KÜLSHAMMER NOTES

G, H linear algebraic groups over k, $\operatorname{char} k = p > 0$.
U maximal unipotent subgroup of H.

Question 1. *Let $\sigma\colon U \to G$ be a representation. Are there only finitely many conjugacy classes of representation $\rho\colon H \to G$ such that $\rho|_U$ is G-conjugate to σ?*

H	G	status
finite	non-reductive	counterexample [Cram]
—"—	GL_n	p doesn't divide $\lvert H\rvert \Rightarrow$ yes [Maschke's Theorem]
—"—	connected, reductive	p doesn't divide $\lvert H\rvert \Rightarrow$ yes [Richardson]
		– embed G in GL_n and study induced map $\operatorname{Hom}(H,G) \to \operatorname{Hom}(H,GL_n)$
finite	GL_n	yes [Representation Theory]
—"—	G	p is good for $G \Rightarrow$ yes [Slodowy]
		– uses Richardson's argument
finite	G_2	counterexample [BMR]
connected, semisimple	reductive (connected, semisimple)	yes [1.]
—"—	non-reductive	yes [2.]

Sketch of proof 1. □

Sketch of proof 2. □