MAIN ARTICLE

A co-flow structure for goal-directed internal change

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Abstract

We describe a co-flow structure that models internal, goal-directed changes to an attribute (e.g., employee loyalty) of fundamental material (e.g., employees). This co-flow accommodates problems not adequately modeled with an existing, generic structure. Our structure builds on the co-flow proposed by Hines, which uses an information delay to model external change to an attribute. We use a first-order information delay to model both external changes to the attribute from the material stock and internal changes from an internal goal for the attribute. We provide an exact, dynamic solution for this co-flow enabling us to precisely describe its equilibrium and non-equilibrium behavior. Several examples are provided and discussed, including a situation where a management program is designed to increase average employee loyalty. In addition, we review applications of traditional and Hines co-flow structures to provide background and to describe our evolutionary path towards design of the new co-flow.

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Additional Supporting Information may be found online in the supporting information tab for this article.

Introduction

System dynamics problems often examine situations where one or more attributes or characteristics are tracked in relation to a given material flow. For example, when modeling changes to the labor force, the average work experience of the employees may be important to include in the model (Sterman, 2000, p. 469). In addition to reflecting the work experience of new hires entering the labor force and losses in work experience due to attrition, work experience can also increase for every year the workers stay in the labor force through experience gained on the job or when management creates specific programs to train them. In each of these examples, we can use a co-flow

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structure to model the attribute. The co-flow relates two stocks: the "fundamental material" stock and its associated attribute stock.

The co-flow is an important component of many modeling projects. It is referred to as one of seven essential building blocks, or generic structures (i.e., external resource production, compounding, draining, stock adjustment, implicit goal-seeking/smoothing, acquisition, and co-flow; Paich, 1985; Long, 1990; Saeed, 1989). The co-flow makes it possible to track one or more attributes of a given material or stock. It has been used to simulate psychological or behavioral attributes of people, including workers in an organization (Cooke, 2003; Rahmandad and Hu, 2010), patients living with a given disease (Fallah-Fini *et al.*, 2013), students learning in schools and organizations (Davidsen *et al.*, 1993; Hines and House, 2001), consumers driving economic growth (Oliva *et al.*, 2003), and soldiers and insurgents at war (Anderson, 2011). The co-flow is also used to deal with problems in manufacturing and product development (Ford and Sterman, 1998; Homer, 1999), the use of non-renewable resources (Wils, 1998), fishery management (Dudley, 2008), project management (Repenning, 2000; Lyneis and Ford, 2007; Lee and Peña-Mora, 2007), and ecosystems (Arquitt and Johnstone, 2004; Fiddaman, 2007).

In this paper, we discuss three types of simulation problems that use a co-flow structure. The first two types have well-known structures, each with two alternative formulations (Hines, 2005; Sterman, 2000; Hu and Keller, 2009): traditional and Hines co-flows, and traditional and Hines co-flows with experience, which we refer to as Type I and Type II, respectively. The third problem is undocumented in the system dynamics literature. To better understand it and its relationship to the other co-flows, we need to distinguish between external and internal changes *to the attribute*. We refer to changes to the attribute due solely to flows into and out of the fundamental stock as external changes; any other changes are defined as internal changes to the attribute. The third problem models both external changes to an attribute from the fundamental material stock and internal changes to the attribute from an internal goal for the attribute. We refer to this problem as the "attribute with goal-directed, internal change" co-flow, or Type III co-flow.

Below we provide an in-depth description of each of these co-flow problems along with their associated model structures and dynamic behavior. We discuss Types I and II co-flows to provide the background and an evolutionary path to the Type III co-flow.

Type I Co-flow: Modeling an Attribute with No Internal Changes

Type I: problem example

Suppose a company continually borrows money at different interest rates and pays off some of its past loans (Hines, 2005). Assume that, once the company

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borrows money at a given interest rate, the interest rate remains fixed until the loan is paid off. If the average interest rate of new loans is known, how can the firm track the interest rate relative to the outstanding debt? Here the outstanding debt is the fundamental material stock. There are only external changes to the attribute stock since its changes only depend on inflows and outflows from the outstanding debt stock. How the interest rate attribute is modeled depends on the modeling approach.

Type I: model structure

When attribute changes come only from sources external to the attribute, such as in response to changes in the fundamental material stock, then one can use the traditional (Warren, 2008[1]) or Hines co-flow (Hines, 1983, 2005; Sterman, 2000). The traditional co-flow (Figure 1a; Table A-1 in Appendix A, supporting information) models the external change to the attribute as a stock (i.e., total attribute) with flows into and out of it. For example, the fundamental material could be "outstanding loans" and the total attribute "loan risk" (McDonald and Dowling, 1993).

The Hines co-flow (Figure 1b; Table A-1 in Appendix A, supporting information), on the other hand, takes a different approach. Although the traditional co-flow has a stock for the total value of the attribute, the fundamental material stock is directly associated with the average value of the attribute and not to the total attribute. Although both the traditional and Hines co-flow include "average attribute", in the traditional co-flow it is treated as an auxiliary variable, whereas in the Hines co-flow it is treated as a stock, and they are calculated differently (see Table A-1, supporting information). In the Hines co-flow, there is no separate outflow from the attribute. Instead, the Hines co-flow uses a first-order information delay to model the bidirectional, external change to the average attribute.

Type I: model dynamics

To illustrate the basic dynamic behavior of Type I co-flows, we made two simulation runs for each co-flow, shown in Figure 1, by first setting the initial values of the stocks to their equilibrium values (Run 1), and then changing their initial values (Run 2).

Table 1 lists the equations for the initial conditions of the stocks that are necessary and sufficient for each stock to be in equilibrium (e.g., constant for all time). It also includes equilibrium values when we set "rate of inflow

[1]Warren devotes all of Chapter 5 to a discussion of resource attributes. He uses the Type I traditional flow to model these attributes; however, he uses an optional, additional "attribute draining process" (see his Worksheet 6a and 6b, pp. 313–317).

