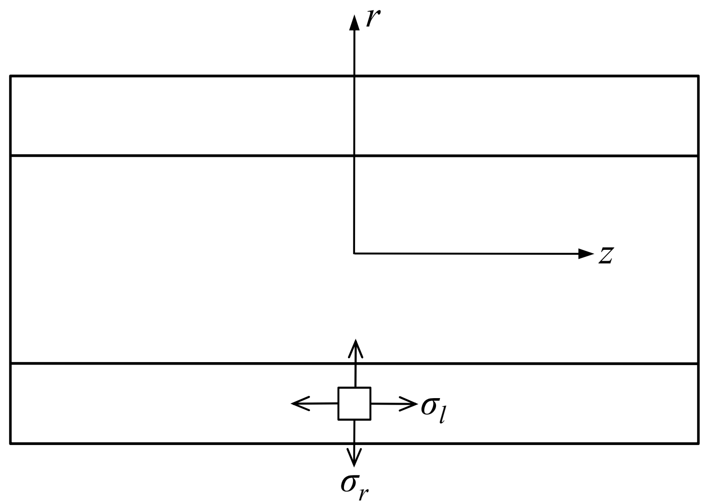
Thick-Walled Cylinders

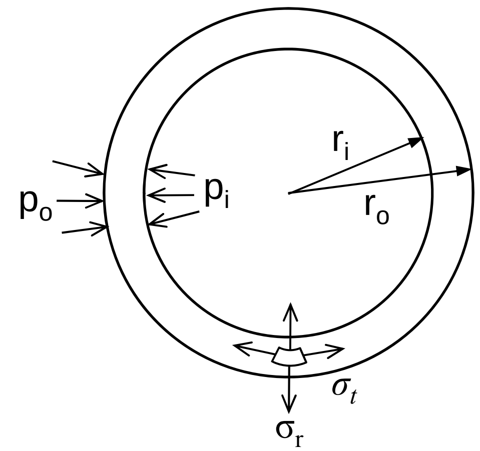
(Notes,3.14)

MAE 316 – Strength of Mechanical Components

Y. Zhu

Cylinders (3.14)



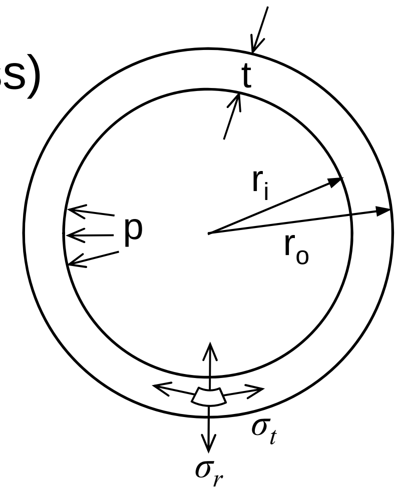


Applications

- $p_i = 0$
  - Submarine
  - Vacuum chamber
  - Shrink fit
  - Buried pipe
- $p_o = 0$
  - Gun barrel
  - Liquid- or gas-carrying pipe
  - Hydraulic cylinder
  - Gas storage tank

Thin-Walled Pressure Vessels (Review)

$$\sigma_t = \frac{pr_i}{t} \text{ (hoop stress)} \qquad \sigma_l = \frac{pr_i}{2t} \text{ (longitudinal stress)}$$



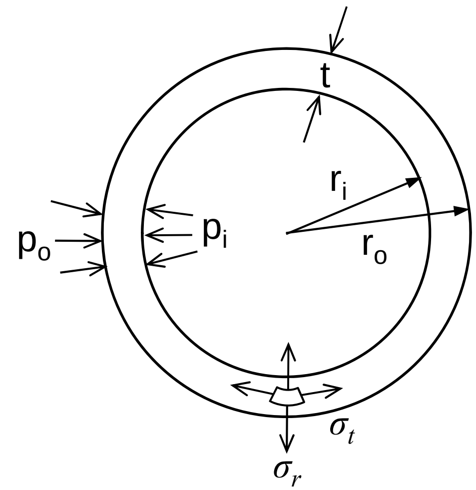
- For a thin-walled pressure vessel, $r_i/t > 10$, so "hoop" stress (σ_t) variation in the radial direction is minimal
- Radial stress (σ_r) is equal to *-p* on the inner surface, zero on the outer surface, and varies in between.
- σ_r is negligible compared to σ_t.

