PAMO Problem Proposals

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1. Find the 2019^{th} natural number n such that $\binom{2n}{n}$ is not divisible by 5.
2. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that
$$f(x^2) - yf(y) = f(x+y)(f(x) - y)$$
for all real numbers x and y.
3. ABC is an acute-angles triangle. The bisectors of angles A and B meet BC and AC at D and E respectively. P is a point on DE such that the distances from P to AC and BC are x and y respectively. Show that the distance from P to AB is $x + y$. (Alternatively, give specific values for x and y and ask to calculate the distance from P to AB.)
4. A pawn is a chess piece which attacks the two squares diagonally in front of it. What is the maximum number of pawns which can be placed on an $n \times n$ chessboard such that no two pawns attack each other? (*Alternatively, ask for the maximum number of pawns which can be placed on an $a \times b$ chessboard.*)
5. A subset S of the set $T_m = \{1, 2, \ldots, m-1\}$ is called *cosy* if for every $x \in S$, either $2x$ or $2x - m$ is also in S. Find the smallest odd number m such that T_m has a cosy subset with exactly 2009 members.
6. Let $ABCD$ be a parallelogram. The angle bisectors of $\angle BAD$ and $\angle BCD$ intersect BD at E and G respectively. Then angle bisectors of $\angle ABC$ and $\angle ADC$ intersect AC at F and H respectively. Prove that
$$AE^2 + BH^2 = AG^2 + BF^2.$$