PART I

Vibrations, Waves, and Sound

The first four chapters lay the groundwork for the rest of the book. After a brief introduction to sound in Chapter 1, we review some basic principles of motion. To understand sound we need to understand motion, including force and acceleration and how they are related by Newton's second law of motion. We need to understand work and energy, force, and pressure.

In Chapter 2 we consider vibrating systems, and in Chapter 3 we consider waves. We compare sound waves to light waves, ocean waves, and other types of waves. We note how all waves can undergo reflection, refraction, interference, and diffraction. Finally, we study resonance.

Readers who have had a good high school or college physics course may be familiar with most of the material covered in Chapters 1–4 and may wish to either skim over it quickly or go on the Chapter 5.

The metric system of units (in particular, the Système International, or SI) is used throughout this book. It is the preferred system for scientific work, and in the future (hopefully) it will come into general use in the United States as it has in the rest of the world. Other systems of units are listed in Appendix A.1.

From Chapter 1 of *The Science of Sound*, Third Edition. Thomas D. Rossing, Richard F. Moore, Paul A. Wheeler.

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CHAPTER

1 What Is Sound?

The word *sound* is used to describe two different things:

1. An auditory sensation in the ear;
2. The disturbance in a medium that can cause this sensation.

By making this distinction, the age-old question, If a tree falls in a forest and no one is there to hear it, does it make a sound? can be answered.

The science of sound, which is called *acoustics*, has become a broad interdisciplinary field encompassing the academic disciplines of physics, engineering, psychology, speech, audiology, music, architecture, physiology, neuroscience, and others. Among the branches of acoustics are architectural acoustics, physical acoustics, musical acoustics, psychoacoustics, electroacoustics, noise control, shock and vibration, underwater acoustics, speech, physiological acoustics, etc.

This book is intended to be an introduction to acoustics, written in appropriate language, primarily for students without college physics and mathematics. A few basic mathematical ideas (such as logarithms) are introduced, and a brief review of algebra is included in the appendix for those who need it. A few basic concepts from physics, such as motion, energy, power, and waves, are introduced as needed.

In this chapter you should learn:

- About sound and sources of sound;
- About distance, speed, and velocity;
- How to represent motion graphically;
- About acceleration, force, and pressure;
- About energy and power.

1.1 ■ WHAT IS A SOUND WAVE?

The world is full of waves: sound waves, light waves, water waves, radio waves, X rays, and others. The room in which you are sitting is being crisscrossed by light waves, radio waves, and sound waves of many different frequencies. Practically all communication depends on waves of some type. Although sound waves are vastly different from radio waves or ocean waves, all waves possess certain common properties. One is that they carry information from one point to another. They also transport energy, as we will learn.

Sound waves travel in a solid, liquid, or gas. Mostly we will focus our attention on longitudinal sound waves in air. Longitudinal means that the back-and-forth motion of air is in the direction of travel of the sound wave (as compared to waves on a rope, in which the back-and-forth motion is perpendicular to the direction of wave travel). As the wave travels through the air, the air pressure changes by a slight amount, and it is this slight change in pressure that allows our ears (or a microphone) to detect the sound. Longitudinal and transverse waves are compared in Fig. 1.1.

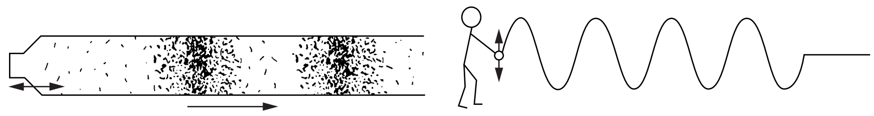


FIGURE 1.1 (a) Longitudinal motion of air molecules in a sound wave created by a loudspeaker; (b) transverse wave motion on a rope shaken up and down at one end.

1.2 ■ SOURCES OF SOUND

Sound can be produced by a number of different processes, which include the following.

1. *Vibrating bodies* When a drumhead or a piano soundboard vibrates, it displaces the air next to it and causes the local air pressure to increase and decrease slightly (Fig. 1.2(a)). These pressure fluctuations travel outward as a sound wave. Vibrating bodies are the most familiar sources of sound.

2. *Changing airflow* When we speak or sing, our vocal folds (cords) alternately open and close so that the rate of air flow from our lungs increases and decreases, resulting in a sound wave. Similarly, a vibrating clarinet reed or the lips of a brass player cause a changing airflow into a clarinet or a trumpet. Air flowing through a screen or a grill produces a sort of hissing noise. An extreme case of changing airflow is a siren, in which holes on a rapidly rotating plate alternately pass and block air from a compressor, resulting in a very loud sound (Fig. 1.2(b)).

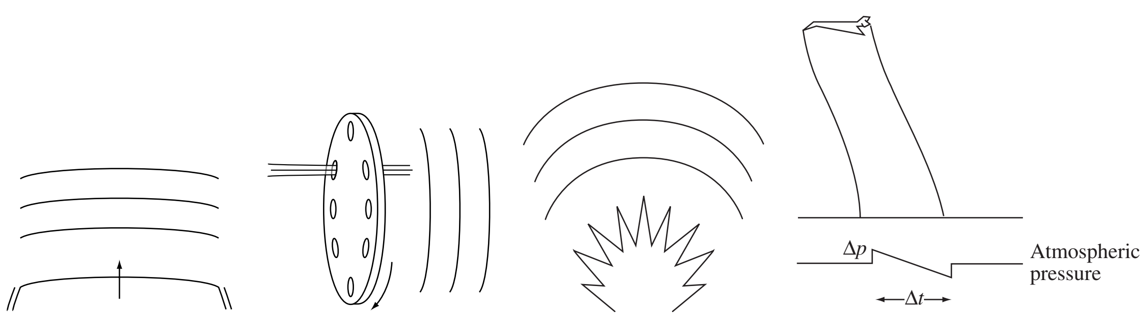


FIGURE 1.2 Some examples of sound sources: (a) a vibrating drumhead causes pressure changes in the air nearby; (b) holes in a rotating siren alternately allow and stop the flow of air; (c) an explosion rapidly heats the air nearby; (d) a supersonic airplane gives rise to shock waves.

3. *Time-dependent heat sources* An electrical spark produces a crackle; an explosion produces a bang due to the expansion of the air caused by its rapid heating (Fig 1.2(c)). Thunder results from rapid heating of air by a bolt of lightning.

4. *Supersonic flow* Shock waves result when a supersonic airplane or a speeding rifle bullet forces air to flow faster than the speed of sound (Fig 1.2(d)). We discuss sonic booms in Chapter 32.

How many sound sources can you think of? Do they fit one of the four categories above? Can you think of any sound sources that do not fit these categories? (There are a few.)

1.3 ■ WANTED AND UNWANTED SOUND

Our environment is filled with sound. Some of those sounds are made by humans, some by animals, some by machines, and some by natural causes, such as the weather. Some sounds, such as music and speech, are desirable (sometimes, at least); others, such as those of heavy vehicles on a street, are generally not. Unwanted sounds are often referred to as *noise*. Control of environmental noise is an important (but all too often overlooked) factor in maintaining environmental quality. Only recently has environmental noise been regulated by governmental agencies.

One difficult problem in controlling environmental noise is disagreement about what sounds are wanted and unwanted. Loud music may appeal to revelers at a party but not to their neighbors. The roar of a motorcycle engine may convey the feeling of power to the owner but insult the ears of bystanders.

Chapters 30–32 discuss environmental noise. In these chapters the reader will learn about the effect of noise on people and how to control noise. Although it is possible to isolate oneself from a noisy environment, it is difficult to do so (the term *soundproofing* could probably be described as a figment of our imagination). Control of noise is by far the best way to control environmental noise.

Before we go on to explore the science of sound and some of its applications, we should consider a few physical principles that we wish to apply in order to understand it. We want to understand such concepts as motion, force, kinetic and potential energy, pressure, and power and how these concepts apply to vibrating systems as well as to sound waves.

Not only do we wish to familiarize ourselves with these concepts, but sometimes we wish to solve problems that apply them, because doing physics is really the best way to understand physics. Physics involves more than solving problems, of course. Doing physics generally includes observation of natural phenomena, careful measurement, analysis of the measurements, formulating theories and laws to explain them, and applying these theories and laws to other situations or phenomena. Although it would be nice to make our own measurements on sound and formulate our own theories, this is not always practical, and so we make use of the measurements of many other scientists. We will, however, apply their theories to specific examples of interest, and that is the essence of most of the numerical exercises in this book.

In solving these numerical exercises or problems we will wish to apply some basic mathematics. Mathematics is the precise language of science, and when scientists talk to each other they often use this language quite freely. However, it is possible to translate most scientific concepts into a more familiar language, and that is what is intended in this book. We will, however, use the language of mathematics when it is especially appropriate.

1.4 ■ DISTANCE, SPEED, AND VELOCITY

For an object that travels at constant speed, the distance traveled is given by the simple formula

$$\text{distance} = \text{speed} \times \text{time}. \tag{1.1}$$

We realize that if we travel at a steady rate of 50 mi/h for 2 h, we will cover 100 mi, as the formula states. The same distance will be covered if our average speed is 50 mi/h for 2 h, even though our actual speed at different times may vary. In fact, the average speed can be defined as distance divided by time:

$$\text{speed} = \frac{\text{distance}}{\text{time}}. \tag{1.2}$$

Many different units are used to express distance: feet, inches, meters, yards, even furlongs. However, because most of the world uses metric units, it is prudent to emphasize the use of the metric system. The meter, then, will be our preferred unit of length, with occasional reference to feet and inches. Conversions between various units appear in the appendix.

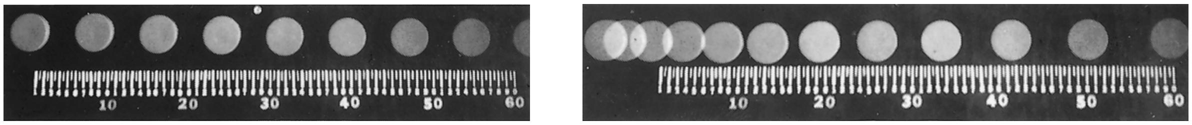
The metric system uses prefixes to denote various powers of ten, as follows:

$$\begin{aligned}
&1 \text{ kilometer} &&= 1 \text{ km} &&= 10^{3} \text{ m} &&= 1000 \text{ m}\\
&1 \text{ centimeter} &&= 1 \text{ cm} &&= 10^{-2} \text{ m} &&= 1/100 \text{ m}\\
&1 \text{ millimeter} &&= 1 \text{ mm} &&= 10^{-3} \text{ m} &&= 1/1000 \text{ m}\\
&1 \text{ micrometer} &&= 1\ \mu\text{m} &&= 10^{-6} \text{ m} &&= 1/1{,}000{,}000 \text{ m}
\end{aligned}$$

These are the principal prefixes used for distance; others appear in the appendix. (The term *micron* is often used in place of *micrometer*.) Note that the first syllable is accented in all of these units, including “kil´ometer” and “mic´rometer.”

Physicists often speak of *velocity* rather than speed. Velocity specifies the direction of motion as well as the speed. In describing motion in one direction, however, no distinction need be made, and in most of the vibrating systems we wish to discuss, the vibrations are in one direction. Speed is correctly defined as the magnitude of velocity (without regard to direction), and we will use the symbol v for speed, because it is customary to do so; furthermore, it reminds us that speed and velocity are closely related.

To find speed of an object, we must measure both the distance traveled and the time of travel. This is often done in the laboratory by photographing an object illuminated by a stroboscopic (“strobe”) lamp that flashes at regular intervals. The photographs in Fig. 1.3 were taken with the lamp flashing ten times per second in a darkened room while the camera shutter remained open. Thus the position of the object was recorded at intervals of 0.1 s. It is easy to see that in Fig. 1.3(a) the speed stays the same, whereas in Fig. 1.3(b) it increases as the object moves from left to right.



(a) (b)

FIGURE 1.3 Stroboscopic pictures of motion: (a) constant speed; (b) changing speed. Both pictures were made with ten flashes per second.

We can easily determine the speed of the object in Fig 1.3(a). During each interval of 0.1 s, it appears to move 7.5 cm, so the speed is

$$v = \frac{7.5 \text{ cm}}{0.1 \text{ s}} = 75 \text{ cm/s} = 0.75 \text{ cm/s}.$$

Since the object moves with constant speed, the average speed and instantaneous speed are the same; this is not the case in Fig 1.3(b).

EXAMPLE 1.1 If the speed limit is posted at 30 mi/h, what is the corresponding limit in meters per second?

Solution

$$30\frac{\text{mi}}{\text{hr}} = \frac{(30 \text{ mi/h})(5280 \text{ ft/mi})(0.305 \text{ m/ft})}{3600 \text{ s/h}}$$
$$= 13.4 \text{ m/s}.$$

EXAMPLE 1.2 A motorist travels 100 mi. The first half of the distance takes one hour; the second half takes $1\frac{1}{2}$ h. What is the average speed for the journey?

Solution

$$v_{\text{av}} = \frac{d}{t} = \frac{100 \text{ mi}}{1 + 1.5 \text{ h}} = 40 \text{ mi/h}.$$

EXAMPLE 1.3 A motorist travels 300 mi. During the first half of the journey his average speed is 60 mi/h, and during the second half his average speed is 30 mi/h. What is his average speed for the entire journey?

Solution

$$t_1 = \frac{150 \text{ mi}}{60 \text{ mi/h}} = 2.5 \text{ h};$$

$$t_2 = \frac{150 \text{ mi}}{30 \text{ mi/h}} = 5.0 \text{ h}.$$

$$v_{av} = \frac{d}{t} = \frac{300 \text{ mi}}{2.5 + 5.0 \text{ h}} = 40 \text{ mi/h}.$$

(Note that the answer is *not* 45 mi/h.)

1.5 ■ GRAPHICAL REPRESENTATION OF MOTION

If a picture is worth a thousand words, a well-constructed graph must be worth at least five thousand, especially when it comes to describing motion. Suppose we wish to represent the changing position of the objects in Fig. 1.3 using two graphs. One coordinate is the position, represented by y, and the other coordinate is the time t. It does not really matter what point is selected as the zero or starting point (called the origin), but it is convenient to take it as the end of the meter stick; alternatively, it could have been the position of the object at the first flash. The graphs in Fig. 1.4 are the result.

Now we will use these graphs to help us determine average and instantaneous speed. To the graphs we add the useful constructions shown in Fig. 1.5. The distance FE is called Δy, a symbol read as "delta y," which means the "change in y." Similarly Δt (line DF) represents the "change in t." In Fig. 1.5(a), the average speed during the time interval from $t = 0.2$ s to $t = 0.4$ s is

$$v_{av} = \frac{\Delta y}{\Delta t} = \frac{15 \text{ cm}}{0.2 \text{ s}} = 0.75 \text{ m/s}.$$

In Fig. 1.5(b) it is

$$v_{av} = \frac{\Delta y}{\Delta t} = \frac{8.4 \text{ cm}}{0.2 \text{ s}} = 42 \text{ cm/s}.$$

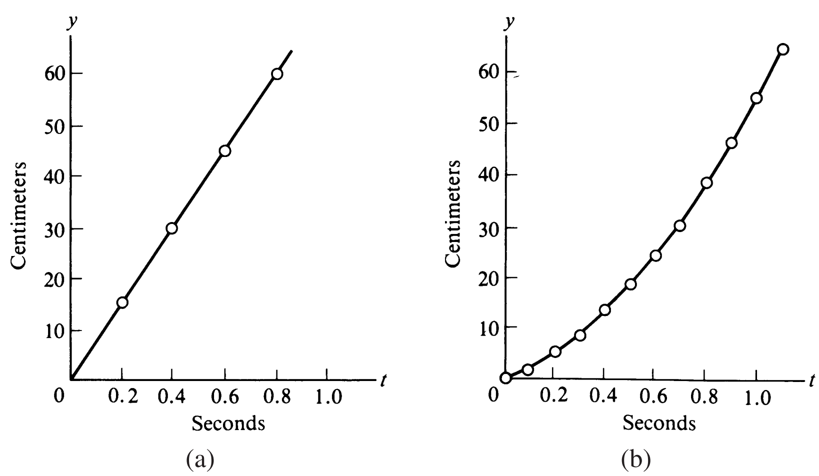


FIGURE 1.4 Graphical representation of the motion shown in Fig. 1.3. The object in (a) moved at a constant speed; the object in (b) changed its speed.

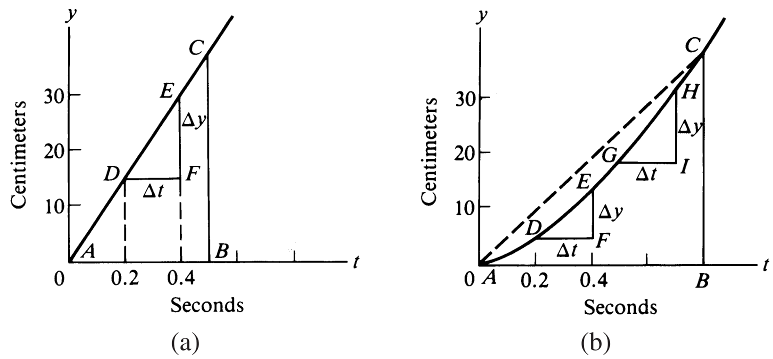


FIGURE 1.5 Curves of uniform and changing motion as in Fig. 1.4 with constructions added for determining speed.

In Fig. 1.5(a), the speed we calculate should be the same regardless of the time interval Δt selected. In fact, the larger time interval $\Delta t = 0.5$ s (represented by the line AB) should be used to improve accuracy:

$$v = \frac{\Delta y}{\Delta t} = \frac{37.5 \text{ cm}}{0.5 \text{ s}} = 75 \text{ cm/s}.$$

This is not true in the case of the changing motion in Fig. 1.5(b). Using the time intervals represented by DF, GI, and AB gives three different values of average speed, because the speed is changing:

$$v_1 = \frac{8.4}{0.2} = 42.0 \text{ cm/s} \qquad (DF),$$

$$v_2 = \frac{12.5}{0.2} = 62.5 \text{ cm/s} \qquad (GI),$$

$$v_3 = \frac{37.5}{0.8} = 46.9 \text{ cm/s} \qquad (AB).$$

If the instantaneous speed at a particular time is to be determined, the time interval Δt must be made very small. As Δt becomes smaller and smaller, the line (chord) DE more and more nearly approaches the slope of the curve at point P as shown in Fig. 1.6. Thus,

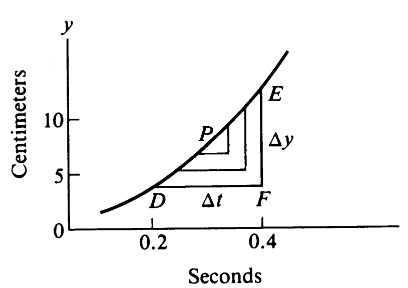


FIGURE 1.6 Shrinking Δy and Δt in order to obtain instantaneous speed.

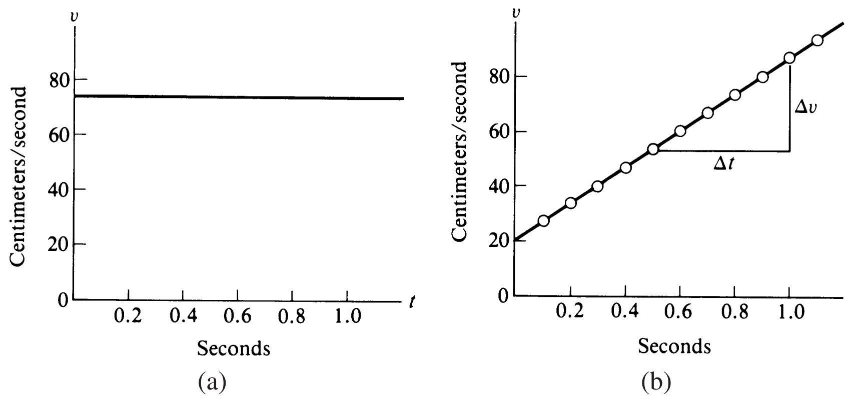


FIGURE 1.7 Speed as a function of time for the objects shown in Figs. 1.3 and 1.4.

instantaneous speed can be interpreted as the *slope* (steepness) of the curve representing position y as a function of time t.

A graph showing speed as a function of time is also useful in describing motion. If the object photographed in Fig. 1.3 had a speedometer attached to it, we would have a photographic record of speed each time the strobe light flashed. These values of speed could then be plotted on a graph of v versus t. However, we can also determine speed from the graphs already drawn, because speed at any time is represented by the slope of the curve showing y versus t at that time. Figure 1.7 shows speed as a function of time for each graph in Fig. 1.4.

1.6 ■ FORCE AND ACCELERATION

Force is a quantity with which we are all familiar; it can be described as a push or a pull. Practically all human activity involves forces: running, lifting, eating, writing, and even standing.

Applying a force to an object may distort the object, change its motion, or both. You may remember Newton's famous law of motion (his second law) tells us that the *acceleration* (change in motion) of an object is equal to the net force F divided by its mass m. Newton's second law of motion can be written as

$$F = ma \quad \text{or} \quad a = \frac{F}{m} \tag{1.3}$$

Clearly a greater force is required to obtain the same acceleration for an object of large mass as compared to an object of small mass.

A force applied to a movable object at rest causes it to accelerate (move with increasing speed) in the direction of the force. This is consistent with Newton's second law of motion as well as with our own experience. Applying a force in the direction of motion tends to increase its speed (we press the "accelerator" pedal to increase the speed of an automobile). Applying a force in a direction opposite to the direction of motion tends to decrease the speed (i.e., produces a negative acceleration, or a *deceleration*). Logically, we could call the brake pedal of an automobile the *decelerator* pedal.

Acceleration refers to a change in speed. When we push down the accelerator pedal in an automobile, we expect the speed to increase. In describing motion, acceleration is defined as the rate of change of speed (just as speed is the rate of change of position). *Average acceleration* is the ratio of change of speed Δv to time interval Δt:

$$a_{\text{av}} = \frac{\Delta v}{\Delta t}. \tag{1.4}$$

Instantaneous acceleration at a particular time can be determined by making Δt and Δv very small, just as we determined instantaneous speed v by making Δt and Δy very small.

In Fig. 1.7(a), the speed remains unchanged in time; in Fig. 1.7(b) it increases at a steady rate. The acceleration in Fig. 1.7(a) is therefore zero; in Fig. 1.7(b) it is

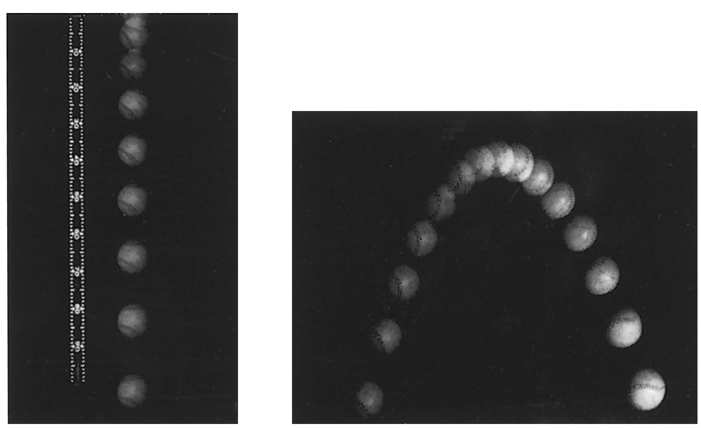
$$a = \frac{35 \text{ cm/s}}{0.5\text{s}} = 70 \text{ cm/s}^2.$$

Note the units for acceleration; the unit *cm* appears to the first power, but the unit *s* appears squared.

An object in *free fall* in the gravitational field of the earth experiences a constant acceleration of 9.8 m/s^2. Thus if it begins with no initial speed (up or down), at the end of the first second it will have a speed of 9.8 m/s; at the end of 2 s its speed will be 19.6 m/s, and so on.

