Test 2

'April' Camp 2020

Time: $4\frac{1}{2}$ hours

1. Two players write pluses or minuses in front of consecutive natural numbers $1, 2, 3, \ldots$ and calculate the sum of all the numbers that have a sign in front of them. This way, the first player starts with writing a sign in front of 1, and this is his sum after the first move. Then the second player writes a sign in front of 2 and calculates the sum of the first two numbers with signs – this is the sum after his first move. After that the first player writes a sign in front of 3 and calculates the sum of three numbers with signs, and so on. When the sum becomes larger than 2019 in absolute value, the game stops. The player who made the last move loses. Which player has a winning strategy?

2. We say that a set S of integers is *rootiful* if, for any positive integer n and any $a_0, a_2, \ldots, a_n \in S$, all integer roots of the polynomial $a_0 + a_1 x + \cdots + a_n x^n$ are also in S. Find all rootiful sets of integers that contain all numbers of the form $2^a - 2^b$ for positive integers a and b.

3. Let $ABCDE$ be a convex pentagon such that $\angle EDC \neq 2 \cdot \angle ADB$ and $CD = DE$. Suppose that a point P is located in the interior of the pentagon such that $AP = AE$ and $BP = BC$. Prove that P lies on the diagonal CE if and only if

$$\operatorname{area}(BCD) + \operatorname{area}(ADE) = \operatorname{area}(ABD) + \operatorname{area}(ABP).$$