + yf(x).$$

3. Determine the greatest positive integer m such that each square of a $m \times m$ array can be painted either red or blue so that not all the squares at the intersection of any two rows and any two columns are the same colour.

4. In a cyclic quadrilateral $ABCD$, let E be the intersection of AD and BC (so that C is between B and E), and F be the intersection of AC and BD. Let M be the midpoint of side CD, and let $N \neq M$ be a point on the circumcircle of ΔABM such that $\frac{AM}{MB} = \frac{AN}{NB}$. Show that E, F and N are collinear.

Each problem is worth 7 points.