

PAMO Stream Test 2

April Camp 2019

Time: $4\frac{1}{2}$ hours

1. Let Γ be the circumcircle of an acute triangle ABC . The perpendicular line to AB passing by C cuts AB in D and Γ again in E . The bisector of the angle C cuts AB in F and Γ again in G . The line GD meets again Γ in H and the line HF meets it again in I . Prove that $AI = EB$.
2. Find all non-negative integers n for which the equation

$$(x^2 + y^2)^n = (xy)^{2018}$$

admits positive integral solutions.

3. Adamu and Afaafa choose, each in his turn, positive integers as coefficients of a polynomial of degree n . Adamu wins if the polynomial obtained has an integer root; otherwise, Afaafa wins. Afaafa plays first if n is odd; otherwise Adamu plays first. Prove that:
 - i) Adamu has a winning strategy if n is odd.
 - ii) Afaafa has a winning strategy if n is even.