

# Intermediate Test 1

Stellenbosch Camp 2019

Time:  $2\frac{1}{2}$  hours

1. If  $x + \frac{1}{x} = 3$ , what is the value of  $x^5 + \frac{1}{x^5}$ ?
2. Given a triangle  $ABC$  and two points  $M$  and  $N$  on sides  $AB$  and  $AC$  respectively. Let  $BN$  and  $CM$  intersect at  $P$ . It is given that the areas of  $\triangle CPN$ ,  $\triangle BPM$  and  $\triangle BPC$  are 4, 6 and 5 respectively. Find the area of  $\triangle ABC$ .  
(Bonus: if you would like an extra mark, use the areas 20, 19, 2019 instead)
3. Find all positive integers  $n$  where the product of the positive factors of  $n$  is  $n^3$ .
4. A set  $T$  of integers is called *broken* if there are integers  $a < b < c$  such that  $a$  and  $c$  are in  $T$ , but  $b$  is not in  $T$ .  
Find the number of broken subsets of  $\{1, 2, \dots, 2019\}$ .
5. Let  $ABC$  denote an equilateral triangle. Let  $M$  and  $N$  denote the midpoints of  $AB$  and  $BC$ , respectively. Let  $P$  be a point outside  $ABC$  such that  $APC$  is isosceles and right-angled at  $P$ . Lines  $PM$  and  $AN$  meet at  $I$ . Prove that  $CI$  is the angle bisector of  $\angle ACM$ .

· --- ,  
 --- ( ' v ' ) ---  
 ' " \ . \_ . / - " ,  
 ^ ^