Intermediate Test 2

Stellenbosch Camp 2019

Time: $2\frac{1}{2}$ hours

1. In triangle $\triangle ABC$, the angle bisector of $\angle BAC$, the perpendicular bisector of AC and the altitude from C to AB are concurrent. Find the value of $\angle BAC$.

2. Find all positive integers n such that $\frac{n^2+8n+51}{n+4}$ is also a positive integer.

3. Prove that for all real numbers x, y and z,

$$x^2 + 5y^2 + z^2 \geq 2y(2x + z)$$

4. The points E and F lie on sides AB and AD, respectively, of a parallelogram $ABCD$ such that $|AB| = 4|AE|$ and $|AD| = 4|AF|$. Prove that BF, DE, and AC are concurrent.

5. The cells of an 8×8 chessboard are all coloured in white. A move consists in inverting the colours of a 1×3 rectangle, either vertical or horizontal (the white cells become black and the black cells become white). Is it possilbe to colour all cells of the chessboard in black in a finite number of moves?