Senior Test 3

April Camp 2019

Time: $4\frac{1}{2}$ hours

1. Determine all pairs (n, k) of distinct positive integers such that there exists a positive integer s for which the numbers of divisors of sn and of sk are equal.

2. Given any set S of positive integers, show that at least one of the following two assertions holds:

   (a) There exists distinct finite subsets F and G of S such that $\sum_{x\in F} 1/x = \sum_{x\in G} 1/x$;

   (b) There exists a positive rational number $r < 1$ such that $\sum_{x\in F} 1/x \neq r$ for all finite subsets F of S.

3. Let k be a positive integer. The organising committee of a tennis tournament is to schedule the matches for $2k$ players so that every two players play once, each day exactly one match is played, and each player arrives to the tournament site the day of his first match, and departs the day of his last match. For every day a player is present on the tournament the committee has to pay 1 coin to the hotel. The organisers want to design the schedule so as to minimise the total cost of all players' stays. Determine this minimum cost.