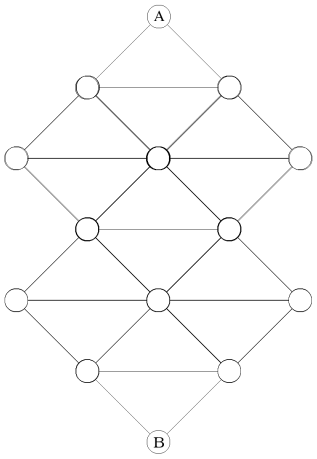
Intermediate Test 3

Stellenbosch Camp 2019

Time: $2\frac{1}{2}$ hours

1. Find the number of paths from A to B, where the only allowed moves are moving down along a diagonal line or moving left or right along a horizontal line, but never crossing the same line twice.



2. Two intersecting circles, C_1 and C_2, have a common tangent which touches C_1 at P and C_2 at Q. The two circles intersect at M and N, where N is nearer to PQ than M is. The line PM meets the circle C_2 again at R. Prove that MQ bisects $\angle PMR$.

3. Emma and Emile play a game on a 2019×2019 board made up of unit grid squares. Emma plays first by placing a knight on one of the squares and thereafter they take turns to place a knight on a square that does not already contain a knight and is not attacked by one of the already placed knights. The first player who cannot do this loses. Can one of the players always guarantee that they will win? If so, which one?

4. Let a, b, and c be positive real numbers such that $b, c \in [1, 2)$ and

$$\frac{a+b}{b(1+c)} + \frac{a+c}{c(1+b)} = 2.$$

Show that a, b, and c are the lengths of the sides of a triangle.

5. Find the least positive integer k such that 2050^{2051} can be written as a sum of k 5th powers.