# PAMO Stream Test 1

April Camp 2019

Time: $4\frac{1}{2}$ hours

1. Let $\Gamma$ be the circumcircle of an acute triangle $ABC$. The perpendicular line to $AB$ passing by $C$ cuts $AB$ in $D$ and $\Gamma$ again in $E$. The bisector of the angle $C$ cuts $AB$ in $F$ and $\Gamma$ again in $G$. The line $GD$ meets again $\Gamma$ in $H$ and the line $HF$ meets it again in $I$. Prove that $AI = EB$.

2. Find all non-negative integers $n$ for which the equation

$$\left(x^2 + y^2\right)^n = (xy)^{2018}$$

admits positive integral solutions.

3. Adamu and Afaafa choose, each in his turn, positive integers as coefficients of a polynomial of degree $n$. Adamu wins if the polynomial obtained has an integer root; otherwise, Afaafa wins. Afaafa plays first if $n$ is odd; otherwise Adamu plays first. Prove that:

   i) Adamu has a winning strategy if $n$ is odd.

   ii) Afaafa has a winning strategy if $n$ is even.