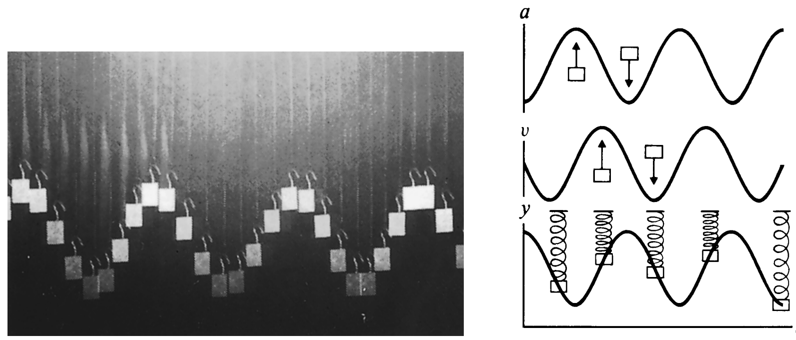
Note that acceleration does not always increase speed. If an object is thrown upward, the acceleration due to gravity acts to slow it down, or decelerates it. Figure 1.8 shows stroboscopic photographs of two objects in free fall. In Fig. 1.8(a), the object is dropped from rest; in Fig. 1.8(b), it is thrown upward, slows down, and then begins its descent.

Figure 1.9 shows a stroboscopic photograph of an object with an acceleration that changes its direction with time. The object, a mass attached to a spring, is executing a type of vibratory motion called *simple harmonic* motion, which will be discussed in Chap-



(a) (b)

FIGURE 1.8 Stroboscopic photographs of two objects in free fall: (a) object dropped from rest; (b) object thrown upward. (Photographs by Christopher Chiaverina.)



(a) (b)

FIGURE 1.9 Vibratory motion in which y, v, and a all change with time.

ter 2. The camera has been turned during the exposure, so that multiple images of the object create a photographic record of position as a function of time. Note that position y, speed v, and acceleration a all change with time.

EXAMPLE 1.4 A bicyclist accelerates from 0 to 10 m/s in 20 s. He then applies the brakes and comes to a stop in 5 s. What is his average acceleration in each case?

Solution

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{10 - 0 \text{ m/s}}{20 \text{ s}} = 0.5 \text{ m/s}^2;$$
$$a_{\text{av}} = \frac{0 - 10 \text{ m/s}}{5 \text{ s}} = -2 \text{ m/s}^2.$$

EXAMPLE 1.5 A ball is thrown upward with a velocity of 15 m/s. How long does it take to reach its maximum height? How long does it take to fall back to the ground (neglect air resistance)?

Solution

$$a = \frac{\Delta v}{\Delta t}, \text{ so } \Delta t = \frac{\Delta v}{a} = \frac{0 - 15 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.53 \text{ s}.$$

1.7 ■ PRESSURE

Newton's second law of motion describes the way in which the net force acting on an object will set that object into motion. So far as that law is concerned, it does not matter whether the net force is due to a single force acting at a point or many forces distributed around the object. In weighing an object, we can consider the entire force of gravity acting at one point, which we appropriately refer to as the *center of gravity* or *center of mass*, even though gravity acts on every part of the object.

There are other times, however, when the distribution of forces is important. For example, you can walk on snow without sinking if you wear snowshoes; on the other hand, if you were to cross a wooden floor in spike heels, you would severely damage the floor. The difference in this case is the *area* over which the same total force is distributed.

It is useful to define a quantity called *pressure* as the force divided by the area over which it is distributed. To be more specific, it is the force acting perpendicular to a surface divided by the area of that surface:

$$p = F_{\perp}/A. \tag{1.5}$$

Because force is measured in newtons, pressure is measured in newtons/meter^2 (N/m^2) (or pounds/square inch in the British system). A 50-kg (110-lb) person standing on spike heels with an area of 20 mm^2 would exert a pressure of nearly 25 million N/m^2 (about 1.8 tons per square inch!) on the portion of the floor under the heel. A pascal (Pa) is often used to express pressure (1 Pa = 1 N/m^2). In most of the world, tire pressure is measured in kPa (kilopascals).

Fluids (liquids and gases) exert forces on the walls of their containers and anything immersed in them. One of the important properties of all fluids at rest, in fact, is that the pressure acts perpendicular to all surfaces (walls of the container as well as immersed objects). The pressure at any point in an open container of fluid (liquid or gas) is determined by the weight of the fluid above that point. For example, the weight of the atmosphere above us results in a pressure of about 10^5 N/m^2 (15 lb/in^2) at sea level; at an altitude of about 5.5 km (3.4 miles), the pressure is only one half as great. The *buoyancy* of an immersed object is due to the excess upward pressure on its bottom surface, as shown in Fig. 1.10.

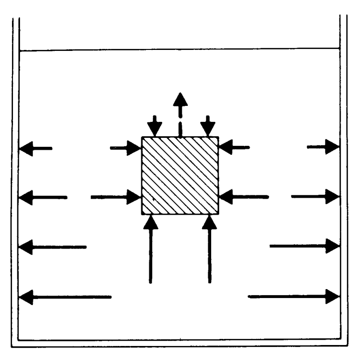


FIGURE 1.10 Pressure in a container of fluid (a) acts on all surfaces; (b) is proportional to depth. Buoyant force (dashed arrow) on the immersed object is due to the excess upward force.

Slow variations in atmospheric pressure (as measured by a barometer) are indicative of changing weather. Sound waves consist of very small but rapid variations in pressure.

EXAMPLE 1.6 What is the total force on your chest wall due to the air outside?

Solution Assume that your chest wall has an area of about 0.5 m^2,

$$F = pA = (10^5 \text{ N/m}^2)(0.5 \text{ m}^2) = 5 \times 10^4 \text{ N}$$

This is equal to about 11,000 lb; why doesn't your chest collapse?

1.8 ■ GRAPHICAL REPRESENTATION OF A SOUND WAVE

In Section 1.4, we learned how the motion of an object can be represented by graphs of distance, speed, or acceleration versus time. The graph tells us the position, speed, or acceleration of the object at each moment.

Similarly, it is useful to make a graph of sound pressure versus time. As the sound wave passes a certain point, the sound pressure rises and falls; a microphone measures this sound pressure. The graph of sound pressure versus time is called the *waveform* of the sound. Connecting the microphone to a *cathode-ray oscilloscope* or a computer allows us to display the waveform (graph of sound pressure versus time) on a screen.

Two sound waveforms are shown in Fig. 1.11. The one in Fig. 1.11(a) is a smoothly varying waveform; the one in Fig. 1.11(b) is a complex waveform (actually a musical sound made by a guitar). You may wish to speak or sing into a microphone connected to an oscilloscope so that you can observe the waveform of your golden voice.

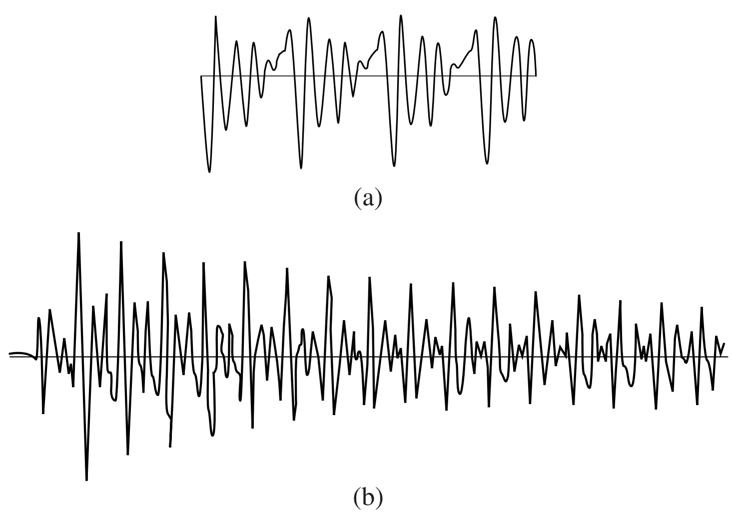


FIGURE 1.11 (a) Waveform (graph of sound pressure versus time) of a simple sound; (b) complex waveform of a musical sound made by a guitar.

1.9 ■ WORK AND ENERGY

The terms work and energy have various meanings in everyday life, but in the language of physics they have very definite meanings. *Work* is done when a force is applied to an object that moves. The work that is done is the product of the average force times the distance moved parallel to the force:

$$\mathcal{W} = Fd. \tag{1.6}$$

Work is expressed in newton-meters, or *joules* (abbreviated J). If a force of one newton causes an object to move one meter, then one joule of work has been done.

The force of gravity on an object (its weight) is given by the formula mg where $g = 9.8\ \text{m/s}^2$ is the acceleration of an object in free fall. Thus if an object falls a vertical distance h, the work done by gravity is:

$$\mathcal{W} = mgh. \tag{1.7}$$

By the same token, raising an object of mass m to a height h requires an amount of work $\mathcal{W} = mgh$.

Energy is perhaps the central idea underlying all branches of science. There are many forms of energy (e.g., mechanical, electrical, thermal, chemical, radiant, nuclear). A great deal of modern technology has as its goal the more efficient conversion of one of these forms of energy into another. For example, our future appears to depend on the development of the technology to convert solar (radiant) energy and the nuclear energy contained in sea water into electrical energy to replace our dwindling supply of oil and gas (chemical energy).

In our study of acoustics, we are concerned mainly with mechanical energy (and, to a lesser extent, electrical energy). Mechanical energy is closely related to work. Work is the transfer of energy. Systems with mechanical energy have the potential to do work. Vibrating systems have mechanical energy; mechanical energy is carried by the moving molecules in a sound wave. Energy, like work, is measured in joules. Sometimes a distinction is made between energy of motion, called *kinetic energy*, and stored energy, called *potential energy*, which is the capacity to do work (by virtue of position, for example).

Without going into a detailed discussion of energy, let us describe the energy of five completely different systems:

1. A baseball flying through the air has energy of motion, or *kinetic energy*. It is obvious that if it strikes another object (a bat, perhaps?) it can do work by virtue of its kinetic energy. The amount of kinetic energy is given by the formula

   $$\text{KE} = \tfrac{1}{2}mv^2, \tag{1.8}$$

   where m is mass and v is its speed.
2. A block of wood lifted to a height h above the floor has *potential energy*, because if it were allowed to fall, it could do work. The amount of potential energy is given by the formula

   $$\text{PE} = mgh, \tag{1.9}$$

   where h is its height above the floor.

3. A spring that has been stretched (or compressed) has potential energy, because if allowed to relax it can do work. If we use the constant K to denote its spring constant ("stiffness"), the potential energy when it is stretched an amount y from its relaxed length is given by the formula

$$\mathrm{PE} = \tfrac{1}{2} K y^2. \tag{1.10}$$

4. A bottle of gas of volume V whose pressure exceeds atmospheric pressure P_0 by a small amount p has potential energy

$$\mathrm{PE} = \frac{1}{2}\frac{V}{P_0} p^2. \tag{1.11}$$

5. A guitar string displaced a small distance y at its midpoint has potential energy

$$\mathrm{PE} = \frac{2T}{L} y^2, \tag{1.12}$$

where T is the tension in the string and L is its length. When the string is released, this energy is changed into kinetic energy, and thereafter the energy changes back and forth between kinetic and potential. The energy of vibrating systems will be discussed in Chapter 2.

Often the analysis of motion is facilitated by considering the way in which one form of energy is converted into another. For example, if we lift a heavy object we do work, and we give the object potential energy (PE $= mgh$). If we allow the object to free-fall, it acquires kinetic energy (KE $= \frac{1}{2}mv^2$). The speed it acquires in falling a distance h can easily be calculated by equating gain of KE to loss of PE without the need to calculate the time of fall:

$$\begin{aligned} \tfrac{1}{2}mv^2 &= mgh \\ v^2 &= 2gh \\ v &= \sqrt{2gh}. \end{aligned} \tag{1.13}$$

In describing vibrating systems in Chapter 2, we will make use of the fact that such systems continually convert potential energy to kinetic energy, and vice versa. Twice during each cycle of oscillation the energy is all kinetic, and twice it is all potential; at other times, the total energy is shared between potential and kinetic forms. As an oscillator slows down due to friction, the mechanical energy decreases because some of it is converted to another familiar form of energy: *heat*.

1.10 ■ POWER

Note that the definition of work (force × distance) says nothing about the time during which work is done. Raising a 2-kg mass to a height of 1 m requires 19.6 J of work whether the task is done in 1 s or 10. If the task were done in 4 s, for example, the average rate at which work is done would be 4.9 joules/second (J/s). The rate at which work is done is called *power*. Power is simply work divided by time:

$$\mathcal{P} = \frac{\mathcal{W}}{t}. \tag{1.14}$$

Power can be expressed in J/s, but this unit is used so frequently that it is given a special name, the *watt* (abbreviated W). Electrical equipment is rated according to the number of watts of electrical power it requires. A 100-W lamp converts electrical energy (to heat and light) at the rate of 100 J/s. The electric company, which sells electrical energy, installs meters that indicate how much electrical energy has been consumed. Instead of using joules (watt-seconds), however, the billing unit is the kilowatt-hour, the amount of work done by one kilowatt of power in one hour. One kilowatt-hour (kWh) equals $1000 \times 60 \times 60 = 3.6 \times 10^6$ J.

In the British system of units, work and energy are measured in *foot-pounds* and power in *horsepower*. One horsepower equals 550 ft = lb/s, which is roughly the rate at which a horse can do work. One horsepower equals 745.7 W.

EXAMPLE 1.7 A guitar string 65 cm long and having a tension of 55 N is displaced 8 mm at its midpoint. How much potential energy does it have?

Solution

$$\mathrm{PE} = \frac{2(55\ \mathrm{N})}{(0.65\ \mathrm{m})}(8 \times 10^{-3}\ \mathrm{m})^2 = 0.11\ \mathrm{J}.$$

EXAMPLE 1.8 How much power is needed to raise a 2-kg mass to a height of 3 m in 15 s?

Solution

$$\begin{aligned}\mathcal{P} &= \frac{\mathcal{W}}{t} = \frac{mgh}{t} = \frac{(2\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(3\ \mathrm{m})}{15\ \mathrm{s}}\\ &= 3.92\ \mathrm{W}.\end{aligned}$$

1.11 UNITS

The preferred system of units for expressing physical quantities is the SI (Système International), or mks (meter-kilogram-second), system. Besides these three basic units, the system uses such units as newtons, joules, and watts, which are derived in a logical manner from the basic units. (For example, a newton equals kilograms × meters/seconds^2.) The SI, or mks, system is described in Appendix A.1.

Another metric system in use, the not as commonly as the mks system, is the cgs system, based on the centimeter, gram, and second. The cgs system is also described in Appendix A.1.

The fps (foot, pound, second) system of units is still very much in use in the United States, although its popularity is declining as we aim toward a conversion to metric units. Besides the basic units of foot, pound, and second, the fps system uses slugs, foot-pounds, horsepower, etc., as described in Appendix A.1.

1.12 ■ SUMMARY

Sound can refer to either an auditory sensation in the ear or the disturbance in a medium that causes this sensation. Sound is carried by waves in a solid, liquid, or gas. Unwanted sound is generally referred to as noise.

The motion of an object can be described by expressing its distance, speed, and acceleration as functions of time. A graphical representation of the motion consists of a plot of distance, speed, or acceleration as a function of time. Two other basic quantities, force and mass, are related to acceleration through Newton's second law of motion: $F = ma$. Pressure is the force per unit area, and is especially important in describing the behavior of objects immersed in a liquid or gas.

Another useful description of an object expresses its kinetic energy (energy of motion) and potential energy (stored energy) as functions of time. Power is the rate at which energy is expanded or the rate at which work is done. The preferred system of units is the mks (SI) system, which uses meters, kilograms, and seconds as its basic units, but also includes derived units such as newtons, joules, watts, and so forth.

REFERENCES AND SUGGESTED READINGS

Appel, K., J. Gastineau, C. Bakken, and D. Vernier (1998). *Physics with Computers*. Portland, Ore.: Vernier Software.

Hall, D. E. (1991). *Musical Acoustics*, 2nd ed. Pacific Grove, Calif.: Brooks/Cole.

Hewitt, P. G. (1993). *Conceptual Physics*, 7th ed. New York: Harper Collins.

Kirkpatrick, L. D., and G. F. Wheeler (1998). *Physics, A World View*, 3rd ed. Forth Worth: Saunders.

Rossing, T. D. (1982). *Acoustics Laboratory Experiments*. DeKalb: Northern Illinois University.

GLOSSARY

acceleration The rate of change of speed or velocity.

coordinates A set of numbers used to locate a point along a line or in space.

force An influence that can deform an object or cause it to change its motion.

gravity The force exerted by the Earth on all objects on or near it.

joule A unit of energy or work; one joule is equal to one newton-meter, also one watt-second.

kinetic energy Energy of motion; the capacity to do work by virtue of that motion; equal to one half mass times velocity (or speed) squared.

mass A measure of resistance to change in motion; equal to force divided by acceleration.

newton A unit of force.

potential energy Stored energy; the capacity to do work by virtue of position.

power The rate of doing work; equal to work or energy divided by time.

pressure Force divided by area.

speed The rate at which distance is covered; equal to distance divided by time.

stroboscope A light that flashes at a regular rate, making possible a photographic record of motion.

watt A unit of power; equal to one joule per second.

work The net force on an object times the distance through which the object moves.

Δ The Greek letter $delta$, denoting change in some quantity.

REVIEW QUESTIONS

1. What are two different meanings of the word *sound*?
2. What is the science of sound generally called?
3. What is the difference between a longitudinal and a transverse wave? Give an example of each.
4. What are four different processes that can produce sound? Give an example of each.
5. What is the difference between *speed* and *velocity*?
6. The slope of a graph of position versus time is equal to what quantity?
7. What three quantities are related by Newton's second law of motion?
8. Describe the motion of an object when no net force is applied.
9. Arrange the following in order from largest to smallest: 0.004 m, 0.4 mm, 4×10^{-5} km, 4×10^{-5} μm.
10. What is the difference between *pressure* and *force*?
11. Compare the pressure on the top and the bottom sides of a thin plate immersed in water.
12. What is the pressure of the atmosphere on our bodies?
13. What is a waveform of a sound?
14. What unit is used to express energy? work?
15. What is *kinetic energy*? *potential energy*?
16. Give a formula for the potential energy of a displaced guitar string and explain each symbol.
17. What is the difference between *power* and *energy*?
18. When you pay your electricity bill, are you paying for power used or for energy used?

QUESTIONS FOR THOUGHT AND DISCUSSION

1. At the same time a rifle is fired in an exactly horizontal position over level ground, a bullet is dropped from the same height. Both bullets strike the ground at the same time. Can you explain why?
2. What are some advantages of using the metric (SI) system of units rather than the English system?
3. In the sixteenth century, Galileo is said to have dropped objects of various weights from the Leaning Tower of Pisa. Since all objects in *free fall* accelerate at 9.8 m/s^2, one would expect them to reach the ground at the same time. Careful observation, however, indicates that an iron ball will strike the ground sooner than a baseball of the same diameter. Can you explain why? Would the same be true on the moon? (The Apollo astronauts actually photographed a free-fall experiment on the moon using a hammer and a feather.)
4. Think of an object comparable in size to each of the following:
(a) 10^7 m; **(b)** 10^3 m; **(c)** 1 m; **(d)** 10^{-3} m; **(e)** 10^{10} m.
5. Does shifting to a lower gear increase the power of an automobile? Explain.
6. Draw a diagram, similar to Fig. 1.10, showing how pressure acts on a floating object.

EXERCISES

1. Letting your classroom serve as the "origin" ($x = 0$, $y = 0$), express the approximate coordinates (x, y) of your place of residence. Let $x =$ the distance east and $y =$ the distance north, as on a map. Use any convenient unit of distance.
2. The speed of a bicycle increases from 5 mi/h to 10 mi/h in the same time that a car increases its speed from 50 mi/h to 55 mi/h. Compare their accelerations.
3. The density of water is 1.00 g/cm^3 and that of ice is 0.92 g/cm^3. What are the corresponding densities in SI units (kg/m^3)?
4. If the speed limit is posted as 55 mi/h, express this in km/h and in m/s (1 mi = 1.61 km).
5. A car accelerates from rest to 50 mi/h in 12 s. Calculate its average acceleration in m/s^2. Compare this to the acceleration of an object in free fall (1 mi/h = 0.447 m/s).
6. An object weighing 1 lb (English units) has a mass of 0.455 kg. Express its weight in newtons and thereby express a conversion factor for pounds to newtons.
7. Express your own mass in kilograms and your weight in newtons.
8. Calculate average speed in each of the following cases:

(a) An object moves a distance of 25 m in 3 s.

(b) A train travels 2 km, the first at an average speed of 50 km/h and the second at an average speed of 100 km/h. (*Note:* The average speed is not 75 km/h.)

(c) A runner runs 1 km in 3 min and a second kilometer in 4 min.

(d) An object dropped from a height of 75 m strikes the ground in 4 s.

9. Estimate the total force on the surface of your body due to the pressure of the atmosphere.

10. Calculate the kinetic energy of a 1500-kg automobile with a speed of 30 m/s. If it accelerates to this speed in 20 s, what average power has been developed?

11. An electric motor, rated at $\frac{1}{2}$ horsepower, requires 450 W of electrical power. Calculate its efficiency (power out divided by power in). What happens to the rest of the power?

12. Calculate the potential energy of:

(a) A 3-kg block of iron held 2 m above the ground;

(b) A spring with a spring constant $K = 10^3$ N/m stretched 10 cm from its equilibrium length;

(c) A 1-L bottle ($V = 1000\ \text{cm}^3$) with a pressure 10^4 N/m^2 above atmospheric pressure ($P = 10^5$ N/m^2).

EXPERIMENTS FOR HOME, LABORATORY, AND CLASSROOM DEMONSTRATION

Home and Classroom Demonstration

1. *Longitudinal waves on a coiled spring (Slinky)* For best results, suspend a Slinky from a long horizontal stick or rod by attaching several strings (about a meter in length). However, a giant Slinky will work satisfactorily on a smooth polished floor in spite of a small amount of friction. Jerk one end of the Slinky in the direction to increase its length and observe the pulse wave that propagates. Produce a small pulse and a large pulse in rapid succession. Does the distance between the two pulses change as they travel down the spring? What does this indicate about the relationship between amplitude (pulse size) and wave speed? Generate a series of waves by smoothly increasing and decreasing its length.

Repeat the experiment with transverse rather than longitudinal pulses and waves.

2. *Siren disk* Blow air through a siren disk. If none is available, you can construct one by drilling regularly spaced holes in a wooden disk attached to a rotator. Note that the pitch of the tone depends upon the speed of rotation of the disk, whereas the loudness is determined by the rate of airflow.

3. *Moving object stroboscopically observed* In a partially or totally dark room, observe a white ball in stroboscopic light. (If none is available, a hand stroboscope can be constructed by cutting slots around the circumference of a disc mounted on a dowel rod with a finger hole for rotating it). Roll the ball on a table or other horizontal surface and compare what you see to Fig. 1.3(a). Roll the ball down an incline and compare what you see to Fig. 1.3(b).

Observe a mass oscillating on the end of a spring, and see if you can make it appear to stand still by adjusting the rate of your stroboscope (either the flashing light or the hand stroboscope).

4. *Moving-object video capture* Make a video recording of a moving object. Use a VCR with a single-frame player or a "frame grabber" to transfer single frames to a computer. Measure the distance the object has moved between successive frames.

5. *Falling object stroboscopically observed* Observe a falling object in stroboscopic light (or with video capture) and compare what you see to Fig. 1.8(a). Toss a ball upward at an angle and compare what you see to Fig. 1.8(b).

6. *U-tube manometer* Attach a length of rubber tubing to a U-shaped glass tube filled with colored water placed in front of a meter stick. The difference in heights of the water in the two sides of the U-tube represents the pressure in cm of water (a unit commonly used by organ builders). To convert cm of water to newtons/meter^2 or pascals (Pa), multiply by 100. Calibrate your lungs by blowing and sucking to obtain 100 cm of water (10^4 Pa) above and below atmospheric pressure. Which is easier to do?

