Senior Test 1 Question 1

- $\text{lcm}(m,n) = \dfrac{mn}{\gcd(m,n)}$ **1 mark**
- Let $m = ax$ and $n = ay$ **1 mark**
- The inequality becomes $a(x+y) \leq a(1+xy)$ **1 mark**
- $(x-1)(y-1) \geq 0$ **1 mark**
- True because $x, y \geq 1$ **1 mark**
- Equality iff $x = 1$ or $y = 1$ **1 mark**
- Equality iff $m \mid n$ or $n \mid m$ **1 mark**

Alternative solution:

- If $m \mid n$ or $n \mid m$ then equality **1 mark**
- Otherwise $m \mid \text{lcm}(m,n)$, so $\text{lcm}(m,n) \geq 2m$ **2 marks**
- $2m \geq m + n$ **1 mark**
- $\text{lcm} + \gcd > \text{lcm} \geq 2m \geq m + n$ **2 marks**
- Tying everything together **1 mark**