Intermediate Test 4

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. How many numbers from 1 to 2018 inclusive can be written as the difference of two perfect squares?

2.

3. Find all functions $f : \mathbb{R} -> \mathbb{R}$ such that for all real numbers x,

$$2f(x) + 3f(1 - x) = x - 4x^3.$$

4. Prove that it is impossible to write a positive integer in every cell of an infinite chessboard, in such a manner that, for all positive integers m, n, the sum of numbers in every $m \times n$ rectangle is divisible by $m + n$.

5.