April Camp 2018: Test 2
PAMO Stream
Time: $4\frac{1}{2}$ hours

1. In two 3×3 grids, positive integers are arranged according to the following condition: if the number n appears in a grid, the number of times it appears in that grid is a multiple of n. Prove that for any such arrangement of numbers in the two grids, one can always find two blocks such that the numbers in those blocks in the first grid are identical, and the numbers in those blocks in the second are also identical (although not necessarily the same as those in the first).

2. Let $a_1, a_2, \ldots, a_n, k$ and M be positive integers such that

$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} = k \quad \text{and} \quad a_1 a_2 \cdots a_n = M.$$

If $M > 1$, prove that the polynomial

$$p(x) = M(x+1)^k - (x+a_1)(x+a_2)\cdots(x+a_n)$$

has no positive roots.

3. Let $ABCDE$ be a convex pentagon such that $AB = BC = CD$, $\angle EAB = \angle BCD$, and $\angle EDC = \angle CBA$. Prove that the perpendicular line from E to BC and the line segments AC and BD are concurrent.