Junior Test 1 Question 1

- $\text{lcm}(m, n) = \dfrac{mn}{\gcd(m, n)}$ **1 mark**

- Let $m = ax$ and $n = ay$ **1 mark**

- The inequality becomes $a(x + y) \leq a(1 + xy)$ **1 mark**

- $(x - 1)(y - 1) \geq 0$ **1 mark**

- True because $x, y \geq 1$ **1 mark**

- Equality iff $x = 1$ or $y = 1$ **1 mark**

- Equality iff $m \mid n$ or $n \mid m$ **1 mark**

Alternative solution:

- If $m \mid n$ or $n \mid m$ then equality   **1 mark**

- Otherwise $m \mid \text{lcm}(m, n)$, so $\text{lcm}(m, n) \geq 2m$ **2 marks**

- $2m \geq m + n$ **1 mark**

- $\text{lcm} + \gcd > \text{lcm} \geq 2m \geq m + n$ **2 marks**

- Tying everything together   **1 mark**