7. *Deciding if pressure in a container depends upon the amount of water in the container* Place the end of the tubing attached to a manometer at various depths in a cylinder of water and show that the pressure (in cm of water) is equal to the depth of the tube below the surface. Repeat with containers of varying size and shape to show that the pressure depends only of the depth below the surface, regardless of the shape of the container or how much water it holds.

8. *Force on a container wall* Blowing a collapsed varnish can back to shape demonstrates the relationship of force to pressure. Measure the area of the large side of the can and multiply by the pressure difference inside and outside (indi-

cated on the manometer) to obtain the net force on the collapsed side of the can.

9. *Lifting a concrete block by blowing into a beach ball* A concrete block can be lifted by blowing air into a beach ball. A manometer indicates the pressure in the ball during and after inflation. (How much blowing pressure would be required to inflate an air mattress if someone is already lying on in?)

10. *Air pressure on a newspaper* Cover most of a thin board on a table with a sheet of newspaper. Strike the end of the board with the fist and note that the inertia of the air mass inhibits movement of the newspaper.

11. *Sound waveforms* Connect a microphone to a cathode-ray oscilloscope or a PC with a sound-input card and display sound waveforms (graphs of sound pressure versus time) on the screen. Speak and sing various vowel sounds. An "oo" sound sung in falsetto voice produces a very smooth waveform, for example. Try various musical instrument sounds.

12. *Human power* Run up a flight of stairs as fast as you can and time yourself with a stopwatch, a watch with a sweep second hand, or a digital watch. Measure the height of the stairway (number of steps times the height of each). Calculate the work done (using Eq. (1.7)) and the average power (using Eq. (1.14)). For your information, 1 hp equals 746 W (How does your power compare to that of a horse?)

Laboratory Experiments

Accelerated Motion (Experiment 1 in *Acoustics Laboratory Experiments*)

Graph Matching (Experiment 1 in *Physics with Computers*)

Picket Fence Free Fall (Experiment 5 in *Physics with Computers*)

Newton's Second Law (Experiment 9 in *Physics with Computers*)

CHAPTER

2 Vibrating Systems

Nature provides many examples of vibrating systems: trees swaying in the wind, atoms in a molecule of water, the motion of the tides, electric current in a flash of lightning, and so on. Although the motion is vastly different in each of these systems, they have several things in common: For one thing, the motion repeats in each regular time interval, which we call the *period* of the vibration; second, some type of *force* constantly acts to restore the system toward its point of equilibrium.

In this chapter you should learn:

- About simple vibrating systems;
- About vibrating systems with two or more masses;
- About vibrations in musical instruments;
- About vibration spectra.

2.1 ■ SIMPLE HARMONIC MOTION

Consider the very simple vibrating system in Fig. 2.1 consisting of a mass m attached to the end of a spring. We assume that the amount of stretch in the spring is proportional to the stretching force (which is true of most springs if they are not stretched too far), so that in order to stretch it a length l, a force Kl is required. The symbol K is the *spring constant*, or stiffness, of the spring.

Because the spring is vertical, the force of gravity on the mass stretches the spring by an amount that remains constant. In the equilibrium position shown in Fig. 2.1(b), the downward force of gravity on the mass (its weight) is just balanced by the upward force exerted by the spring; therefore, the system is in equilibrium. The description of the motion is simplified if we specify the displacement of y of the mass from the equilibrium position and the net force F that acts on the mass. The relationship between F and y is easily shown to be

$$F = -Ky. \tag{2.1}$$

The minus sign in this equations reminds us that when the mass is below its equilibrium position (y is negative), the net force will be upward (F is positive), as shown in Fig. 2.1(a). Thus the force F could be called a *restoring force*, which always acts in a direction to restore m to its equilibrium position. When the restoring force is proportional to the displacement, as it is in most vibrating systems we study, the motion is given a special name: *simple harmonic motion*. For a system in simple harmonic motion, the period

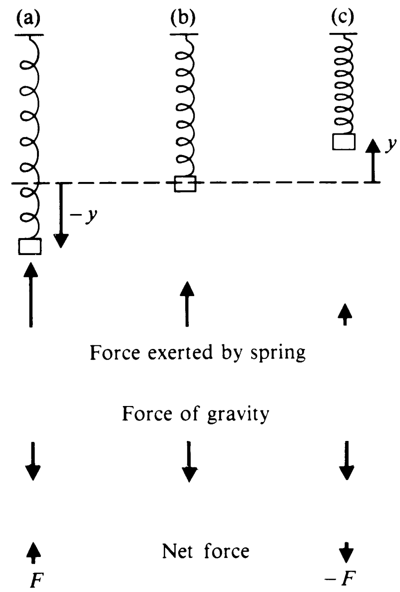


FIGURE 2.1
A simple vibrator consisting of mass and spring. In (b) the upward force exerted by the spring and the force of gravity balance each other, and the net force F on the mass is zero.

is independent of the amplitude (size) of the vibration. The *frequency* f of vibration is the number of oscillations per second, which is obviously the reciprocal of the period T of one vibration:

$$f = 1/T. \tag{2.2}$$

It is customary to use a unit called the *hertz* (abbreviated Hz) to denote cycles per second. In the case of the vibrating mass-spring system, the frequency of vibration is given by the formula

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}. \tag{2.3}$$

Note that to double the frequency of vibration, the mass m may be reduced to one-fourth its original size, or the spring constant K may be made four times larger.

EXAMPLE 2.1 Suppose a certain spring stretches 0.10 m when loaded with 2 kg. What is its spring constant? At what frequency will it vibrate when loaded with 2 kg? 0.5 kg?

Solution At rest $Kl = mg$, so

$$K = \frac{mg}{l} = \frac{2(9.8)}{0.10} = 196 \text{ N/m} \quad \text{(newtons per meter)}.$$

Its frequency of vibration when $m = 2$ kg is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{(2)(3.14)}\sqrt{\frac{196}{2}} = \frac{\sqrt{98}}{6.28} = 1.6 \text{ Hz}.$$

When loaded with 0.5 kg, the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{(2)(3.14)}\sqrt{\frac{196}{0.5}} = \frac{\sqrt{392}}{6.28} = 3.2 \text{ Hz}.$$

A strobe photograph of a mass-spring system was shown in Fig. 1.9. Also shown was a graph of position y as a function of time. A graph of speed v versus time can be made by taking the slope of that graph at every time t. Graphs of y and v as functions of time are shown in Fig. 2.2. Mathematicians refer to curves shaped like these as *sinusoidal* or *sine* curves. The maximum value of y is called the *amplitude*.

2.2 ■ ENERGY AND DAMPING

The formula for the kinetic energy KE of a moving mass was given in Section 1.9:

$$\text{KE} = \tfrac{1}{2}mv^2, \tag{2.4}$$

where m is mass and v is speed. Similarly, the potential energy PE of a spring, stretched or compressed a distance y from its equilibrium length, was given as

$$\text{PE} = \tfrac{1}{2}Ky^2. \tag{2.5}$$

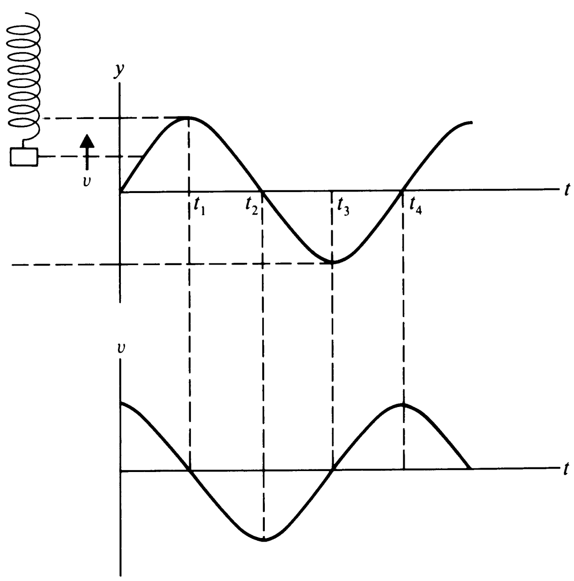


FIGURE 2.2 Graphs of simple harmonic motion: (a) displacement versus time; (b) speed versus time. Note that speed reaches its maximum when displacement is zero and vice versa.

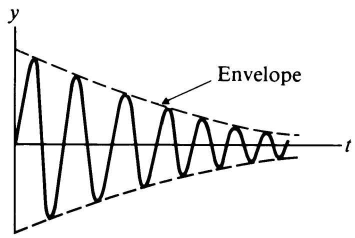


FIGURE 2.3 Displacement of a damped vibrator whose amplitude decreases with time.

From the graphs in Fig. 2.2, it is clear that v^2 reaches its maximum value when y^2 is zero, and vice versa. Thus, the total mechanical energy is constantly changing from kinetic to potential to kinetic. At times t_1, and t_3, potential energy is a maximum, and at t_2 and t_4, kinetic energy is a maximum.

Any real vibrating system tends to lose mechanical energy as a result of friction and other loss mechanisms. Unless the energy is renewed in some way, the amplitude of the vibrations will decrease with time, as shown in Fig. 2.3. In many vibrating systems, a certain fraction (usually small) of the energy is lost during each cycle of vibration; the result is a curve that decreases in amplitude in the manner shown in Fig. 2.3. The dashed curve, which indicates the change in amplitude with time, is called the *envelope*, or decay curve. A vibrating system whose amplitude decreases in this way is said to be *damped*, and the rate of decrease is the *damping constant*.

2.3 ■ SIMPLE VIBRATING SYSTEMS

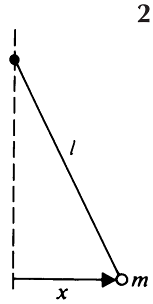


FIGURE 2.4 A simple pendulum.

Besides the mass-spring system already described, the following are examples of systems that vibrate in simple harmonic motion:

1. *Pendulum (small angle).* A simple pendulum, consisting of a mass m attached to a string of length l (see Fig. 2.4), vibrates in simple harmonic motion, provided that $x \ll l$. Assume that the mass of the string is much less than m, the frequency of vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}, \tag{2.6}$$

where g is the acceleration due to gravity. Note that the frequency does not depend on the mass.

2. *A spring of air.* A piston of mass m, free to move in a cylinder of area A and length l, vibrates in much the same manner as a mass attached to a spring (see Fig. 2.5). The spring constant of the air in the cylinder is determined by its compressibility and turns out to be $K = \gamma pA/l$, so the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma pA}{ml}}, \tag{2.7}$$

where p is the gas pressure, A is the area, m is the mass of the piston, and γ is a constant that is 1.4 for air.

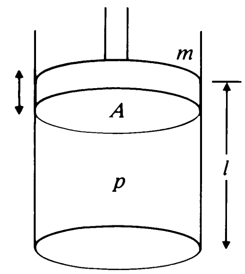


FIGURE 2.5
A piston free to vibrate in a cylinder.

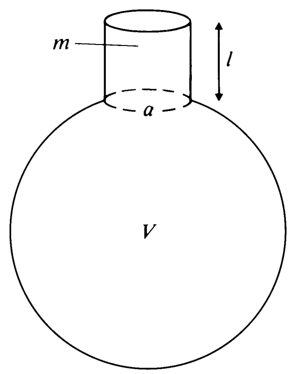


FIGURE 2.6
A Helmholtz resonator.

3. *A Helmholtz resonator*. Another common type of air vibrator, illustrated in Fig. 2.6, is often called a Helmholtz resonator, after H. von Helmholtz (1821–1894), who used it to analyze musical sounds. The mass of air in the neck now serves as the piston, and the air in the larger volume V as the spring. The frequency of vibration is

$$f = \frac{v}{2\pi}\sqrt{\frac{a}{Vl}}, \tag{2.8}$$

where a is the area of the neck, l is its length, V is the volume of the resonator, and v is the speed of sound ($v \simeq 344$ m/s).

The Helmholtz resonator can be thought of as having a mass m and a spring constant K that are

$$m = \rho a l \quad \text{and} \quad K = \frac{\rho a^2 v^2}{V}, \tag{2.9}$$

where ρ is the density of air.

Helmholtz resonators can have a variety of shapes and sizes. For example, blowing air across an empty pop bottle causes the air in its neck to vibrate at a fairly low frequency.

Note that the smaller the neck area a, the lower the frequency of vibration, which may seem a little surprising at first glance.

EXAMPLE 2.2 A small flask consists of a sphere 9.8 cm in diameter plus a neck 3 cm in diameter and 10 cm long. At what frequency will it resonate?

Solution

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}(3.14)(0.049\text{ m})^3$$
$$= 4.93 \times 10^{-4}\text{ m}^3;$$
$$a = \pi r^2 = 3.14(0.015)^2 = 7.07 \times 10^{-4}\text{ m}^2.$$
$$f = \frac{v}{2\pi}\sqrt{\frac{a}{Vl}} = \frac{344\text{ m/s}}{2(3.14)}\sqrt{\frac{7.07 \times 10^{-4}\text{ m}^2}{(4.93 \times 10^{-4}\text{ m}^3)(0.10\text{ m})}}$$
$$= 207\text{ Hz}.$$

2.4 ■ SYSTEMS WITH TWO OR THREE MASSES

The vibrating systems considered in the preceding section have one thing in common: A single coordinate is sufficient to describe their motion. In other words, they have one *degree of freedom*. In this section, we will consider vibrators with two or more degrees of freedom. Such systems have more than one natural *mode* of vibration, and the different modes will generally have different frequencies.

Consider the system consisting of two masses and three springs shown in Fig. 2.7. The system has two "normal," or independent, modes, as shown in Fig. 2.7(a) and 2.7(b). In one mode, the masses move in the same direction; in the other, they move in opposite directions. Assuming equal masses and springs with the same stiffness K, the frequencies of the two modes are

$$f_a = \frac{1}{2\pi}\sqrt{\frac{K}{m}}, \qquad f_b = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}. \tag{2.10}$$

Note that mode (a) has the same frequency as the simple mass-spring system shown in Fig. 2.1, whereas mode (b) has a frequency that is 1.7 times that of mode (a).

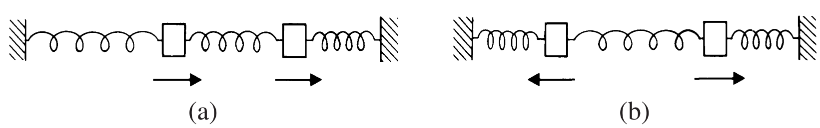
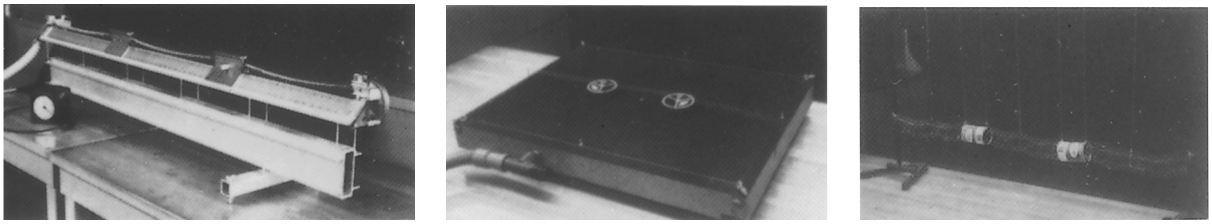


FIGURE 2.7 Modes of vibration of a two-mass vibrator. The mode shown in (a), in which the masses move in the same direction, will have the lower frequency.



(a) (b) (c)

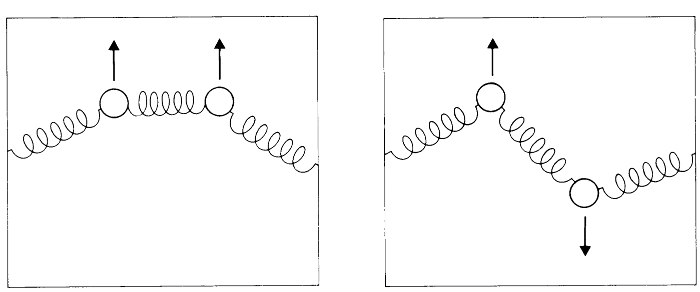
FIGURE 2.8 Two-mass vibrators using (a) a linear air track; (b) an air table; (c) masses and springs hung from an overhead rod.

Modes (a) and (b) are virtually independent of each other. That is, the system can vibrate in mode (a) with minimal excitation of mode (b), and vice versa. If one sets the system into oscillation by giving the two masses a push or pull, the resulting motion will nearly always be a combination of modes (a) and (b). There are many recipes for combining these two modes in different proportions, and thus many ways in which the system can vibrate.

A great deal about the physics of vibration can be learned from watching the motion of a two-mass vibrator. Many physics laboratories have linear air tracks or air tables, on which objects move on a film of air with negligible friction. These are ideal for studying two-mass oscillators. Another convenient arrangement is to hang the masses on long cords from the ceiling or an overhead rod. The cords must be as long as possible to minimize the tendency of the masses to swing like pendulums. These three arrangements are shown in Fig. 2.8.

On the linear air track shown in Fig. 2.8(a), the masses are constrained to move in one direction only. In the systems shown in Fig. 2.8(b) and 2.8(c), however, the masses can move at right angles to the springs as well. Vibrations in this direction are called *transverse* vibrations, whereas vibrations in the direction of the springs are called *longitudinal* vibrations. Some systems vibrate only in transverse modes, some only in longitudinal. The air column of a musical wind instrument, for example, vibrates longitudinally, whereas the membrane of a drum vibrates transversely. A violin string normally vibrates transversely, although longitudinal vibrations (which sound like squeaks or squeals) are occasionally excited by the bowing of unskilled players.

In addition to their two modes of longitudinal vibration, the two-mass systems shown in Figs. 2.8(b) and 2.8(c) have two modes of transverse vibration, which are shown in Fig. 2.9. In the mode of lower frequency, the masses move in the same direction; in the



(a) (b)

FIGURE 2.9 Modes of transverse vibration of a two-mass system. (a) In the mode of lower frequency, masses move in the same direction; (b) in the mode of higher frequency, masses move in opposite directions.

mode of higher frequency, they move in opposite directions. This behavior is similar to the longitudinal modes shown in Fig. 2.7.

Adding a third mass to the systems of Fig. 2.7 adds additional modes of vibration. In the case of the linear vibrator in Fig. 2.7(a), which vibrates only longitudinally, a third mode of longitudinal vibration appears. The three independent modes of vibration are those shown in Fig. 2.10. The systems in Figs. 2.7(b) and 2.7(c) can vibrate transversely as well; in addition to the three modes of longitudinal vibration in Fig. 2.10, they will have the three independent modes of transverse vibration shown in Fig. 2.11.

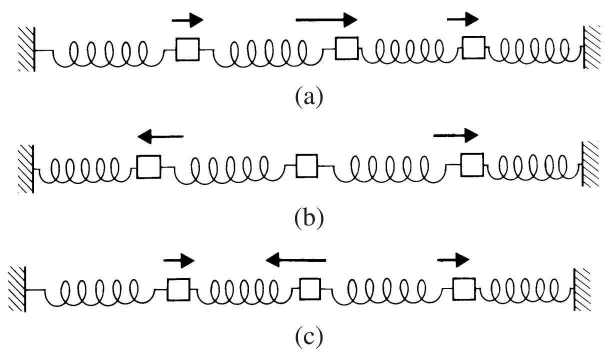


FIGURE 2.10 Independent modes of longitudinal vibrations of a three-mass vibrator.

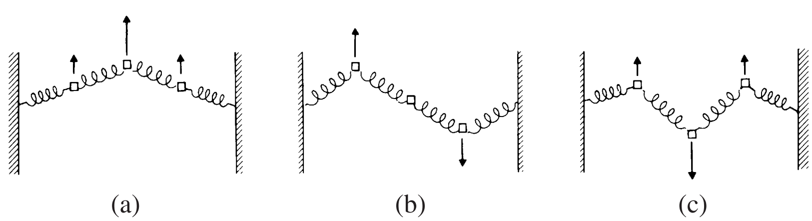


FIGURE 2.11 Independent modes of transverse vibration of a three-mass oscillator.

The independent modes shown in Figs. 2.7, 2.9, 2.10, and 2.11 are often called the *normal* modes of the vibrating systems. Getting the system to vibrate in a single normal mode requires special care. It is perhaps best done by driving the system at the frequency of the desired mode (this phenomenon, called *resonance*, will be discussed in Chapter 4). Carefully displacing the masses by the proper amounts and releasing them will also cause the system to vibrate in a single mode.

EXAMPLE 2.3 The vibrating system in Fig. 2.7 consists of two 0.5-kg masses and three springs having spring constants of 50 N/m. Find the frequencies of its vibrational modes.

Solution

$$f_a = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{50\ \text{N/m}}{0.5\ \text{kg}}} = 1.59\ \text{Hz};$$

$$f_b = \frac{1}{2\pi}\sqrt{\frac{3(50)}{0.5}} = 2.76\ \text{Hz}.$$

2.5 ■ SYSTEMS WITH MANY MODES OF VIBRATION

In the case of the mass-spring vibrating systems, each new mass added one new mode of longitudinal vibration and one new mode of transverse vibration, if the system was able to vibrate it transversely. In general, a system of N masses of the type shown in Fig. 2.8(b) or 2.8(c) will have N longitudinal and N transverse modes of vibration. If the masses were free to move in all three coordinate directions, there would be $2N$ transverse modes of vibration and N longitudinal modes, where N is the number of masses. The number of frequencies associated with the transverse modes may be only N, however, because corresponding modes in two directions usually have the same frequency.

The transverse modes of vibration for 1, 2, 3, 4, 5, and 24 masses are sketched in Fig. 2.12. Note that in each case the number of transverse modes equals the number of masses. There are an equal number of longitudinal modes, but they are more difficult to represent in a diagram. In each case, the mode of highest frequency is the one in which adjacent masses move in opposite directions.

Note that as the number of masses increases, the system takes on a wavelike appearance. In fact, a vibrating guitar string can be thought of as a mass-spring system with very large N. The propagation of waves on a string will be considered in Chapter 3.

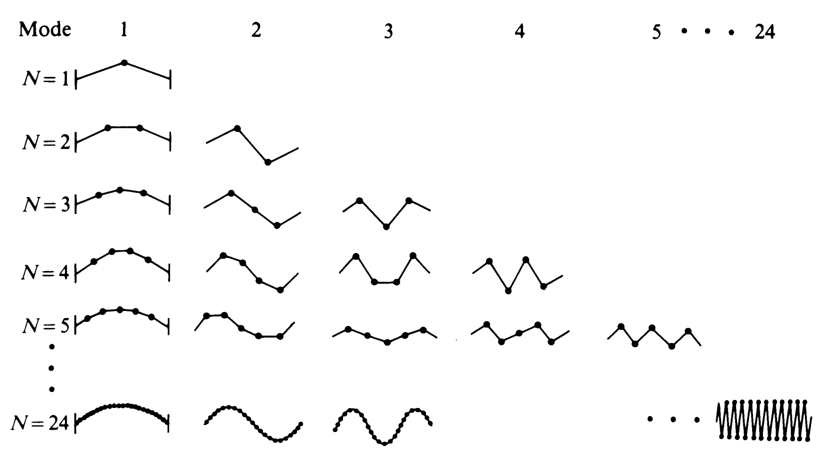


FIGURE 2.12 Modes of transverse vibration for mass-spring systems with different numbers of masses. A system with N masses has N modes.

2.6 ■ VIBRATIONS IN MUSICAL INSTRUMENTS

All musical sound is generated by some type of vibrating system, whether it is a string on a violin, the air column of a trumpet, the head of a drum, or the voice coil of a loudspeaker. Often the vibrating system consists of two or more vibrators that work together, such as the reed and air column of a clarinet, the strings and sounding board of a piano, or the strings and body of a guitar. The acoustics of musical instruments is the subject of Part 3, but a brief description of several common musical vibrators will be made in closing this chapter on vibrating systems.

