# MAPS OF 1-COHOMOLOGIES

**Proposition 1.** *Let $f : K' \to K$, $K'$ an algebraic group also acting on $V$ by group automorphisms. Then $\alpha \circ f$ is a 1-cocycle from $K' \to V$.*

*Proof.* □

The map $\zeta : Z^1(K, V) \to Z^1(K', V)$ defined by

$$\zeta(\alpha) \quad = \quad \alpha \circ f$$

is a map of 1-cocycles.

**Proposition 2.** *Let $g : V \to V'$ be an equivariant homomorphism. Then $g \circ \alpha$ is a 1-cocycle from $K \to V'$.*

*Proof.* □

The map $\xi : Z^1(K, V) \to Z^1(K, V')$ defined by

$$\xi(\alpha) = g \circ \alpha$$

is a map of 1-cocycles. More generally, we have the following Definition.

**Definition 1.** *Define the map*

$$Z^1(\zeta, \xi) : Z^1(K, V) \to Z^1(K', V')$$

*by*

$$Z^1(\zeta, \xi)(\alpha) = g \circ \alpha \circ f.$$

Then, for example,

$Z^1(\zeta, \mathrm{id}) = \zeta$, where $\mathrm{id} : V \to V$ is the identity map, and

$Z^1(\mathrm{id}, \xi) = \xi$, where $\mathrm{id} : H \to H$ is the identity map.

**Proposition 3.** *The map $Z^1(\zeta, \xi)$ descends to give a well-defined map*

$$H^1(\zeta, \xi) : H^1(K, V) \to H^1(K', V')$$

*of 1-cohomologies, defined by*

$$H^1(\zeta, \xi)\left(\psi(\alpha)\right) = \left(\psi' \circ Z^1(\zeta, \xi)\right)(\alpha),$$

*where $\psi : Z^1(K, V) \to H^1(K, V)$ and $\psi' : Z^1(K', V') \to H^1(K, V)$ are the canonical projections. Moreover, the following diagram commutes.*

$$\begin{array}{ccc} Z^1(K,V) & \xrightarrow{Z^1(\zeta,\xi)} & Z^1(K',V') \\ \downarrow{\scriptstyle \psi} & & \downarrow{\scriptstyle \psi'} \\ H^1(K,V) & \xrightarrow{H^1(\zeta,\xi)} & H^1(K',V') \end{array}$$

*Proof.* □