Intermediate Test 2

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. The magical country of Vertexia, there are a certain number of cities. Each pair of cities is connected by exactly one magical portal, and it turns out that there are 5050 portals in total. How many cities are there in this wonderful land?

2. Let M be the midpoint of side AB of equilateral triangle ABC, and let points N, S and K divide side BC into four equal segments. Given that P is the midpoint of CM, prove that $\angle MNB = \angle KPN = 90°$.

3. Find all real numbers x such that

$$(x+3)^4 + (x+5)^4 = 4.$$

4. Find all positive integers m, n such that

$$1 + 5 \cdot 2^m = n^2.$$

5. Show that if we are given 50 segments on the real line, then either there are 8 of them which are pairwise disjoint or 8 of them with a common point.

   A segment is defined as a closed interval $[a, b] = \{x \in \mathbb{R} \ : \ a \leq x \leq b\}$.