COMMUTATIVE DIAGRAM

$$
\begin{array}{ccccccccc}
\cdots & \longrightarrow & \mathrm{Hom}(\Gamma,P)_\sigma & \xrightarrow{z} & Z^1(\Gamma,V)_\sigma & \xrightarrow{Z^1(\iota,\mathrm{id})} & Z^1(\Gamma',V)_{\sigma^\iota} & \xleftarrow{z^\iota} & \mathrm{Hom}(\Gamma',P)_{\sigma^\iota} & \longleftarrow & \cdots \\
 & & \downarrow{\scriptstyle \phi} & & \downarrow{\scriptstyle \psi} & & \downarrow{\scriptstyle \psi^\iota} & & \downarrow{\scriptstyle \phi^\iota} & & \\
\cdots & \longrightarrow & \mathrm{Hom}(\Gamma,P)_\sigma/V & \xrightarrow{h} & H^1(\Gamma,V)_\sigma & \xrightarrow{H^1(\iota,\mathrm{id})} & H^1(\Gamma',V)_{\sigma^\iota} & \xleftarrow{h^\iota} & \mathrm{Hom}(\Gamma',P)_{\sigma^\iota}/V & \longleftarrow & \cdots \\
 & & \downarrow{\scriptstyle \widetilde{\phi}} & & \downarrow{\scriptstyle \widetilde{\psi}} & & \downarrow{\scriptstyle \widetilde{\psi^\iota}} & & \downarrow{\scriptstyle \widetilde{\phi^\iota}} & & \\
\cdots & \longrightarrow & \mathrm{Hom}(\Gamma,P)_\sigma/VC_L(\sigma) & \xrightarrow{\widetilde{h}} & H^1(\Gamma,V)_\sigma/C_L(\sigma) & \xrightarrow{\widetilde{H^1}(\iota,\mathrm{id})} & H^1(\Gamma',V)_{\sigma^\iota}/C_L(\sigma) & \xleftarrow{\widetilde{h^\iota}} & \mathrm{Hom}(\Gamma',P)_{\sigma^\iota}/VC_L(\sigma) & \longleftarrow & \cdots
\end{array}
$$