

Senior Problem Pool

Algebra

1. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x^2) - yf(y) = f(x+y)(f(x) - y)$ for all real numbers x and y .
2. Suppose that a , b , and c are real numbers such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}.$$

Show that for every odd natural number n , we have that

$$\frac{1}{a^n} + \frac{1}{b^n} + \frac{1}{c^n} = \frac{1}{(a+b+c)^n}.$$

Combinatorics

Geometry

Number Theory

1. Suppose that $2n+1$ and $3n+1$ are both squares. Show that 40 divides n .
2. Does there exist a natural number n such that

$$1^{2018} + 2^{2018} + \dots + n^{2018}$$

is prime?

3. Suppose that a, b , and N are relatively prime integers. Show that there is an integer m such that a and $b+mN$ are relatively prime.
4. Find all natural numbers n such that there exist n distinct natural numbers a_1, a_2, \dots, a_n such that $a_i \mid a_{a_i+1}$ for $i = 1, 2, \dots, n$, and such that $a_n = 2018n$.

Miscellaneous