## Senior Test 1 Question 1

- $lcm(m,n) = \frac{mn}{\gcd(m,n)}$  **1 mark**
- Let m = ax and n = ay 1 mark
- The inequality becomes  $a(x+y) \le a(1+xy)$  1 mark
- $(x-1)(y-1) \ge 0$  **1 mark**
- True because  $x, y \ge 1$  1 mark
- Equality iff x = 1 or y = 1 1 mark
- Equality iff  $m \mid n \text{ or } n \mid m$  1 mark

## Alternative solution:

- If  $m \mid n$  or  $n \mid m$  then equality **1 mark**
- Otherwise  $m \mid \operatorname{lcm}(m, n)$ , so  $\operatorname{lcm}(m, n) \geq 2m$  **2 marks**
- $2m \ge m + n$  1 mark
- $lcm + gcd > lcm \ge 2m \ge m + n$  2 marks
- Tying everything together 1 mark