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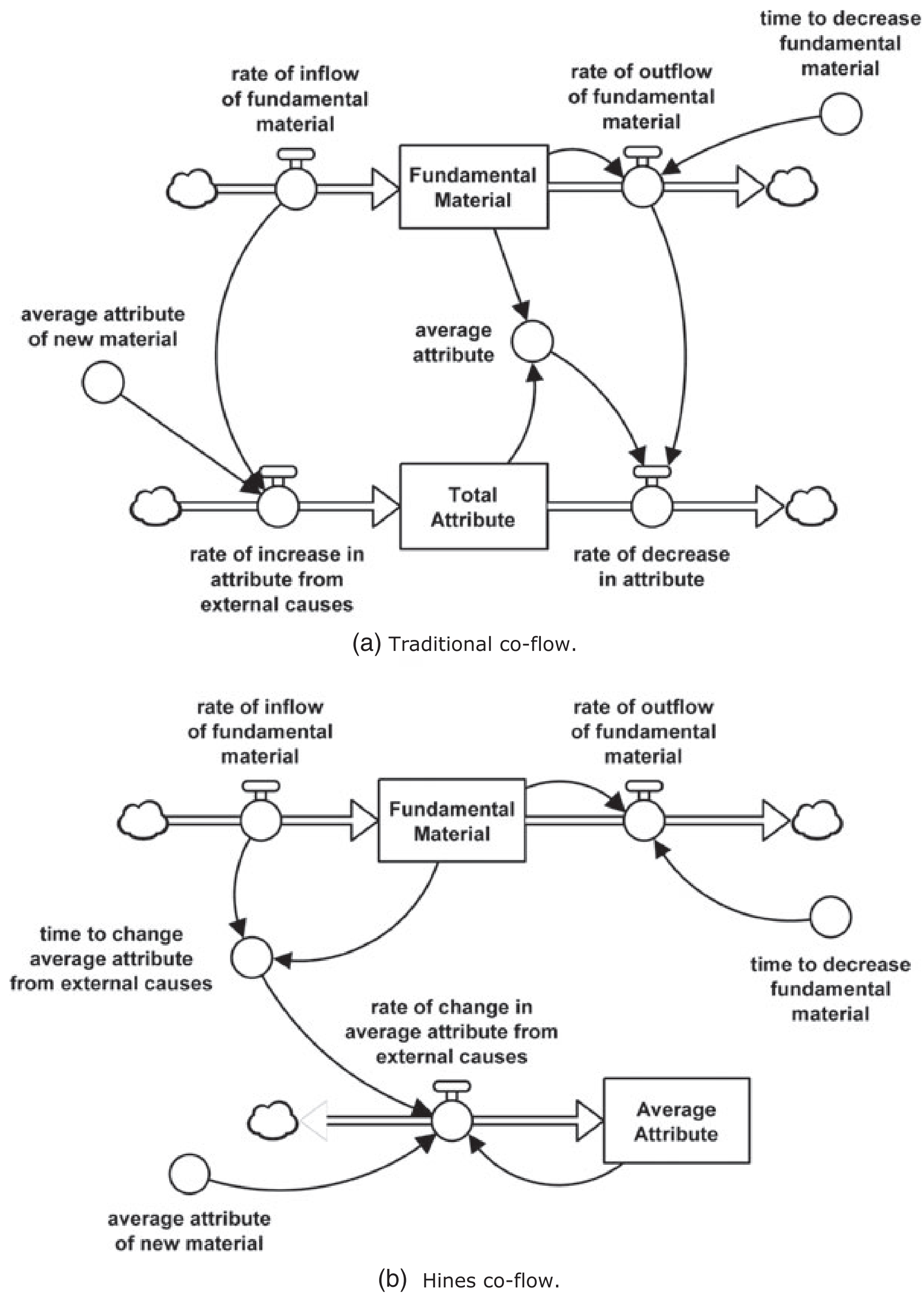

Fig. 1. (a) Traditional co-flow. (b) Hines co-flow

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Table 1. Initial conditions and values for the traditional and Hines co-flows (Type I)

Initial value of stock	Initial fundamental material (traditional and Hines)	Initial total attribute (traditional)	Initial average attribute (Hines)
Type I: traditional	Time to decrease fundamental material × Rate of inflow of fundamental material (material) = 500	Rate of increase in attribute from external causes × Time to decrease fundamental material (attribute) = 25,000	n/a
Type I: Hines	Same as Type I: traditional	n/a	Average attribute of new material (attribute/material) = 50

of fundamental material" = 50 material/months, "average attribute of new material" = 50 attribute/material, and "time to decrease fundamental material" = 10 months.

Note in Table 1 that the condition for the initial value of the total attribute for the traditional co-flow and the condition for the initial average attribute in the Hines co-flow do not depend on the value of the fundamental material. This means that if the initial value of the fundamental material is not its equilibrium value and the initial value of the total attribute and average attribute are set to their equilibrium values for their respective co-flows, the latter two stocks will remain in equilibrium.

Figures 2–4 illustrate the simulation runs and Table 2 summarizes the results. In the table, "global equilibrium" means that both stocks in each co-flow are in equilibrium at the same time.

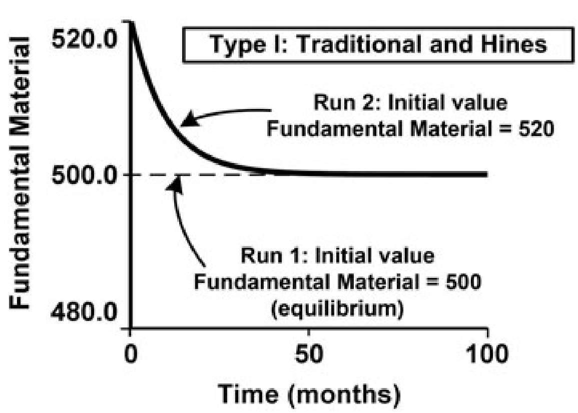

Fig. 2. Plots of fundamental material for run 1 and 2 for Type I co-flows

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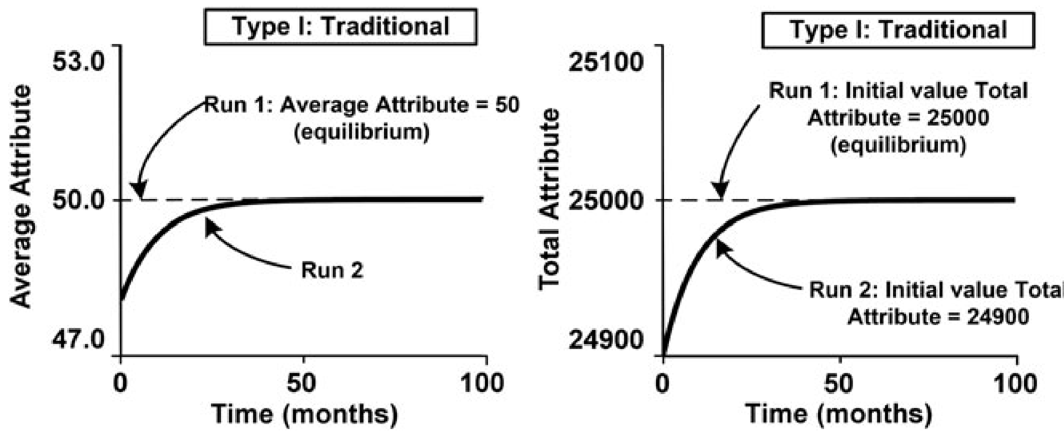
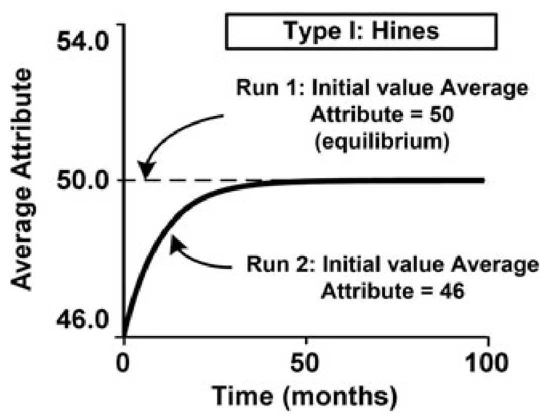


