

PAMO Stream Test 4

April Camp 2019

Time: $4\frac{1}{2}$ hours

1. Find all quadruplets (a, b, c, d) of positive integers such that

$$\left(1 + \frac{1}{a}\right) \left(1 + \frac{1}{b}\right) \left(1 + \frac{1}{c}\right) \left(1 + \frac{1}{d}\right) = 4.$$

2. Akello divides a square up into finitely many white and red rectangles, each (rectangle) with sides parallel to the sides of the parent square. Within each white rectangle, she writes down the value of its width divided by its height, while within each red rectangle, she writes down the value of its height divided by its width. Finally, she calculates x , the sum of these numbers. If the total area of the white rectangles equals the total area of the red rectangles, what is the least possible value of x she can get?
3. Let ABC be a triangle with $AB \neq AC$. The incircle of ABC touches the sides BC , CA , AB at X , Y , Z respectively. The line through Z and Y intersects BC extended in X' . The lines through B that are parallel to AX and AC intersect AX' in K and L respectively. Prove that $AK = KL$.