AFEM: Axisymmetric Project Verification Tests

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Chapter 1

Introduction

This document compiles several types of closed form verification tests that can be compared against in the finite element solutions. It provides several examples and the closed form solutions for these examples.

Chapter 2

Example 1: Uniaxial Stress on Bar

This example performs a simple uniaxial stress test on an axisymmetric bar. An example of a mesh that could be applied to this problem

2.1 Closed form solution

Chapter 3

Example 2: Pressure Applied to Simply Supported Circular Plate

http://www.roymech.co.uk/Useful$_{T}ables/Mechanics/Plates.html$

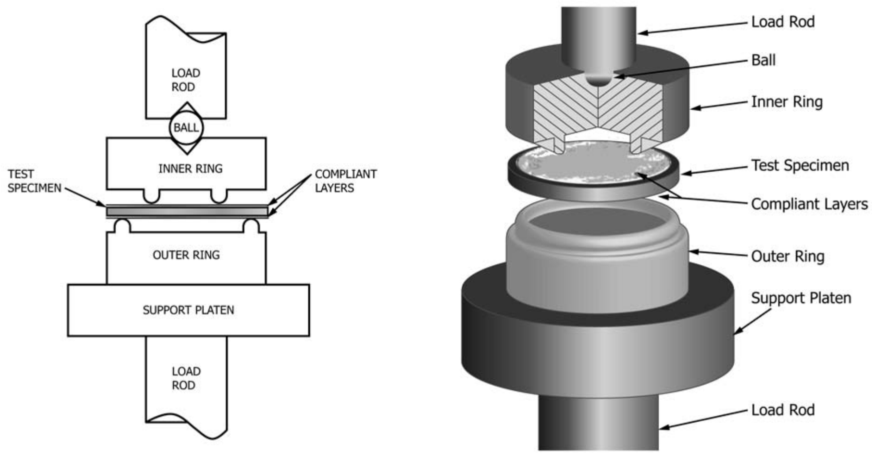


Figure 3.1:

3.1 Closed form solution

See Ansys Verification Problems p55

The equation for the displacement of the center of the plate is as follows:

$$\delta = \frac{3F(1-\nu^2)D_L^2}{8\pi E h^3}\left(\frac{D_S^2}{D_L^2}\left[1+\frac{(1-\nu)(D_s S^2 - D_L^2)}{2(1+\nu)D^2}\right] - \left(1+\ln\frac{D_s}{D_L}\right)\right) \tag{3.1}$$

Vitmar, F. F., and Pukh, V. P., "Method of Determining Sheet Glass Strength," Zavodskava Laboratoriya, Vol. 29, No. 7, 1963, pp. 863-867.

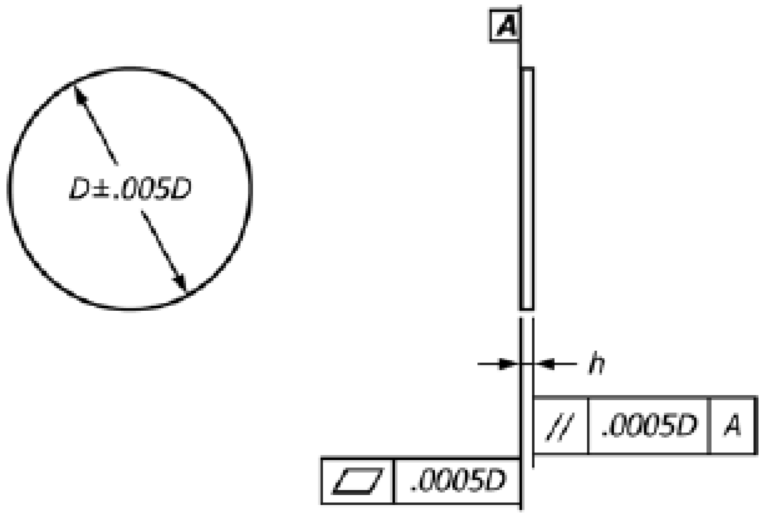
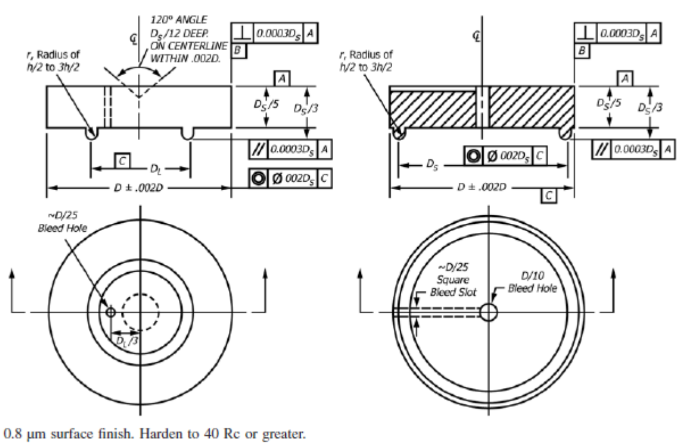


Figure 3.2:



NOTE 1—0.4 to 0.8 µm surface finish. Harden to 40 Rc or greater.