Fig. 3. Plots of average attribute and total attribute for run 1 and 2 for Type I traditional co-flow

Fig. 4. Plots of average attribute for run 1 and 2 for Type I Hines co-flow



Type II co-flow: modeling an attribute with a constant rate of internal change

Type II: problem example

Suppose a firm is interested in the average work experience of its employees and how that experience changes as employees enter and leave the firm. The firm defines work experience as the experience the employee brings to the firm (i.e., external changes to work experience) and the years of experience[2] each employee gains just by spending time in the firm (i.e., internal

[2]Although in most cases the years of experience accrued in a given time period will be the same as the number of years in that time period, in some situations this may not be the case. For example, a training program may confer more than one year of experience per year of training. Thus, for the co-flow with experience, the units for the rate of gain in total attribute from internal causes are years of experience per year.

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Table 2. Summary of behavior of Type I co-flows

	Stocks	Global equilibrium	Individual stock equilibrium	Away from equilibrium
Type I: traditional	Fundamental material Total attribute	Both stocks can be in equilibrium at the same time and remain in equilibrium if they start in equilibrium. If the stocks start in equilibrium, the average attribute is also in equilibrium since it equals the ratio of the two stocks.	Each stock can be in equilibrium if the other is not	Each stock that does not start in equilibrium approaches its equilibrium value as time goes to infinity. The average attribute is not in equilibrium if either stock is not in equilibrium.
Type I: Hines	Fundamental material Average attribute	Both stocks can be in equilibrium at the same time and remain in equilibrium if they start in equilibrium	Each stock can be in equilibrium if the other is not	Each stock that does not start in equilibrium approaches its equilibrium value as time goes to infinity

changes to work experience). If the “average experience of new hires” and “rate of experience gain from being in the company” are known and constant, how can the firm track work experience? Here the number of employees is the fundamental material stock. How the work experience attribute is modeled depends on the modeling approach.

Type II: model structure

When there is internal change in the attribute and this change occurs at a steady or constant rate, one can use the “co-flow with experience” or “Hines co-flow with experience” (Hines, 2005). To model the steady rate of internal change in the attribute, each form of the co-flow adds a constant or steady state flow to the attribute stock representing the rate of internal change of the attribute (Figure 5; Table A-2 in Appendix A, supporting information). Both formulations have a term for the “rate of increase in the average attribute from internal causes”. In the traditional co-flow with experience, this variable does not flow directly into the average attribute variable but instead is first converted to the rate of increase in the total attribute. In the Hines co-flow with experience, this rate flows directly into the average attribute variable. The “time to change average attribute from external causes” is a dilution time related to the time it takes the “average attribute of new material” to dilute the average attribute.

Type II: model dynamics

As we did for Type I co-flows, we generated two simulation runs for each co-flow in Figure 5 by first setting the initial values of the stocks to their

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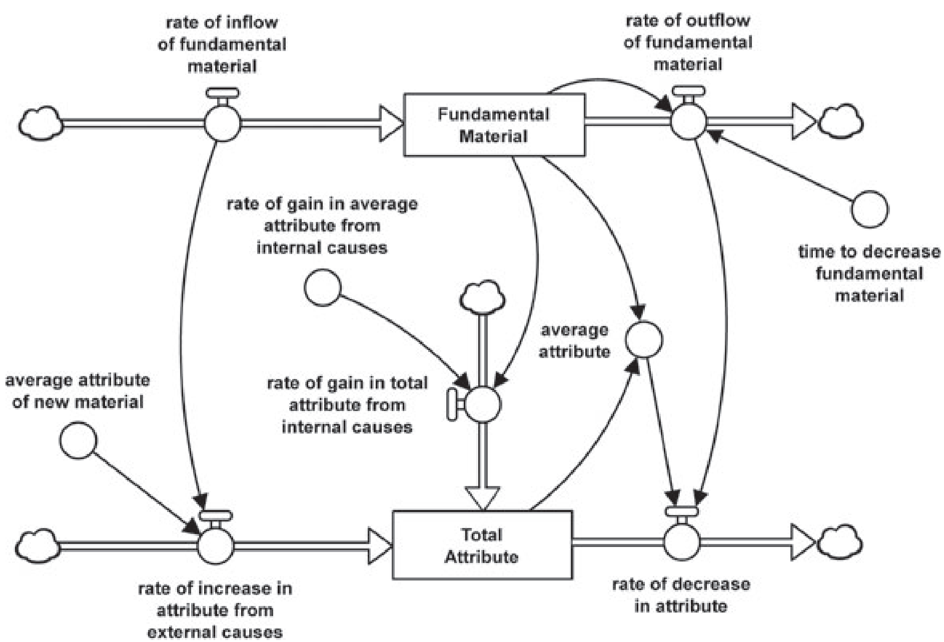
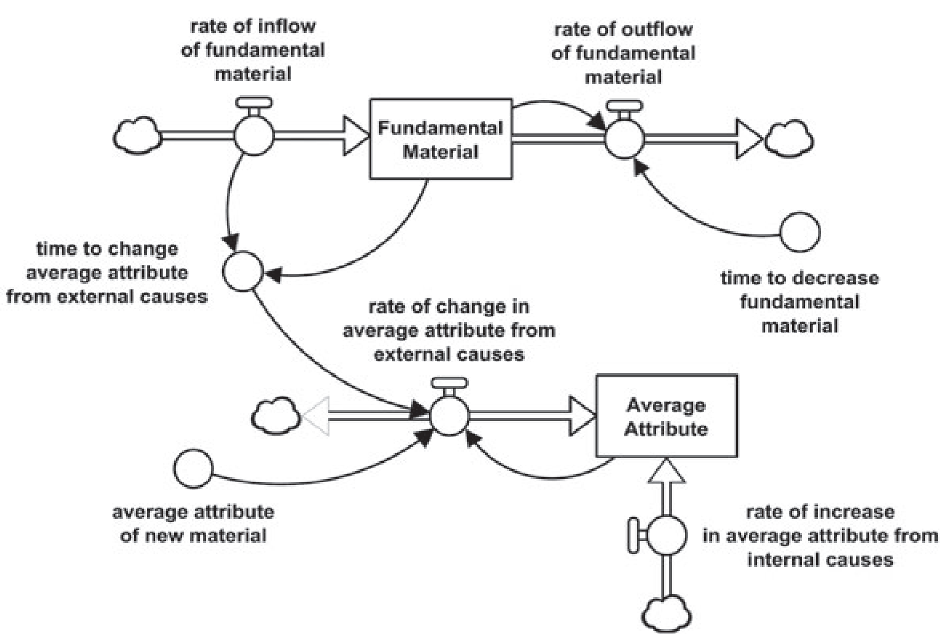


(a) Traditional co-flow with experience.



