# Some Intermediate Algebra problems

### The coaches

### December 2019

1. Compute $\sqrt{2019 \times 2020 \times 2021 \times 2022 + 1}$.

2. Find all $n$ for which $n^4 + 4$ is prime.

3. Find all integer values $n$ and $m$ that satisfy $n^4 + 2n^3 + 2n^2 + 2n + 1 = m^2$.

4. Prove that $2\sqrt{n+1} - 2 < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} < 2\sqrt{n} - 1$ for all $n$ where $n$ is a Natural number greater than 1.

5. It is given that:

$$1x_1 + 4x_2 + 9x_3 + ... + 2019^2 x_{2019} = 1$$

$$4x_1 + 9x_2 + 16x_3 + ... + 2020^2 x_{2019} = 2$$

$$9x_1 + 16x_2 + 25x_3 + ... + 2021^2 x_{2019} = 3$$

   What is the value of $16x_1 + 25x_2 + 36x_3 + ... + 2022^2 x_{2019}$ ?

6. Find all real values of $x$ that satisfy $x^4 + 4xk^3 + 3k^4 = 0$, where $k$ is some fixed real number.

7. What is the least possible value of $(x+1)(x+2)(x+3)(x+4) + 2019$ where $x$ is a real number?

8. Let $a, b$ and $c$ be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011$$

$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$$

   What is the value of $a$?

9. Find all functions $f : \mathbb{Q} \to \mathbb{Q}$ such that

$$f(x^2) + f(x + 2y) = (x + 1)f(x) + 2f(y)$$

for all rational numbers $x$ and $y$.