Figure 3.3:

Figure 15.1: Flat Circular Plate Problem Sketch

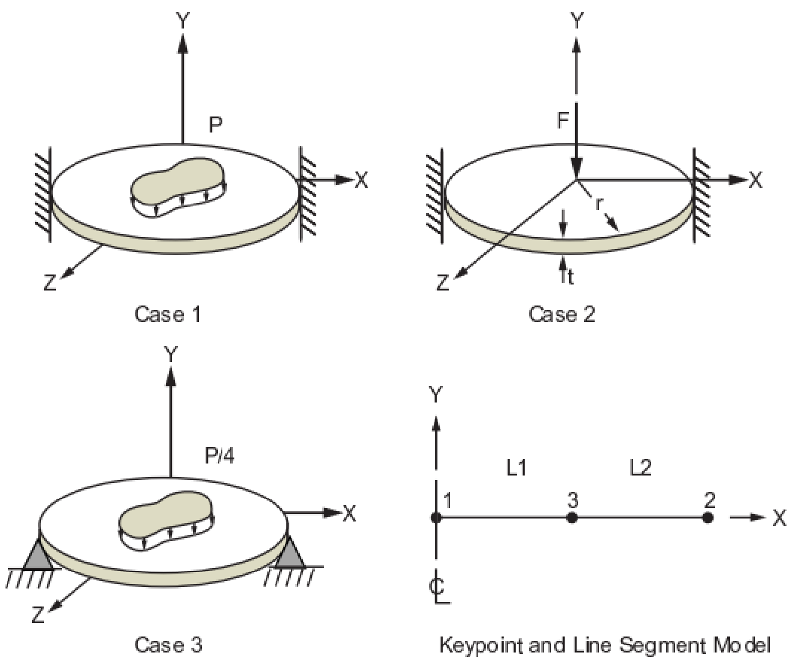


Figure 3.4:

Chapter 4

Example 3: Thick/thin walled pressure vessel

Abaqus verification 1.3.4

4.1 Closed form solution

The radial displacement of a thick walled pressure vessel at radius r is:

$$u(r) = \frac{1-\nu}{E}\frac{(r_i^2 p_i - r_o^2 p_o)r}{r_o^2 - r_i^2} + \frac{1+\nu}{E}\frac{(p_i - p_o)r_i^2 r_o^2}{(r_o^2 - r_i^2)r} \tag{4.1}$$

Chapter 5

Pressure vessel with hemispherical end-cap

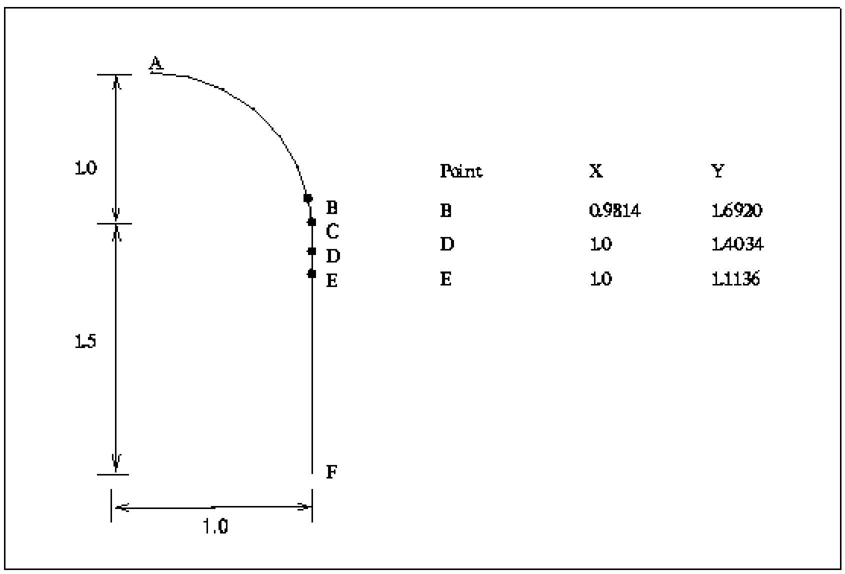


Figure 5.1:

Chapter 6

Belleville Washer

See Ansys Verification Problems p73.

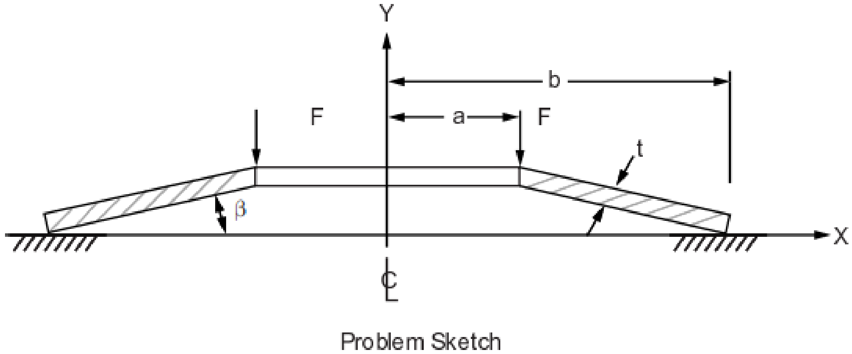


Figure 6.1:

Chapter 7

Method of manufactured solutions

Chapter 8

How do we construct the elements

we need to fix the pyfem2 element in a couple of places.

- Fix the B matrix so that we have the r term and shape function in the 3rd row of the B matrix. (Like we did in Homework 7)
- Fix the integrand of the K stiffness to include the r term $B^T EBrJ_\omega$
- Similarly fix the F term in the forcing function portion of the code.

The types of elements that we're going to focus on in this project is the reduced integration.
Secondary to those, we can optionally figure out the full integration.
Listed out the order of attack for the elements we develop could be:

- Full integration (maybe easiest to implement)
- Reduced integration (down to one gauss point)
- Selectively reduced integration ????
- Reduced integration with hourglass control ???

The key for our project is to get the testing framework up and running for several types of loading conditions and provide something for elements to be tested against. This testing framework is the most likely thing that Fuller will re-use in pyfem2. That's most likely what he wants.