(b) Hines co-flow with experience.

Fig. 5. (a) Traditional co-flow with experience. (b) Hines co-flow with experience

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equilibrium values (Run 1), and then changing the initial values (Run 2). Table 3 lists the equations for the initial conditions of the stocks that are necessary and sufficient for each stock to be in equilibrium (e.g., constant for all time). We set the same values for "rate of inflow of fundamental material", "average attribute of new material", and "time to decrease fundamental material" as in the Type I simulations. In addition, we set "rate of gain in average attribute from internal causes" = 1.1 attribute/(months × material).

Note in Table 3 that the condition for the initial value of the total attribute for the traditional co-flow and the condition for the initial average attribute in the Hines co-flow, unlike Type I co-flows, do depend on the value of the fundamental material. This means that if the initial value of the fundamental material is not its equilibrium value and the initial value of the total attribute and average attribute are set to their equilibrium values for their respective co-flows, the latter two stocks will not be in equilibrium.

The plots of the fundamental material in Figure 2 for Type I co-flows are the same for Type II co-flows. Figures 6 and 7 illustrate the simulation runs and Table 4 summarizes the results.

When the Type I co-flow is in equilibrium, the average attribute for the traditional co-flow is equal to the average attribute for the Hines co-flow. Similarly, when Type II co-flow is in equilibrium, the average attribute for the traditional co-flow is equal to the average attribute for the Hines co-flow.

Table 3. Initial conditions and values for the traditional and Hines co-flows (Type II)

Initial value of stock	Initial fundamental material	Initial total attribute	Initial average attribute
Type II: traditional	Time to decrease fundamental material × rate of inflow of fundamental material (material) = 500 (same as Type I: traditional)	Time to decrease fundamental material × (Fundamental material × Rate of gain in average attribute from internal causes + Rate of increase in attribute from external causes) (attribute) = 30,500	n/a
Type II: Hines	Same as Type II: traditional	n/a	((Rate of inflow of fundamental material × Average attribute of new material + Fundamental material × Rate of increase in average attribute from internal causes)/Rate of inflow of fundamental material) (attribute/material) = 61

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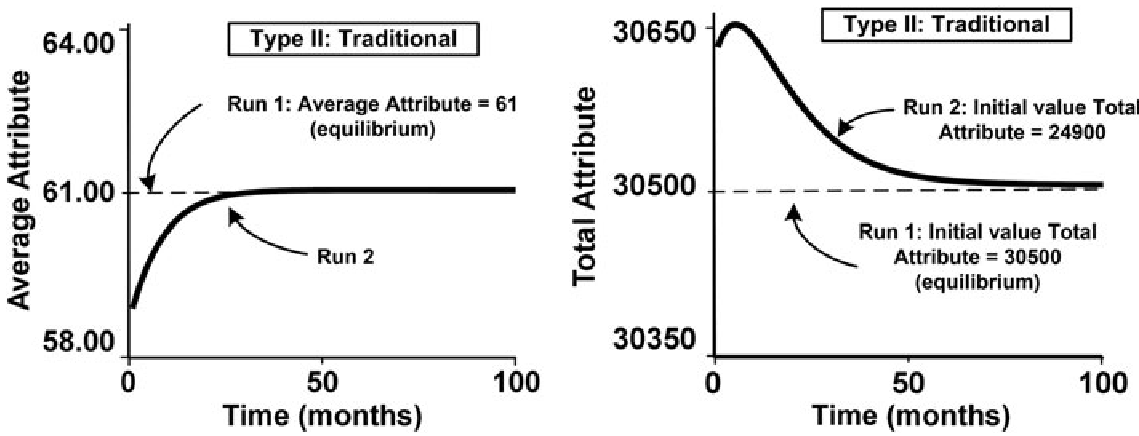
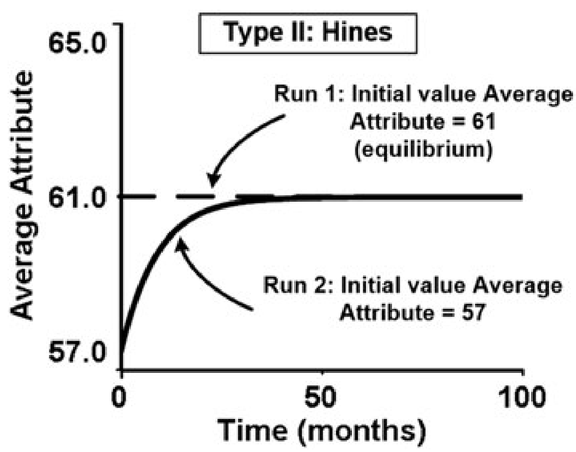


Fig. 6. Plots of average attribute and total attribute for run 1 and 2 for Type II co-flow

Fig. 7. Plot of average attribute for runs 1 and 2 for Type II Hines co-flow



Type III co-flow: modeling an attribute with goal-directed internal change

Type III: problem example

Suppose a government agency is interested in the loyalty of its employees and how that loyalty changes as employees enter and leave the agency. The firm defines loyalty as the loyalty the employees bring to the agency and the loyalty the agency instills in its employees through internal programs whose goal is to raise loyalty to a particular level. You might wonder how anyone could be loyal to a government agency before they work there. There could be many

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Table 4. Summary of behavior of Type II co-flows

	Stocks	Global equilibrium	Individual stock equilibrium	Away from equilibrium
Type II: traditional	Fundamental material Total attribute	Both stocks can be in equilibrium at the same time and remain in equilibrium if they start in equilibrium. If the stocks start in equilibrium, the average attribute is also in equilibrium since it equals the ratio of the two stocks.	Total attribute stock cannot be in equilibrium unless the fundamental stock is in equilibrium	Each stock that does not start in equilibrium approaches its equilibrium value as time goes to infinity. The average attribute is not in equilibrium if either stock is not in equilibrium.
Type II: Hines	Fundamental material Average attribute	Both stocks can be in equilibrium at the same time and remain in equilibrium if they start in equilibrium	Average attribute cannot be in equilibrium unless the fundamental stock is in equilibrium	Each stock that does not start in equilibrium approaches its equilibrium value as time goes to infinity

reasons. An employee's family may have happily worked there, or the employee may have seen the agency, like the U.S. Secret Service or U.S. Federal Bureau of Investigation (FBI), as fulfilling his or her life's passion. However, the realities of government life may diminish that initial loyalty, and the agency may find it necessary to create programs to encourage loyalty and a sense of esprit de corps. If the "average loyalty of hires" and the internal loyalty goal of the agency are known and constant, how can the agency track loyalty in the model.? Here the number of employees is the fundamental material stock. To cover this situation and similar ones, this paper describes using an information delay in a co-flow structure to model internal changes to attributes that move toward a goal (e.g., a desired level of loyalty).

