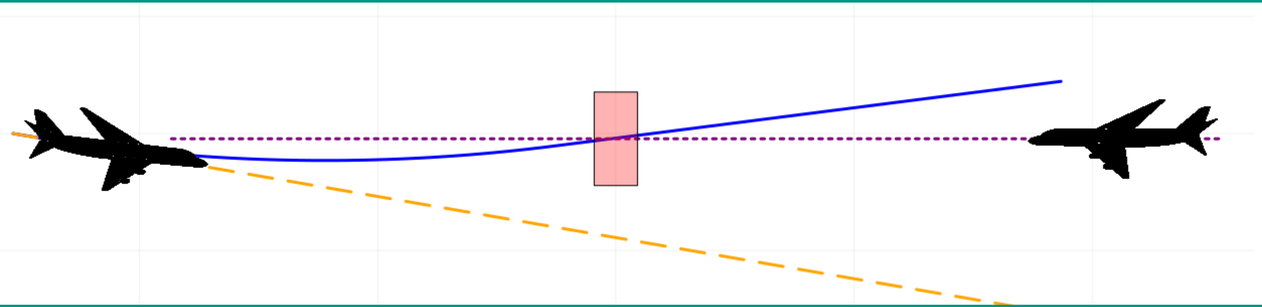
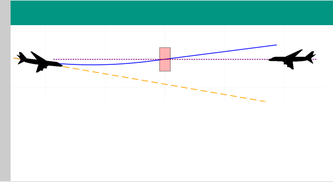


Of Good Demons and Bad Angels:

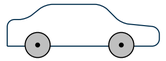
Guaranteeing Safe Control under Finite Precision

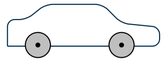
FMCAD 2025

Samuel Teuber, Debasmita Lohar, Bernhard Beckert | 2025



Motivation: Neural Network Control Systems





1. Cars
2. Constant Velocity
3. Differential Equations

Motivation: Neural Network Control Systems

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$$p'_{rel} = v_{\text{rel}}$$
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How can we prove the **infinite-time horizon** safety of a strategy g?


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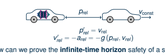
Safe if cars start far enough apart (depends on B).



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1. Simple Case: Just brake

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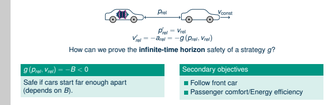
Secondary objectives

- Follow front car
- Passenger comfort/Energy efficiency



1. Continuous Time
2. Infinite-Time Safety

Motivation: Neural Network Control Systems

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$\Rightarrow$ We leverage **Differential Dynamic Logic** to verify NN Control Systems

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Differential Dynamic Logic by Example

We will use $d\mathcal{L}$ as the tool to prove the safety of cyber-physical systems.

[Platzer 2008]

We will use $d\mathcal{L}$ as the tool to prove the safety of cyber-physical systems.

[Platzer 2008]

1. Real World programs $\neq$ hybrid programs
2. *Envelope* instead of concrete strategy
3. Need for monitoring

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hybrid program

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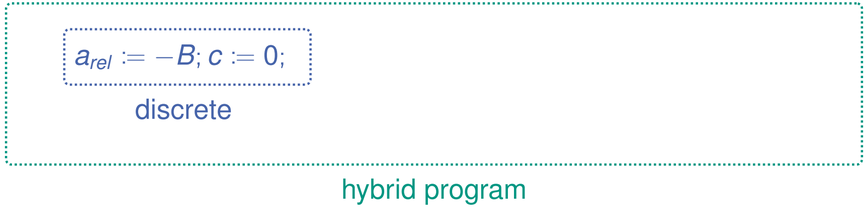

[Platzer 2008]

We will use $d\mathcal{L}$ as the tool to prove the safety of cyber-physical systems.
$a_{rel} := -B; c := 0;$
discrete
hybrid program
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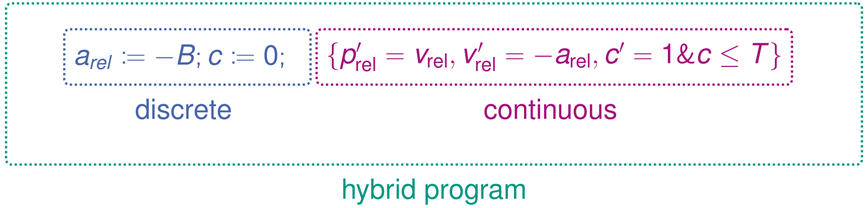

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$a_{rel} := -B; c := 0;$ | $\{p'_{\text{rel}} = v_{\text{rel}}, v'_{\text{rel}} = -a_{\text{rel}}, c' = 1 \& c \leq T\}$

discrete continuous

hybrid program

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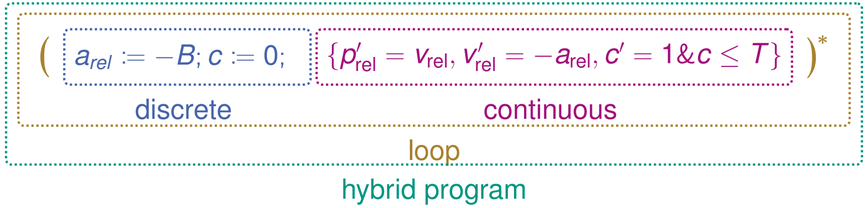

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$(a_{rel} := -B; c := 0; \{p'_{\text{rel}} = v_{\text{rel}}, v'_{\text{rel}} = -a_{\text{rel}}, c' = 1 \& c \leq T\})^*$

discrete continuous loop hybrid program

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We will use $d\mathcal{L}$ as the tool to prove the safety of cyber-physical systems.

$$\Big[\Big(\underbrace{a_{rel} := -B; c := 0;}_{\text{discrete}} \underbrace{\{p'_{\text{rel}} = v_{\text{rel}}, v'_{\text{rel}} = -a_{\text{rel}}, c' = 1 \& c \leq T\}}_{\text{continuous}}\Big)^*\Big]$$

loop

hybrid program

after every program run

[Platzer 2008]

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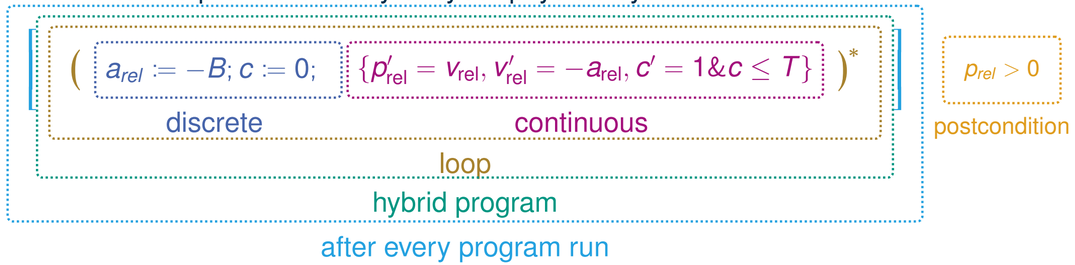

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$p_{rel} > \frac{v_{rel}^2}{2B} \rightarrow$ precondition

$[$ $($ $a_{rel} := -B; c := 0;$ discrete $\{p'_{\text{rel}} = v_{\text{rel}}, v'_{\text{rel}} = -a_{\text{rel}}, c' = 1 \& c \leq T\}$ continuous $)^*$ $]$

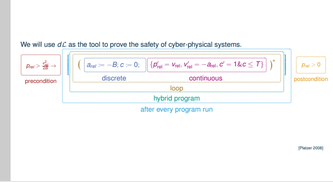
loop

hybrid program

after every program run

$p_{rel} > 0$ postcondition

[Platzer 2008]



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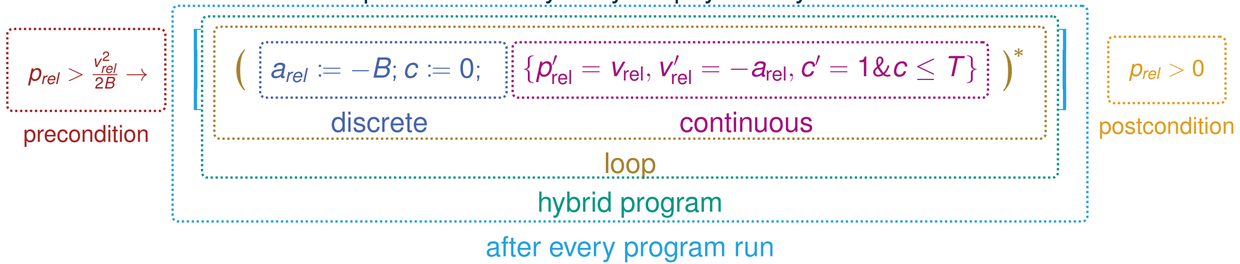

We can prove safety through a proof calculus.

