

# Intermediate Test 1

Stellenbosch Camp 2018

Time:  $2\frac{1}{2}$  hours

1. Let  $x$  be a real number such that

$$x + \frac{1}{x} = 3.$$

Find the value of

$$x^5 + \frac{1}{x^5}.$$

2. How many different permutations of the word INTERCONNECTION are there? (Interchanging two letters that are the same does not count as a different word.)
3. Let  $ABC$  be a triangle. Prove that the internal angle bisector of the angle  $\angle ABC$  and the perpendicular bisector of the line segment  $AC$  intersect on the circumcircle of triangle  $ABC$ .
4. Prove that  $m + n \leq \gcd(m, n) + \text{lcm}(m, n)$  for all positive integers  $m, n$ . When does equality occur?
5. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x + y)) = x + f(y)$$

for all  $x, y \in \mathbb{R}$ .

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