Intermediate Test 3

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. For any real number x, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x, and let $\{x\} = x - \lfloor x \rfloor$ be the fractional part of x. Find all real numbers a, b, and c such that

$$\begin{aligned} \lfloor a \rfloor + \{b\} &= -2.3, \\ \lfloor b \rfloor + \{c\} &= 8.9, \qquad \text{and} \\ \lfloor c \rfloor + \{a\} &= 23.4. \end{aligned}$$

2. Lets $PQRS$ be a cyclic quadrilateral such that $PQ = QR$, $PR = SR$ and $PQ \parallel SR$. Let ASB be a tangent at S where A lies on PQ. If $\angle BSR = 72°$, find the value of $\angle RPQ$.

3. Let n be a positive integer. Find the last digit of

$$n^{2018} + (n+1)^{2018} + \cdots + (n+99)^{2018}.$$

4. Let ABC be a triangle, and let the midpoint of AC be M. The circle tangent to BC at B and passing through M meets the line AB again at P. Prove that $AB \times BP = 2BM^2$.

5. Show that for all positive real numbers x and y,

$$\frac{x^2}{x+2y} + \frac{y^2}{2x+y} \geq \frac{x+y}{3}.$$