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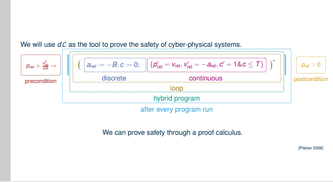

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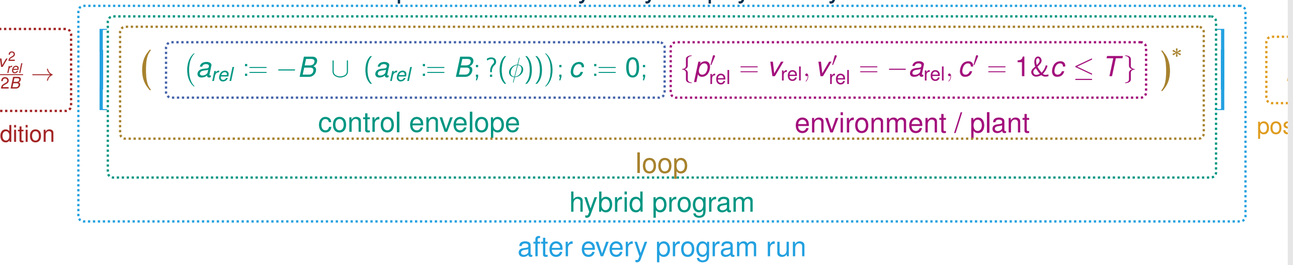

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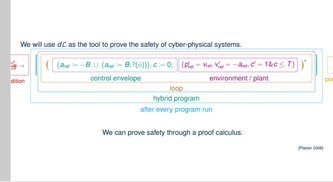

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Verifiably Safe AI via Logically Linked Envelopes

$$\left(\alpha_{\text{ctrl}} \;;\alpha_{\text{plant}}\right)^{*} \quad \text{Safe}$$

g 

[NeurIPS'24]

1. Nondeterministic Mirror
2. VerSAILLE
3. NN Verification
4. Mirror: Refinement Proof
5. Safety guarantee

Verifiably Safe AI via Logically Linked Envelopes

$$\left(\alpha_{\text{ctrl}} \; ; \alpha_{\text{plant}}\right)^* \quad \text{Safe}$$

$$\left(\underbrace{\alpha_g}_{} \; ; \alpha_{\text{plant}}\right)^*$$

Nondeterministic Mirror:
$\alpha_g := \text{mirror}\,(g)$

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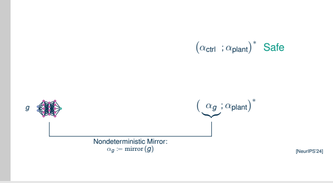

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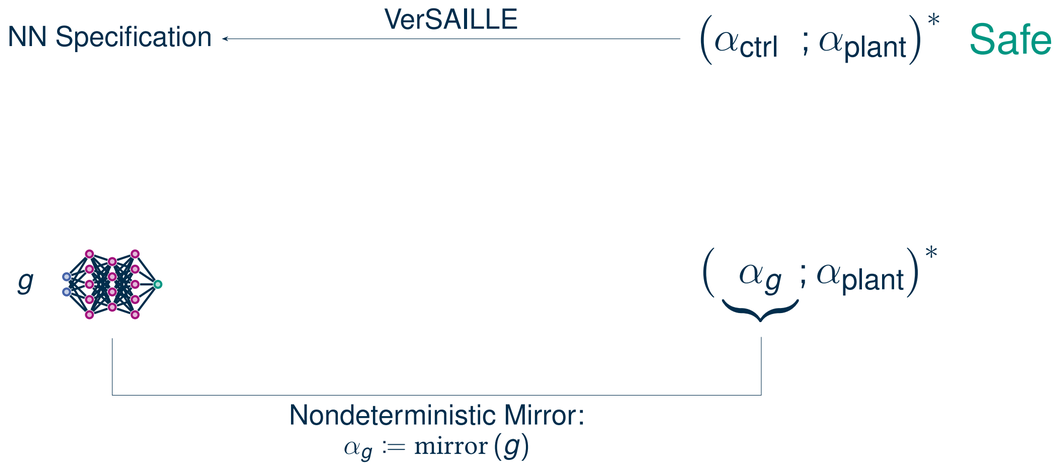

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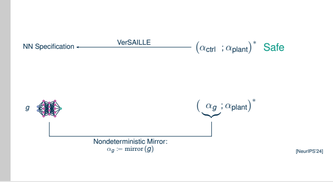

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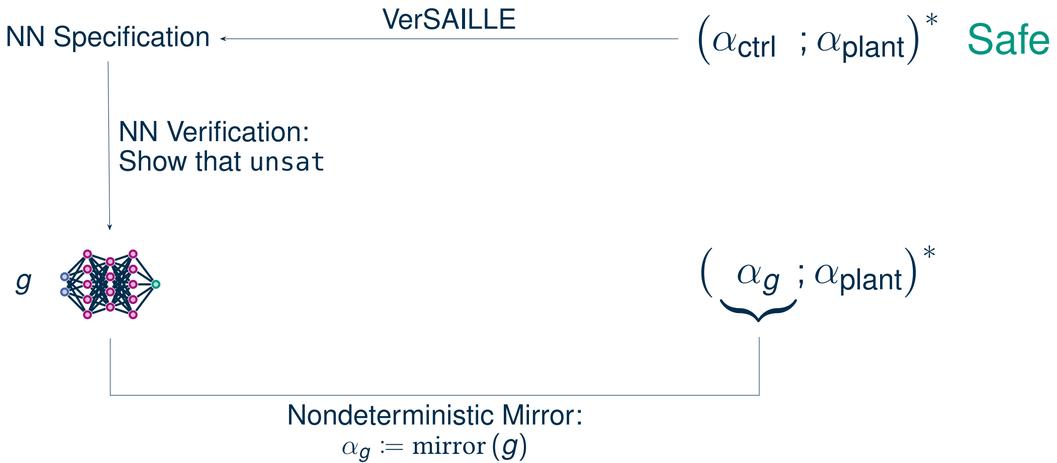

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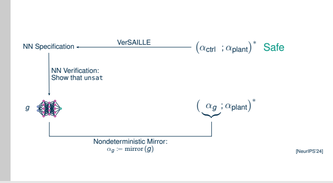

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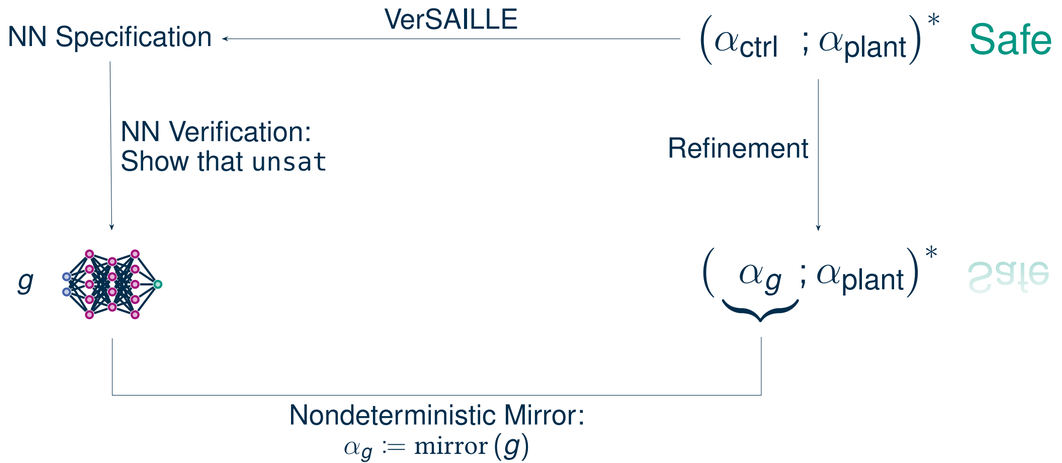

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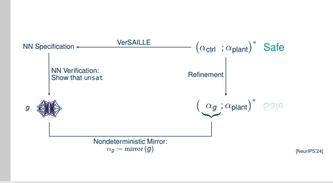

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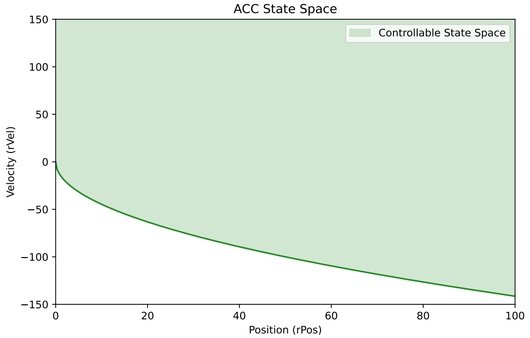
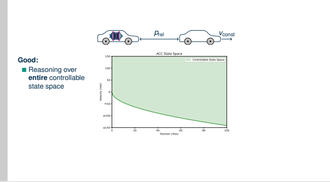
VerSAILLE: The Good

Good:

- Reasoning over **entire** controllable state space





VerSAILLE: The Good

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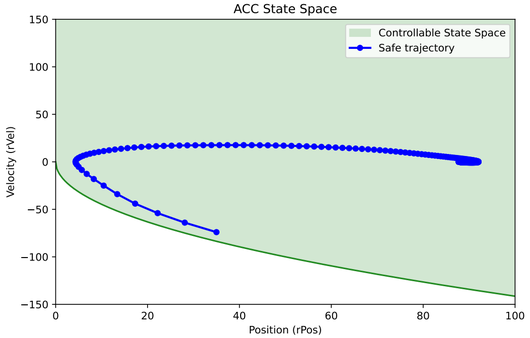
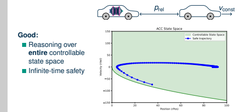
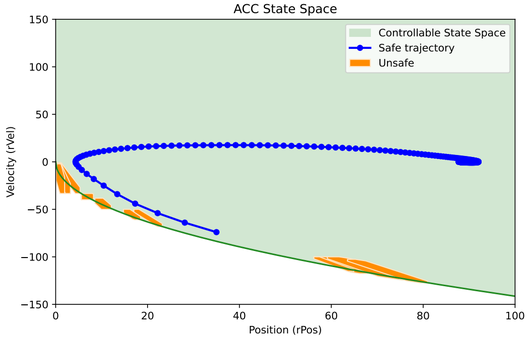


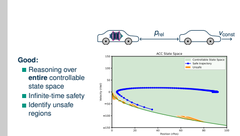


VerSAILLE: The Good

Good:

- Reasoning over **entire** controllable state space
- Infinite-time safety
- Identify unsafe regions





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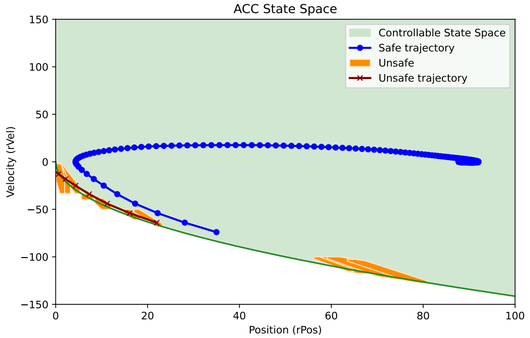
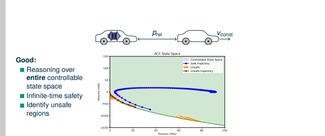
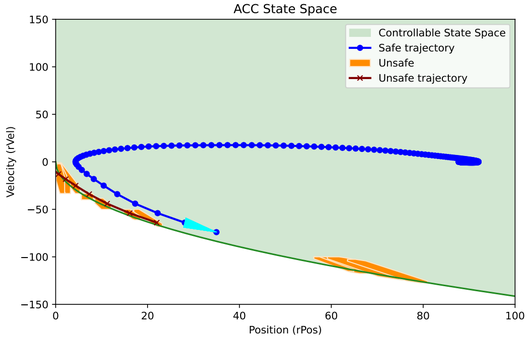




VerSAILLE: The Good and the Bad

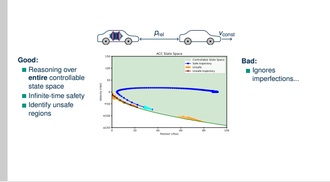
Good:

- Reasoning over **entire** controllable state space
- Infinite-time safety
- Identify unsafe regions



Bad:

- Ignores imperfections...



