# Senior Test 3

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. Let $ABC$ be a triangle, and let the midpoint of $AC$ be $M$. The circle tangent to $BC$ at $B$ and passing through $M$ meets the line $AB$ again at $P$. Prove that $AB \times BP = 2BM^2$.

2. Show that for all positive real numbers $x$ and $y$,
$$\frac{x^2}{x+2y} + \frac{y^2}{2x+y} \geq \frac{x+y}{3}.$$

3. Consider an 8x8 grid of squares. On each square is a lightbulb which is initially switched off. A move consists of choosing a square and either the verticle or horizonal direction, and toggling the lightbulb on that square and it's immediate neighbours in the chosen direction. For clarity this means that usually 3 bulbs are flipped unless the square is on an edge in which case 2 bulbs may be flipped. After some amount of moves a single bulb is switched on (the other 63 are off). Determine which of the 64 bulbs can possibly be on.

4. Determine whether or not there is a positive integer $m$ such that
$$(m+1)^3 + (m+2)^3 + \cdots + (2m)^3$$
is a square.

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