

Junior Test 1 Question 1

- $\text{lcm}(m, n) = \frac{mn}{\text{gcd}(m, n)}$ 1 mark
- Let $m = ax$ and $n = ay$ 1 mark
- The inequality becomes $a(x + y) \leq a(1 + xy)$ 1 mark
- $(x - 1)(y - 1) \geq 0$ 1 mark
- True because $x, y \geq 1$ 1 mark
- Equality iff $x = 1$ or $y = 1$ 1 mark
- Equality iff $m \mid n$ or $n \mid m$ 1 mark

Alternative solution:

- If $m \mid n$ or $n \mid m$ then equality 1 mark
- Otherwise $m \mid \text{lcm}(m, n)$, so $\text{lcm}(m, n) \geq 2m$ 2 marks
- $2m \geq m + n$ 1 mark
- $\text{lcm} + \text{gcd} > \text{lcm} \geq 2m \geq m + n$ 2 marks
- Tying everything together 1 mark