Type III: rationale and model structure

Some internal change in an attribute is best modeled by goal-directed behavior using negative feedback. As an example, consider the need to manage the levels of depression among psychiatric patients at an inpatient clinic. The fundamental material is patients, who enter and leave the clinic. The attribute modeled is the patients' average level of depression. There is an internal "program" providing treatment to target the patients' depression. The program's "goal" is to bring the average level of depression of patients down to zero. Here, the rate of reduction in average depression does not arise from the flow of patients into and out of the clinic. And, it does not take the

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form of a linear increase. Thus the existing mechanisms to model change in an attribute (i.e., Type I and II) are not appropriate to model this kind of change, and an alternative approach is needed.

Another example of an internal change not adequately modeled by existing mechanisms is people's beliefs about cars and climate change. Here, the fundamental material stock is people seeking to purchase a new car. The average attribute is their average belief about the effect of car choices on climate change. Political and advocacy groups seek to influence these beliefs (i.e., the program or policy) and have a desired average belief (i.e., the goal). The rate of change in the actual average belief espoused by car buyers may be influenced by political and advocacy groups (i.e., negative feedback). Some attributes, such as levels of depression, beliefs about cars and climate change, or other psychological states, may adjust in response to negative feedback.

In previous work, information delays have been used to model changes to social and psychological "soft"[3] variables such as perceptions, attitudes, and dissonance (Hunter *et al.*, 1976). Here, we propose using an information delay to model internal change to an attribute as it moves toward a goal because the attribute is typically associated with a non-conserved flow (e.g., a flow into a psychological variable). If your anger is modeled as a stock, a flow into it is a "non-conserved flow" because no one loses anger if you get angry.

In the co-flow with internal change, the goal can be a desired level, such as the level of a particular belief that management desires employees to endorse. The employees' average belief adjusts toward the goal. The smoothing closes the gap between the goal and the value of the average attribute. Thus, in the co-flow with internal change, we replace the inflow found in the "Hines co-flow with experience" (Type II) with an information delay, which incorporates both a goal and negative feedback (see Figure 8; Table A-3 in Appendix A, supporting information). By including this smoothing, we add to the subtlety and variety of social processes that can be integrated into system dynamics models. For example, we might want to include a variable like quality of experience as an attribute, which is qualitatively different from the concept of experience used in Type II models.

We based our approach on Hines' Type I structure, which represents the attribute stock as an "average value" as opposed to a "total quantity". The attribute is typically a soft variable and Hines' Type I structure handles soft variables better (Levine, 2000; Levine and Nguyen, 2000) than the traditional Type I co-flow. For example, the range of a soft variable usually needs to be limited, for example, from 0 to 100 (Gambardella and Lounsbury, 2017). Often, a modeler using a traditional co-flow process needs to employ techniques to maintain this range. Yang *et al.* (2017) modeled General Electric's (GE) innovative process using a traditional Type I co-flow structure as part of a

[3]We use the term "soft variable" principally because it is connected with social sciences, not because it is intrinsically unmeasurable (see Gambardella and Lounsbury, 2017).

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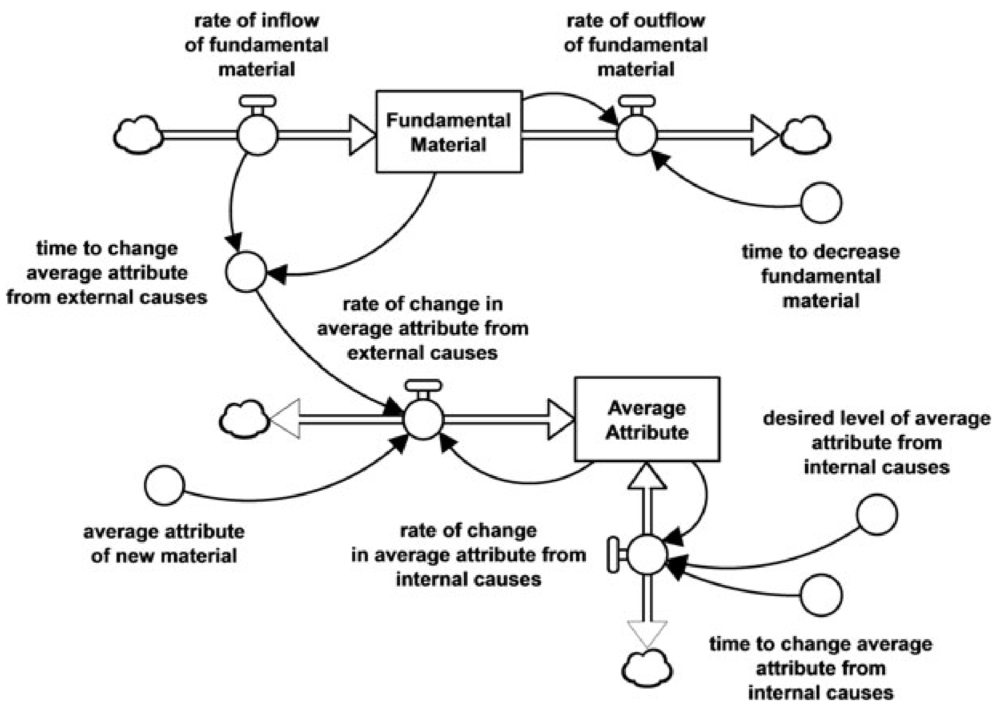

Fig. 8. Exponential smoothing to model both external and internal changes to an attribute (Type III)

larger model. In their co-flow, which modeled GE's continuous improvement and process innovation environment, "adopted ideas" is the fundamental material and "learning ability" the total attribute. They added additional model variables to limit[4] the maximum value of "learning ability" to 100. As we will show later, approaches that limit the upper limit of the attribute are not needed for Hines-based Type I and Type III co-flows.

Whereas the Hines co-flow with experience (Type II) is applicable when the rate of change is linear, the co-flow with internal change is applicable when the rate of change is responsive to negative feedback. In the Type III co-flow there are two information delays: the existing one used to model external change to the average attribute and the new one used to model internal change to the average attribute.

Type III: behavior and analysis

To understand the behavior of the Type III co-flow, we derived an exact solution from the differential equations associated with it. A concise notation for

[4]See Gambardella and Lounsbury (2017) for another example of this and other limit approaches.

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the variables in the co-flow is illustrated in Figure 9 and listed as follows. We use this notation in the rest of the paper.