1. *Vibrating string*. The vibrating string can be thought of as the limit of the mass-spring system (see Fig. 2.11) when the number of masses becomes very large. The string itself has mass and elasticity or "springiness." There will be many modes of vibration, and their frequencies turn out to be very nearly whole-number multiples of the frequency of the lowest or *fundamental* mode. When the higher modes have frequencies that are whole number multiples of the fundamental frequency, we call them *harmonics*. Several modes of a vibrating string are illustrated in Fig. 2.13. The guitar (Fig. 2.14), for example, uses vibrating strings.

2. *Vibrating membrane*. Drumheads are membranes of leather or plastic stretched across some type of tensioning hoop or frame. A membrane can be thought of as a two-

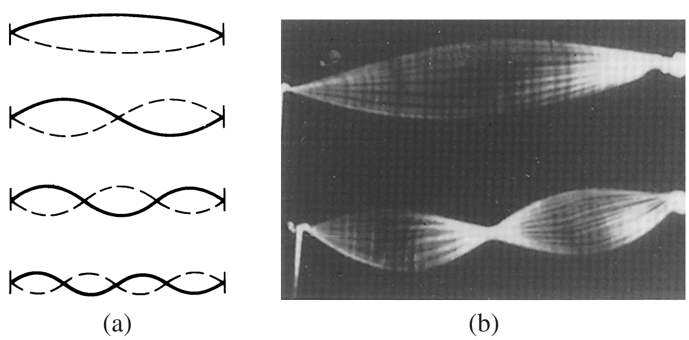


FIGURE 2.13
(a) Modes of a vibrating string; (b) strobe picture of a string vibrating in its lowest two modes.

FIGURE 2.14
A guitar. Coupling between strings, wood plates, and enclosed air leads to many modes of vibration, which will be discussed in Chapter 10.

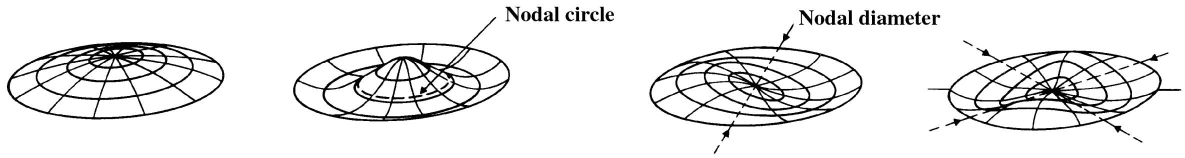


FIGURE 2.15 Modes of circular membrane. The first two modes have circular symmetry; the second two do not. (From Morse and Ingard, 1968.)

dimensional string in that its restoring force is due to tension applied from the edge. A membrane, like a string, can be tuned by changing the tension. Membranes, being two-dimensional, can vibrate in many modes that are not normally harmonic. Four modes of vibration of a circular membrane are illustrated in Fig. 2.15. The first two have circular symmetry; the second two have *nodal* lines (indicated by the arrows), which act as pivots for a rocking motion. Three familiar examples of drums that use vibrating membranes to produce sound are shown in Fig. 2.16.

3. *Vibrating bar*. Many percussion instruments use vibrating bars as sound sources. The stiffness of a bar provides the restoring force when it bends, so no tension need be applied. Thus the ends may be free, as they are in most percussion instruments, or clamped (see Fig. 2.17). The frequencies of the vibrational modes of a uniform bar with free ends (as

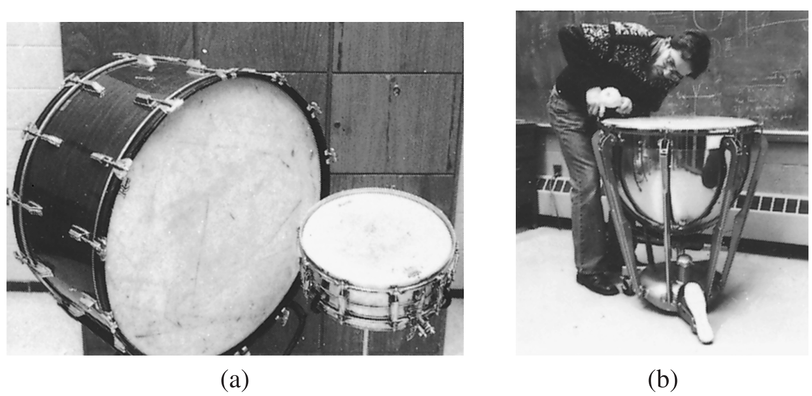


FIGURE 2.16 (a) Bass drum and snare drum; (b) timpani.

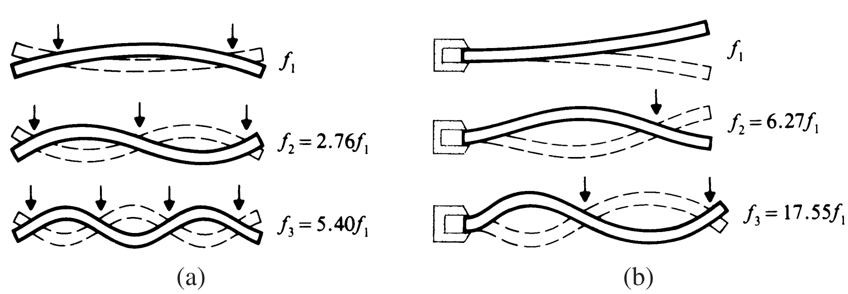


FIGURE 2.17 Nodes of vibrating bars: (a) both ends free; (b) one end clamped. Arrows locate the nodes.

FIGURE 2.18
A xylophone.
(Courtesy of
J. C. Deagan Co.)

the bars of a glockenspiel, for example) have the ratios 1 : 2.76 : 5.40 : 8.93, etc., which are nowhere near harmonic. The bars of marimbas, xylophones (see Fig. 2.18), and other instruments, however, have been shaped to have a quite different set of mode frequency ratios. Bars can also vibrate longitudinally, but longitudinal modes are not normally used in musical instruments.

4. *Vibrating plate*. Vibrating plates, like vibrating bars, depend on their own stiffness for the necessary restoring force. Plates have many modes of vibration, some exhibiting great complexity.

An interesting way to study the modes of vibration of plates is through the use of Chladni patterns, first described by E. F. F. Chladni in 1787. Particles of salt or sand are sprinkled on a vibrating plate, which is then excited to vibrate in one of its normal modes. The particles, agitated by the vibrations, tend to collect along nodal lines, where the vibrations are minimal. Chladni patterns of a circular plate are shown in Fig. 2.19.

Three musical instruments that use vibrating plates are shown in Fig. 2.20.

5. *Tuning fork*. A tuning fork consists of two bars joined together at one end. Thus the modes of vibration will resemble those of a bar clamped at one end, as illustrated in Fig. 2.17(b). Tuning forks are very convenient standards of frequency; once adjusted, they maintain their frequency for a long time. The frequency of a tuning fork may be raised by shortening its length or by removing material near the ends of the prongs. The frequency

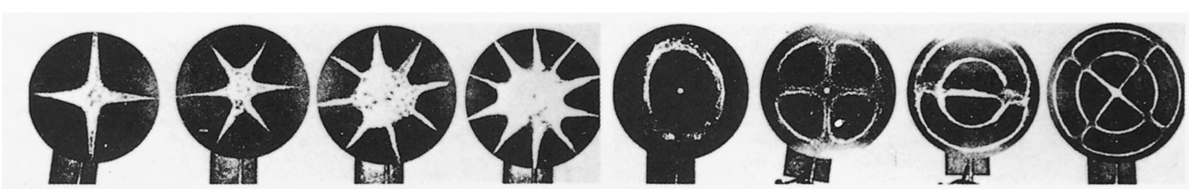
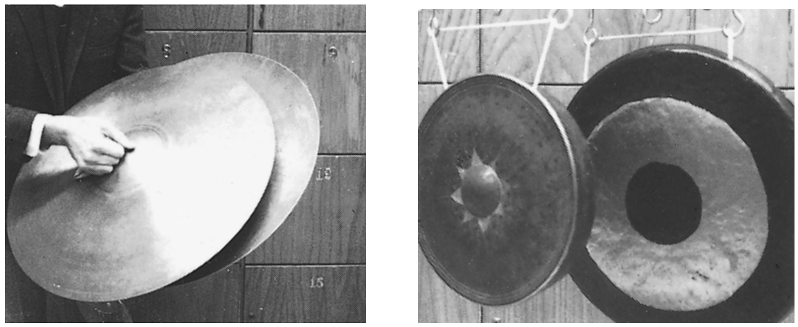
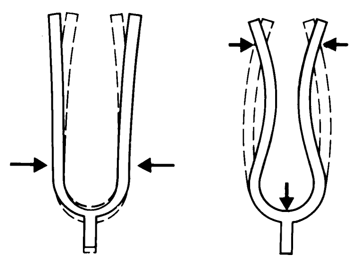


FIGURE 2.19 Chladni patterns of a circular plate. The first four have two, three, four, and five nodal lines but no nodal circles: the second four have one or two nodal circles. (Rossing, 1977).



(a) (b)

FIGURE 2.20
(a) Cymbals;
(b) gong (left) and
tam tam (right).



(a) (b)

FIGURE 2.21
Vibrations of a tuning fork:
(a) principal mode;
(b) "clang" mode, which occurs at a higher frequency than the principal mode.

can be lowered by removing material near the base of the prongs, which decreases the stiffness.

As shown in Fig. 2.21, two modes of vibration of the tuning fork are the principal mode and the "clang" mode, which occurs at a much higher frequency (nearly three octaves higher in a typical fork). In their normal motion, the bars pivot about two nodes marked by arrows, causing the handle to move up and down. Thus, if the handle is pressed against another object (e.g., a table top), it may cause that object to act as a sounding board. (In a noisy environment, the handle may be touched to one's forehead in order to conduct sound directly to the inner ear.)

6. *Air-filled pipes*. The vibrational behavior of a column of air, as found in an organ pipe or the bore of a trumpet, can be compared to that of the air spring we discussed in Section 2.3, but is better understood by considering the sound waves within it. Thus we leave the discussion of this type of musical vibrator to Chapter 4.

2.7 ■ COMPLEX VIBRATIONS: VIBRATION SPECTRA

In the preceding three sections, we have considered vibrating systems that can vibrate in several different modes. Each of these modes has a different frequency,* and hence it

*Occasionally two different modes of vibration will have the same natural frequency; they are then called *degenerate* modes. These are rare in musical instruments, however.

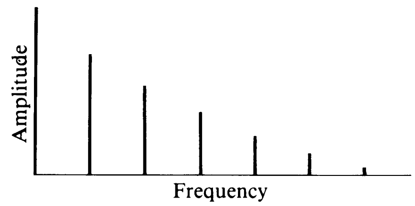


FIGURE 2.22 The vibration spectrum of a plucked string. The spectrum is a recipe that tells us the frequency and amplitude of each mode of vibration that is excited. In this case, the frequencies are harmonics (multiples) of the fundamental, but this will not be so in some vibrating systems.

can be excited individually by some type of driving force at that frequency, as shown in Figs. 2.9–2.19.

More commonly, however, when a vibrating system is excited, it vibrates in several modes at once. A description of its vibrational motion therefore requires a "recipe," which tells us the amplitude and frequency of each of the modes that have been excited. Such a recipe is called the *spectrum* of the vibration. A vibration spectrum of a plucked string is shown in Fig. 2.22. Spectra of this type will appear frequently throughout this book.

When we observe a vibrating system with several modes, we often wish to determine its vibration spectrum. Electronic instruments called *spectrum analyzers* enable us to do this in the laboratory. Spectrum analysis is also called Fourier analysis in honor of the mathematician Joseph Fourier (pronounced "four-yay"), who pioneered in the mathematics of spectrum analysis. The Fourier analysis of sound waves will be discussed in Chapter 7, and examples will be given in the chapters on musical instruments.

2.8 ■ SUMMARY

Vibrational motion repeats itself in a regular interval of time called the period. Vibrating systems have some type of force acting to restore the system toward its point of equilibrium. In the case of simple harmonic motion, this force is proportional to the displacement.

In a vibrating system, the total mechanical energy changes from kinetic to potential to kinetic during each cycle of vibration. The rate at which the total energy decreases depends on the damping forces. Some systems can vibrate in several independent modes. The actual vibratory motion may be a combination of these modes.

All musical sound is generated by some type of vibrating system. Common vibrators include strings, membranes, bars, plates, and air columns. The familiar tuning fork is combination of two bars vibrating in opposite directions, for example.

REFERENCES AND SUGGESTED READINGS

Appel, K., J. Gastineau, C. Bakken, and D. Vernier (1998). *Physics with Computers*. Portland, Ore.: Vernier Software.

Chladni, E. F. F. (1787). "Entdeckungen über die Theorie des Klanges," translated excerpts in R. B. Lindsay, *Acoustics: Historical and Philosophical Development*. Stroudsburg, Pa.: Dowden, Hutchinson and Ross, 1973.

Hewitt, P. G. (1993). *Conceptual Physics*. New York: Harper Collins.

Morse, P. M., and K. U. Ingard (1968). *Theoretical Acoustics*. New York: McGraw-Hill.

Rossing, T. D. (1982). *Acoustics Laboratory Experiments* (Northern Illinois University, DeKalb).

Rossing, T. D. (1976, 1977). "Acoustics of Percussion Instruments," *The Physics Teacher* **14**:546, and **15**:278.

Rossing, T. D., and N. H. Fletcher (1995). *Principles of Vibration and Sound*. New York: Springer-Verlag.

Rossing, T. D., D. A. Russell, and D. E. Brown (1992). "On the acoustics of tuning forks," *American J. Physics* **60**:620–626.

GLOSSARY

amplitude Maximum displacement from rest.

damping Loss of energy of a vibrator, usually through friction.

envelope Time variation of the amplitude (or energy) of a vibration.

frequency The number of vibrations per second; expressed in hertz (Hz).

fundamental mode The mode of lowest frequency.

harmonics Modes of vibration whose frequencies are whole-number multiples of the frequency of the fundamental mode.

Helmholtz resonator A vibrator consisting of a volume of enclosed air with an open neck or port.

longitudinal vibration Vibration in which the principal motion is in the direction of the longest dimension.

node, or nodal line A point or line where minimal motion takes place.

normal modes Independent ways in which a system can vibrate.

period The time duration of one vibration; the minimum time necessary for the motion to repeat.

simple harmonic motion Smooth, regular vibrational motion at a single frequency such as that of a mass supported by a string.

spectrum A "recipe" that gives the frequency and amplitude of each component of a complex vibration.

spring constant ("stiffness") The strength of a spring; restoring force divided by displacement.

transverse vibration Vibration in which the principal motion is at right angles to the longest dimension.

waveform Graph of some variable (e.g., position of an oscillating mass or sound pressure) versus time.

REVIEW QUESTIONS

1. What is meant by the period of a vibration? How is it related to the frequency?
2. In what units is the spring constant of a spring expressed?
3. What is meant by simple harmonic motion?
4. Doubling the distance a spring is stretched increases the restoring force by what factor?
5. Doubling the distance a spring is stretched increases its potential energy by what factor?
6. How does the frequency of a simple pendulum change when its mass is doubled?
7. How does the frequency of a Helmholtz resonator change when its volume is doubled? when the radius of its neck is doubled?
8. How many modes of longitudinal vibration does a two-mass system have? how many modes of transverse vibration?
9. How many modes of longitudinal vibration does a four-mass system have? how many modes of transverse vibration?
10. Describe the lowest mode of vibration in a vibrating string.
11. What is a node? Describe a node in a vibrating string. Describe a node in a vibrating membrane.
12. How many nodes are there in the lowest mode of a bar with free ends?
13. Describe the first two vibrational modes of a tuning fork.
14. What is a spectrum of vibration? In what sense is it a recipe?

QUESTIONS FOR THOUGHT AND DISCUSSION

1. Present an argument to show that the maximum kinetic energy of a mass-spring vibrator is equal to the maximum potential energy. Does the total mechanical energy remain constant throughout a cycle?
2. A damped vibrator is found to decrease its amplitude by one-half every 30 s. What is its amplitude at the end of 5 min? In theory will it every stop vibrating? Will it in practice? Explain. (*Hint:* $(\frac{1}{2})^{10} = \frac{1}{1024} \simeq 0.001$.)
3. With the help of Figs. 2.10 and 2.12, make a diagram of the four independent longitudinal modes of vibration for a four-mass vibrator.
4. To excite a tuning fork in its principal mode of vibration with a minimum of "clang" sound, where should you strike it? Of the four microphone positions $A, B, C,$ and D in Fig. 2.23, which will best pick up the sound of the fork? Why?

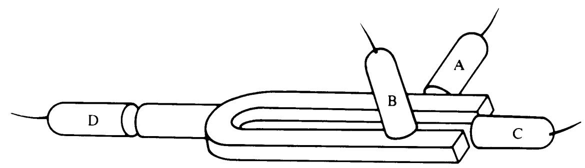


FIGURE 2.23

5. Why is the center arrow in Fig. 2.10(a) larger than the other two arrows?
6. Which Chladni patterns in Fig. 2.19 most nearly correspond to the second and fourth diagrams in Fig. 2.15? In what way are they different?

EXERCISES

1. Hanging a mass of 1 kg on a certain spring causes its length to increase 0.2 m.
   (a) What is the spring constant K of that spring?
   (b) At what frequency will this mass-spring system oscillate?
2. Copy the graphs of displacement and velocity shown in Fig. 2.2, and draw graphs of kinetic energy and potential to the same scale of time.
3. Most grandfather clocks have a pendulum that ticks (makes half a vibration) each second. What length of pendulum is required? (The value of g was given in Chapter 1 as 9.8 m/s^2.)
4. A bass-reflex loudspeaker enclosure (see Fig. 19.16) is essentially a Helmholtz resonator. Given the following parameters, what resonance frequency might be expected? $V = 0.5$ m^3, $a = 0.02$ m^2, $l = 0.05$ m, speed of sound $v = 343$ m/s at $T = 20$°C.
5. Calculate the maximum potential energy of the mass-spring system described in Problem 1 if its maximum displacement is 5 cm.
6. In the two-mass system shown in Fig. 2.7, each mass is 2 kg and each spring constant $K = 100$ N/m. Calculate the frequencies of modes (a) and (b).
7. Equation (2.3) for the frequency of a simple mass-spring vibrator assumes that the mass of the spring is much smaller than that of the load and thus can be neglected. This will not always be the case. The formula can be refined by letting m be the mass of the load plus one-third the mass of the spring. Suppose that the spring in the example in Section 2.1 has a mass of 100 g (K was found to be 196 N/m). Calculate the vibration frequencies with loads of 0.5 kg and 2 kg, and compare them to those given in the example.

EXPERIMENTS FOR HOME, LABORATORY, AND CLASSROOM DEMONSTRATION

Home and Classroom Demonstration

1. *Simple vibrating system: dependence of frequency on mass* Load a spring with several different masses and determine its frequency by counting oscillations during some appropriate time interval (such as a half-minute). What happens to the frequency when the mass is doubled? What happens when it is quadrupled?

2. *Simple vibrating system: dependence of frequency on spring constant* Determine the frequency of the simple vibrating system using several different spring constants. The spring constant can be determined by loading the spring with different masses and noting its static deflection, but this may not be necessary. Connecting two identical springs in series reduces the spring constant by half, whereas connecting them in parallel doubles the spring constant. What happens to the frequency when the spring constant is doubled? What happens when it is halved?

3. *Pendulum: dependence of frequency on mass* Determine the frequency of a simple pendulum by counting oscillations during some appropriate time interval. What happens to the frequency when the mass is doubled? (Be careful not to change the length.) What happens when it is quadrupled?

4. *Pendulum: dependence of frequency on length* Determine the frequency of a simple pendulum by counting oscillations. What happens to the frequency when the length is doubled? What happens when it is halved?

5. *Matching a pendulum to a mass-spring vibrator* Begin with a spring having an unloaded length L_0. Load it with a mass M so that is length increases by L. Verify that a pendulum of length L has the same natural frequency as the mass-spring system. Can you show why this is so? (*Hint:* What is the spring constant?)

6. *Soda-bottle resonator* Blow across the top of an empty soda bottle to determine its natural frequency (acting as a Helmholtz resonator). Fill the bottle half full of water and determine the frequency change.

7. *Modes of a vibrating string* Pluck a stretched string and note its frequency (pitch). Touch the string lightly at its center for a moment to damp out the fundamental and note the frequency. Repeat by touching it at one-third its length.

8. *Chladni patterns* Support a square plate at its center, sprinkle fine sand or salt on it, and bow the edge with a violin or cello bow. By touching the plate at different places and varying the bowing position, interesting vibration patterns can be obtained. Alternatively, a current-carrying coil can be positioned near a small permanent magnet attached to the plate in order to apply a sinusoidal force at a single frequency. Varying the frequency causes the plate to vibrate in its various modes. Repeat with a circular plate (the latter method was used to make the Chladni patterns in Fig. 2.19).

Laboratory Experiments

Simple harmonic motion (Experiment 2 in *Acoustics Laboratory Experiments*)

Vibrating strings (Experiment 4 in *Acoustics Laboratory Experiments*)

Vibrations of bars and plates (Experiment 5 in *Acoustics Laboratory Experiments*)

Stroboscopic measurements (Experiment 8 in *Acoustics Laboratory Experiments*)

Back and forth motion (Experiment 2 in *Physics with Computers*)

Pendulum periods (Experiment 14 in *Physics with Computers*)

Simple harmonic motion (Experiment 15 in *Physics with Computers*)

Energy in simple harmonic motion (Experiment 17 in *Physics with Computers*)

CHAPTER

3 Waves

Section 1.1 gave us a brief introduction to sound waves. We learned that sound waves in air are longitudinal waves; that is, the back-and-forth motion of the air is in the direction of travel of the sound wave. Many other types of waves, such as light waves and radio waves, are transverse waves. Although sound waves are very different from light waves or ocean waves, all waves possess certain common properties. In this chapter, some of these common properties will be discussed, along with some particular properties of sound waves.

In this chapter you should learn:

- About progressive waves and standing waves;
- About sound waves;
- About reflection of waves at a boundary;
- About refraction of waves;
- About interference and diffraction;
- About the Doppler effect.

3.1 ■ WHAT IS A WAVE?

One of the first properties noted about waves is that they can transport energy and information from one place to another through a medium, but the medium itself is not transported. A disturbance, or change in some physical quantity, is passed along from point to point as the wave propagates. In the case of light waves or radio waves, the disturbance is a changing electric and magnetic field; in the case of sound waves, it is a change in pressure and density. But in either case, the medium reverts to its undisturbed state after the wave has passed.

All waves have certain things in common. For example, they can be reflected, refracted, or diffracted, as we shall see later in this chapter. All waves have energy, and they transport energy from one point to another. Waves of different types propagate with widely varying speeds, however. Light waves and radio waves travel 3×10^8 m (186,000 mi) in 1 s, for example, whereas sound waves travel only 344 m/s. Water waves are still slower, traveling only a few feet in a second. Light waves and radio waves can travel millions of miles through empty space, whereas sound waves require some material medium (gas, liquid, or solid) for propagation.

From Chapter 3 of *The Science of Sound*, Third Edition. Thomas D. Rossing, Richard F. Moore, Paul A. Wheeler.

3.2 ■ PROGRESSIVE WAVES

Suppose that one end of a rope is tied to a wall and the other end is held, as shown in Fig. 3.1. If the end being held is moved up and down f times per second, a wave with a frequency f will propagate down the rope, as shown. (When it reaches the tied end, a reflected wave will return, but we will ignore this for the moment.) The wave travels at a speed v that is determined by the mass of the rope and the tension applied to it.

If we were to observe the wave carefully (a photograph might help), we would note that the crests or troughs of the wave are spaced equally; we call this spacing the wavelength λ (Greek letter lambda).

It is not difficult to see that the wave velocity is the frequency times the wavelength:

$$v = f\lambda. \tag{3.1}$$

That is, if f waves pass a certain point each second and the crests are λ meters apart, they must be traveling at a speed of $f\lambda$ meters per second.

