Intermediate Test 4

Stellenbosch Camp 2018

Time: $2\frac{1}{2}$ hours

1. How many numbers from 1 to 2018 inclusive can be written as the difference of two perfect squares?

2.

3.

4. Prove that it is impossible to write a positive integer in every cell of an infinite chessboard, in such a manner that, for all positive integers m, n, the sum of numbers in every $m \times n$ rectangle is divisible by $m + n$.

5. Let A_1, A_2, A_3 be three points in the plane, and for convenience, let $A_4 = A_1$, $A_5 = A_2$. For $n = 1, 2$ and 3, suppose that B_n is the midpoint of A_nA_{n+1} and suppose that C_n is the midpoint of A_nB_n. Suppose that A_nC_{n+1} and B_nA_{n+2} meet at D_n and that A_nB_{n+1} and C_nA_{n+2} meet at E_n. Calculate the ratio of the area of triangle $\triangle D_1D_2D_3$ to the area of triangle $\triangle E_1E_2E_3$.