Example

Exercise. *If $h : \mathbb{R}^n \to \mathbb{R}$ is convex, and if $\theta : \mathbb{R} \to \mathbb{R}$ is convex and increasing, then $f(x) = \theta(h(x))$ is convex. That is, for any $x, y \in \mathbb{R}^n$ and $\lambda \in [0, 1]$,*

$$(\theta \circ h)(\lambda x + (1 - \lambda)y) \leq \lambda(\theta \circ h)(x) + (1 - \lambda)(\theta \circ h)(y).$$

Proof. Let $h : \mathbb{R}^n \to \mathbb{R}$ be convex and $\theta : \mathbb{R} \to \mathbb{R}$ be convex and increasing.
Take $x, y \in \mathbb{R}^n, \lambda \in [0, 1]$. By convexity of h, we have

$$h(\lambda x + (1 - \lambda)y) \leq \lambda h(x) + (1 - \lambda)h(y),$$

and therefore:

$$\begin{aligned}
\theta[h(\lambda x + (1 - \lambda)y)] &\leq \theta[\lambda h(x) + (1 - \lambda)h(y)] && \text{by monotonicity of } \theta \\
&\leq \lambda\theta(h(x)) + (1 - \lambda)\theta(h(y)). && \text{by convexity of } \theta
\end{aligned}$$

□