It is possible to propagate either transverse or longitudinal waves in solids, but in general only longitudinal waves propagate through gases and liquids. Figure 3.2 illustrates the propagation of longitudinal and transverse waves in a mass-spring system. This system is also a large-scale model (in one dimension) of a solid crystal and illustrates ways in which vibrations may propagate in a solid.

In a solid, longitudinal waves travel at a speed represented by the formula

$$v = \sqrt{\frac{E}{\rho}}, \tag{3.2}$$

where ρ is the density of the solid and E is called the elastic modulus (*Young's modulus*). Note that the speed of longitudinal waves in a solid bar is independent of its dimensions. This is not so for transverse waves, whose speed is dependent on the dimensions. For a wire or string, the transverse wave velocity (speed) is

$$v = \sqrt{\frac{T}{\mu}}, \tag{3.3}$$

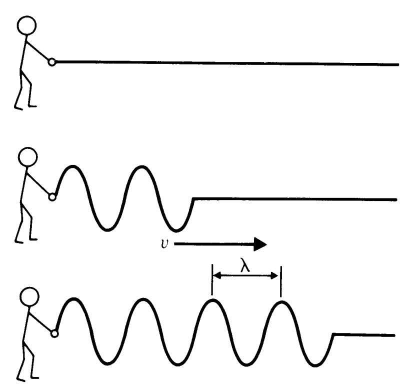


FIGURE 3.1
A traveling wave generated by moving the end of a rope.

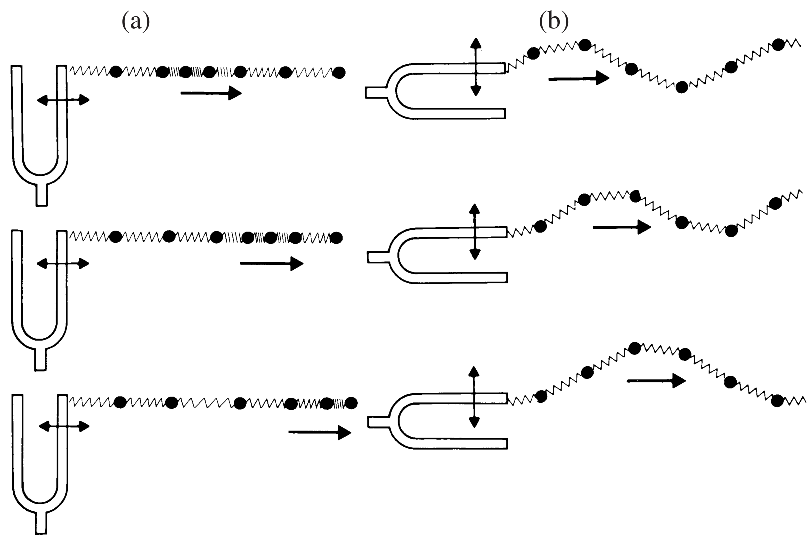


FIGURE 3.2
Wave motion in a one-dimensional array:
(a) longitudinal waves;
(b) transverse waves.

where T is the tension and μ is the mass per unit of length. In a stiff rod or bar, the velocity of transverse waves varies with frequency, and so a simple formula cannot be written. In general, longitudinal waves travel much faster than transverse waves do in solids. The speed of longitudinal (sound) waves in aluminum, for example, is 5000 m/s (about 3 mi/s).

EXAMPLE 3.1 The density of steel is 7700 kg/m^3 and Young's elastic modulus is 19.5×10^{10} N/m^2. What is the speed of longitudinal waves in a steel glockenspiel bar? How does this compare with the speed of longitudinal vibrations (sound waves) in air?

Solution

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{19.5 \times 10^{10}\ \text{N/m}^2}{7.7 \times 10^3\ \text{kg/m}^3}} = 5032\ \text{m/s}.$$

(This is $\frac{5032}{343} = 14.7$ times the speed of longitudinal (sound) waves in air.)

EXAMPLE 3.2 What tension would a steel wire 1 mm in diameter require in order that the transverse and longitudinal wave speeds are equal?

Solution

$$v = \sqrt{\frac{T}{\mu}}, \quad \text{so} \quad T = \mu v^2$$

$$\rho(\pi r^2)v^2 = 7700(3.14)(5 \times 10^{-4})^2(5032)^2$$
$$= 1.53 \times 10^5 \text{ N}$$

(far greater than the breaking force).

3.3 ■ IMPULSIVE WAVES; REFLECTION

Suppose that the rope in Fig. 3.1 is given a single impulse by quickly moving the end up and down. The impulse will travel at the wave speed v and will retain its shape fairly well as it moves down the rope, as illustrated in Fig. 3.3.

The question arises as to what happens when the incident pulse reaches the end of the rope. Careful observation shows that a pulse reflects back toward the sender. This reflected pulse is very much like the original pulse, except that it is upside down. If the end of the rope were left free to flop like the end of a whip, the reflected pulse would be right side up, as illustrated in Fig. 3.4(b). Photographs of impulsive waves on a long string with fixed and free ends are shown in Figs. 3.5 and 3.6. Note that the reflected pulse in Fig. 3.5 is upside down. This is called a *reversal of phase*. In Fig. 3.6 the *phase* of the reflected wave remains the same as that of the original wave.

It is instructive to tie the rope at the base of a mirror (see Fig. 3.7). Then the mirror image of the pulse (generated by your image) travels at the same speed and arrives at the end of the rope at the same time as the actual pulse, and appears to continue on as the reflected pulse. (To make the sense of the pulse correct, two mirrors can be used to form

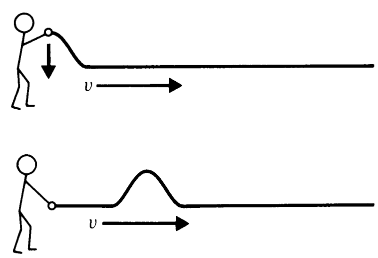


FIGURE 3.3 An impulsive wave generated by moving the end of a rope.

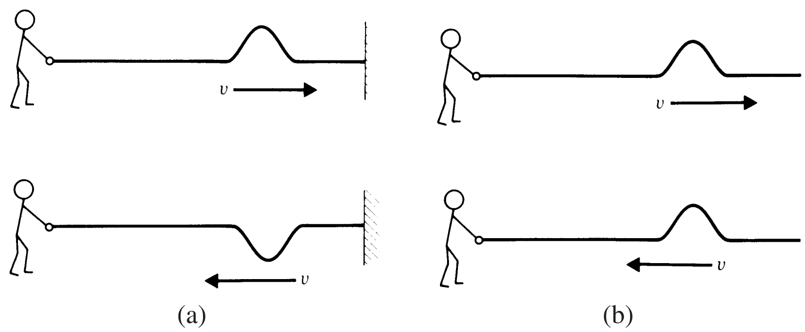


FIGURE 3.4 Reflection of an impulsive wave (a) at a fixed end; (b) at a free end.

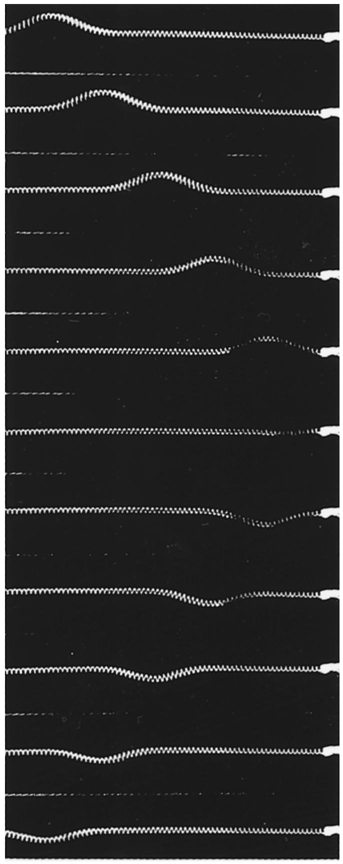


FIGURE 3.5 An impulsive wave in a long spring. The pulse travels left to right and reflects back to the left as in Fig. 3.4(a). (From *PSSC Physics*, 2nd ed., 1965, D. C. Heath & Co. with Education Development Center, Newton, Mass.)

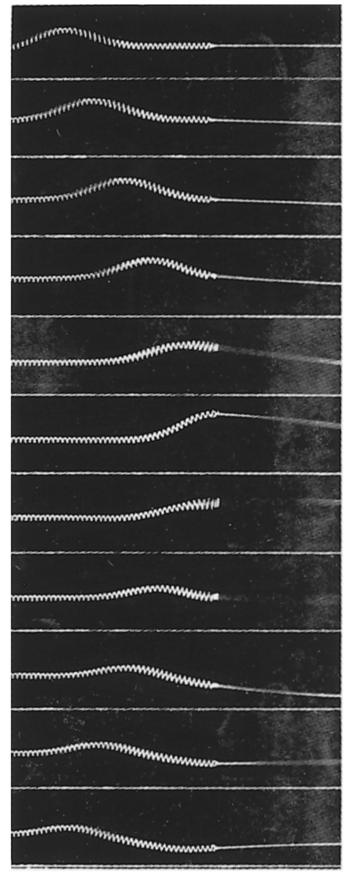


FIGURE 3.6 An impulsive wave in a spring showing reflection at a free end (actually a very light thread). Compare the reflected pulse to that of Fig. 3.5. (From *PSSC Physics*, 2nd ed., 1965, D. C. Heath & Co. with Education Development Center, Newton, Mass.)

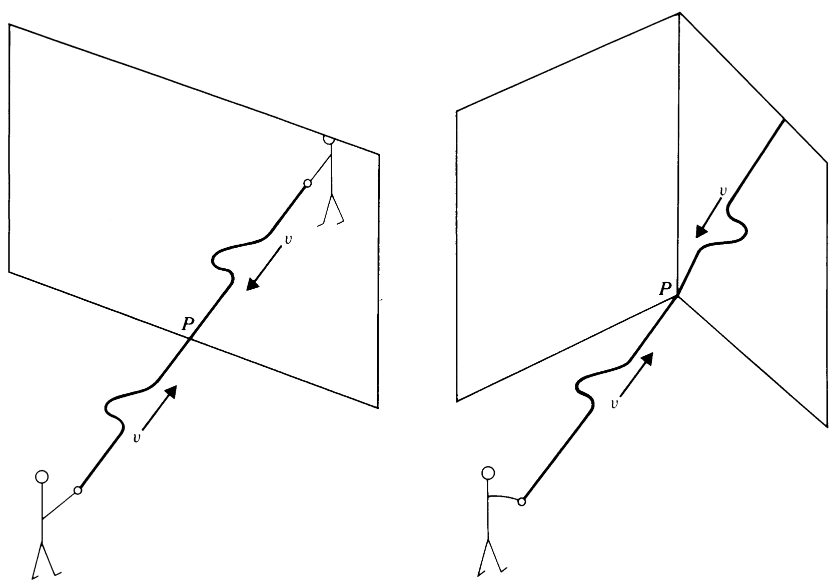


FIGURE 3.7 The mirror image of an impulsive wave approaching a point of reflection P. In a plane mirror, the two pulses have the same sense, but in a corner mirror the two pulses have opposite sense (just as the incident and reflected pulses on the rope with a fixed end).

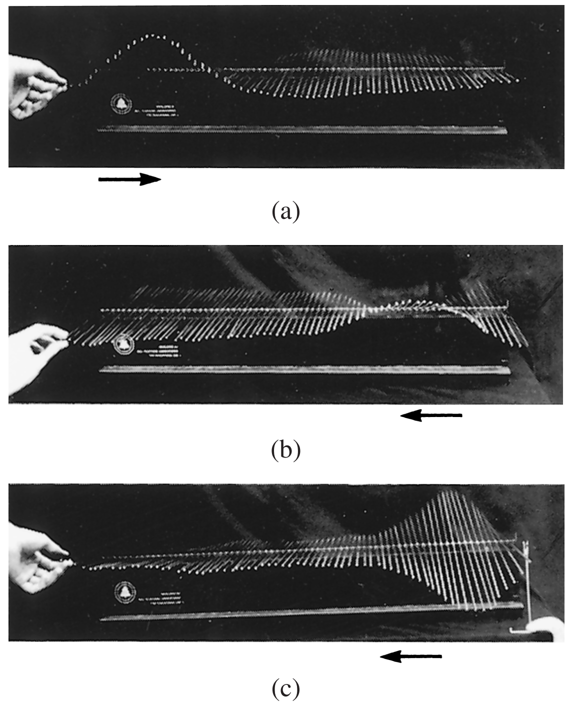


FIGURE 3.8
Wave propagation on a "wave machine": (a) incident pulse; (b) reflection at a free end; (c) reflection at a fixed end. (Photographs by Craig Anderson.)

a corner reflector, but this is really not necessary to achieve the sensation of the reflected pulse coming from a virtual source and meeting the original pulse at the point of reflection.)

One can think of the reflected wave on the rope as coming from an imaginary source, whether or not a mirror is there to show a reflected image. If the rope is tied to a solid object at its far end, the deflection must be zero at all times, even when the pulse arrives; this requires that the pulse be met by a pulse of opposite sense, as shown in Fig. 3.4(a). If the end is free, it snaps like a whip, momentarily doubling its displacement when the pulse arrives. This is equivalent to the arrival of a pulse with the same sense, which then continues as the reflected pulse shown in Fig. 3.4(b).

Several interesting properties of waves can be studied with a wave machine developed at the Bell Laboratories, which consists of a long array of rods attached to a wire. Waves travel slowly on this machine; hence they can be observed rather easily. Reflection of a pulse at free and fixed ends is illustrated in the photographs of the wave machine in Fig. 3.8.

3.4 ■ SUPERPOSITION AND INTERFERENCE

An interesting feature of waves is that two of them, traveling in opposite direction, can pass right through each other and emerge with their original identities. The *principle of linear superposition* describes this behavior. For wave pulses on a rope or spring, for example, the displacement at any point is the sum of the displacements due to each pulse by itself. The wave pulses shown in Fig. 3.9 illustrate the principle of superposition. If the pulses have the

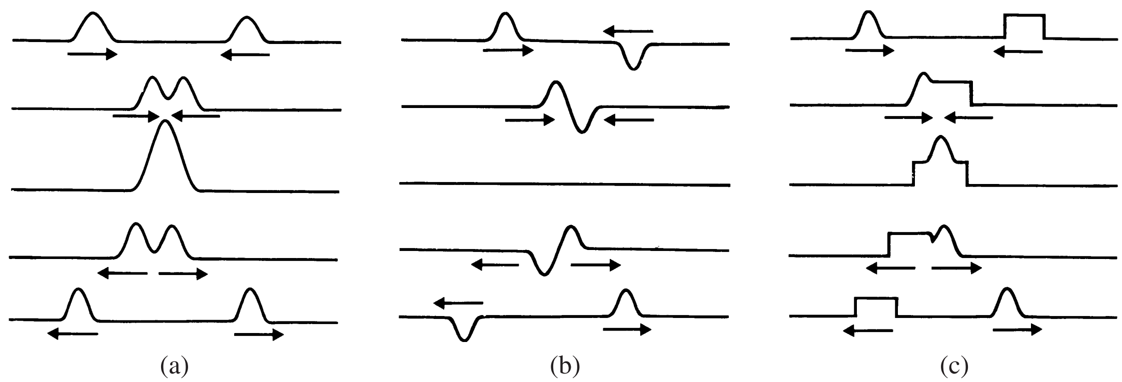


FIGURE 3.9 The superposition of wave pulses that travel in opposite directions: (a) pulses in the same direction; (b) pulses in opposite directions; (c) pulses with different shapes.

same sense, they add; if they have the opposite sense, they subtract when they meet. These are examples of *interference* of pulses. The addition of two similar pulses (3.9a) is called *constructive interference*; the subtraction of opposing pulses (3.9b) is called *destructive interference*.

Suppose that both ends of a rope (or the wave machine shown in Fig. 3.8) are shaken up and down at the same frequency, so that continuous waves travel in both directions. Continuous waves interfere in much the same manner as the impulsive waves we have just considered. If two waves arrive at a point when they have opposite sense, they will interfere destructively; if they arrive with the same sense, they will interfere constructively. Under these conditions, the waves do not appear to move in either direction, and we have what is called a *standing wave*.

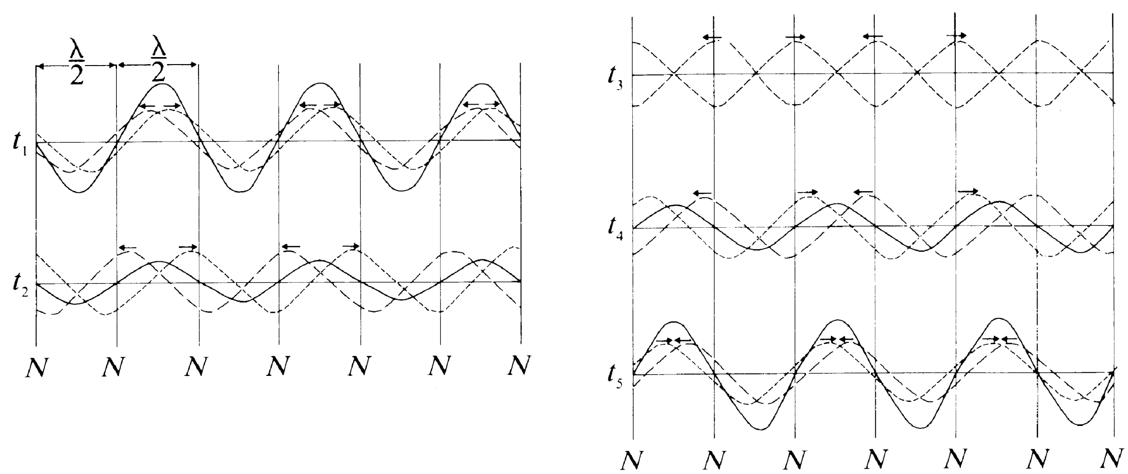


FIGURE 3.10 Interference of two identical waves in a one-dimensional medium. At times t_1 and t_5 there is constructive interference, and at t_3 there is destructive interference. Note that at points marked N, the displacement is always zero.

In the case of two identical waves (same frequency and amplitude) traveling in opposite directions on a rope or a spring, there will be alternating regions of constructive and destructive interference, as shown in Fig. 3.10. The points of destructive interference that always have zero displacement are called nodes; they are denoted by N in Fig. 3.10. Between the nodes are points of constructive interference, where displacement is a maximum; these are called antinodes. At the antinodes, the displacement oscillates at the same frequency as in the individual waves; the amplitude is the sum of the individual wave amplitudes.

Note that the antinodes in Fig. 3.10, formed by the interference of two identical waves, are one-half wavelength apart. Because these points of maximum displacement do not move through the medium, the configuration is called a standing wave. Standing waves result whenever waves are reflected back to meet the oncoming waves. The case illustrated in Fig. 3.10, in which the forward and backward waves have the same amplitude, is a special case that leads to total interference. If the two incident waves do not have the same amplitude, the nodes will still be points of minimum but not zero displacement.

3.5 ■ SOUND WAVES

Sound waves are longitudinal waves that travel in a solid, liquid, or gas. To aid in your understanding of sound waves, consider a large pipe or tube with a loudspeaker at one end. Although sound waves in this tube are similar in many respects to waves on a rope, they are more difficult to visualize, because we cannot see the displacement of the air molecules as the sound wave propagates down the tube.

Suppose we consider first a single pulse, as we did in the case of the rope. An electrical impulse to the loudspeaker causes the speaker cone to move forward suddenly, compressing the air directly in front of it *very* slightly (even a very loud sound results in a pressure increase of less than 1/10,000 atmospheric pressure). This pulse of air pressure travels down the tube at a speed of about 340 m/s (more than 700 mi/h). It may be absorbed at the far end of the tube, or it may reflect back toward the loudspeaker (as a positive pulse of pressure or a negative one), depending on what is at the far end of the tube.

Reflection of a sound pulse for three different end conditions is illustrated in Fig. 3.11. If the end is open, the excess pressure drops to zero, and the pulse reflects back as a negative

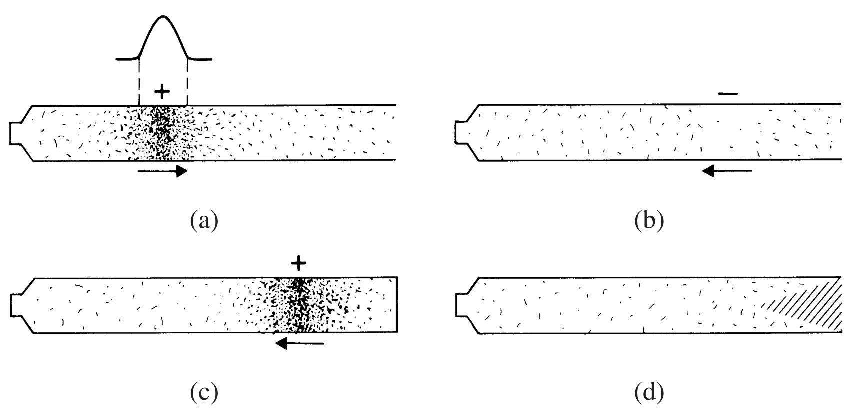


FIGURE 3.11 Reflection of a sound pulse in a pipe: (a) incident pulse; (b) reflection at an open end; (c) reflection at a closed end; (d) no reflection from absorbing end.

pulse of pressure, as shown in Fig. 3.11(b); this is analogous to the fixed-end condition illustrated in Figs. 3.5 and 3.8(b).* If the end is closed, however, the pressure builds up to twice its value, and the pulse reflects back as a positive pulse of pressure; this condition, shown in Fig. 3.11(c), is analogous to the free-end reflection of Figs. 3.6 and 3.8(c). If the end is terminated with a sound absorber, there is virtually no reflected pulse. Such a termination is called *anechoic*, which means "no echo."

The speed of sound waves in an ideal gas is given by the formula

$$v = \sqrt{\frac{\gamma RT}{M}}, \tag{3.4}$$

where T is absolute temperature, M is the molecular weight of the gas, and γ and R are constants for the gas. For air, $M = 2.88 \times 10^{-2}$, $R = 8.31$, and $\gamma = 1.4$, so $v = 20.1\sqrt{T}$. The absolute temperature T is found by adding 273 to the temperature on the Celsius scale. At $t = 21°\text{C}$, for example, $T = 294$ K, so $v = 344$ m/s. At Celsius zero, $v = 331$ m/s. Over the range of temperature we normally encounter, the speed of sound increases by about 0.6 m/s for each Celsius degree, and an approximate formula for the speed of sound is sufficiently accurate:

$$v = 331.3 + 0.6t \text{ m/s}, \tag{3.5}$$

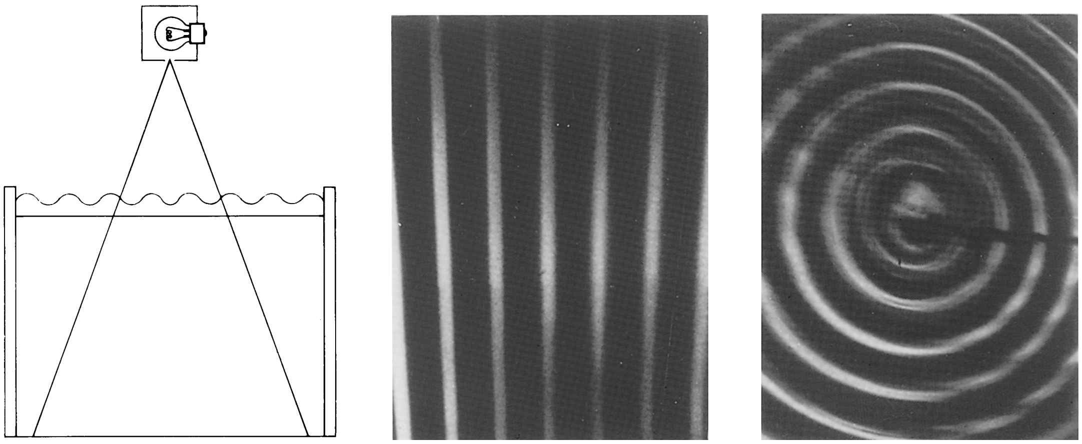
where t is the temperature in degrees on the Celsius scale. The speed of sound in an ideal gas is independent of atmospheric pressure. In air the change in speed with pressure change is generally too small to measure.

