January Monthly Problem Set

Due: February 2019

1. Does there exist a natural number n such that

$$1^{2018} + 2^{2018} + \cdots + n^{2018}$$

is prime?

2. Fix a natural number $n \geq 2$. Find the smallest constant C such that

$$\sum_{1 \leq i < j \leq n} x_i x_j (3x_i^2 + x_j^2)(x_i^2 + 3x_j^2) \leq C \left(\sum_{i=1}^{n} x_i \right)^6$$

for all non-negative real numbers $x_1, x_2, \ldots, x_n$. For this value of C, when does equality occur?