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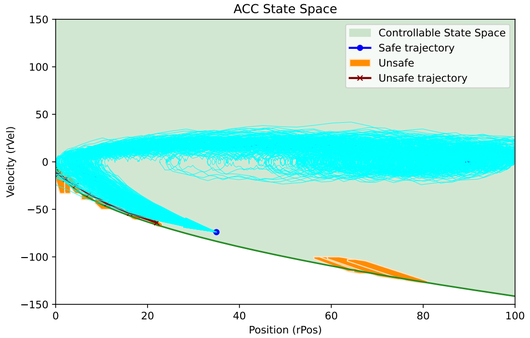
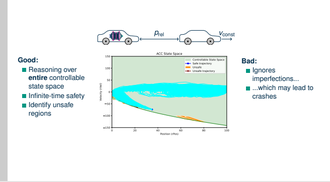


Bad:

- Ignores imperfections...
- ...which may lead to crashes



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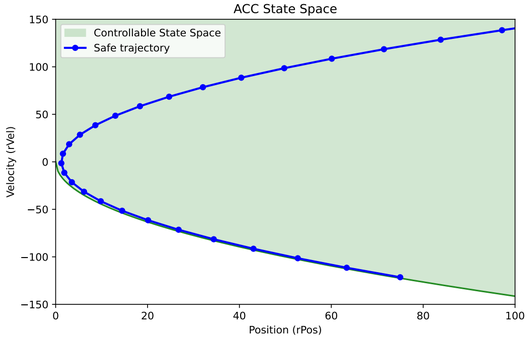
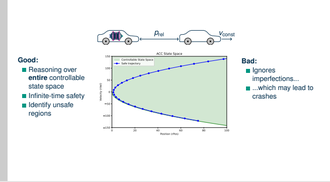
- Identify unsafe regions



Bad:

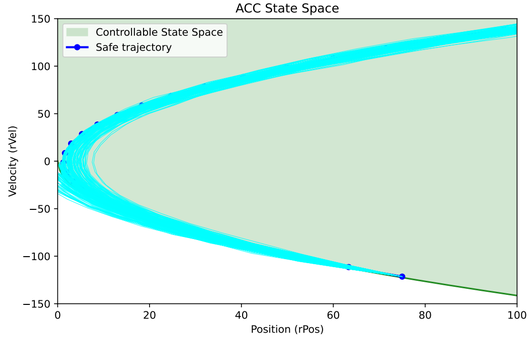
- Ignores imperfections...
- ...which may lead to crashes



VerSAILLE: The Good and the Bad

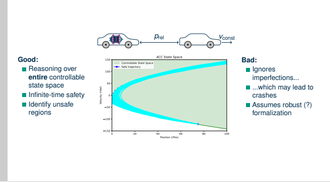
Good:

- Reasoning over **entire** controllable state space
- Infinite-time safety
- Identify unsafe regions



Bad:

- Ignores imperfections...
- ...which may lead to crashes
- Assumes robust (?) formalization

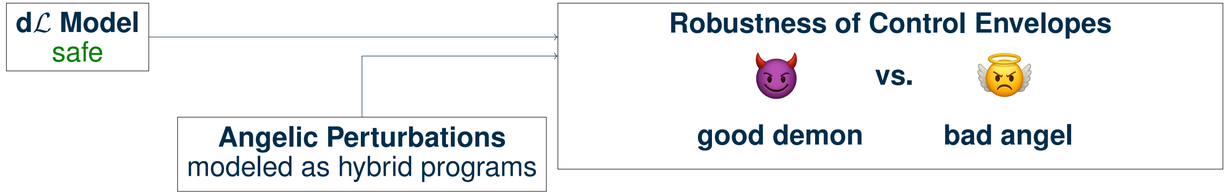


Contribution

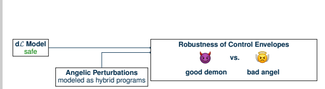
Angelic Perturbations
modeled as hybrid programs

Angelic Perturbations
modeled as hybrid programs

Contribution



└Contribution



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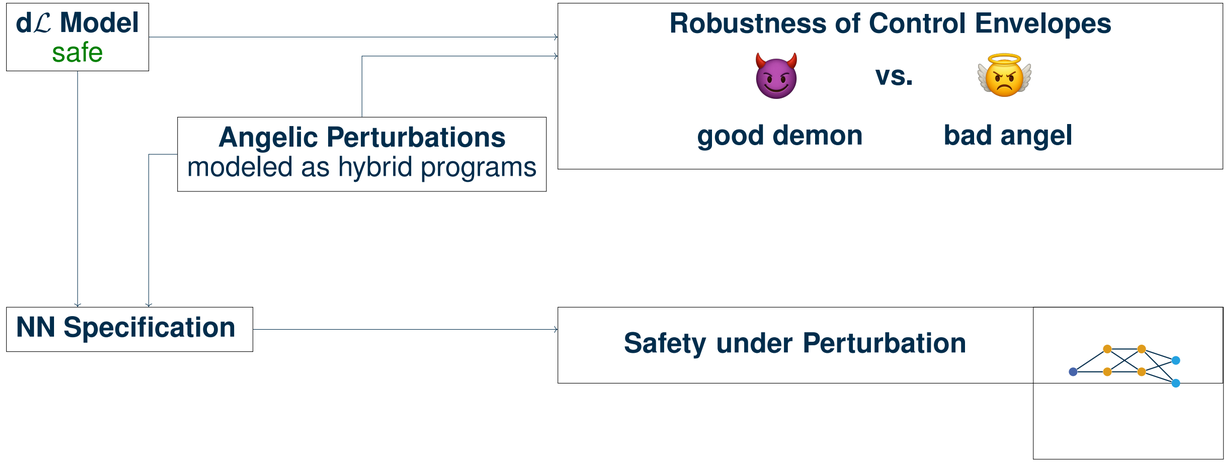

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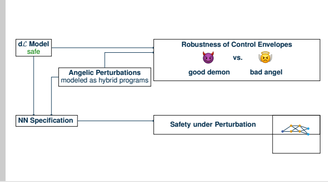

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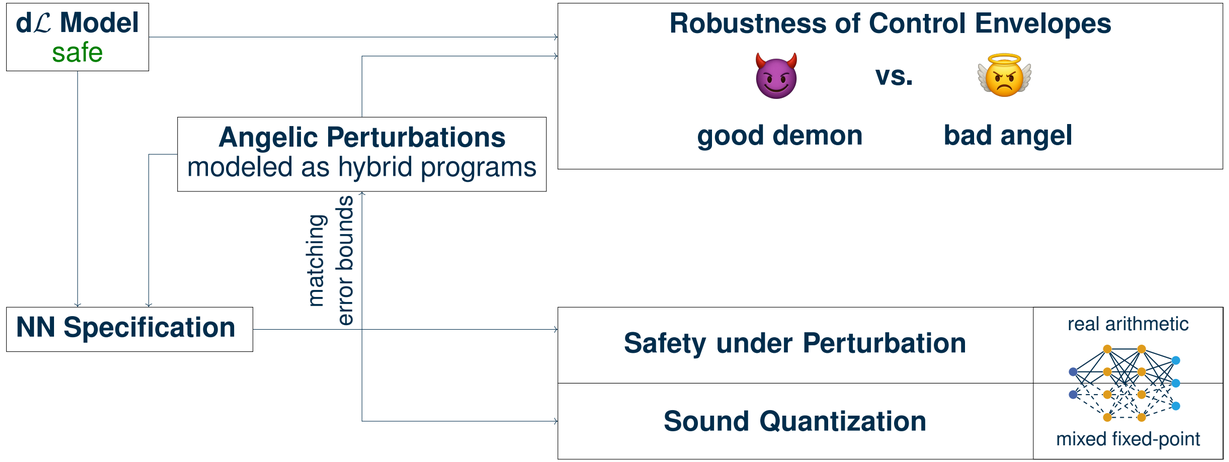

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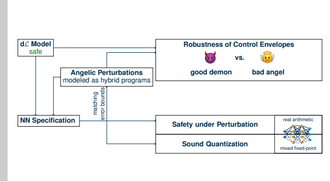
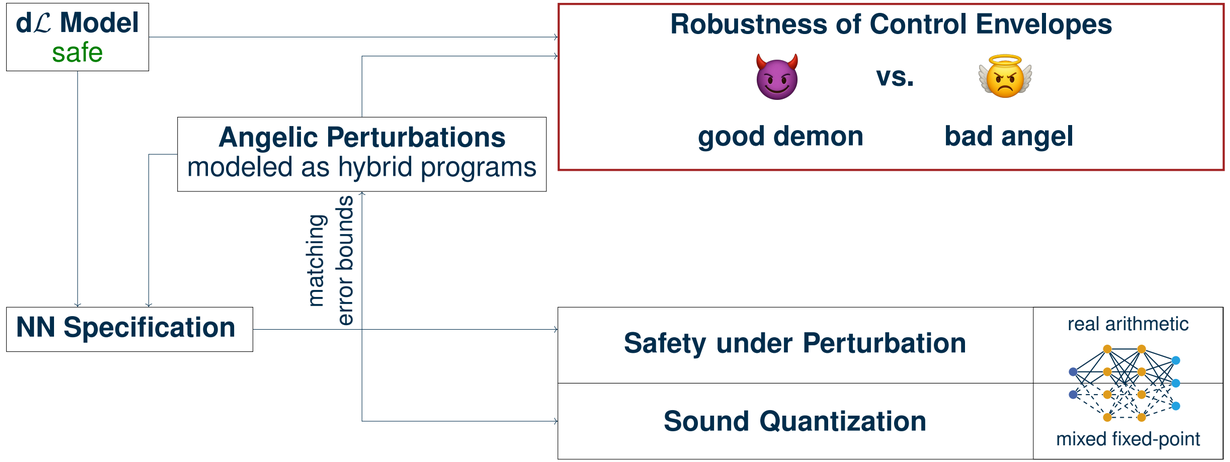


