Beginner Test 2

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. Find all real numbers x such that

$$3x^2 + 5x - 2 = 0.$$

2. Compute the prime factorisation of $3^{16} - 2^{16}$.

3. For any real number x, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x, and let $\{x\} = x - \lfloor x \rfloor$ be the fractional part of x. Find all real numbers a, b, and c such that

$$\begin{aligned} \lfloor a \rfloor + \{b\} &= -2.3, \\ \lfloor b \rfloor + \{c\} &= 8.9, \qquad \text{and} \\ \lfloor c \rfloor + \{a\} &= 23.4. \end{aligned}$$

4. Lets $PQRS$ be a cyclic quadrilateral such that $PQ = QR$, $PR = SR$ and $PQ \parallel SR$. Let ASB be a tangent at S where A lies on PQ. If $\angle BSR = 72^\circ$, find the value of $\angle RPQ$.

5. Show that for all real numbers x and y,

$$9x^2 \geq 12xy - 4y^2.$$

6. Consider an 8×8 chessboard with the bottom left 3×3 squares occupied by cute little frogs. Each cute frog can jump over any frog adjacent to it, vertically or horizontally, to land on another square two squares away from where it started. Is it possible by repeating this operation to get all the cute frogs to the top right 3×3 squares?