Thick-Walled Cylinders (3.14)

- For thick-walled pressure vessels

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$



- Maximum shear stress $\tau_{\max} = \frac{1}{2}(\sigma_t - \sigma_r)$

- If the ends of the cylinder are capped, must include longitudinal stress.

$$\sigma_l = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

Thick-Walled Cylinders

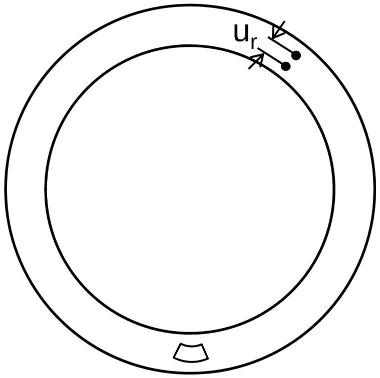
- Examples of <u>closed</u> cylinders include pressure vessels and submarines.
- Examples of <u>open</u> cylinders include gun barrels and shrink fits.

- Radial displacement of a thick-walled cylinder

$$u_r = \frac{1-\nu}{E}\frac{(r_i^2 p_i - r_o^2 p_o)r}{r_o^2 - r_i^2} + \frac{1+\nu}{E}\frac{(p_i - p_o)r_i^2 r_o^2}{(r_o^2 - r_i^2)r}$$

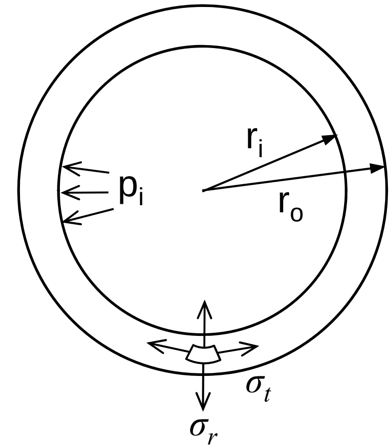
$\mathrm{E} = \text{Young's modulus}$

$\nu = \text{Poisson's ratio}$



Thick-Walled Cylinders (3.14)

- Special case: Internal pressure only (p_o = 0)



$$\boxed{\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) \quad \& \quad \sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)}$$

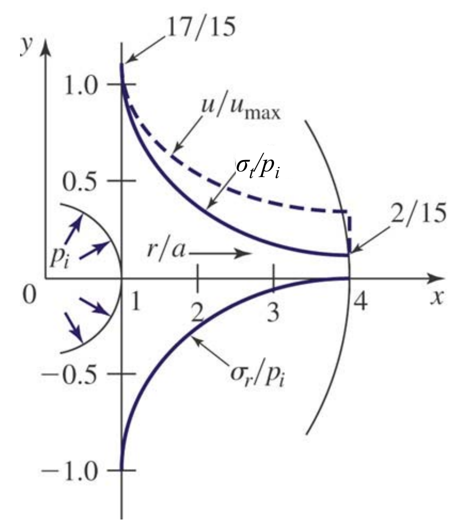
$$(\sigma_r)_{\max} = -p_i \quad @ \quad r = r_i$$

$$(\sigma_t)_{\max} = p_i \frac{(r_i^2 + r_o^2)}{(r_o^2 - r_i^2)} \quad @ \quad r = r_i$$

$$\boxed{u_r = \frac{p_i r_i^2 r}{E(r_o^2 - r_i^2)}\left[(1-\nu) + (1+\nu)\frac{r_o^2}{r^2}\right]}$$

$$(u_r)_{r=r_i} = \frac{p_i r_i}{E}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + \nu\right)$$

$$(u_r)_{r=r_o} = \frac{2 p_i r_i^2 r_o}{E(r_o^2 - r_i^2)}$$



Thick-Walled Cylinders

- Compare previous result with thin-walled pressure vessel case (p_o = 0)

$$\sigma_t = p_i \frac{(r_i^2 + r_o^2)}{(r_o^2 - r_i^2)} \quad @ \quad r = r_i$$

$$r_o = r_i + t$$

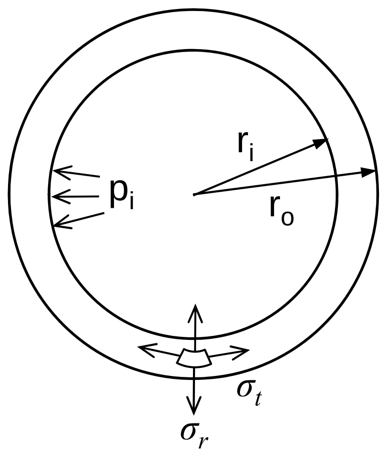
$$t = r_o - r_i$$

$$\sigma_t = p_i \frac{(r_i^2 + r_o^2)}{(r_o + r_i)(r_o - r_i)} = p_i \frac{\left[r_i^2 + (r_i + t)^2\right]}{(r_i + t + r_i)t}$$

