Intermediate February Monthly Problem Set

Due: 28 February 2019

1. Sophie had to solve a math problem in the class. While cleaning the blackboard, she accidentally erased a part of her problem as well: the text that remained on board was $37 \cdot (72 + 3x) = 14{*}{*}45$, where $*$ marks an erased digit. Show that Sophie can still solve her problem, knowing that x is an integer.

2. On the side CD of square $ABCD$ point E is chosen such that $\angle ABE = 60°$. Point F is chosen on line AB such that $BE = BF$ and point A is between F and B. Let M be the intersection of lines EF and AD.

   a) Find $\angle BME$.

   b) The bisector of angle CBE intersects CD at N. Find the angles of triangle BMN.

3. Let there be $n \geq 2$ real numbers such that none of them is greater than the arithmetic mean (normal average) of the other numbers. Prove that all the numbers are equal.

4. Consider the isosceles triangle ABC with $\angle A = 100°$. Let BD be the angle bisector of $\angle ABC$ with D a point on AC. Let E be a point on BD such that $BE = BC$ and where D is between points B and E. Let F be a point on BC with F between B and C such that $AB = BF$. Prove that the lines AC and EF are perpendicular.

5. The teacher gave Emma four distinct integers and asked Emma to calculate the greatest common divisor of every two of these numbers. She got the answers 1, 2, 3, 4, 5 and N where $N > 5$. What is the smallest possible value of N?

6. A table consisting of 9 rows and 2001 columns is filled with integers 1, 2, $\ldots$, 2001 in such a way that each of these integers occurs in the table exactly 9 times and the integers in any column differ by no more than 3. Find the maximum possible value of the minimal column sum (sum of the numbers in one column).

7. Let n be a positive integer. Both n and n^2 only contain the digits 1, 2 and 3 (not necessarily all of them). Determine all possible values of n.

8. The (English language version of the) game of Scrabble™ consists of 100 tiles, each containing either a letter from A to Z (some letters occur more than once), except for two blank tiles; see `the relevant Wikipedia page` for the exact distribution of multiplicities of each letter.

   In a solo game of Scrabble, the player starts by choosing seven tiles from the 100 available tiles at random. What is the probability that the player picks up exactly two vowels?

Email submission guidelines

- Email your solutions to `samf.training.assignments@gmail.com`.
- Submit each question in a single separate PDF file (with multiple pages if necessary), with your name and the question number written on each page.
- If you take photographs of your work, use a document scanner such as CamScanner to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.