

Senior Test 3 Question 4

- $1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$ 1 mark
- So we want $\frac{(2m)^2(2m+1)^2}{4} - \frac{m^2(m+1)^2}{4} = x^2$ 1 mark
- Equivalent to $m^2(3m+1)(5m+3) = 4x^2$ and so $(3m+1)(5m+3)$ is a square 1 mark
- $\gcd \leq 4$, so both factors are squares or each is twice a square 2 marks
- Both squares is impossible because 3 is not a q.r. modulo 5 1 mark
- Both twice a square is impossible since 2 is not a q.r. modulo 3 1 mark