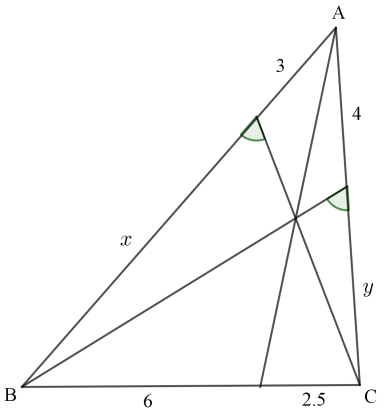
Intermediate Test 4

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. How many numbers from 1 to 2018 inclusive can be written as the difference of two perfect squares?

2. Solve for lengths x and y in the following diagram:



3. Find all functions $f : \mathbb{R} -> \mathbb{R}$ such that for all real numbers x,

$$2f(x) + 3f(1 - x) = x - 4x^3.$$

4. Prove that it is impossible to write a positive integer in every cell of an infinite chessboard, in such a manner that, for all positive integers m, n, the sum of numbers in every $m \times n$ rectangle is divisible by $m + n$.

5. An exam with k questions is presented to n students. A student fails the exam if they get less than half the answers right. We say that a question is easy if more than half of the students get it right. Decide if it is possible that

   (a) All students fail even though all the questions were easy.

   (b) No student fails even though no question was easy.