$$\text{for t} << \text{r}_\text{i} \ \ \text{(thin} - \text{walled)}$$

$$\sigma_t = p_i \frac{\left[r_i^2 + r_i^2\right]}{(2r_i)t} = p_i \frac{2r_i^2}{(2r_i)t}$$

$$\sigma_t = \frac{p_i r_i}{t} \quad \text{(inside)}$$



Thick-Walled Cylinders

- Continued…

$$\sigma_t = \frac{2p_i r_i^2}{(r_o^2 - r_i^2)} \quad @ \quad r = r_o$$

$$r_o = r_i + t$$

$$t = r_o - r_i$$

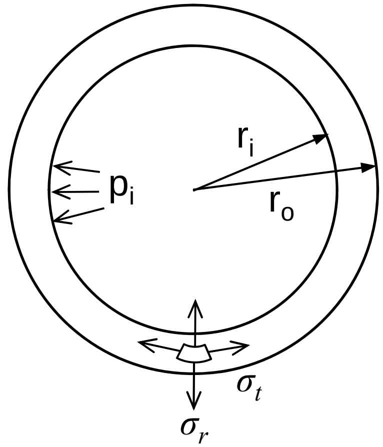
$$\sigma_t = \frac{2p_i r_i^2}{(r_o + r_i)(r_o - r_i)} = \frac{2p_i r_i^2}{(r_i + t + r_i)t}$$

$$\text{for t} << \text{r}_\text{i} \ \text{(thin} - \text{walled)}$$

$$\sigma_t = \frac{2p_i r_i^2}{(2r_i)t}$$

$$\sigma_t = \frac{p_i r_i}{t} \quad \text{(outside)}$$

$$\therefore \ \sigma_t \ \text{same on inside and outside}$$



Thick-Walled Cylinders

- Special case: External pressure only (p_i = 0)

$$\boxed{\sigma_r = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right) \;\; \& \;\; \sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)}$$

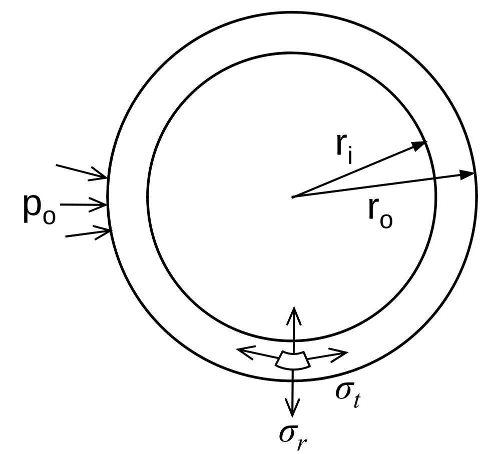
$$(\sigma_r)_{\max} = -p_o \quad @ \quad r = r_o$$

$$(\sigma_t)_{\max} = -\frac{2 p_o r_o^2}{(r_o^2 - r_i^2)} \quad @ \quad r = r_i$$

$$\boxed{u_r = -\frac{p_o r_o^2 r}{E(r_o^2 - r_i^2)}\left[(1-\nu) + (1+\nu)\frac{r_i^2}{r^2}\right]}$$

$$(u_r)_{r=r_i} = -\frac{2 p_o r_o^2 r_i^2}{E(r_o^2 - r_i^2)}$$

$$(u_r)_{r=r_o} = -\frac{p_o r_o}{E}\left(\frac{(r_o^2 + r_i^2)}{(r_o^2 - r_i^2)} - \nu\right)$$



Example 1

Find the tangential, radial, and longitudinal stress for a pipe with an outer diameter of 5 inches, wall thickness of 0.5 inches, and internal pressure of 4000 psi.

Example 2

Find the maximum allowable internal pressure for a pipe with outer radius of 3 inches and wall thickness of 0.25 inches if the maximum allowable shear stress is 4000 psi.