$M(t)$ = fundamental material
$A(t)$ = average attribute
m_1 = rate of inflow of fundamental material (non-zero constant)
$m_2(t)$ = rate of outflow of fundamental material
t_3 = time to decrease fundamental material (non-zero constant)
$x_1(t)$ = rate of change of average attribute from external causes
$x_2(t)$ = rate of change of average attribute from internal causes
y_1 = average attribute of new material (constant)
y_2 = desired level of average attribute from internal causes (constant)
$t_1(t)$ = time to change average attribute from external causes (i.e., dilution time)
t_2 = time to change average attribute from internal causes (non-zero constant)

Combining the equations for the co-flow (see Tables A-1–A-3, supporting information) with the concise definition of the variables yields the following equations:

$$\frac{\mathrm{d}M(t)}{\mathrm{d}t} = m_1 - m_2 = m_1 - \frac{M(t)}{t_3}, \text{where } m_2 = \frac{M(t)}{t_3} \tag{1}$$

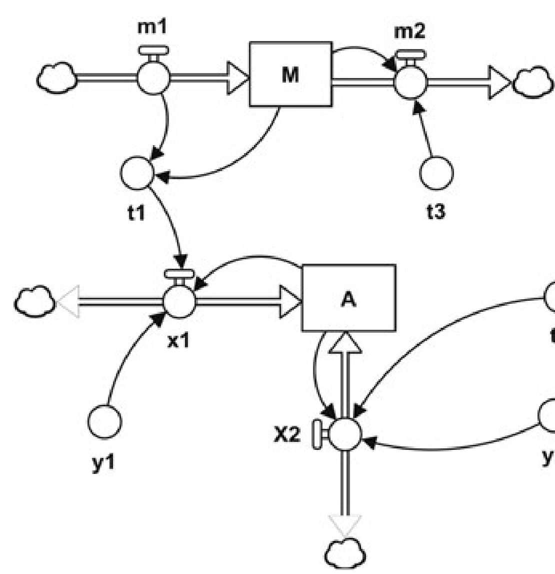

Fig. 9. Type III co-flow with abbreviated notation

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$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = x_1 + x_2 = \frac{[y_1 - A(t)]}{t_1} + \frac{[y_2 - A(t)]}{t_2}$$
$$= -\left(\frac{M(t) + m_1 \cdot t_2}{M(t) \cdot t_2}\right) \cdot A(t) + \left(\frac{y_1 \cdot m_1 \cdot t_2 + y_2 \cdot M(t)}{M(t) \cdot t_2}\right) \tag{2}$$

where $t_1 = \frac{M(t)}{m_1}$.

Equation (3) is the solution to Eq. (1) when we choose the value of the stock $M(t)$ at $t = 0$, $M(0)$, as the initial condition:

$$M(t) = (M(0) - M_E) \cdot e^{-t/t_3} + M_E \tag{3}$$

where $M_E \equiv m_1 \cdot t_3$. If $M(0) = M_E$ in Eq. (3), then $M(t)$ will be constant for all time. Hence M_E is the value of $M(t)$ at equilibrium.[5] There are several consequences that follow from Eq. (3). If $M(t)$ does not start in equilibrium (i.e., $M(0) \neq M_E$), then it will only reach equilibrium in the limit as $t \to \infty$. These results are also true for Type I and Type II co-flows since all three co-flows share the same structure for the fundamental material stock.

Equation (4) is the solution to Eq. (2) when we choose the value of the stock $A(t)$ at $t = 0$, $A(0)$, as the initial condition:

$$A(t) = \frac{y_2 \cdot (M(0) - M_E) \cdot \left(1 - e^{-t/t_2}\right) \cdot e^{-t/t_3} + [A(0) \cdot M(0) - M_E \cdot A_E] \cdot e^{-t/t_2} \cdot e^{-t/t_3} + M_E \cdot A_E}{(M(0) - M_E) \cdot e^{-t/t_3} + M_E} \tag{4}$$

where

$$A_E = y_1 \cdot \alpha + y_2 (1 - \alpha), \quad \alpha = \frac{t_2}{t_3 + t_2} < 1 \tag{5}$$

As $t \to \infty$, the average attribute $A(t)$ approaches the constant A_E, its equilibrium value. A necessary condition for $A(t)$ to be in equilibrium is $\frac{\mathrm{d}A(t)}{\mathrm{d}t} = 0$; however, this is not a sufficient condition since applying this condition in Eq. (2) yields a value of $A(t)$ that depends on time:

$$A(t) = \left[\frac{y_1 \cdot t_2 + y_2 \cdot t_1(t)}{t_1(t) + t_2}\right] = y_1 \cdot \gamma(t) + y_2 (1 - \gamma(t)) \tag{6}$$

where $\gamma(t) = \frac{t_2}{t_1 + t_2} = \frac{m_1 \cdot t_2}{M(t) + m_1 \cdot t_2} < 1$.

[5]This is also the value of M(t) obtained when setting $\frac{\mathrm{d}M(t)}{\mathrm{d}t}$ to zero.

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In Eq. (6), $A(t)$ is constant only if $M(t)$ is a constant for all time, i.e., when $M(t)$ is in equilibrium. In that case $M(t) = m_1 \cdot t_3 \equiv M_E$ and $A(t)$ becomes A_E. Now suppose $M(t)$ is in equilibrium, then Eq. (4) becomes

$$A(t) = (A(0) - A_E) \cdot e^{-t/t_2} \cdot e^{-t/t_3} + A_E \tag{7}$$

We make several observations about Eq. (7). First, t_3 represents the dilution time since $t_3 = t_1$. Second, A(t) approaches equilibrium in the limit as $t \to \infty$, or is at equilibrium when $A(0) = A_E$. Third, the further the initial value of $A(t)$ is from its equilibrium value (i.e., the larger the value of $|A(0) - A_E|$) the longer it will take $A(t)$ to be within a given percentage of A_E (e.g., the time t when $\left|\frac{A(t) - A_E}{A_E}\right| \leq k$, where k is a constant and $k < 1$).

To summarize the general results about equilibrium:

i If both $M(t)$ and $A(t)$ start in equilibrium, i.e., $A(0) = A_E$ and $M(0) = M_E$, then they will remain in equilibrium.
ii If $M(t)$ and $A(t)$ do not start in equilibrium, each will reach equilibrium only when $t \to \infty$.
iii For any finite time, $M(t)$ can be in equilibrium when $A(t)$ is not in equilibrium, but $A(t)$ cannot be in equilibrium unless $M(t)$ is in equilibrium.

Properties (i) and (ii) also hold for Types I and II Hines co-flows, and property (iii) holds just for the Hines Type II co-flow. When $y_2 = 0$ and $t \to \infty$, the Type III co-flow reduces to the Hines co-flow (Type I). In this case $M(t)$ satisfies Eq. (3) and $A(t)$ satisfies the equation

$$A(t) = \frac{\left(A(0) \cdot M(0) - y_1 \cdot M_E\right) \cdot e^{-t/t_3} + M_E \cdot y_1}{(M(0) - M_E) \cdot e^{-t/t_3} + M_E} \quad \text{(Hines co-flow, Type I)} \tag{8}$$

Additionally, if M(t) is in equilibrium, Eq. (8) becomes

$$A(t) = \left(A(0) - y_1\right) \cdot e^{-t/t_3} + y_1 \quad \text{(Hines co-flow, Type I)} \tag{9}$$

It is interesting that under the conditions $y_2 = 0$ and $t \to \infty$, which yield a Hines Type I co-flow, property (iii) no longer holds. Instead, $A(t)$ can be in equilibrium when $M(t)$ is not in equilibrium.