Contribution



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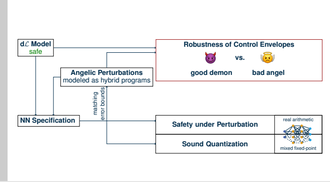

Control Envelope Robustness

What we know:

$$\text{pre} \rightarrow \left[\left(\text{ctl}; \ \text{env} \right)^* \right] \text{post}$$

What we know:
$\text{pre} \rightarrow \left[\left(\text{ctl}; \ \text{env} \right)^* \right] \text{post}$

Control Envelope Robustness

What we know:

$$\text{pre} \rightarrow \left[\; (\; \text{ctl}; \;\; \text{env} \;)^* \; \right] \text{post}$$

What we want:

$$\text{pre} \rightarrow \left[\; (\; \text{sense}; \;\; \text{ctl} \;\; ; \;\; \text{act}; \;\; \text{env} \;)^* \; \right] \text{post}$$

What we know:
pre → [(ctl; env)*] post
What we want:
pre → [(sense; ctl ; act; env)*] post

Control Envelope Robustness

What we know:

$$\text{pre} \rightarrow \left[\left(\text{ctl}; \ \text{env} \right)^* \right] \text{post}$$

What we want:

$$\text{pre} \rightarrow \left[\left(\text{sense}; \ \text{ctl} \ ; \ \text{act}; \ \text{env} \right)^* \right] \text{post}$$

$$\begin{aligned}
\text{sense} &\equiv \varepsilon_p := *; ?\left(|\varepsilon_p| \leq 10^{-3}\right); \\
& \quad p_{\text{rel}}^{\text{ctrl}} := p_{\text{rel}}^{\text{env}} + \varepsilon_p \\
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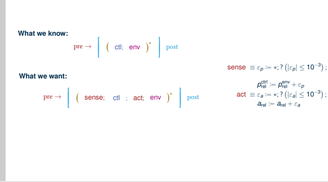

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😈 Angel resolves nondeterminism

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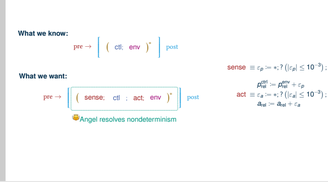

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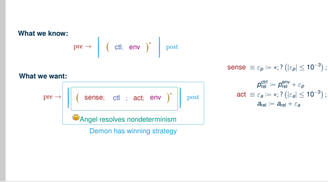

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What we want:

Demon resolves nondeterminism

$$\text{pre} \rightarrow \left[\left(\text{sense}; \ (\text{ctl})^d; \ \text{act}; \ \text{env} \right)^* \right] \text{post}$$

Angel resolves nondeterminism

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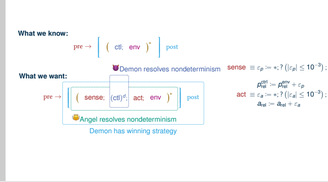

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Demon has winning strategy

Angelic Perturbation

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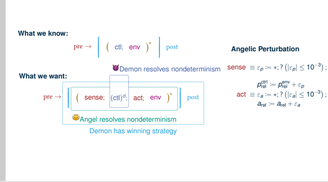

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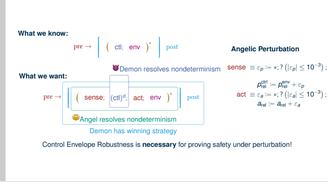
Control Envelope Robustness is **necessary** for proving safety under perturbation!



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Control Envelope Robustness: Observations

Liveness as Special Case

$$\text{sense} \equiv \textbf{skip}$$
$$\text{act} \equiv \textbf{skip}$$

Liveness as Special Case
sense ≡ skip
act ≡ skip

Control Envelope Robustness: Observations

Liveness as Special Case

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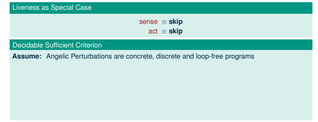
Decidable Sufficient Criterion

Assume: Angelic Perturbations are concrete, discrete and loop-free programs



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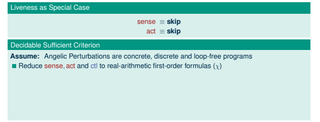

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- For invariant inv check that: (based on insights from Prebet et al. 2024)

$$\begin{aligned} \forall \bar{x}_0\ \forall x_1\ \exists \bar{x}_2\ \forall \bar{x}_3\ \big(&\text{inv}\,(\bar{x}_0) \land \chi_{\text{sense}}\,(\bar{x}_0, \bar{x}_1) \rightarrow \\ &(\chi_{\text{ctl}}\,(\bar{x}_1, \bar{x}_2) \land (\chi_{\text{act}}\,(\bar{x}_2, \bar{x}_3) \rightarrow \chi_{\text{ctl}}\,(\bar{x}_0, \bar{x}_3)))\big) \end{aligned}$$

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Can be simplified for additive actuator perturbations

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∀x̄0 ∀x1 ∃x̄2 ∀x̄3 (inv(x̄0) ∧ χsense(x̄0, x̄1) → (χctl(x̄1, x̄2) ∧ (χact(x̄2, x̄3) → χctl(x̄0, x̄3))))
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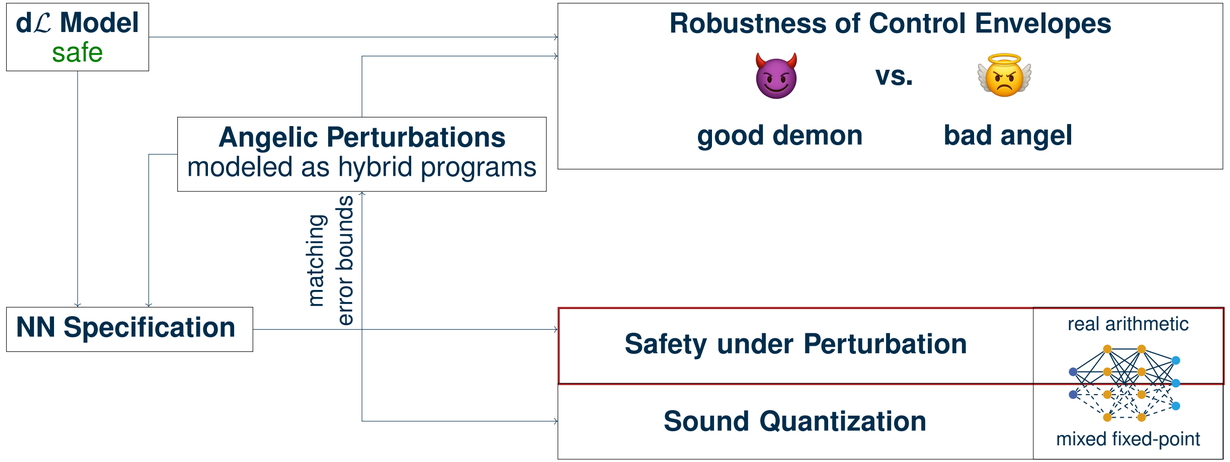
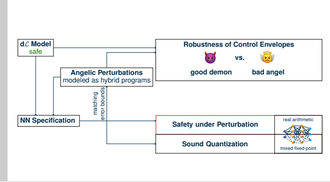
Contribution





Perturbed VerSAILLE

- Recall: VerSAILLE derives an NN specification from the dL model
- We extend this specification generation for **safety under angelic perturbations**

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Perturbed VerSAILLE

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All Control Actions Safe

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Phrased as counterexample search:

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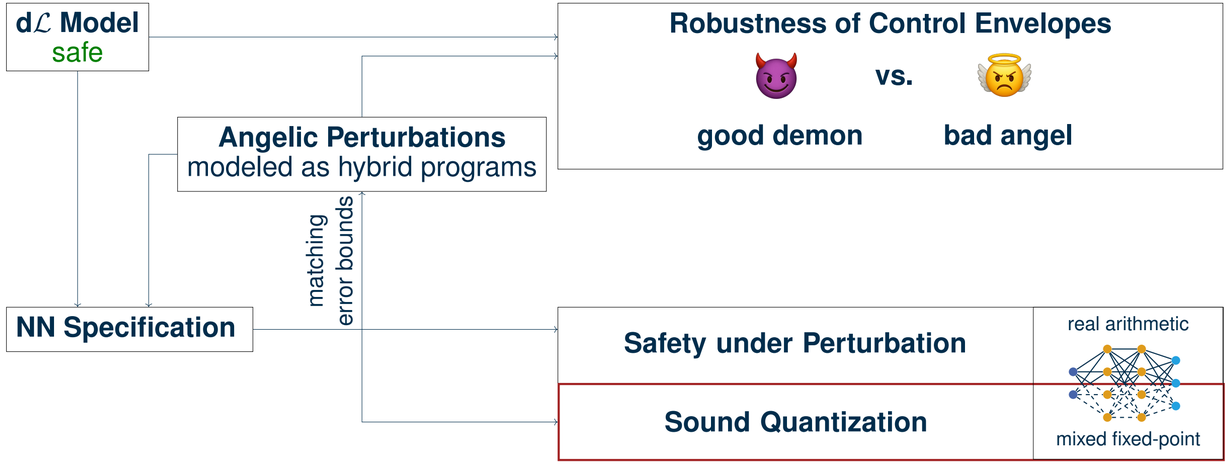
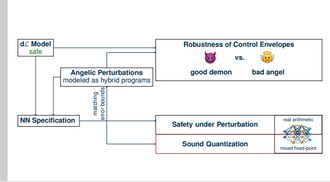
Contribution





