

April Camp 2018: Test 1

IMO Stream

Time: $4\frac{1}{2}$ hours

1. A rectangle R with odd integer side lengths is divided into small rectangles with integer side lengths. Prove that there is at least one among the small rectangles whose distances from the four sides of R are either all odd or all even.
2. Determine all integers $n \geq 2$ with the following property: for any integers a_1, a_2, \dots, a_n whose sum is not divisible by n , there exists an index $1 \leq i \leq n$ such that none of the numbers

$$a_i, a_i + a_{i+1}, \dots, a_i + a_{i+1} + \dots + a_{i+n-1}$$

is divisible by n . (We let $a_i = a_{i-n}$ where $i > n$.)

3. Let $ABCC_1B_1A_1$ be a convex hexagon such that $AB = BC$, and suppose that the line segments AA_1 , BB_1 and CC_1 have the same perpendicular bisector. Let the diagonals AC_1 and A_1C meet at D , and denote by ω the circle ABC . Let ω intersect the circle A_1BC_1 again at $E \neq B$. Prove that lines BB_1 and DE intersect on ω .