

January Monthly Problem Set

Due: February 2019

1. Find all functions $f : \mathbb{N} \rightarrow \mathbb{N}$ such that

$$f(mn) = f(\gcd(m, n))f(\text{lcm}(m, n))$$

for all $m, n \in \mathbb{N}$.

2. Does there exist a natural number n such that

$$1^{2018} + 2^{2018} + \cdots + n^{2018}$$

is prime?

3. Fix a natural number $n \geq 2$. Find the smallest constant C such that

$$\sum_{1 \leq i < j \leq n} x_i x_j (3x_i^2 + x_j^2)(x_i^2 + 3x_j^2) \leq C \left(\sum_{i=1}^n x_i \right)^6$$

for all non-negative real numbers x_1, x_2, \dots, x_n . For this value of C , when does equality occur?