NN Synthesis: Mixed Precision Tuning

Daisy Darulova et al. 2018

- Can assign mixed precision to variables given bound on output error
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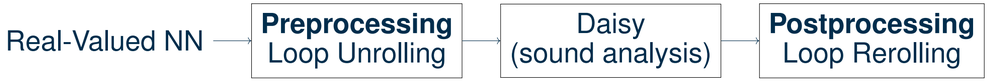

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    2 inputs / 256 ReLU nodes / approx. 12k parameters
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ACC (disc.)	0.01	59s	1m	2.01h	579
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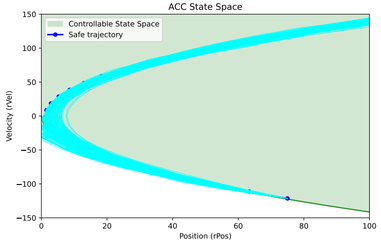

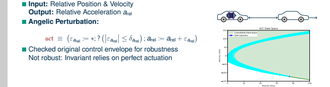

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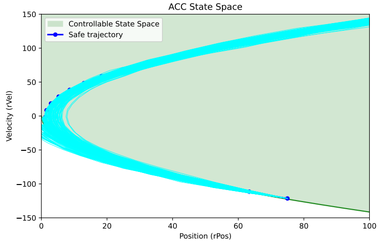

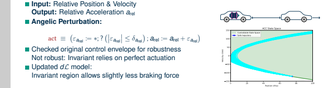

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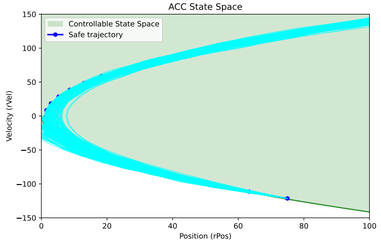

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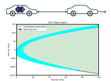

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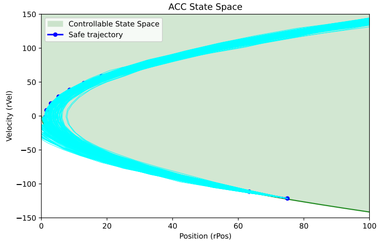

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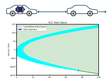

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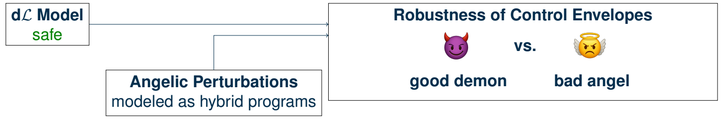

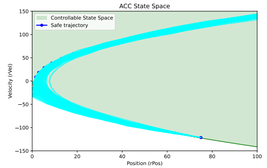

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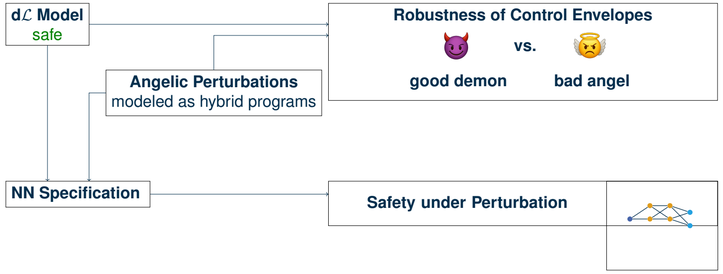

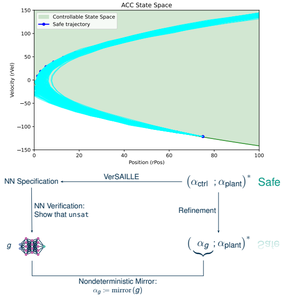

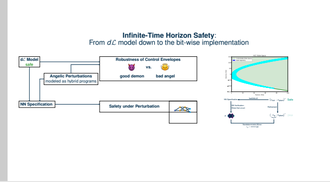

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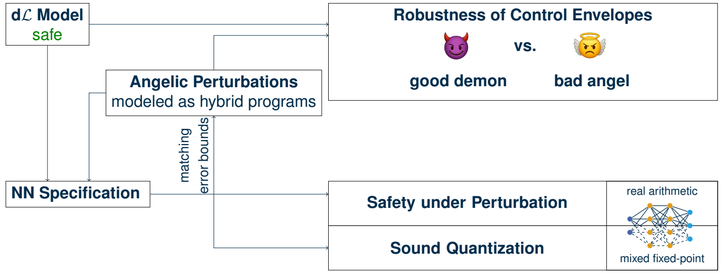

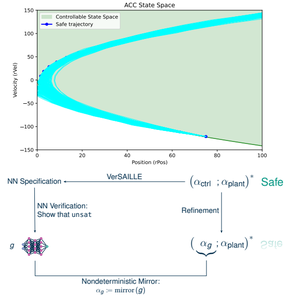

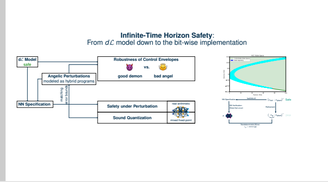

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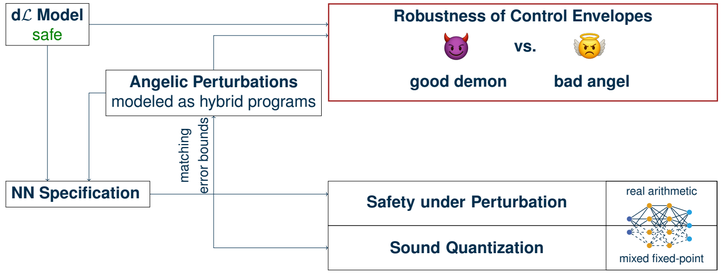

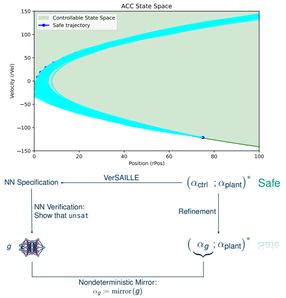

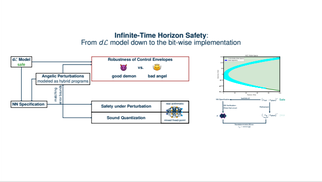

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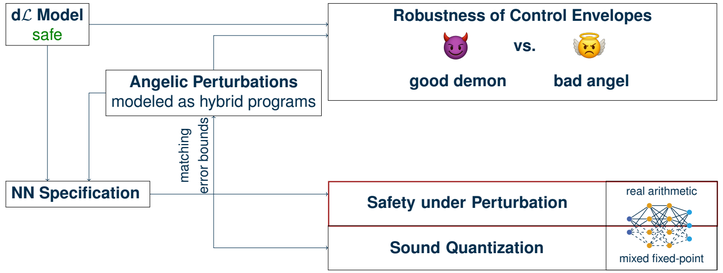

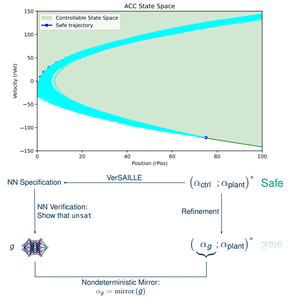

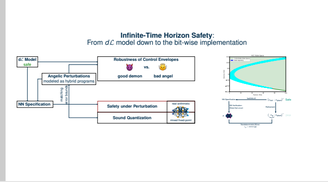

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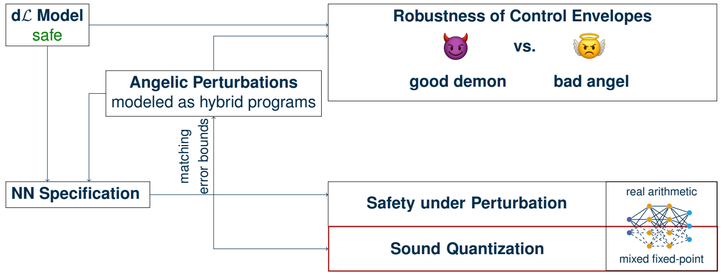

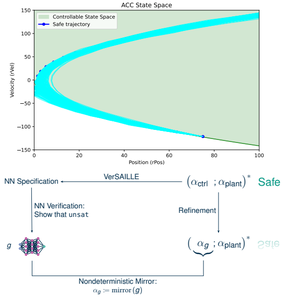

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Backup

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ModelPlex

Input: Safe Control Envelope in $d\mathcal{L}$

$$p_{\text{rel}} > \frac{v_{\text{rel}}^2}{2B} \rightarrow \left[(a_{\text{rel}} := -B; c := 0; \{p'_{\text{rel}} = v_{\text{rel}}, v'_{\text{rel}} = a_{\text{rel}}, c' = 1 \& c \leq T\})^*\right] p_{\text{rel}} > 0$$

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1. Monitor Formula: If satisfied $\rightarrow$ implementation matches behavior of hybrid program
2. Controller: Usually abstract $\Rightarrow$ envelope
3. More complex programs

ModelPlex

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ModelPlex creates a **controller monitor formula**:

$$a^+_{\text{rel}} = -B$$

Satisfaction during concrete run implies correct controller behavior.
Equally applicable for more complicated controllers (nondeterminism, conditions, ...)

