suggested problems

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1. A crowd of marching band members are getting ready for a parade. The grand marshal tried to arrange them in rows of 10, 9, 8, 7, 6, 5, 4, 3 and 2, but in every case there was a missing person in the last row. Finally, the fed up grand marshal ordered them to march in single file. If the total number of people did not exceed 5000, how many were there?

2. A chemist received a shipment of ten bottles of a certain medication. Each bottle contains one thousand pills but, due to an error, the pills in one bottle are each 10 milligrams too much. The pharmacist, Miss Tree, has a scale that can measure the mass of an object. She realises that she can use the scale only once and find the faulty bottle. How does she do it? (Assume she knows how much a bottle *should* weigh.)

   Six months later, ten more bottles of the same drug are delivered. Miss Tree is told that a worse mistake has been made: this time the order included any number of bottles that were 10 milligrams too heavy. What is the smallest number of weighings required to identify every faulty bottle?

3. A carpenter can use his table saw to cut a large wooden cube into 64 unit cubes by making 9 cuts. But if he rearranges the pieces before each cut, only six cuts are necessary. Prove that it can't be done in fewer than six cuts.

4. An n-sided die has the numbers 1 to n marked on each face. When rolled, the die comes to rest showing one of these numbers on its upper surface. If two such die are thrown together, in how many outcomes is the maximum of the two rolls an even integer?

5. A certain military organisation consists of a navy, an infantry and an air force. There are two officers and 2 commanders in each division. A committee needs to be formed, containing three officers and 3 commanders. It must also contain two leaders from each division. How many committees can be formed with this requirement?

6. There are n identical cars at points on a circular track. Among all of them, they have just enough petrol for one car to complete a lap. Show that there is a car which can complete a lap by collecting petrol from the other cars along the way.

7. Show that the cubes of the roots of $x^3 + ax^2 + bx + ab = 0$ are roots of $x^3 + a^3x^2 + b^3x + a^3b^3 = 0$.

8. If a and b are respectively the arithmetic means of the squares and cubes of all numbers less than n and relatively prime to it (including 1), prove that
$$n^3 - 6an + 4b = 0$$

9. Out of n straight lines whose lengths are $1, 2, 3, \ldots, n$ centimetres respectively, four are chosen so that a quadrilateral may be formed. Show that the number of ways to do this so that a circle may be inscribed in the quadrilateral is
$$\frac{1}{48}\left(2n(n-2)(2n-5) - 3 + 3(-1)^n \right)$$