Intermediate Test 1

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. Lets x be a real number such that
$$x + \frac{1}{x} = 3.$$
Find the value of
$$x^5 + \frac{1}{x^5}.$$

2. How many different permutations of the word INTERCONNECTION are there? (Interchanging two letters that are the same does not count as a different word.)

3. Let ABC be a triangle. Prove that the internal angle bisector of the angle $\angle ABC$ and the perpendicular bisector of the line segment AC intersect on the circumcircle of triangle ABC.

4. Prove that $m + n \leq \gcd(m, n) + \text{lcm}(m, n)$ for all positive integers m, n. When does equality occur?

5. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that
$$f(f(x + y)) = x + f(y)$$
for all $x, y \in \mathbb{R}$.