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Verifiably Safe AI via Logically Linked Envelopes

Theorem (Soundness)

Assume:

- *g is a (piece-wise Noetherian) neural network*
- *C* $\equiv \left(\phi \rightarrow \left[(\alpha_{ctrl}; \alpha_{plant})^*\right] \psi\right)$ *is a valid d$\mathcal{L}$ contract*
- $\mathrm{controller}$ *is a controller monitor (ModelPlex)*
- $\mathrm{invariant}$ *is an inductive invariant*

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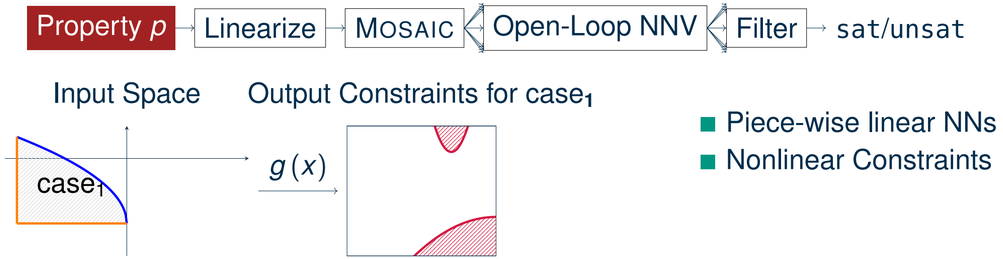
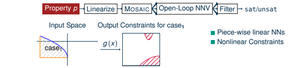
Mosaic: Efficient and Complete NN Verification for Nonlinear Properties



- Piece-wise linear NNs
- Nonlinear Constraints



1. Piece-wise linear NNs!

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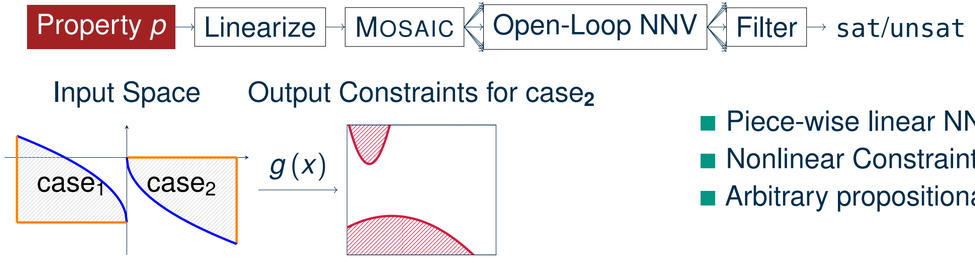
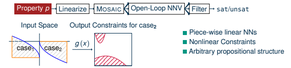


- Piece-wise linear NNs
- Nonlinear Constraints
- Arbitrary propositional structure



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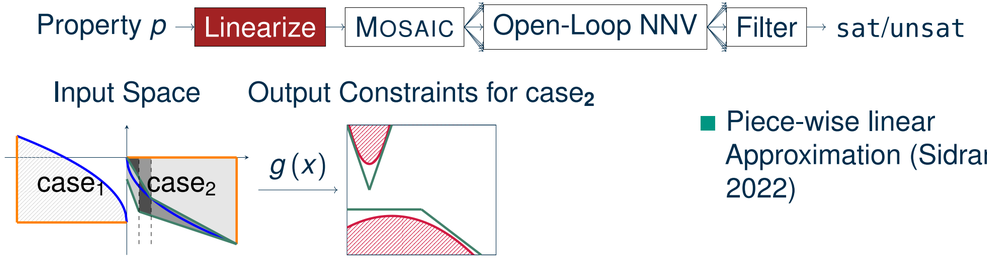
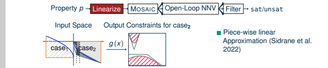


- Piece-wise linear Approximation (Sidrane et al. 2022)



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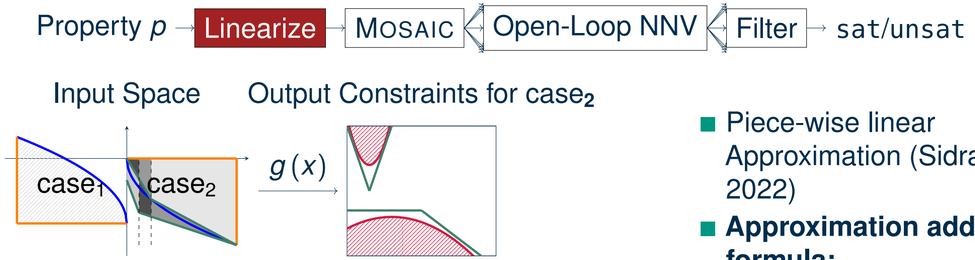

- Piece-wise linear Approximation (Sidrane et al. 2022)
- **Approximation added to formula:**

$$p \wedge c \rightarrow c_o$$
$$\wedge\, c_u \rightarrow c$$

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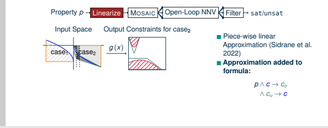
Backup



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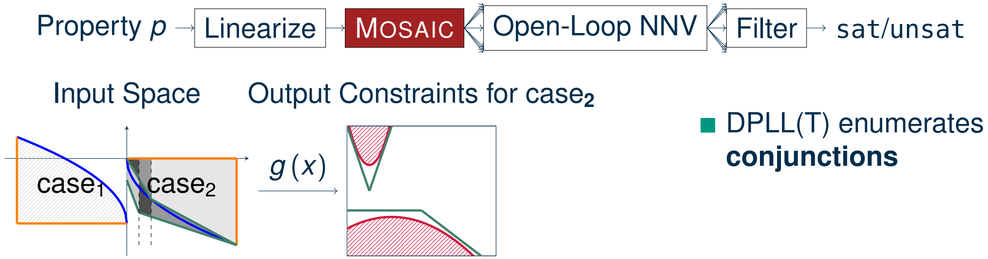
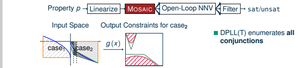
Mosaic: Efficient and Complete NN Verification for Nonlinear Properties



- DPLL(T) enumerates **all conjunctions**



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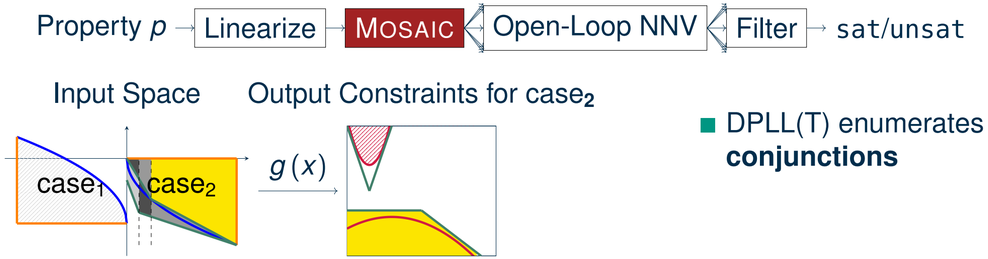

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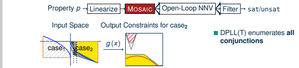

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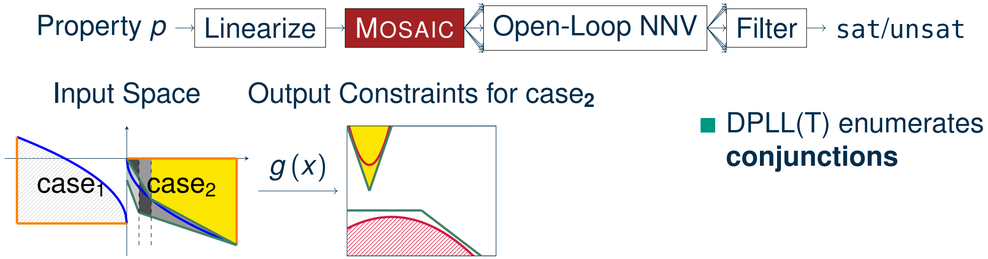

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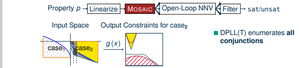

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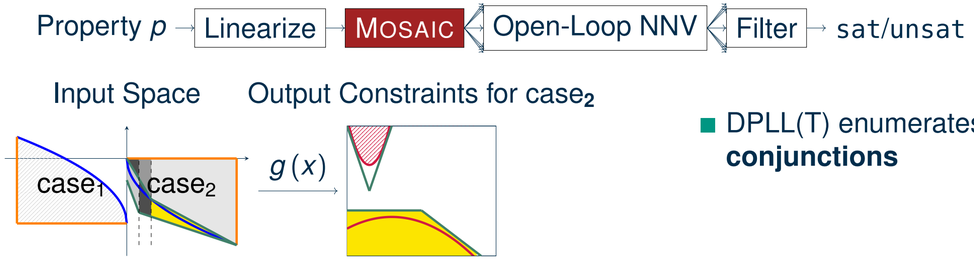

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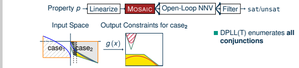

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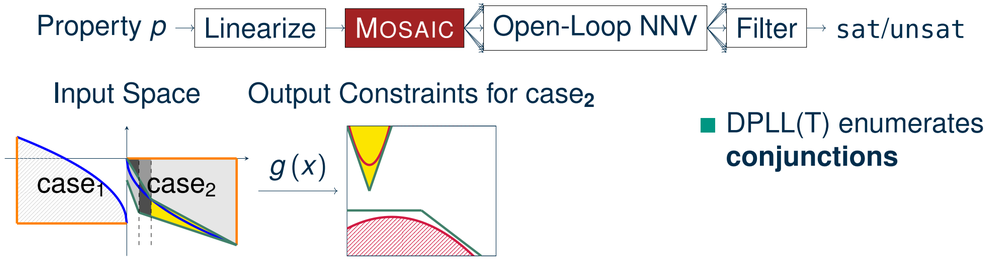

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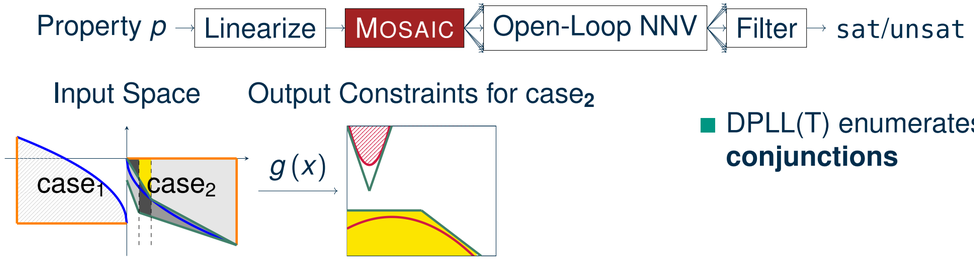

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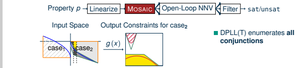

