April Camp 2018: Test 1
PAMO Stream
Time: $4\frac{1}{2}$ hours

1. Suppose that ω is the circumcircle of the triangle ABC with $AC > AB$. Let X and Y be two points on AC and circle ω respectively, such that $CX = CY = AB$. (The points A and Y lie on different sides of the line passing through B and C). The line XY intersects ω for the second time in point P. Show that $PB = PC$.

2. A rectangle R with odd integer side lengths is divided into small rectangles with integer side lengths. Prove that there is at least one among the small rectangles whose distances from the four sides of R are either all odd or all even.

3. Determine all integers $n \geq 2$ with the following property: for any integers $a_1, a_2, \ldots, a_n$ whose sum is not divisible by n, there exists an index $1 \leq i \leq n$ such that none of the numbers
$$a_i,\ a_i + a_{i+1},\ \ldots,\ a_i + a_{i+1} + \ldots a_{i+n-1}$$
is divisible by n. (We let $a_i = a_{i-n}$ where $i > n$.)