DN-2019-1 *South African Mathematics Olympiad 2019 Problem 5*

Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that

$$f\left(a^3\right) + f\left(b^3\right) + f\left(c^3\right) + 3f(a+b)f(b+c)f(c+a) = (f(a+b+c))^3$$

for all integers a, b, and c.