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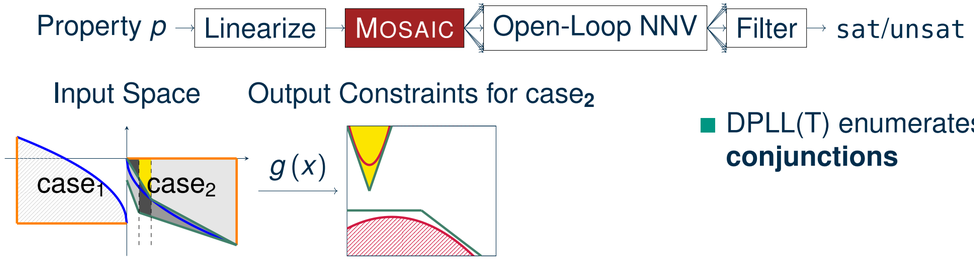

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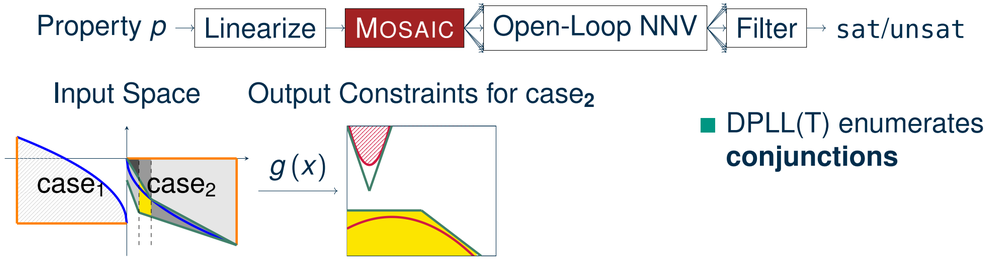

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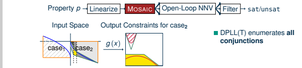

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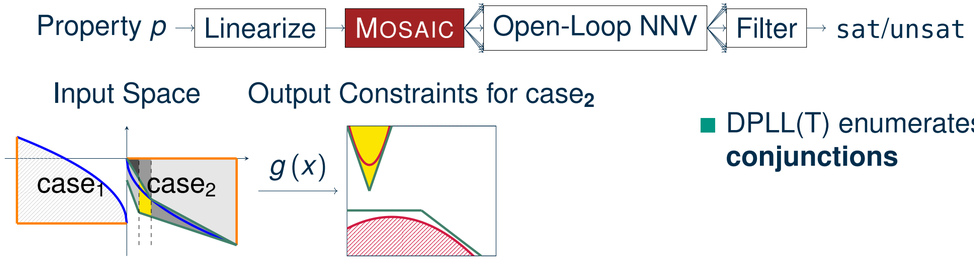

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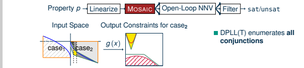

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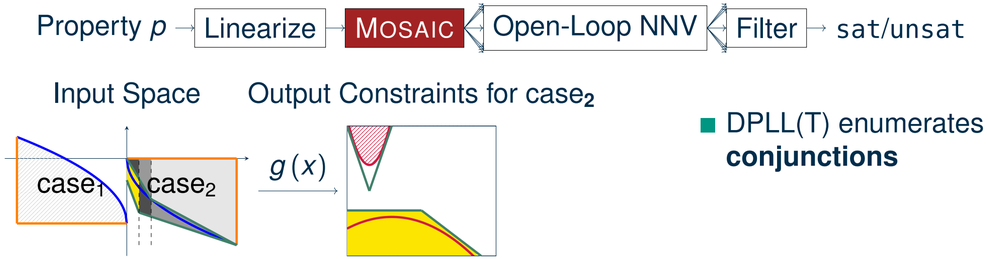

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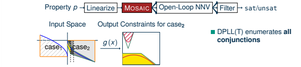

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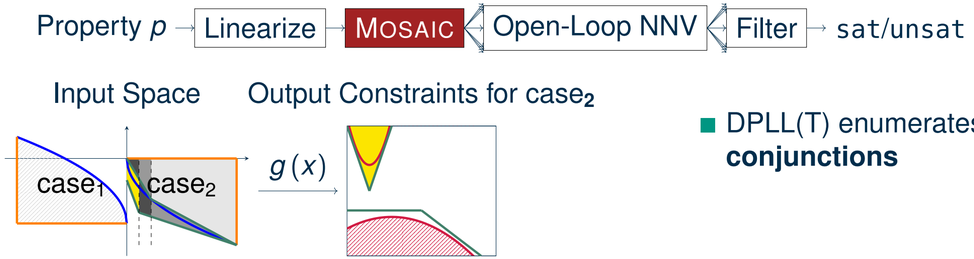

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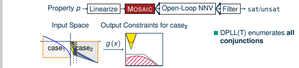

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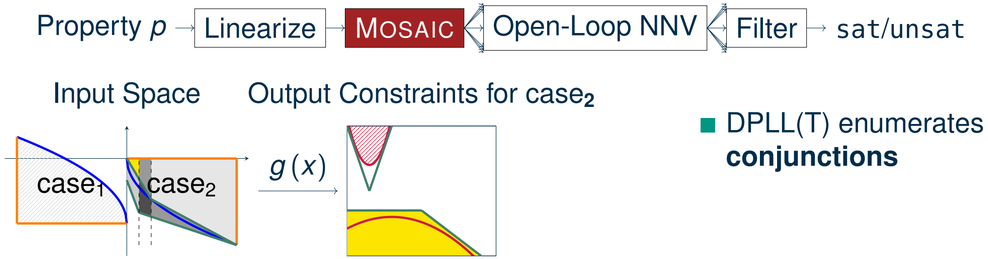

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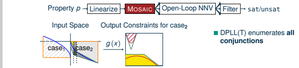

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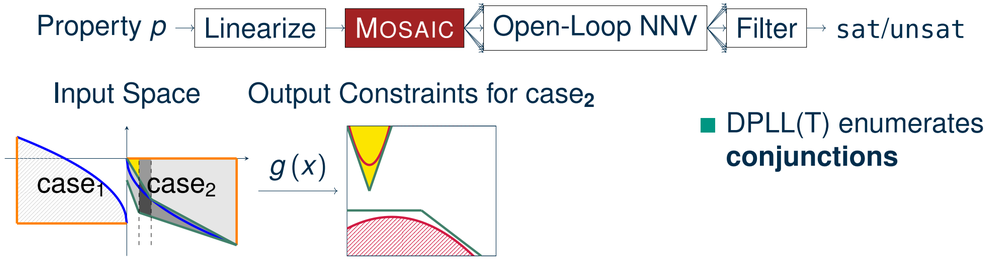

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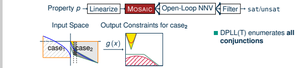

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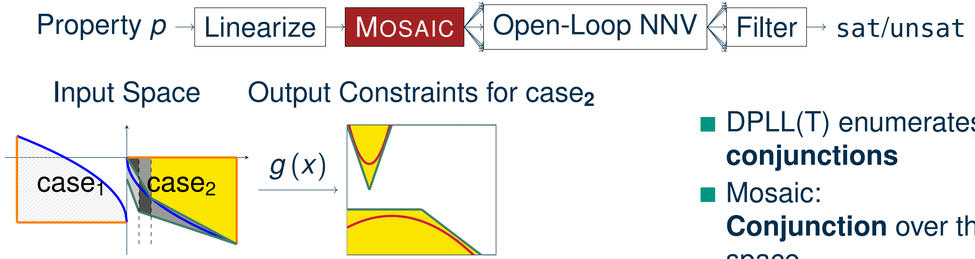
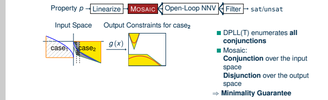


- DPLL(T) enumerates **all conjunctions**
- Mosaic:
  Conjunction over the input space
  Disjunction over the output space

⇒ **Minimality Guarantee**

References

Backup



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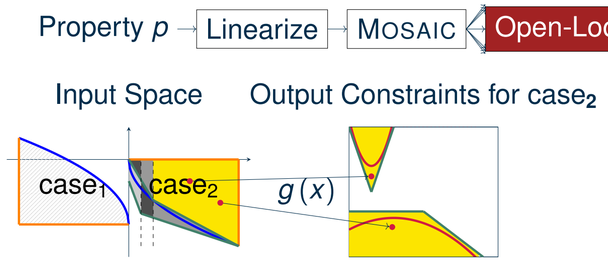
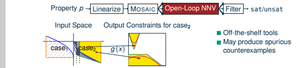


- Off-the-shelf tools
- May produce spurious counterexamples



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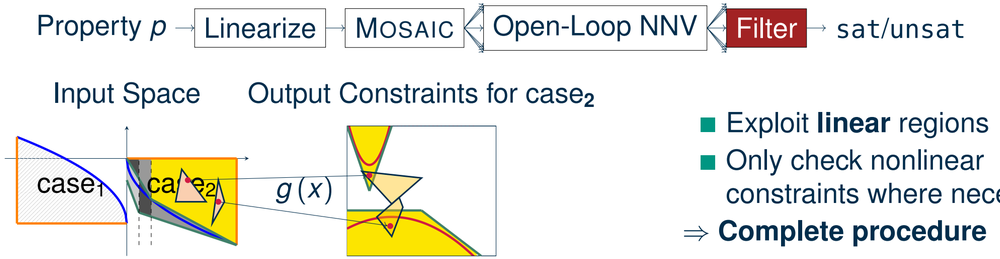
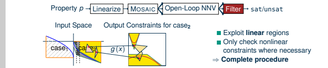


- Exploit **linear** regions
- Only check nonlinear constraints where necessary

$\Rightarrow$ **Complete procedure**



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Complexity of Mosaic

We assume:
- An NN with N ReLU nodes
- A property with M atomic constraints and I input variables

Then we get the following complexities:
- M atomic constraints: $\mathcal{O}\left(2^M\right)$ NNV queries
- Naive encoding via SMT for N ReLU nodes: $\mathcal{O}\left(2^{2^{N+I}}\right)$
- With Mosaic: $\mathcal{O}\left(2^{N+2^I}\right)$

Overall:

$$\mathcal{O}\left(2^{M+2^{N+I}}\right) \text{ vs. } \mathcal{O}\left(2^{M+N+2^I}\right)$$

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Evaluation: Vertical Airborne Collision Avoidance

Prev. Adv.	Status	Time	CE regions	First CE
DNC	**safe**	0.35 h	—	—
DND	**safe**	0.28 h	—	—
DES1500	**unsafe**	5.45 h	49,428	0.04 h
CL1500	**unsafe**	5.18 h	34,658	0.08 h
SDES1500	**unsafe**	4.05 h	5,360	0.97 h
SCL1500	**unsafe**	4.89 h	11,323	0.36 h
SDES2500	**unsafe**	3.66 h	5,259	1.39 h
SCL2500	**unsafe**	4.45 h	7,846	0.53 h

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Evaluation: Comparison with SMT

Evaluated on two Adaptive Cruise Control properties (one satisfiable, one unsatisfiable).
NNs with 256 ReLU nodes.