Sound waves travel much faster in liquids and solids than they do in gases. The speed of sound in several materials is given in Table 3.1.

TABLE 3.1 Speed of sound in various materials

Substance	Temperature (°C)	Speed (m/s)	Speed (ft/s)
Air	0	331.3	1,087
Air	20	343	1,127
Helium	0	970	3,180
Carbon dioxide	0	258	846
Water	0	1,410	4,626
Methyl alcohol	0	1,130	3,710
Aluminum	—	5,150	16,900
Steel	—	5,100	16,700
Brass	—	3,480	11,420
Lead	—	1,210	3,970
Glass	—	3,700–5,000	12–16,000

*In an actual tube with an open end, a little of the sound will be radiated; most of it, however, will be reflected as shown.



(a) (b) (c)

FIGURE 3.12 (a) A ripple tank for projecting an image of water waves; (b) straight waves on a ripple tank; (c) circular waves on a ripple tank. (Photographs by Christopher Chiaverina.)

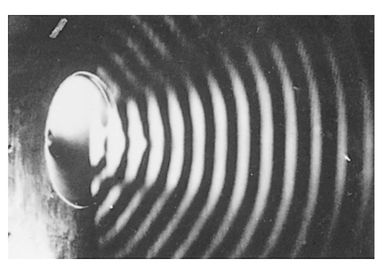
3.6 ■ WAVE PROPAGATION IN TWO AND THREE DIMENSIONS

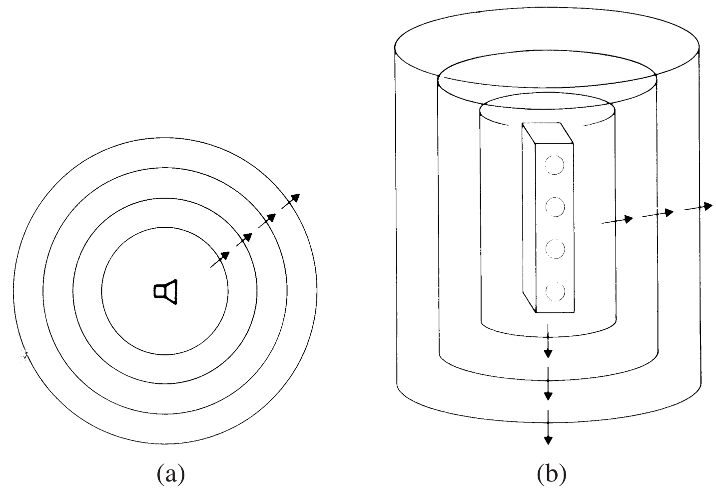
Thus far we have considered only waves that travel in a single direction (along a rope or in a pipe, for example). One-dimensional waves of this type are a rather special case of wave motion. More often, waves travel outward in two or three dimensions from a source.

Water waves are a familiar example of two-dimensional waves. Many wave phenomena, in fact, can be studied conveniently by means of a ripple tank in the laboratory. A ripple tank uses a glass-bottom tray filled with water; light projected through the tray forms an image of the waves on a large sheet of paper, as shown in Fig. 3.12.

Three-dimensional waves are difficult to make visible. An ingenious technique has been used to photograph three-dimensional wave patterns (though not the actual waves) at the Bell Laboratories and elsewhere. A tiny microphone and a neon lamp together scan the sound field in a dark room while a camera lens remains open in a time exposure. The brightness of the neon lamp is controlled by an amplifier, so that bright streaks appear at wave crests, as shown in Fig. 3.13.

FIGURE 3.13 The pattern of sound waves from a loudspeaker produced by scanning with a microphone and neon lamp. (From Kock 1971.)





(a) (b)

FIGURE 3.14 Sound wave patterns from (a) a single small loudspeaker (*point* source); (b) a column of loudspeakers (*cylindrical* source).

Different types of sources radiate different kinds of patterns. A point source or a source that is spherically symmetric radiates spherical waves. A line source or a source with cylindrical symmetry radiates cylindrical waves. A large flat source radiates plane waves. Real sound sources are never true point sources, line sources, or flat sources, however; what we may have in real life are sources that approximate one of these geometries.

A source that is very small compared to a wavelength of sound approximates a point source and emits nearly spherical waves. A small enclosed loudspeaker will radiate nearly spherical waves at low frequency, as shown in Fig. 3.14(a). A column of small loudspeakers may resemble a line source at low frequency and emit cylindrical waves, as shown in Fig. 3.14(b). Symmetrical radiation patterns of this type can be observed outside, away from reflecting objects, or in an anechoic (echo-free) room.

3.7 ■ THE DOPPLER EFFECT

Ordinarily, the frequency of the sound waves that reach the observer is the same as the frequency of vibration of the source. There is a notable exception, however, if either the source or the observer is in motion. If they are moving toward each other, the observed frequency is greater than f_s; if they are moving apart, the observed frequency is lower than f_s. This apparent frequency shift is called the *Doppler effect*.

The Doppler effect is explained quite simply with the aid of Fig. 3.15. Suppose that a source S emits 100 waves per second. An observer at rest, O, will count 100 waves per second passing him or her. However, an observer O' moving toward the source will count more waves because he or she "meets" them as he or she moves, just as the driver of an automobile meets oncoming traffic. The apparent frequency (the rate at which the observer meets waves) will be

$$f' = f_s \frac{v + v_o}{v}, \tag{3.6}$$

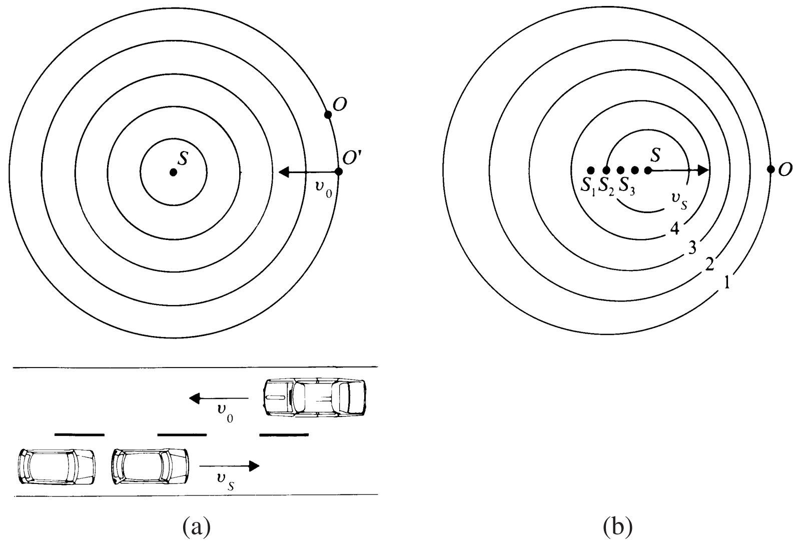


FIGURE 3.15
The Doppler effect: (a) observer moving toward the sound source; (b) source moving toward the observer.

where f_s is the frequency of the source, v_o is the speed of the observer, and v is the speed of sound. Note that after the observer passes the sound source, v_o must be subtracted from v. Thus the frequency drops abruptly as the observer passes the source.

There is also a Doppler effect if the source is in motion. You have probably observed a drop in pitch or frequency of the noise as a truck or car passes by while you are standing at the side of the road. The case of the moving source is shown in Fig. 3.15(b). The source emitted the wave numbered 1 when it was at position S_1, number 2 when at S_2, etc. The wave fronts resemble spheres with centers constantly shifting to the right as the source moves. Thus the observer O receives waves at a greater rate than he or she would from a stationary source. If the speed of the source is v_s, the apparent frequency will be

$$f = f_s \frac{v}{v - v_s}. \tag{3.7}$$

Note that if the source moves directly toward the observer, the frequency will drop *abruptly*, not gradually, as the source passes by.

The Doppler effect is also used by astronomers to determine the motion of a distant star. By carefully analyzing the spectra of starlight, astronomers can detect shifts in frequency (Doppler shifts) due to the motion of the stars toward or away from us. Because the universe appears to be expanding, most distant starts exhibit a "red shift"; that is, their frequency is shifted toward the low-frequency (red) end of the visible spectrum. It is remarkable how much we know about our universe just by analyzing the light from distant stars, some of which was radiated millions of years ago and has been traveling through space ever since!

EXAMPLE 3.3 An automobile horn emits a tone with a frequency of 440 Hz. What is the apparent frequency when the automobile approaches an observer at 55 mi/h (25 m/s) and what is the apparent frequency when it recedes at this same speed?

Solution

$$f' = f_s \frac{v}{v - v_s} = 440 \frac{343}{343 - 25} = 475 \text{ Hz};$$

$$f' = 440 \frac{343}{343 + 25} = 410 \text{ Hz}.$$

Note that the pitch drops by 14% (more than two semitones on the musical scale).

EXAMPLE 3.4 A police radar speed gun transmits microwaves having a frequency of 9600 MHz. What is the upward shift in frequency for waves reflected from an automobile traveling at 55 mi/h (25 m/s)?

Solution View the automobile as a mirror moving at 25 m/s; the "image" of the source appears to move at twice this speed, or 50 m/s.

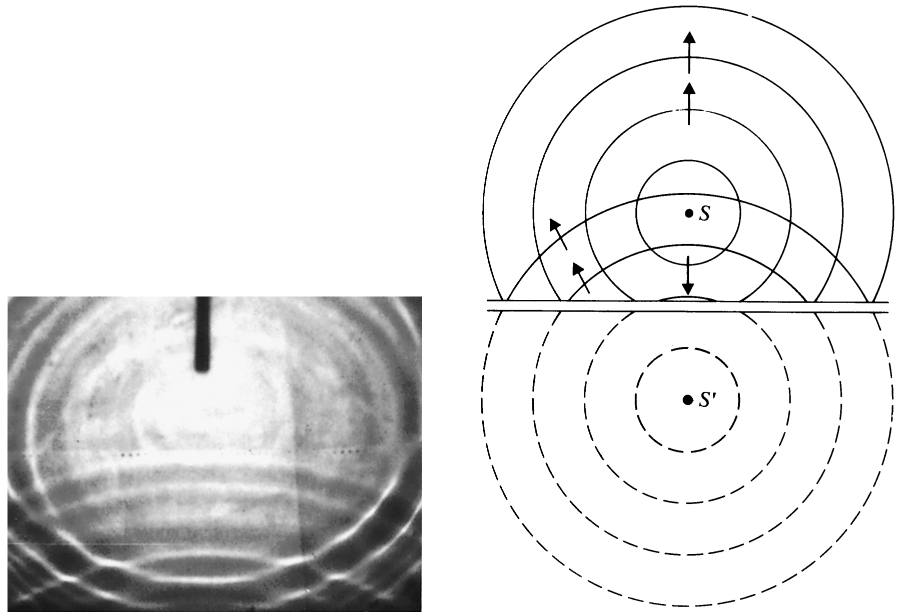
$$\begin{aligned}\Delta f = f' - f_s &= f_s \frac{v}{v - v_s} - f_s \\ &= 9.6 \times 10^9 \frac{3 \times 10^8}{3 \times 10^8 - 50} - 9.6 \times 10^9 \\ &\cong 9.6 \times 10^9 \left(1 + \frac{50}{3 \times 10^8} - 1\right) \\ &= 1600 \text{ Hz}.\end{aligned}$$

(Although this frequency shift is only one part in six million, it can readily be measured—as many speeding motorists know—by mixing together the transmitted and reflected microwaves.)

3.8 ■ REFLECTION

The reflection of wave pulses of one dimension on a rope or in a tube was discussed in Sections 3.3 and 3.5. Waves of two or three dimensions undergo similar reflections when they reach a barrier. The reflection of light waves by a mirror is a phenomenon familiar to all of us, as is the echo that results from clapping one's hands some distance away from a large wall, which reflects the sound waves back to the source. Figure 3.16(a) shows the reflection of water waves from a straight barrier. Note that the spherical reflected waves appear to come from a point behind the barrier. This point, which is called the *image* is denoted by S' in Fig. 3.16(b). It is the same distance from the reflector as the source S is.

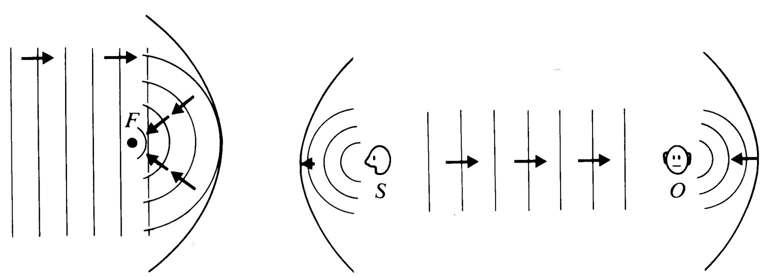
Reflection of waves from a curved barrier can lead to the *focusing* of energy at a point, as shown in Fig. 3.17. A curious case of sound focusing, which occurs in "whispering galleries," is shown in Fig. 3.17(b). Sound originating from a source S is reflected by a curved barrier, beamed to a second curved barrier, and focused at O.



(a) (b)

FIGURE 3.16 Reflection of waves from a barrier: (a) waves on a ripple tank from a point source; (b) reflected waves appear to originate from image S'.

Well-known examples of whispering galleries are found in the Museum of Science and Industry in Chicago and in the National Capitol in Washington. The curved ceilings of certain auditoriums, such as the Mormon Tabernacle in Salt Lake City, make it possible to transmit whispers between selected spots. However, the focusing of sound by curved walls is frequently detrimental to the acoustics of auditoria, as we will discuss in Chapter 23.



(a) (b)

FIGURE 3.17 Reflection of waves by a curved barrier: (a) incoming waves are focused at F by a curved reflector; (b) whispering gallery in which two curved reflectors beam sound from source S to observer O with great efficiency.

3.9 ■ REFRACTION

When the speed of waves changes, a phenomenon called *refraction* occurs, which can result in a change in the direction of propagation or a bending of the waves. The change of speed may occur abruptly as the wave passes from one medium to another, or it may

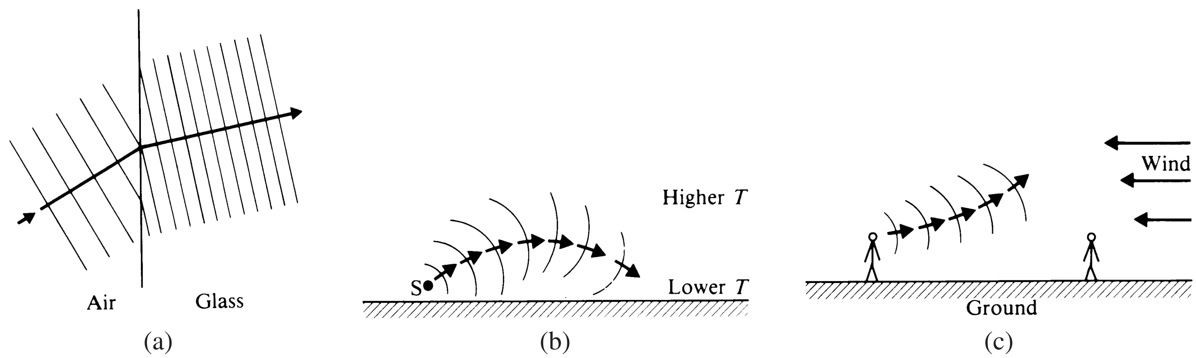


FIGURE 3.18 Refraction of waves: (a) light waves passing from air to glass; (b) sound waves in the atmosphere when temperature varies with height; (c) sound traveling against the wind.

change gradually if the medium changes gradually. These two situations are illustrated in Fig. 3.18.

The situation illustrated in Fig. 3.18(b), which sometimes occurs during the cool evening hours, causes sounds to be heard over great distances. Because the speed of sound increases with temperature (Section 3.5), the sound travels faster some distance above the ground where the temperature is greater. This results in a bending of sound downward as shown. Sound that would ordinarily be lost to the upper atmosphere is refracted back toward the ground.

Figure 3.18(c) shows why it is difficult to be heard when yelling against the wind. (It is *not* because the wind blows the sound waves back; even a strong wind has a speed much less than that of sound). Refraction results because the wind speed is less near the ground than it is some distance above it. Because the speed of sound with respect to the air (in this case, moving air) remains the same, the ground speed of the sound changes with altitude. The resulting refraction causes some of the sound to miss its target.

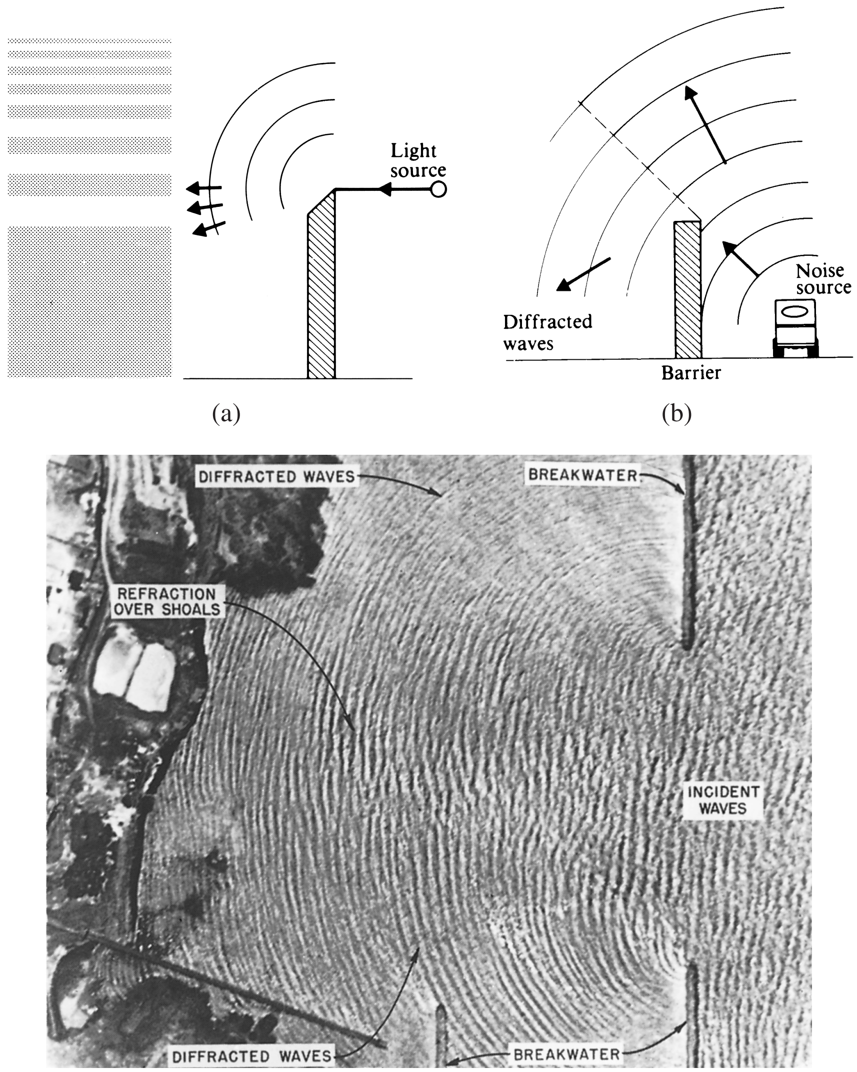
3.10 ■ DIFFRACTION

When waves encounter an obstacle, they tend to bend around the obstacle. This is an example of a phenomenon known as *diffraction*. Diffraction is also apparent when waves pass through a narrow opening and spread out beyond it. Examples of the diffraction of water waves, light waves, and sound waves are shown in Figs. 3.19, 3.20, and 3.21.

An important point to remember is that it is the size of the opening in relation to the wavelength that determines the amount of diffraction. A loudspeaker 0.2 m (8 in.) in diameter, for example, will distribute sound waves of 100 Hz ($\lambda = 3.4$ m) in all directions, but waves of 2000 Hz ($\lambda = 0.2$ m) will be much louder directly in front of the speaker than at the sides, because diffraction will be minimal.

3.11 ■ INTERFERENCE

In Section 3.4, we pointed out that interference between incident and reflected waves leads to standing waves. Standing waves exist in a room due to interference between waves



(c)

FIGURE 3.19 Diffraction of waves by a barrier: (a) shadow of a straight edge magnified to show diffraction of light; (b) diffraction of sound waves allows noise to "leak" around a wall; (c) ocean waves in a harbor. (Photograph (c) courtesy of University of California at Berkeley.)

reflected from the ceiling, walls, and other surfaces; these can be observed by moving one's head around while a pure tone is played through a loudspeaker.

Waves from two identical sources provide another example of interference. Constructive and destructive interference lead to minima and maxima in certain directions, as shown in Fig. 3.22. The interference patterns are determined by the spacing of the two sources compared to a wavelength.

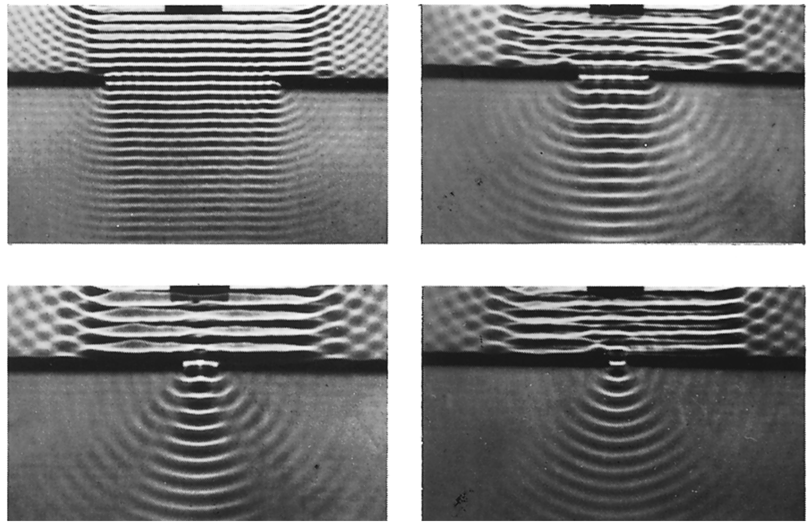


FIGURE 3.20 Diffraction of water waves passing through openings of various sizes. The narrower the opening (compared to the wavelength), the greater the diffraction. (Courtesy of Film Studio, Educational Development Center.)

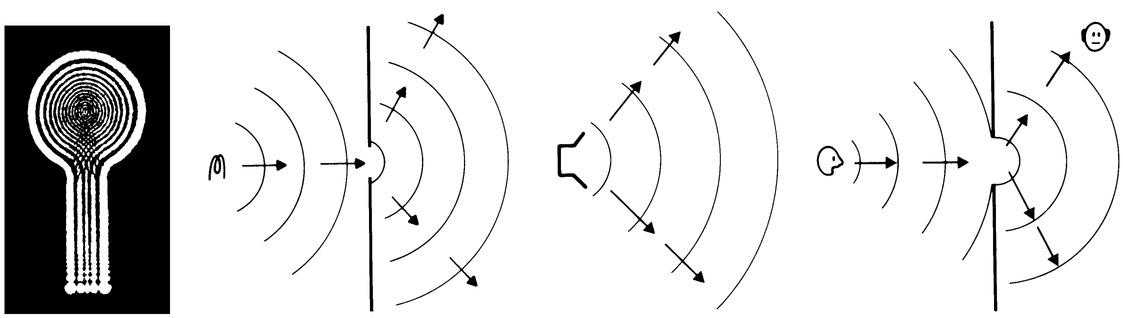
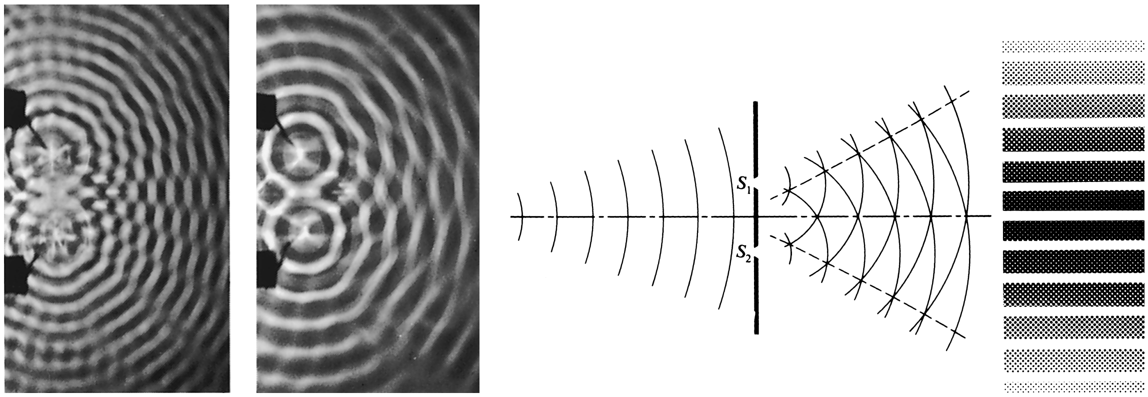


FIGURE 3.21 Diffraction of waves passing through narrow openings: (a) light waves passing through a keyhole; (b) light waves through a very narrow slit; (c) sound waves from a loudspeaker; (d) diffraction allows sound to be heard behind a doorway.