Having an exact solution to a model is a good way to check the accuracy of simulation software and the correctness of the model, and to provide an

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additional check[6] on the validity of the solution (see Appendix B, supporting information).

Type III: quantifying the range of the average attribute

In many situations, we want to limit the range of $A(t)$, especially if it is a psychological or sociological variable (e.g., from 0 to 100; Gambardella and Lounsbury, 2017). As mentioned earlier, often a modeler needs to employ techniques to limit this range. However, those techniques are not needed for the Type III co-flow.[7] In Appendix C (supporting information), we show that if $\beta = \max\{A(0), y_1, y_2\}$, $A(0) \geq 0$, and $y_i \geq 0$ for i = 1, 2, then $0 \leq A(t) \leq \beta$, where β is a constant (e.g., $\beta = 100$).

Type III: loyalty example

To illustrate the Type III co-flow, we assume it is the model a company uses to examine strategies to increase or maintain the loyalty of its employees. Figure 10 illustrates a model of employees working for Acme Parts Company. Company employees represent the fundamental material stock, with "average loyalty" as its associated attribute. New hires bring with them a level of loyalty to the company and existing employees have a level of loyalty. We use several fictitious scenarios to illustrate the behavior of the Type III co-flow and then comment on embedding the co-flow in a larger model. We assume loyalty is measured with a (fictitious) loyalty index that ranges from 0 to 100.

Scenario 1: new hires have higher loyalty than existing employees

In one division of its company, Acme faces a problem. Suppose at time $t = 0$ in this division the average loyalty of new hires ($y_1 = 40$) is greater than the average loyalty of existing employees ($A(0)$ = 30), and the number of employees, $M(t)$, is in equilibrium. If Acme does nothing, after a very long time ($t \to \infty$) the average loyalty will settle to $y_1 = 40$, which follows from Eq. (9) of the Hines co-flow, Type I. Acme cannot afford to wait that long and initiates a project to raise employee loyalty more quickly. Acme hires a contractor to create and implement a loyalty program. This contractor brings a reliable and tested means to measure company loyalty and monitor progress. At time t = 0, the contractor institutes an internal loyalty program with the desired goal

[6]A primary check is to substitute the solution back into the original differential equations. However, it is useful to start with simulation software, especially when the primary check requires extensive calculations.

[7]This is also true for the Type I Hines co-flow since it is a special case of the Type III co-flow. However, it is not true for the average attribute in the traditional or Hines co-flows with experience.

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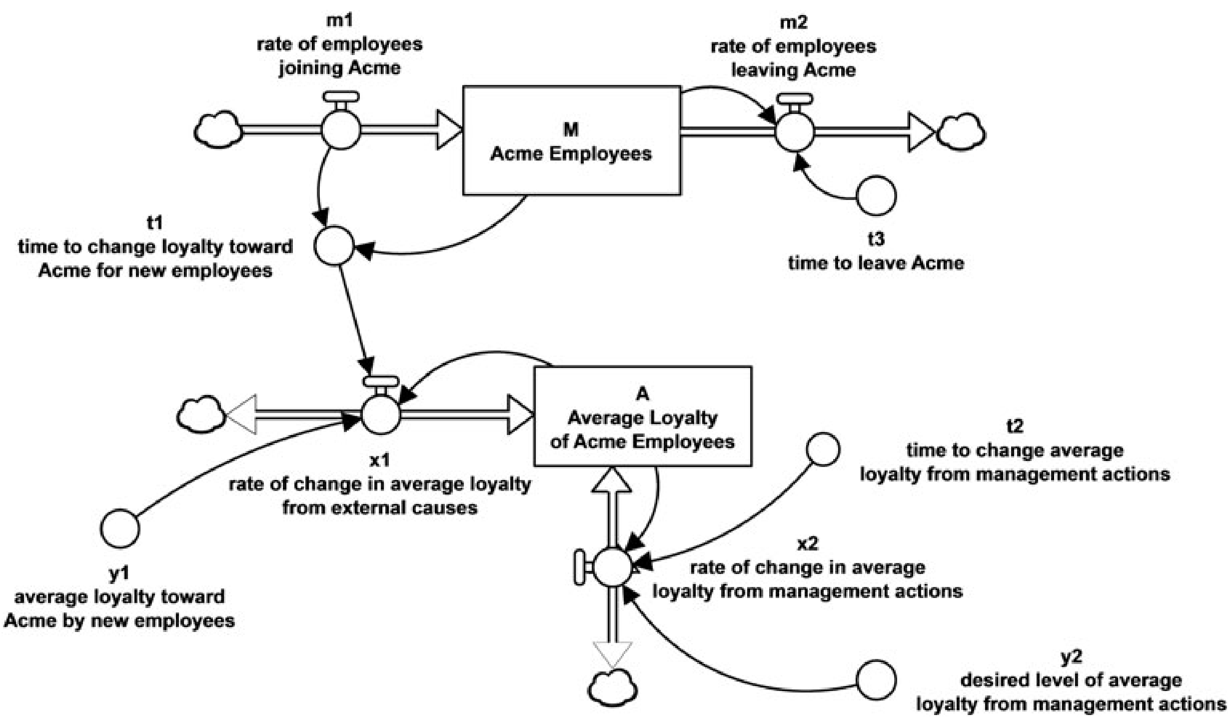

Fig. 10. Modeling changes to average loyalty using Type III co-flow

$y_2 = y_1 = 40$. It follows from Eq. (5) that the equilibrium value of $A(t)$ is $A_E = y_1 = y_2 = 40$. Then, Eq. (7) becomes

$$A(t) = (30 - 40)\cdot e^{-t/t_2}\cdot e^{-t/t_3} + 40 \tag{10}$$

Since $A(t)$ will only approach 40 in the limit of $t \to \infty$, Acme, who is short of funds, only requires the contractor to get to within 1 percent of it. If $t_3 = 10$ months and $t_2 = 2$ months, then using Eq. (10) we can determine how long it will take Acme to reach within 1 percent of its goal. Solving for time in the inequality $0.01 > \left|\frac{A(t)-40}{40}\right|$ yields $t > 5.37$ months. If we did not have a loyalty program and used the Hines co-flow in Eq. (9) to compute this time, we would get $t > 32.2$ months.

When Acme received the estimate of 5.37 months from the contractor, they wondered how much more the goal y_2 would need to be for $A(t)$ to reach 40 at $t = 5.37$ months. An internal goal of 40.5 would raise the average loyalty to 40 in 5.37 months, which follows from Eqs (5) and (7).

