PAMO Test 1

2018

Time: $4\frac{1}{2}$ hours

1. We consider a square $ABCD$ and a point E on the segment CD. The bisector of $\angle EAB$ cuts the segment BC in F. Prove that $BF+DE = AE$.

2. On a 50×50 chessboard, we put, in the lower left corner, a die whose faces are numbered from 1 to 6. By convention, the sum of digits on two opposite side os the die equals 7. Adama wants to move the die to the diagonally opposite corner using the following rule: at each step, Adama can roll the die only on to its right side, or to its top side. We suppose that whenever the die lands on a square, the number on its bottom face is printed on the square. By the end of these operations, Adama wants to find the sum of the 99 numbers appearing on the chessboard. What are the maximum and minimum possible values of this sum?

3. Find the maximum and minimum of the expression

$$\max(a_1, a_2) + \max(a_2, a_3, + \cdots + \max(a_{n-1}, a_n) + \max(a_n, a_1),$$

where $(a_1, a_2, \ldots, a_n)$ runs over the set of permutatons of $(1, 2, \ldots, n)$.