Press and Shrink Fits

(3.16)

MAE 316 – Strength of Mechanical Components

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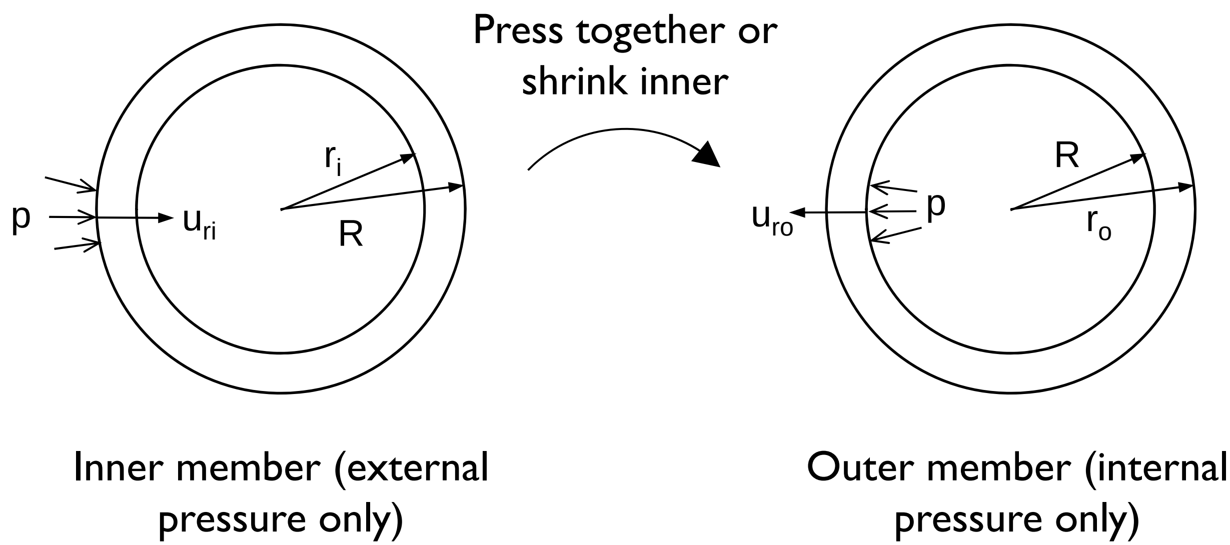
Press and Shrink Fits (3.16)



- Assume inner member has slightly larger outer radius than inner radius of outer member.
- *R* is the shared radius between the two pieces before they are pressed together.
- Interference pressure will develop upon assembly.

Press and Shrink Fits (3.16)

$$u_{ri} = -\frac{pR}{E_i}\left[\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right] \text{ (inner)}$$

$$u_{ro} = \frac{pR}{E_o}\left[\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right] \text{ (outer)}$$

For compatibility

$$|u_{ro}| + |u_{ri}| = \delta$$

$$\delta = pR\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]$$

- Once δ is known we can calculate p, or vice versa.
- Typically, δ is very small, approximately 0.001 in. or less.

Press and Shrink Fits (3.16)

- If the materials are the same:
  - $E = E_i = E_o$
  - $\upsilon = \upsilon_i = \upsilon_o$

$$\delta = \frac{2pR^3}{E}\left[\frac{(r_o^{\,2} - r_i^{\,2})}{(r_o^{\,2} - R^2)(R^2 - r_i^{\,2})}\right]$$

- If the inner member is not hollow, $r_i = 0$.

$$\delta = \frac{2pR}{E}\left[\frac{r_o^{\,2}}{(r_o^{\,2} - R^2)}\right]$$

Example 3

A solid shaft is to be press fit into a gear hub. Find the maximum stresses in the shaft and the hub. Both are made of carbon steel ($E = 30\times10^6$ psi, $\nu = 0.3$).

- Solid shaft
  - $r_i = 0$ in, $R = 0.5$ in. (nominal)
  - Tolerances: $+2.3\times10^{-3}/+1.8\times10^{-3}$ in.
- Gear hub
  - $R = 0.5$ in. (nominal), $r_o = 1$ in
  - Tolerances: $+0.8\times10^{-3}/0$ in.

Example 4

A bronze bushing 50 mm in outer diameter and 30 mm in inner diameter is to be pressed into a hollow steel cylinder of 100 mm outer diameter. Determine the tangential stresses for the steel and bronze at the boundary between the two parts.

- E_b = 105 Gpa
- E_s = 210 Gpa
- ν = 0.5
- radial interference δ = 0.025 mm