

PAMO Test 3

2018

Time: $4\frac{1}{2}$ hours

1. Find all integers a , b and c such that $a + b + c = abc$.
2. Let $ABCDE$ be a regular pentagon, and F some point on the arc AB of the circumcircle of $ABCDE$. Show that

$$\frac{FD}{FE + FC} = \frac{FB + FA}{FD} = \frac{-1 + \sqrt{5}}{2},$$

and that $FD + FB + FA = FE + FC$.

3. Let $n \geq 1$ be an integer, and a_0, a_1, \dots, a_{n-1} be real numbers such that $1 \geq a_{n-1} \geq a_{n-2} \geq \dots \geq a_1 \geq a_0 \geq 0$. We assume that λ is a real root of the polynomial $x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$. Prove that $|\lambda| \leq 1$.