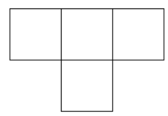
Junior Test 2

Stellenbosch Camp 2019

Time: 4 hours

1. Tile an 8×8 chessboard with T-shaped tetrominoes, which look as follows:



2. Prove that for all $a, b > 0$,
$$\frac{a}{b} + \frac{b}{a} \geq 2.$$

3. In $\triangle ABC$ let $\angle ACB = 90°$, $AC = 1$ and $AB = 2$.

   Let M be the midpoint of AB and D the intersection of the angle bisector of $\angle CAB$ and BC.

   Prove that $AB \perp CM$.

4. Find the first number which appears in all 3 the following arithmetic progressions:
$$21, 34, 57, 70, ...$$
$$33, 37, 41, 45, ...$$
$$42, 75, 108, 141, ...$$

5. There are 7 people A, B, C, D, E, F, and G sitting in a row. B wants to sit next to C and E wants to sit next to F. How many different seating arrangements are there?

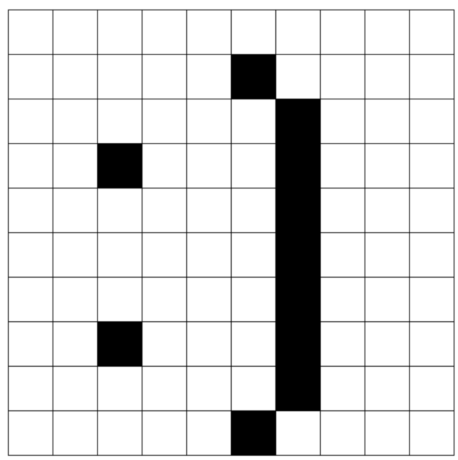
6. Given $\triangle ABC$, with $AB < AC$, let D be the point where the angle bisector of $angleBAC$ intersects the circumcircle of $\triangle ABC$. Let P and Q be the altitudes dropped onto the extensions of AB and AC. Prove that $PB = QC$.

7. What are the last two digits of $7^{7^{7^{7}}}$?

8. Prove that for all $a, b, c, d > 0$,

$$(a+b+c+d)^4 \geq abcd \times 4^4.$$

9. Given the below colouring, is it possible to invert the colours of rows or the colours of columns in some order to achieve a completely white board?



10. Let n be a positive integer greater than 2. Let r_1 be the smallest odd divisor of n greater than 1 and let r_2 be the largest odd divisor of n. Find all n such that

$$n = 5r_1 + 3r_2.$$

```
      .___,
   ___('v')___
   `"-\._./-"'
       ^ ^
```