January Monthly Problem Set

Due: 18 January 2019

1.

2.

3.

4.

5.

6. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$f(mn) = f(\gcd(m, n))f(\text{lcm}(m, n))$$

for all $m, n \in \mathbb{N}$.

7. Does there exist a natural number n such that

$$1^{2018} + 2^{2018} + \cdots + n^{2018}$$

is prime?

8. Fix a natural number $n \geq 2$. Find the smallest constant C such that

$$\sum_{1 \leq i < j \leq n} x_i x_j (3x_i^2 + x_j^2)(x_i^2 + 3x_j^2) \leq C \left(\sum_{i=1}^{n} x_i \right)^6$$

for all non-negative real numbers $x_1, x_2, \ldots, x_n$. For this value of C, when does equality occur?

Email submission guidelines

- Email your solutions to `samf.training.assignments@gmail.com`.
- Submit each question in a single separate PDF file (with multiple pages if necessary), with your name and the question number written on each page.
- If you take photographs of your work, use a document scanner such as CamScanner to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.