# Senior Test 1

## April Camp 2019

## Time: $4\frac{1}{2}$ hours

1. Let $\mathbb{Q}_{>0}$ denote the set of all positive rational numbers. Determine all functions $f : \mathbb{Q}_{>0} \to \mathbb{Q}_{>0}$ satisfying

$$f\left(x^2 f(y)^2\right) = f(x)^2 f(y)$$

for all $x, y \in \mathbb{Q}_{>0}$.

2. Let $n > 1$ be a positive integer. Each cell of an $n \times n$ table contains an integer. Suppose that the following conditions are satisfied:

   (i) Each number in the table is congruent to 1 modulo $n$;

   (ii) The sum of numbers in any row, as well as the sum of numbers in any column is congruent to $n$ modulo $n^2$.

   Let $R_i$ be the product of the numbers in the $i$-th row, and $C_j$ be the product of the numbers in the $j$-th column. Prove that the sums $R_1 + \cdots + R_n$ and $C_1 + \cdots + C_n$ are congruent modulo $n^4$.

3. A point $T$ is chosen inside a triangle $ABC$. Let $A_1$, $B_1$, and $C_1$ be the reflections of $T$ in $BC$, $CA$, and $AB$, respectively. Let $\Omega$ be the circumcircle of the triangle $A_1B_1C_1$. The lines $A_1T$, $B_1T$, and $C_1T$ meet $\Omega$ again at $A_2$, $B_2$, and $C_2$, respectively. Prove that the lines $AA_2$, $BB_2$, and $CC_2$ are concurrent on $\Omega$.