3.12 ■ SUMMARY

We are surrounded by waves of many types (light waves, radio waves, sound waves, water waves, etc.). These quite different types of waves have many properties in common. All carry energy; all can be reflected, refracted, and diffracted; interference leads to regions of minimum and maximum amplitude. However, the speeds at which these waves travel varies widely. Waves can be classed as transverse or longitudinal depending on the direction of the vibrations. Sound waves are longitudinal vibrations of molecules that result in pressure fluctuations. The speed of sound waves in air increases with temperature.

Some wave phenomena can be understood best by considering wave propagation in one dimension and extending the ideas to two- and three-dimensional waves.



(a) (b)

FIGURE 3.22 Interference of waves from two identical sources: (a) water waves in a ripple tank; (b) light waves from two slits illuminated by the same light source. (From *PSSC Physics*, 2nd ed., 1965, D. C. Heath & Co. with Education Development Center, Newton, Mass.)

REFERENCES AND SUGGESTED READINGS

Appel, K., J. Gastineau, C. Bakken, and D. Vernier (1998). *Physics with Computers*. Portland, Ore.: Vernier Software.

Baez, A. V. (1967). *The New College Physics*. San Francisco: Freeman.

Haber-Schaim, U., J. H. Dodge, R. Gardner, and E. A. Shore (1991). *PSSC Physics*, 7th ed. New York: Kendall/Hunt.

Hewitt, P. G. (1993). *Conceptual Physics*, 7th ed. New York: Harper Collins.

Kirkpatrick, L. D., and G. F. Wheeler (1995). *Physics: A World View*, 2nd ed. Philadelphia: Saunders.

Kock, W. E. (1971). *Seeing Sound*. New York: Wiley.

Physical Science Study Committee (1965). *Physics*, 2nd ed. Boston: D. C. Heath.

Project Physics—Text (1975). New York: Holt, Rinehart and Winston.

Rossing, T. D. (1982). *Acoustics Laboratory Experiments* DeKalb: Northern Illinois University.

GLOSSARY

absolute temperature The temperature (in kelvins) on a scale that has its zero at the lowest attainable temperature (−273°C); absolute temperature is found by adding 273 to the Celsius temperature.

amplitude The maximum displacement from equilibrium in a wave or a vibrating system.

anechoic Echo free; an anechoic room is one whose walls, ceiling, and floor are covered with sound-absorbing material, usually in the shape of wedges.

diffraction The spreading out of waves when they encounter a barrier or pass through a narrow opening.

Doppler effect The shift in apparent frequency when the source or observer is in motion.

impulsive wave A brief disturbance or pressure change that travels as a wave.

interference The interaction of two or more identical waves, which may support (constructive interference) or cancel (destructive interference) each other.

longitudinal wave A wave in which the vibrations are in the direction of propagation of the wave; *example:* sound waves in air.

reflection An abrupt change in the direction of wave propagation at a change of medium (by waves that remain in the first medium).

refraction A bending of waves when the speed of propagation changes, either abruptly (at a change of medium) or gradually (e.g., sound waves in a wind of varying speed).

standing wave A wavelike pattern that results from the interference of two or more waves; a standing wave has regions of minimum and maximum amplitude called nodes and antinodes.

superposition The motion at one point in a medium is the sum of the individual motions that would occur if each wave were present by itself without the others.

transverse wave A wave in which the vibrations are at right angles to the direction of propagation of the wave; *example:* waves on a rope.

waveform The graph of some variable (e.g., wave displacement, sound pressure) versus time.

wavelength The distance between corresponding points on two successive waves.

Young's modulus An elastic modulus of a solid; the ratio of force per unit area to the stretch it produces.

REVIEW QUESTIONS

1. How many times faster do light waves travel as compared to sound waves?
2. Compare the speed of longitudinal (sound) waves in an aluminum rod 10 mm in diameter with a similar rod 5 mm in diameter.
3. Doubling the frequency of a sound wave multiplies the wavelength by what factor?
4. Compare the phases of an incident and reflected pulse on a rope with a fixed end and then with a free end.
5. What must be true of the sizes and shapes of two pulses that meet on a rope in order for them to cancel each other (interfere destructively)?
6. Compare the speed of sound in air, water, and steel.
7. How does the speed of sound in air change with temperature?
8. How does the speed of sound in air change with atmospheric pressure?
9. Describe the sound of a car horn as the moving car passes an observer standing at roadside.
10. What is the cause of the "red shift" observed in the spectra of distant stars?
11. Why are curved walls sometimes detrimental to concert hall acoustics?
12. Why is it difficult to be heard when you shout into a strong wind?
13. Why is it possible to hear around a corner but not to see around a corner?
14. Why do you see a flash of lightning seconds before you hear the thunder associated with it?
15. Why do low frequency sound waves from a subwoofer spread out in all directions, but high-frequency sound waves from a tweeter travel pretty much straight ahead?

QUESTIONS FOR THOUGHT AND DISCUSSION

1. Although ocean waves are often described as transverse waves, the motion of a small bit of water is actually in a circle. Why could strictly transverse waves not exist on a water surface?
2. Mine operators carefully select the right atmospheric conditions for blasting operations in order to minimize community disturbance. What atmospheric conditions would be optimum?
3. **(a)** Will a larger pulse (with more energy) overtake a smaller pulse as they travel down a rope?
   (b) Will a baseball thrown with more energy overtake a baseball with less energy?
   (c) Does a loud sound travel faster than a softer sound?
4. A camera lens is made of glass in which light travels slower than it does in air. Could you construct a lens for sound by filling a balloon with carbon dioxide?
5. Would you expect interference effects from two loudspeakers in a room?
6. Compare the speed of sound at altitude of a jet airplane with the speed at ground level.
7. At some high altitude the density of air will equal the density of helium gas at sea level. What would you expect the speed of sound to be at this altitude?

EXERCISES

1. Electromagnetic waves travel through space at a speed of 3×10^8 m/s. Find the frequency of the following. (1 nm $= 10^{-9}$ m)
   (a) radio waves with $\lambda = 100$ m
   (b) waves of red light ($\lambda = 750$ nm)
   (c) waves of violet light ($\lambda = 500$ nm)
   (d) microwaves with $\lambda = 3$ cm (used in police radar)
2. Two trumpet players tune their instruments to exactly 440 Hz. Find the difference in the apparent frequencies due to the Doppler effect if one plays his or her instrument while marching away from an observer and the other plays while marching toward the observer. Is this enough to make them sound out of tune? (Assume 1 m/s as a reasonable marching speed.)
3. How much will the velocity of sound in a trumpet change as it warms up (from room temperature to body temperature, for example)? If the wavelength remains essentially the same (the expansion in length will be very small), by what percentage will the frequency change?
4. At what frequency does the wavelength of sound equal the diameter of the following? (1 in. $= 0.0254$ m)
   (a) a 15-in. woofer
   (b) a 3-in. tweeter
5. A nylon guitar string has a mass per unit length of 8.3×10^{-4} kg/m and the tension is 56 N. Find the speed of transverse waves on the string.
6. The audible range of frequencies extends from approximately 50 to 15,000 Hz. Determine the range of wavelengths of audible sound.
7. The distance from the bridge to the nut on a certain guitar is 63 cm. If the string is plucked at the center, how long will it take the pulse to propagate to either end and return to the center? (Use the speed calculated in Problem 5.)
8. Find the speed of sound in miles per hour at 0°C. This is called Mach 1. A supersonic airplane flying at Mach 1.5 is flying at 1.5 times this speed. Find its speed in miles per hour.
9. A thunderclap is heard 3 s after a lightning flash is seen. Assuming that they occurred simultaneously, how far away did they originate?
10. The density of aluminum is 2700 kg/m^3 and Young's elastic modulus is 7.1×10^{10} N/m^2 (Pa). Compare the speed of longitudinal waves in aluminum to those in steel (See Example 3.1).
11. Compare the speed of sound calculated from Eqs. (3.4) and (3.5) when $t = 30$°C.

EXPERIMENTS FOR HOME, LABORATORY, AND CLASSROOM DEMONSTRATION

Classroom Demonstrations

Waves in one dimension

1. *Waves on a rope* A rope is stretched across the front of the room and one end is fastened to a door handle or other fixed point. The other end is held in one hand, and the other hand strikes it quickly to create a pulse. The speed of this pulse is shown to increase as the tension increases. The phase of the reflected pulse on the rope is seen to be reversed (see Fig. 3.4(a)).

2. *Wave machine* A pulse is sent down a wave machine of the type developed at Bell Laboratories (see Fig. 3.8). Reflection at a fixed end again reverses the impulse, whereas reflection at a free end maintains the same orientation. When the wave machine is terminated with a dashpot, no reflection occurs.

3. *Standing waves on a rope* The rope (in Experiment 1) is moved up and down rhythmically to produce a standing wave pattern. Vary the frequency to produce one, two, and three loops. Have a student grab the rope at a nodal point to show that waves still propagate "through" his or her hand. An elastic rope attached to an electromagnetic wave driver (Pasco SE9409 and WA9753, for example) is particularly convenient for this demonstration. Projecting a transparency similar to Fig. 3.10 (preferably with the three waves in different colors) at the same time is a great help to the students in understanding standing waves.

4. *Standing waves on a wave machine* Standing waves are generated on the wave machine by moving the hand up and down rhythmically, by attaching a motorized driver, or by using an electromagnetic driver. Compare the standing waves that result from a fixed end and a free end.

5. *Wave reflection at an interface* Wave machines generally include two sections having different wave speeds. Attach two

sections together, and show the difference when a pulse originates in the slower medium and the faster medium.

6. *Waves in a ripple tank* Wave images can be projected onto a screen by means of an overhead projector or a large mirror. In a small class, projection onto the ceiling or a white paper on the floor may be satisfactory. The class should be shown how single pulses, plane waves, and circular waves propagate, reflect (at straight and curved reflectors), and refract. Diffraction at a slit and two-source interference should also be demonstrated. More complex phenomena may be demonstrated by means of film or videotape.

7. *Standing waves in a room* A three-dimensional pattern of standing waves can be demonstrated by driving a loudspeaker with a sine wave of about 1000 Hz and having the students move their heads. Ask them to estimate the distance between adjacent maxima (these will be only approximately a half-wavelength apart in a three-dimensional standing wave). Repeat the experiment at other frequencies. They may be surprised to discover a maximum (of sound pressure) in each corner of the room.

Little is to be gained by using two loudspeakers. Two-source interference should probably not be demonstrated in a classroom.*

8. *Diffraction by a slit* Diffraction can best be demonstrated with light waves. A vertical line source (straight-filament lamp or fluorescent tube) is viewed through a slit formed by two fingers; as the spacing is narrowed a diffraction pattern appears. Slits of varying size can be ruled on smoked glass or an old photographic negative and passed around. Best of all are photographic negative slits made by photographing black lines of various widths.

Students should be reminded that the visible spectrum extends less than one octave, whereas the audible spectrum extends more than seven octaves; also, acoustics wavelengths are 30 thousand to 20 million times larger than optical wavelengths.

9. *Diffraction grating* Although multislit diffraction gratings for sound are rare, the diffraction of light by an inexpensive diffraction grating is so impressive that it is worth distributing such gratings in class so that students can view a vertical filament white lamp (showcase bulb), a bare fluorescent tube, and various gas discharge lamps (neon is particularly striking). Inexpensive diffraction glasses of various types are also very striking. (The Arbor Scientific catalog, for example, lists three different types.)

10. *Doppler effect with a moving source* A "Doppler ball," consisting of a small sound source and battery in a Styrofoam ball, can be thrown about the room to illustrate the Doppler effect. Twirling a small loudspeaker on the end of its electrical cable creates an impressive change in sound, but it is a little confusing because the Doppler effect is often overpowered by variation in sound-pressure level, generation of beats, etc.

11. *Recorded Doppler effect* Make a tape recording of trucks passing on an expressway or a train whistle on a passing train (better yet, ask students to make them). Try to estimate the speed of the passing vehicle from the change in pitch as it passes. Emphasize that the change is abrupt as the vehicle passes rather than gradual as the distance to the observer changes. The record set "The Science of Sound" (Folkways) has a good recording of racing cars, as does the video "Physics at the Indy 500" (AAPT).

Laboratory Experiments

Sinusoidal motion: The oscilloscope (Experiment 3 in *Acoustics Laboratory Experiments*)

Wave propagation: The ripple tank (Experiment 6 in *Acoustics Laboratory Experiments*)

Speed of sound (Experiment 9 in *Acoustics Laboratory Experiments*)

Speed of sound (Experiment 24 in *Physics with Computers*)

*See T. D. Rossing, "Acoustic demonstrations in lecture halls: A note of caution," *American J. Physics* **44**, 1220 (1976).

CHAPTER 4

Resonance

Consider a simple mechanical system: a child in a swing (Fig. 4.1). The swing has a natural frequency that is determined by its length (as the pendulum described in Section 2.3). If the swing is given a small push at the right time in each cycle, its amplitude gradually increases. This is an example of *resonance*. The swing receives only a small amount of energy during each push, but provided this amount is larger than the energy lost during each cycle (due to friction and air drag), the *amplitude* of swing increases.

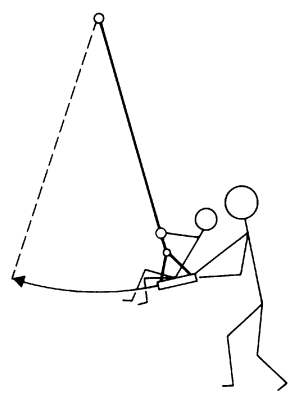


FIGURE 4.1
An example of resonance: a child in a swing.

In this chapter you should learn:

- About resonance in vibrating systems;
- How amplitude and phase change near a resonance;
- About standing waves as resonances in a pipe or a vibrating string;
- About partials, harmonics, and overtones;
- About acoustic impedance.

4.1 ■ RESONANCE OF A MASS-SPRING VIBRATOR

Consider a mass-spring system similar to the one discussed in Section 2.1. Suppose that the spring is attached to a crank, as shown in Fig. 4.2. Let the crankshaft revolve at a frequency f and let the natural frequency of the mass-spring system be f_0. If f is varied slowly, the amplitude A of the mass is observed to change, reaching its maximum $A_{\max}$ when $f = f_0$.

From Chapter 4 of *The Science of Sound*, Third Edition. Thomas D. Rossing, Richard F. Moore, Paul A. Wheeler.

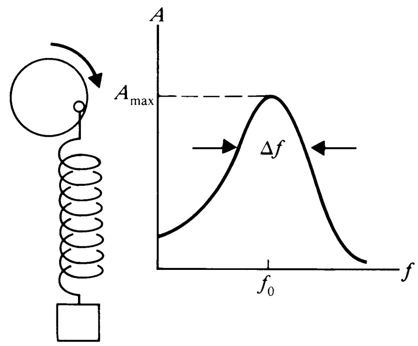


FIGURE 4.2 Resonance of mass-spring vibrator driven at a frequency f.

The mass is forced to vibrate at the frequency f of the crank, but when f matches f_0, the natural frequency of the system, resonance occurs. At resonance the maximum transfer of energy occurs, and the amplitude builds up to a value A_{max} determined by the friction in the system. The graph of amplitude A as a function of frequency f, shown in Fig. 4.2, is nearly symmetrical around its peak, with a width Δf often called the *linewidth*. The linewidth is usually measured at an amplitude of 71% of $A_{max}(A_{max}/\sqrt{2})$.

Just as A_{max} depends on the rate of energy loss (due to friction or *damping*), so Δf also depends on energy loss. For a heavily damped system, Δf is large, and A_{max} is small. For a system with little loss, a "sharp" resonance with small Δf and large A_{max} occurs. Engineers define a quantity $Q = f_0/\Delta f$ to characterize the sharpness of a resonance. (The use of the letter Q comes from the term "quality factor" used to describe electrical circuits. A high-Q circuit is one with a sharp resonance; a low-Q circuit has a broad resonance curve.)

The linewidth Δf and the Q associated with a resonance are intimately related to the damping constant and the decay curve of a vibrator described in Section 2.2. A vibrator that loses its energy slowly will have a sharp resonance, and a vibrator that loses its energy rapidly will have a broad resonance. If the vibrator is set into motion and left to vibrate freely, its decay time is directly proportional to the Q of its resonance.

4.2 ■ PHASE OF DRIVEN VIBRATIONS

If we carefully observe the direction of motion of the crank and the mass, we note an interesting phenomenon. At low frequencies, far below resonance, the two move in the same direction. At frequencies far above resonance, however, they move in opposite directions.

We describe this phenomenon by using the term *phase*, which may be thought of as a specification of the starting point of a vibration. At low frequencies, the entire system follows the motion of the crank, and the spring hardly stretches at all. As the frequency of the crank increases, however, it is more difficult to move the mass, and thus it begins to lag behind the driving force supplied by the crank. At resonance, the mass is one-fourth cycle behind the crank, although its amplitude builds up to its maximum value. As the crank frequency is increased still further, the phase difference becomes greater until finally the mass is one-half cycle behind the crank; that is, the mass and the crank move in opposite directions, as shown at the right in Fig. 4.3. The higher the Q, the more abrupt is this

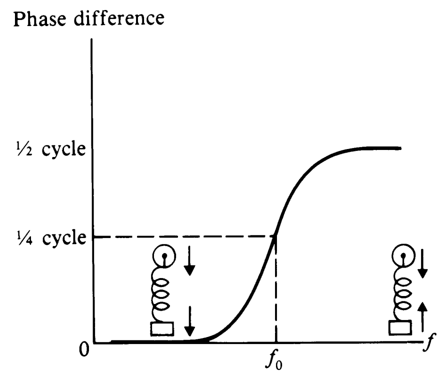


FIGURE 4.3 Phase difference between crank and mass in a driven mass-spring system. At resonance they differ in phase by one-fourth of a cycle.

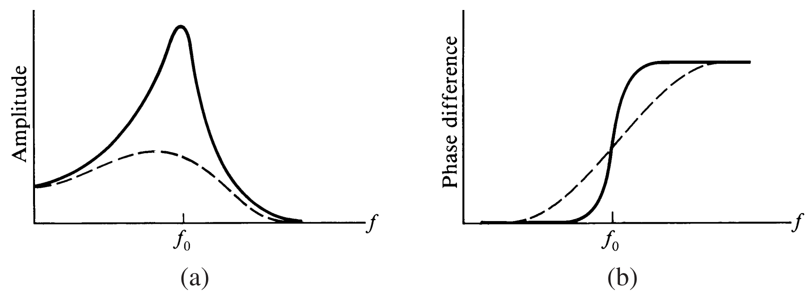


FIGURE 4.4 (a) Response and (b) phase difference for vibrators with more (dashed curve) and less (solid curve) damping.

transition from *in phase* to *opposite phase*. A vibrator with a lot of damping, on the other hand, exhibits a gradual change of phase, as shown in Fig. 4.4.

4.3 ■ STANDING WAVES ON A STRING

In Section 3.4, we learned how interference between two waves traveling in opposite directions leads to standing waves. We also learned how reflection occurs when waves or pulses reach the boundary of the medium in which they propagate. We now combine these two ideas, and show how the modes of vibration or resonances of acoustical systems can be interpreted as waves propagating back and forth between the boundaries.

A simple and familiar example of such a system is a string of length L with both ends fixed as shown in Fig. 4.5. In its fundamental mode (that is, the standing wave with the lowest frequency and the longest wavelength), the string vibrates as shown in Fig. 4.5(a). The wavelength λ can be seen to be twice the string length, so the frequency is $f_1 = v/2L$. In the second mode, shown in Fig. 4.5(b), the wavelength λ equals the string length, so $f_2 = v/L = 2f_1$. Continuing to higher modes, we find that they have frequencies $3f_1$, $4f_1$, etc. The frequency of the nth mode will be

$$f_n = n\frac{v}{2L} = nf_1. \qquad (4.1)$$

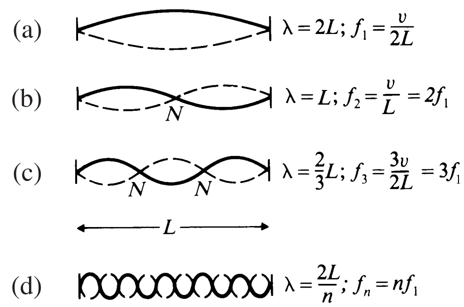


FIGURE 4.5 Modes of vibration or resonances of a vibrating string as standing waves. The nodes are denoted by N. Note that the frequencies are harmonics of the fundamental frequency f_1.

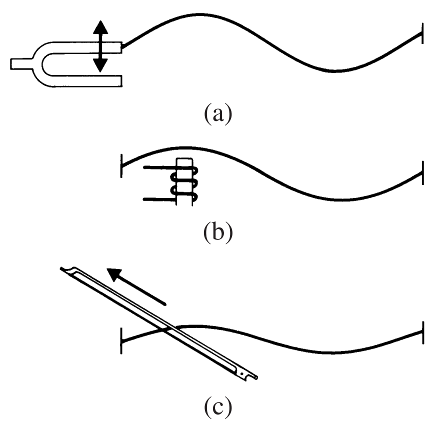


FIGURE 4.6 Three ways to drive a string at one of its resonances: (a) a tuning fork; (b) an electromagnet; (c) a violin bow.

Substituting the expression for wave speed given in Section 3.2 gives the following expression for the modes of a vibrating string:

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}, \tag{4.2}$$

where T is tension and μ is mass per unit length.

If a string is driven at the frequency of one of its natural modes, resonance can occur. There are many ways in which to apply the driving force, three of which are illustrated in Fig. 4.6. The magnetic drive shown in Fig. 4.6(b) works only for a string of steel or other magnetic material. The moving violin bow applies a rather complicated driving force (to be described in Chapter 10), which has components at several different frequencies. The string may also be driven by *electromagnetic force*, even when the string is made of a nonmagnetic metal, by placing a permanent magnet near the string and passing an alternating current of the desired frequency through the string.

EXAMPLE 4.1 A steel guitar string with a diameter of 0.3 mm and 65 cm long has a tension of 100 N. Find the frequencies of its first three modes of vibration. The density of steel is 7700 kg/m^3.

