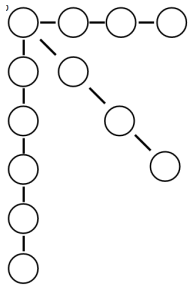
Junior Test 1

Stellenbosch Camp 2019

Time: $2\frac{1}{2}$ hours

1. a) The numbers 1 through 12 must be filled into the circles such that the sum along each line is 32. Do it!



b) The numbers 1 through 12 are arranged in the circles such that each sum is S. What are the possible values of S?

2.

All boxes in a 3×3 table are occupied by zeroes. Suppose that we can choose any 2×2 sub-table and increase all the numbers in it by 1.

Example:

0	0	0
0	0	0
0	0	0

Possible next two moves:

1	1	0
1	1	0
0	0	0

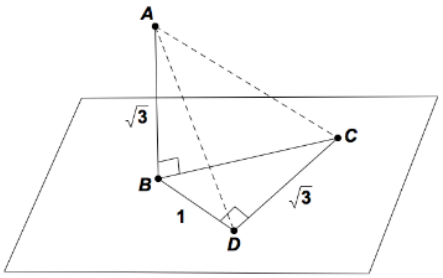
1	1	0
1	2	1
0	1	1

Prove that we cannot obtain the table below using these operations.

4	9	5
10	18	12
6	13	7

3.

Triangle BCD lies in the plane, and A is a point outside the plane. AB is perpendicular to the plane and BD is perpendicular to CD. What is $A\widehat{D}C$ in degrees?



4. You are a citizen of Amestris, living your life as an aspiring Mathematician, but ho! One day, you find yourself in a conundrum! You found a compass on sale, but you cannot make the exact value with the \$7 and \$11 that Amestris trades in.

   What is the maximal value of the compass?

5. If $x + \frac{1}{x} = 3$, what is the value of $x^5 + \frac{1}{x^5}$?

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