Senior Test 2

April Camp 2019

Time: $4\frac{1}{2}$ hours

1. Let $n \geq 3$ be an integer. Prove that there exists a set S of $2n$ positive integers satisfying the following property: For every $m = 2, 3, \ldots, n$ the set S can be partitioned into two subsets with equal sums of elements, with one of the subsets of cardinality m.

2. A circle ω of radius 1 is given. A collection T of triangles is called *good* if the following conditions both hold:

   (i) each triangle from T is inscribed in ω;

   (ii) no two triangles from T have a common interior point.

   Determine all positive real numbers t such that, for each positive integer n, there exists a good collection of n triangles, each of perimeter greater than t.

3. Determine all functions $f : (0, \infty) \to \mathbb{R}$ satisfying

$$\left(x + \frac{1}{x}\right) f(y) = f(xy) + f\left(\frac{y}{x}\right)$$

   for all $x, y > 0$.