PAMO Test 2

2018

Time: $4\frac{1}{2}$ hours

1. Abimbola plays heads or tails with a coin. A sequence of tosses is said to be admissable if he gets an odd number of consecutive "heads" followed by one "tail". (For instance, HHHHTTHHTHT is admissable, but THHHHTTHHT is not admissable.) How many admissable tosses of length n do we have?

2. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(f(x) + y) = f(x^2 - y) + 4f(x)y$ for all real numbers x and y.

3. For which prime numbers p can we find three positive integers n, x and y such that $p^n = x^3 + y^3$?