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Scenario 2: new hires have lower loyalty than existing employees

In another division of its company, Acme faces a different problem. Suppose at time $t = 0$ the average loyalty of new hires ($y_1 = 30$) is less than the average loyalty of existing employees ($A(0) = 40$), the internal goal y_2 is 45, and the number of employees, $M(t)$, is in equilibrium. Also, assume the time to leave Acme, t_3, is 10 months, and the time to change average loyalty from management actions, t_2, is 5 months. For this situation, using Eq. (5) we can see that the average loyalty $A(t)$ is in equilibrium: $A_E = 40$. Acme wants to raise the equilibrium value of $A(t)$, A_E, from 40 to 44 in 3 months and needs to evaluate approximately equal bids for this work from two contractors, each with a different approach:

- Contractor A offers a loyalty program that keeps the internal goal y_2 at 45 while changing t_2 from 5 months to 5/7 of a month.
- Contractor B offers a loyalty program that keeps t_2 at 5 months while changing the internal goal from y_2 at 45 to 51.

Which contractor should they choose? Figure 11 illustrates the results of both approaches. Both would take effect in the third month. Acme chose Contractor A since their approach offered a faster convergence to the new goal.

Suppose that Acme wants to raise the equilibrium value of $A(t)$ from 40 to 46 in 3 months. Then, Contractor A's approach of keeping the internal goal y_2 at 45 while changing t_2 would not work because the values of A_E are

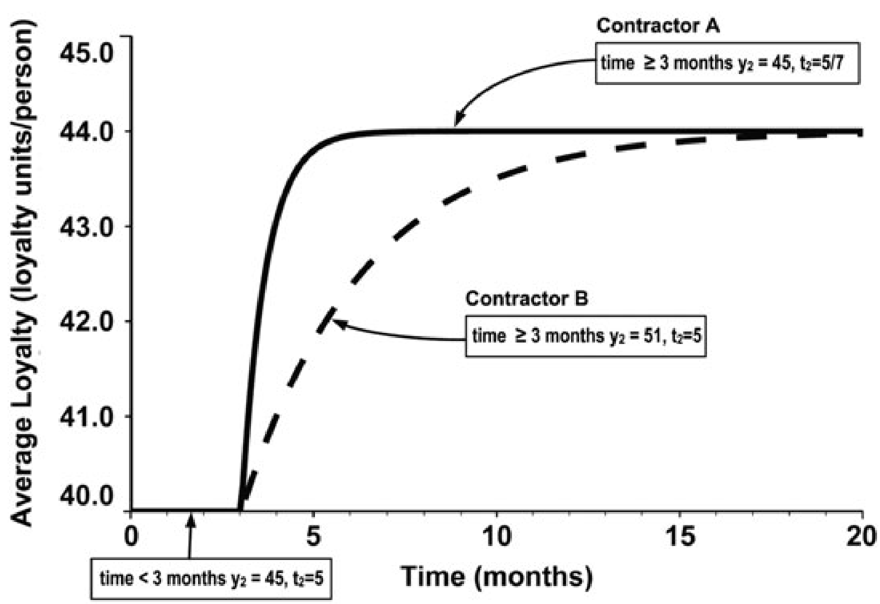

Fig. 11. Comparison of the results of contractor approaches

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restricted to lie between y_1 = 30 and y_2 = 45 (see Appendix C, supporting information).

In this example, Acme used the Type III model to ask questions needed to select a better contractor. This is before bringing the program online. Feedback loops in the model can be useful in initial design of programs as well as when monitoring and perhaps modifying programs after they start.

In this example, we explored the behavior of the Type III co-flow when $M(t)$ was in equilibrium. In Appendix D (supporting information), we illustrate some interesting properties of this co-flow when $M(t)$ is not initially in equilibrium.

Sometimes attributes in a model have an effect on the fundamental stocks of the model. In the Acme loyalty example, for instance, the average employee loyalty to the company could affect the rate at which employees leave the company. The more loyalty the employees feel, the more slowly they may leave the company. Thus, in this case, one may want to include an effect of average loyalty on the rate of employees leaving the company. Furthermore, to the degree that including this relationship changes the accumulation of employees working for the company, the company may want to change their hiring rate. Thus one may want to include an effect of the rate of employees leaving the company on the rate of employees joining the company. By incorporating feedback from the average attribute into the fundamental material in this way, one brings the effects of policy, management actions, and programs into the model dynamics.

Discussion

One purpose of this paper was to describe a new way to model internal changes to attributes in co-flow structures. Specifically, we showed that for a class of attributes, internal changes, like external changes, can be modeled with a first-order exponential smoothing. Depending on how the attribute changes, first-order exponential smoothing may or may not be the optimal way to model the information delay (Sterman, 2000, p. 432). Modelers need to work with content experts to select the form of delay that is most appropriate for the attribute.

Another purpose of this paper was to show the value of using both mathematical analysis and simulations in understanding the equilibrium and non-equilibrium behavior of co-flows. Using both approaches for Type III co-flows, we were able to:

- specify the exact conditions for equilibrium (see Type III: behavior and analysis);
- illustrate how knowing the exact solution can help choose between alternative approaches to implementing a program (see Type III: loyalty example);

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- limit the range of the "average attribute" by limiting the range of the two goals and the initial condition of the "average attribute". This is especially important if the "average attribute" is a psychological or sociological variable (see Appendix C, supporting information);
- understand non-equilibrium behavior (see Appendix D, supporting information).

Although it is not realistic to find exact solutions for higher-order system dynamics models, a limited mathematical analysis may be useful for gaining insight into model behavior and for identifying scenarios to simulate.

Conceptually, our Type III co-flow has two negative loops and two goals[8] (i.e., y_1 and y_2). The "eroding goals" archetype (Senge, 1990) has a similar structure (for related examples see Clauset and Gaynor, 1982; Levine and Doyle, 2002). This archetype appears in systems when there is a gap between performance generated by short-term actions and a long-term goal. In this situation, management has two options. They can either adjust the long-term goal to be closer to the performance achieved by the short-term actions, or they can apply a longer-term fix that targets the cause of the gap (see Barlas and Yasarcan, 2006, 2008, for a comprehensive model of goal dynamics in organizations). In the context of failing to achieve long-term goals, it is hypothesized that management will adjust the long-term goal downward to be closer to actual performance (i.e., the loop representing the symptomatic fix dominates; Braun, 2007). In a Type III co-flow, by contrast, the goals both contribute to determine the final value of the attribute at equilibrium with the model attaining a level at equilibrium that is intermediate to the external and internal goals. In this way, our Type III co-flow broadens the scope of application for this important generic structure.

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[8]Actually, y_1 is an external constraint rather than a goal. Mathematically, it acts like a goal in a smooth.

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Supporting information

Additional supporting information may be found in the online version of this article at the publisher's web-site.

The e-companion, provided as supporting information on the journal's web site, contains all the Stella Architect (version 1.1.2) models used for the simulations in this paper and a working version of the model in Figure . It also contains a document with supporting material, which includes instructions for reproducing the reported results in the paper, derivations of key equations, and the following appendices to this paper:

Appendix A: equations for the co-flows

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