Tool	`ACC_Large`		`ACC_Large` retrained	
	Status	Time	Status	Time
Mathematica	MO	—	MO	—
dReal	TO	—	TO	—
Z3	unknown	510s	unknown	1793s
Z3++	unknown	2550s	unknown	2269s
cvc5	TO	—	TO	—
MathSAT	TO	—	TO	—
ours	**sat**	**87s**	**unsat**	**124s**

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Evaluation: Comparison with Closed-Loop Techniques

Attempt to prove bounded safety on part of Adaptive Cruise Control NN:

Tool	Nonlinearities	Evaluated Configurations	Time (s)	Share of State Space	Result
NNV	no	4	711	0.009%	**safe for 0.1s**
JuliaReach	no	4	—	0.009%	unknown
CORA	yes	10	—	0.009%	unknown
POLAR	poly. Zono.	12	—	0.009%	unknown
ours	polynomial	1	**124**	100.000%	**safe for** ∞

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Infinte-time horizon: *k*-induction?

Conceptual comparison to NNV

- Attempted to show invariance w.r.t. nonlinear loop invariant
- Overapproximation **can lead to wrong results!**

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Tool | Nonlinearities | Evaluated Configurations | Time (s) | Share of State Space | Result
NNV | no | 4 | 711 | 0.009% | safe for 0.1s
JuliaReach | no | 4 | — | 0.009% | unknown
CORA | yes | 10 | — | 0.009% | unknown
POLAR | poly. Zono. | 12 | — | 0.009% | unknown
ours | polynomial | 1 | 124 | 100.000% | safe for ∞
Infinte-time horizon: k-induction?
Conceptual comparison to NNV
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Evaluation: Performance of Mosaic

Comparison to DNNV

- DNNV: No support for nonlinear properties
  Enumerates all **propositionally feasible assumptions**

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- Compare efficiency by counting

Property	# Conjunctions	**# Queries**	# Feasible Conjunctions	# SMT calls
ACC	2.4k	20	86	261
ACC (Fallback)	5.1k	15	72	235
ACAS (DNC)	117.5M	1.7k	9.9k	11.4k
ACAS (DND)	88.9M	1.8k	10.4k	12.0k
ACAS (DES1500)	451.3B	12.5k	58.8k	66.4k
ACAS (CLI1500)	374.4B	13.1k	62.5k	70.4k
ACAS (SDES1500)	9.1T	18.6k	64.1k	75.8k
ACAS (SCLI1500)	18.2T	21.8k	76.0k	88.5k
ACAS (SDES2500)	39.0T	19.0k	66.7k	78.5k
ACAS (SCLI2500)	19.4T	18.6k	67.7k	79.8k

References

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Comparison to DNNV
DNNV: No support for nonlinear properties
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Compare efficiency by counting
Property | # Conjunctions | # Queries | # Feasible Conjunctions | # SMT calls
ACC | 2.4k | 20 | 86 | 261
ACC (Fallback) | 5.1k | 15 | 72 | 235
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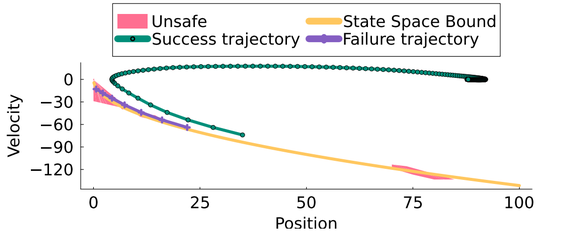
ACAS (DND) | 88.9M | 1.8k | 10.4k | 12.0k
ACAS (DES1500) | 451.3B | 12.5k | 58.8k | 66.4k
ACAS (CLI1500) | 374.4B | 13.1k | 62.5k | 70.4k
ACAS (SDES1500) | 9.1T | 18.6k | 64.1k | 75.8k
ACAS (SCLI1500) | 18.2T | 21.8k | 76.0k | 88.5k
ACAS (SDES2500) | 39.0T | 19.0k | 66.7k | 78.5k
ACAS (SCLI2500) | 19.4T | 18.6k | 67.7k | 79.8k

Evaluation: Adaptive Cruise Control

Verification Property

No crash ($p_{\text{rel}} > 0$)

- Verification on full state space
- Infinite Time-Horizon Safety Guarantee



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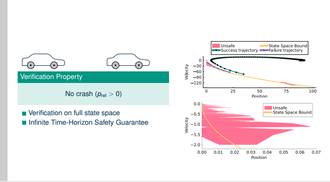
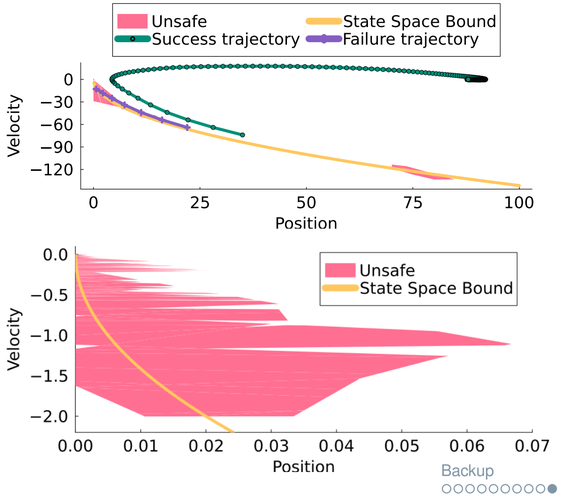


Evaluation: Adaptive Cruise Control

Verification Property

No crash ($p_{\text{rel}} > 0$)

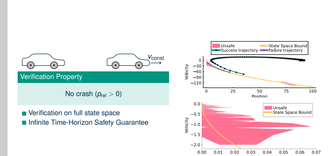
- Verification on full state space
- Infinite Time-Horizon Safety Guarantee



References

KIT

Backup

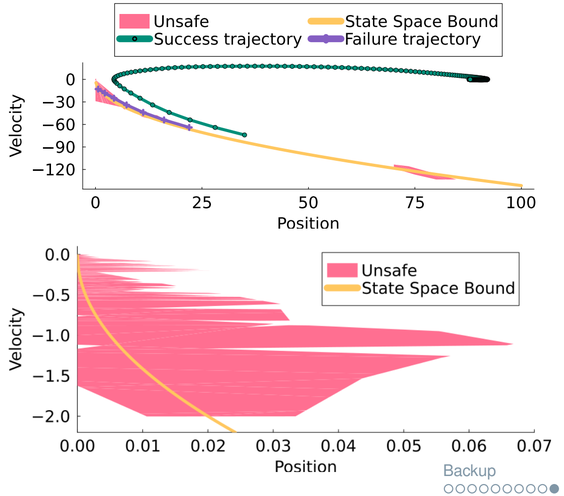


Evaluation: Adaptive Cruise Control

Verification Property

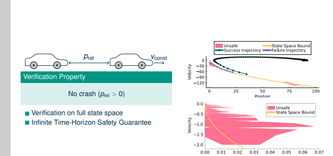
No crash ($p_{\text{rel}} > 0$)

- Verification on full state space
- Infinite Time-Horizon Safety Guarantee



References

KIT

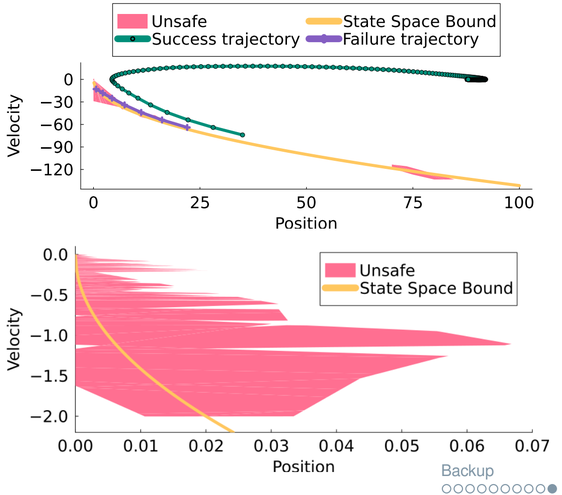


Evaluation: Adaptive Cruise Control

Verification Property

No crash ($p_{\text{rel}} > 0$)

- Verification on full state space
- Infinite Time-Horizon Safety Guarantee



References

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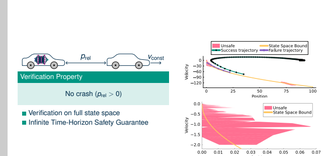
Backup

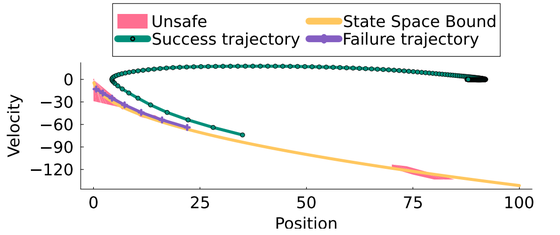


Evaluation: Adaptive Cruise Control

Verification Property

No crash ($p_{\text{rel}} > 0$)

- Verification on full state space
- Infinite Time-Horizon Safety Guarantee
- Some experiments on retraining with counterexample regions



References

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