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Abstract

1 Problem 1, Bulgaria Round 3, 1975

Let n be an odd natural number and a_1, a_2, \dots, a_n a permutation of the numbers $1, 2, \dots, n$. Show that the value $(a_1 - 1) * (a_2 - 2) * \dots * (a_n - n)$ is an even number.

2 Problem 1, Spanish MO, 1995

Consider all sets A of one hundred different natural numbers with the property that any three elements $a, b, c \in A$ (not necessarily different) are the sides of a non-obtuse triangle. Denote by $S(A)$ the sum of the perimeters of all such triangles. Compute the smallest possible value of $S(A)$.

3 Putnum 2002, A2

Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

4 Putnum 2003, A1

Let n be a fixed positive integer. How many ways are there to write n as a sum of positive integers,

$$n = a_1 + a_2 + \dots + a_k,$$

with k an arbitrary positive integer and $a_1 \leq a_2 \leq \dots \leq a_k \leq a_1 + 1$? For example, with $n = 4$ there are four ways: 4, 2+2, 1+1+2, 1+1+1+1.