Solution

$$\mu = \pi r^2 \rho = \pi (1.5 \times 10^{-4})^2 (7700)$$
$$= 5.44 \times 10^{-4} \text{ kg/m};$$
$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.65)}\sqrt{\frac{100}{5.44 \times 10^{-4}}} = 330 \text{ Hz};$$
$$f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}} = \frac{2}{2(0.65)}\sqrt{\frac{100}{5.44 \times 10^{-4}}} = 660 \text{ Hz } (= 2f_1);$$
$$f_3 = 3f_1 = 990 \text{ Hz}.$$

4.4 ■ PARTIALS, HARMONICS, AND OVERTONES

It is appropriate at this point to clarify nomenclature in order to avoid confusion. We will use the term *harmonics* to refer to modes of vibration of a system that are whole-number multiples of the fundamental mode and also to the sounds that they generate. (It is customary to stretch the definition a bit so that it includes modes that are *nearly* whole-number multiples of the fundamental: 2.005 times the fundamental rather than 2, for example.) Thus we say that the modes of an ideal vibrating string are harmonics of the fundamental, but the modes of a real string are usually so close to being whole-number multiples that we also speak of them as harmonics. Note that the term *first harmonic* refers to the fundamental.

Many vibrators do not have modes that are whole-number multiples of the fundamental frequency, however, and the term *overtones* is used to denote their higher modes of vibration. Harmonics are therefore described as overtones whose frequencies are whole-number multiples of the fundamental frequency. Minor confusion arises, however, from the fact that the term harmonics includes the fundamental, but the term overtones does not. Thus the second harmonic is the first overtone, the third harmonic is the second overtone, and so forth.

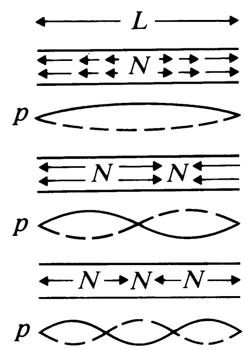
There is another term in common use that refers to modes of vibration of a system or the components of a sound: *partials*. Partials include all the modes or components, the fundamental plus all the overtones, whether they are harmonics or not. The term *upper partials* excludes the fundamental and thus is a synonym of overtones, but use of the former will be avoided in this book.

The actual motion of a vibrating string is a combination of the various modes of vibration. The way in which these modes or partials combine is given by the *spectrum* of the vibration. A vibration spectrum is like a recipe that specifies the relative amplitudes of the partials. Similarly, the spectrum of a sound specifies the amplitudes of its partials, as we will discuss in Chapter 7.

4.5 ■ OPEN AND CLOSED PIPES

The reflection of sound pulses at open and closed ends of pipes was described in Section 3.5. At an open end, a pulse of positive pressure reflects back as a negative pulse; at a closed end, it reflects as a positive pulse. These two end conditions can be used to arrive at the resonances for open and closed pipes.

The motion of the vibrating air in a pipe is a little harder to visualize than the transverse vibrations of a string, because the motion of the air is longitudinal. The displacement of the air is greatest at an *open* end, but the pressure variation is maximum at a *closed end*. A pressure-sensitive microphone inserted into the tube will pick up the most sound at the points where the pressure variations above and below atmospheric pressure are maximum. Thus, in Figs. 4.7 and 4.8, both the air motion and the pressure variations are shown.



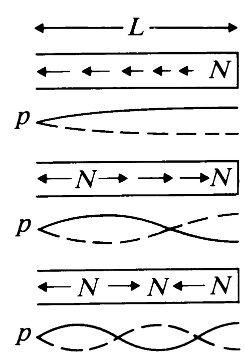
$$\lambda = 2L;\ f_1 = \frac{v}{2L}$$

$$\lambda = L;\ f_2 = \frac{v}{L} = 2f_1 \tag{4.3}$$

$$\lambda = \frac{2}{3}L;\ f_3 = \frac{3v}{2L} = 3f_1$$

$$f_n = nf_1$$
$$(n = 1,\ 2,\ 3\ .\ .\ .)$$

FIGURE 4.7 Modes of vibration or resonances of an open pipe. At the open ends the pressure is equal to atmospheric pressure. The resulting modes include both odd-numbered and even-numbered harmonics. Minimum displacement occurs at the nodes denoted by N.



$$\lambda = 4L;\ f_1 = \frac{v}{4L}$$

$$\lambda = \frac{4}{3}L;\ f_3 = \frac{3v}{4L} = 3f_1 \tag{4.4}$$

$$\lambda = \frac{4}{5}L;\ f_5 = \frac{5v}{4L} = 5f_1$$

$$f_n = nf_1$$
$$(n = 1,\ 3,\ 5\ .\ .\ .)$$

FIGURE 4.8 Modes of vibration or resonances of a closed pipe. At the closed end, the air motion is minimum but the pressure is maximum. The resulting modes include odd-numbered harmonics only. Minimum displacement occurs at the nodes denoted by N.

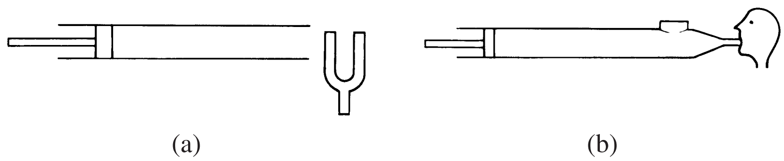


FIGURE 4.9 Resonance of a closed tube demonstrated by (a) holding a tuning fork near one end; (b) blowing on an organ pipe.

In an actual pipe, the pressure variations do not drop to zero right at the open end of the pipe, but rather a small distance beyond. Thus, the pipe appears to have an acoustic length that is slightly greater than its physical length. For a cylindrical pipe of radius r, the additional length, called the *end correction*, is $0.61r$. Twice this amount should be added to the length of a pipe with two open ends to obtain its acoustic length.

Resonance of a tube can be demonstrated by placing a tuning fork near one end of it, as shown in Fig. 4.9(a). A piston at the closed end makes it possible to change the resonance frequency. Blowing through a closed ("stopped") organ pipe may excite several of its resonances. Gentle blowing excites the lowest mode, but blowing much harder causes the pipe to vibrate in its first overtone, which for a closed pipe is the third harmonic ($f_3 = 3f_1$), a musical twelfth above the lowest mode.

In Chapter 15, which deals with speech production, we will be interested in resonances of the human vocal tract that allow us to enunciate various vowel sounds. There, we will consider not only reflections from open and closed ends but from constrictions and changes in pipe diameter as well. At every such discontinuity, a portion of the sound wave is reflected, thus leading to standing waves and resonances. If reflections occur at several places along the pipe, the resonances (in the vocal tract, they are called *formants*) can become rather complex.

EXAMPLE 4.2 Find the first three modes of vibration of a pipe 0.75 m long with open ends (neglect end corrections).

Solution

$$f_1 = \frac{v}{2L} = \frac{343}{2(0.75)} = 229 \text{ Hz};$$

$$f_2 = \frac{2v}{2L} = \frac{2(343)}{2(0.75)} = 457 \text{ Hz};$$

$$f_3 = \frac{3v}{2L} = \frac{3(343)}{2(0.75)} = 658 \text{ Hz}.$$

EXAMPLE 4.3 Find the first three modes of vibration of a pipe 0.75 m long with one open end and one closed end (neglect end correction).

Solution

$$f_1 = \frac{v}{4L} = \frac{343}{4(0.75)} = 114 \text{ Hz};$$

$$f_3 = \frac{3v}{4L} = \frac{3(343)}{4(0.75)} = 343 \text{ Hz};$$

$$f_5 = \frac{5v}{4L} = \frac{5(343)}{4(0.75)} = 572 \text{ Hz}.$$

4.6 ■ ACOUSTIC IMPEDANCE

A quantity that acoustical engineers find very useful is *acoustic impedance* Z_A. It is defined as the ratio of sound pressure p to volume velocity U and is measured in acoustic ohms:

$$Z_A = p/U. \tag{4.5}$$

The volume velocity U is the amount of air that flows through a specified area per second due to passage of a sound wave. In the case of sound propagating in a tube, the specified area would be the cross-sectional area of the tube.

In the case of plane waves propagating in a tube, the acoustic impedance can be found by using the formula $Z_A = \rho v/S$, where ρ is the density of the air (1.15 kg/m^3 at room temperature), v is the speed of sound, and S is the cross-sectional area of the tube. Hence $Z_A \simeq 400/S$, with S measured in square meters. We will find the numerical value of Z_A much less important than the fact that it varies inversely with area S. Thus when there is a constriction, a change in diameter, or a sidebranch in a tube, the impedance change at that point leads to reflection of sound waves.

Acoustic impedance is analogous to electrical impedance, which is the ratio of voltage to electrical current (see Chapter 18). In this case, the voltage is the forcing function that causes current to flow. In the case of a sound wave, sound pressure is the forcing function that causes air flow with volume velocity U. In the chapters on wind instruments, use will be made of acoustic input impedance, which is the ratio of pressure to volume velocity at the input (mouthpiece) of a wind instrument.

4.7 ■ HELMHOLTZ RESONATOR

The Helmholtz resonator was described as a vibrating system in Section 2.3. As a resonator, it has many applications in acoustics. Only a few of them will be described.

1. Before the invention of microphones, amplifiers, and spectrum analyzers, Helmholtz resonators were used to study vibrating objects and analyze complex sounds. In Fig. 4.10(a) the modes of vibration of a bell are being probed by a Helmholtz resonator having a second opening through which resonant sound can be heard by the investigator.

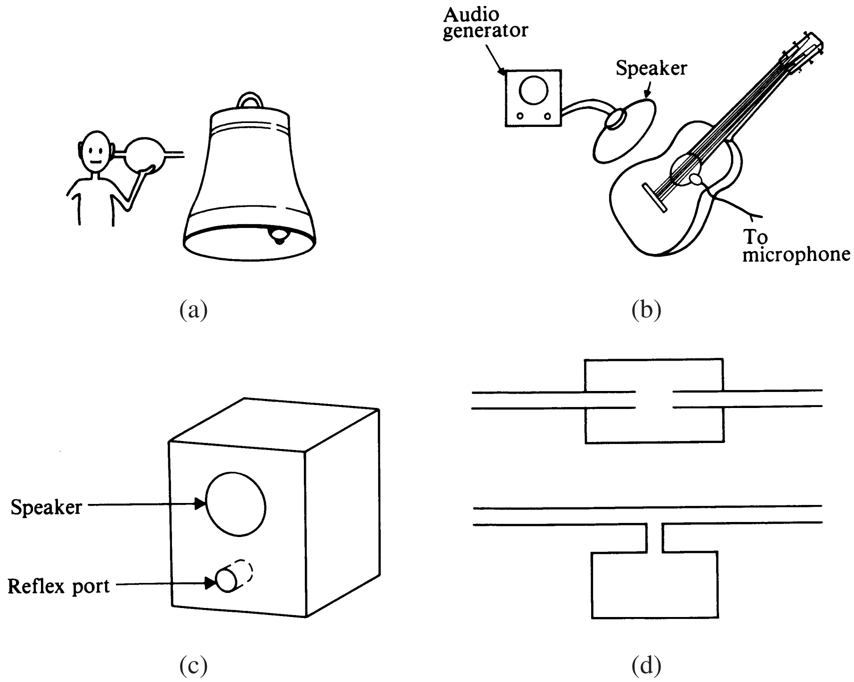


FIGURE 4.10 Some examples of Helmholtz resonators: (a) analysis of vibrations of a bell; (b) air resonance of a guitar body; (c) bass reflex loudspeaker cabinet; (d) two types of side-branch mufflers.

2. The main air resonance of a violin or guitar is essentially a Helmholtz resonance. The frequency can be determined by blowing across the f-holes of a violin or the sound hole of a guitar and listening for the pitch of the resonance. A more precise method is to insert a microphone inside the instrument and generate sound outside by means of a loudspeaker and audio generator.
3. Bass reflex loudspeaker cabinets are designed so that radiation from the back of the speaker cone excites the Helmholtz resonance of the cabinet and appears at the reflex port in phase with the front of the speaker.
4. Some automobile mufflers make use of side branches that absorb sound at their resonance frequency, as shown in Fig. 4.10(d).

4.8 ■ SINGING RODS AND WINEGLASSES

In addition to the bending modes of a bar shown in Fig. 2.17, there are several other ways in which a bar or rod can vibrate. These include longitudinal vibrations (similar to sound waves) and torsional (or twisting) modes.

Sound waves traveling in air are longitudinal waves; the air molecules vibrate back and forth in the same direction the wave is propagating. Similar longitudinal (compressional) waves can propagate in solids and liquids, resulting in longitudinal standing waves or normal modes of vibration. In a bar or rod, longitudinal waves travel at a speed $v = \sqrt{\frac{E}{\rho}}$, where E is Young's elastic modulus for the material and ρ is its density. Note that the wave speed does not depend on frequency or the diameter of the rod.

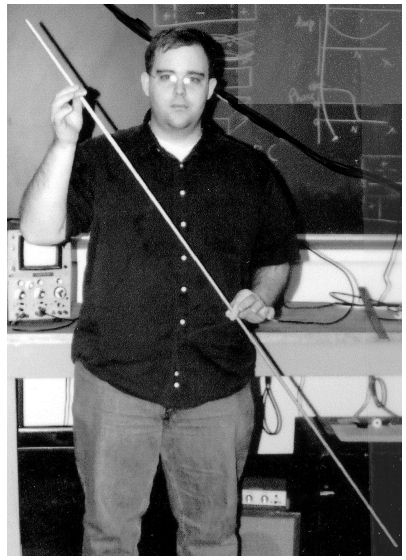


FIGURE 4.11
Stroking an aluminum rod with the fingers to excite longitudinal resonances.

In a bar or rod with free ends, the fundamental mode will have a node at its center, and the maximum vibration occurs at the ends just like the open pipe in Fig. 4.7. Its frequency is the wave speed divided by the wavelength (twice the rod length): $f_1 = \frac{n}{2L}\sqrt{\frac{E}{\rho}}$. The next mode has two nodes at $\frac{1}{4}L$ and $\frac{3}{4}L$ like the second mode in Fig. 4.7, the third mode has three nodes, and so on. Note that only the odd-numbered modes have nodes at the center. The modal frequencies are

$$f_n = \frac{n}{2L}\sqrt{\frac{E}{\rho}} \tag{4.6}$$

In most bars or rods, the longitudinal modes of vibration occur at much higher frequencies than the transverse modes.

Stroking an aluminum rod with the fingers to excite longitudinal resonances, as shown in Fig. 4.11, can create rather loud sounds, and stroke rods are actually used as percussion instruments. Further discussion of longitudinal and transverse vibrations in rods is given in Chapter 13.

Can a singer break a wineglass? We have never seen it happen, but perhaps it is possible. If the singer sings a note at precisely the frequency of the lowest mode of vibration of the wineglass (which can be determined by tapping it lightly on the rim or by running one's finger around the rim), it will certainly excite a resonance in the glass. The problem is that the sound power of the singing voice is small to start with, and only a small part of the total power is contained in the lowest partial or fundamental. Breaking a wineglass with sound (as in the TV commercials) is usually done with an amplifier and a loudspeaker to produce ample sound at the resonance frequency.

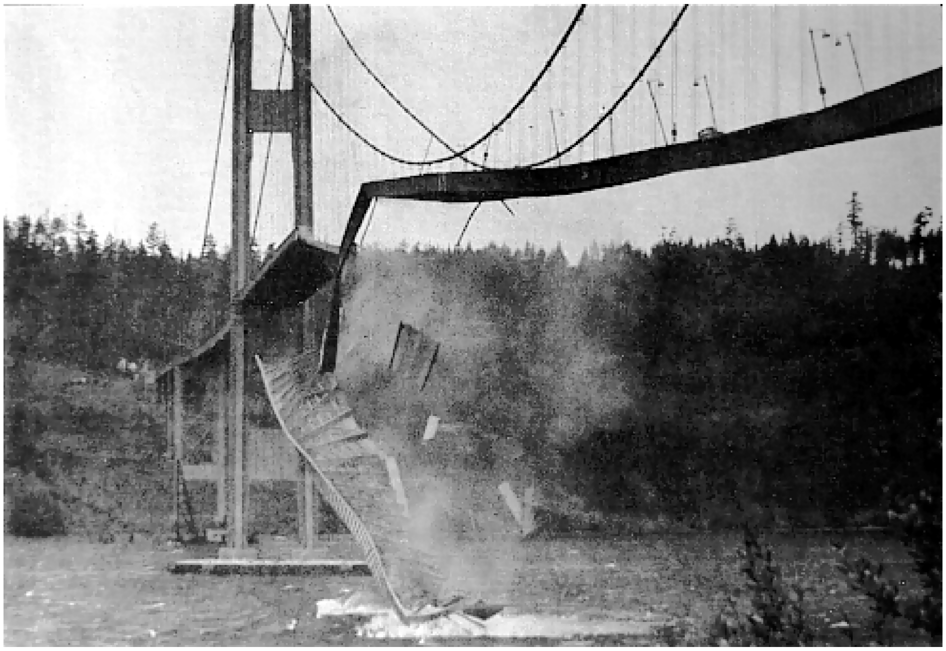


FIGURE 4.12 Collapse of the Tacoma Narrows Bridge (1940) is a dramatic example of self-excited oscillation. The wind excited the bridge in one of its vibrational modes.

4.9 ■ SELF-EXCITATION

A self-excited oscillator is one in which a steady motion, flow of air, or electric current is modulated and a part of the modulated flow is then fed back in such a way as to excite the oscillator. One example, a bowed violin string, will be discussed in Chapter 10. A dramatic example is the Tacoma Narrows Bridge, which collapsed in 1940. A torsional (twisting) mode that the designers had apparently overlooked (and for which they failed to provide adequate damping) was excited by the wind, and it just kept building up in amplitude until the bridge broke into pieces, as shown in Fig. 4.12.

4.10 ■ SYMPATHETIC VIBRATIONS: SOUNDBOARDS

The amount of sound radiated by a vibrating system depends on the amount of air it displaces as it moves (the volume velocity defined in Section 4.6). A vibrating string or the narrow prongs of a tuning fork displace very little air as they vibrate; thus they radiate a small amount of sound. The moving cone of a loudspeaker and the vibrating membrane of a drum, on the other hand, radiate sound rather efficiently.

It is possible to increase the sound radiation from a tuning fork by pressing its handle against a wood plate or tabletop, so that the tuning fork forces the large wood area to vibrate. The vibrations of the wood, called *sympathetic vibrations*, may or may not occur at a frequency near a resonance of the wood plate, but nevertheless they amplify the sound because of the large surface set into vibration.

Violins, guitars, cellos, lutes, and other string instruments depend almost completely on sympathetic vibrations of the wood sounding box for radiation of their sound. Most of the sound radiation in these instruments comes from sympathetic vibrations of the top plate, which is driven by the vibrating strings through the bridge. The top plate has many resonances of its own distributed throughout the playing range, and these resonances, to a large part, determine the quality of the instrument. Sympathetic vibration of the wood also sets the air inside the instrument into vibration, so that sound is radiated from the f-holes (violin) or sound hole (guitar). String instruments will be discussed in Chapter 10.

Pianos and harpsichords have large soundboards with many resonances of their own closely spaced throughout the playing range. Vibrations are transmitted from the string to the soundboard through the bridge as in the violin or guitar. We will discuss pianos and harpsichords in Chapter 14.

4.11 ■ SUMMARY

Resonance occurs when a force applied to a vibrating system oscillates with a frequency at or near the natural frequency of the system. Linewidth, Q, and maximum response are ways to describe the sharpness of the resonance, which in turn depends on the amount of damping in the system. The phase difference between the vibrating object and the driving force changes near the frequency resonance.

Normal modes of vibration of strings, pipes, and similar systems may be described as standing waves. Standing waves, which consist of waves propagating back and forth between the boundaries, lead to resonances in vibrating systems. Acoustic impedance, which is the ratio of sound pressure to volume velocity, is a quantity useful in the analysis of acoustical systems. Since sound radiation depends on air displacement, radiation from a vibrating string is greatly enhanced by the sympathetic vibration of a soundboard.

REFERENCES AND SUGGESTED READINGS

Carpenter, D. R., Jr., and R. B. Minnix (1993). *The Dick and Rae Physics Demo Notebook*. Lexington, Va.: Dick and Rae, Inc.

French, A. P. (1966). *Vibrations and Waves*. New York: Norton. (See Chapter 4.)

Hewitt, P. G. (1993). *Conceptual Physics*, 7th ed. New York: Harper Collins.

Kirkpatrick, L. D., and G. F. Wheeler (1995). *Physics: A World View*, 2nd ed. Philadelphia: Saunders.

Meiners, H. F. (1985). *Physics Demonstration Experiments*. Malabar, Fla.: R. E. Krieger.

Lord Rayleigh (J. W. Strutt) (1894). *The Theory of Sound*, Vol. 2, 2nd ed. London: Macmillan. Reprinted by Dover, New York, 1945. Section 253b.

Rossing, T. D. (1982). *Acoustics Laboratory Experiments*. DeKalb: Northern Illinois University.

Rossing, T. D., and N. H. Fletcher (1995). *Principles of Vibration and Sound*. New York: Springer-Verlag.

GLOSSARY

acoustic impedance A measure of the difficulty of generating flow (in a tube, for example); it is the ratio of the sound pressure to the volume velocity due to a sound wave.

amplitude The height of a wave; the maximum displacement of a vibrating system from equilibrium.

damping Energy loss in a system that slows it down or leads to a decrease in amplitude.

electromagnetic force The force that results from the interaction of an alternating electric current with a magnetic field.

fundamental The mode of vibration (or component of sound) with the lowest frequency.

harmonic A mode of vibration (or a component of a sound) whose frequency is a whole-number multiple of the fundamental frequency.

Helmholtz resonator A vibrator consisting of a volume of enclosed air with an open neck or port.

linewidth The width Δf of a resonance curve, usually measured at 71% of its maximum height; a measure of the sharpness of a resonance (a sharp resonance is characterized by a small linewidth).

overtone A component of a sound with a frequency greater than the fundamental frequency.

partial A component of a sound; includes the fundamental plus the overtones.

phase difference A measure of the relative positions of two vibrating objects at a given time; also the relative positions, in a vibration cycle, of a vibrating object and a driving force.

Q A parameter that denotes the sharpness of a resonance; $Q = f_0/\Delta f$, where f_0 is the resonance frequency and Δf is the linewidth.

resonance When a vibrator is driven by a force that oscillates at a frequency at or near the natural frequency of the vibrator, a relatively large amplitude results.

soundboard A sheet of wood or other material that radiates a substantial amount of sound when it is driven in sympathetic vibration by a vibrating string or in some other manner.

spectrum A recipe for vibratory motion (or sound) that specifies the relative amplitudes of the partials.

sympathetic vibration One vibrator causing another to vibrate at the same frequency (which may or may not be a resonance frequency). An example is a piano string causing the bridge and soundboard to vibrate at the string's frequency.

REVIEW QUESTIONS

1. Write a definition of *resonance* and give several examples.
2. In Fig. 4.4, do the solid curves or the dashed curves represent a higher Q? Explain.
3. What does n equal in Fig. 4.5(d)? How many wavelengths equal L?
4. Distinguish between partials, harmonics, and overtones.
5. If you blow over the ends of two pipes, one with the other end closed and one with it open, which pipe will give the tone of lower pitch? approximately how much lower?
6. A pipe with one open and one closed end has its lowest resonance at 200 Hz. What are the frequencies of its next two resonances?
7. What is acoustic impedance?
8. In order to lower the Helmholtz resonances of a guitar, would you make the sound hole larger or smaller?
9. To excite a singing rod in its fundamental mode, where should you hold it? Where should you stroke it?
10. Can a singer break a wineglass by singing loudly?
11. What is the main function of a piano soundboard?

QUESTIONS FOR THOUGHT AND DISCUSSION

1. If a child in a swing is pushed with the same impulsive force in each cycle, will the amplitude increase by the same amount in each cycle?
2. List as many examples of Helmholtz resonators as you can other than those given in Section 4.7. Are the resonances sharp or broad?
3. Attach a mass to a spring, as in Fig. 2.1 or 4.2, and determine the approximate resonance frequency by moving the top of the spring up and down by hand. Then move it at frequencies below and above resonance, and carefully describe the force exerted on your hand in each case.
4. Does the end correction given in Section 4.5 lower all harmonics of a pipe proportionally, or does it result in the overtones going out of tune? An exact expression for the end correction shows that it varies slightly with wavelength. Does that change your answer?

EXERCISES

1. A particular vibrator has a resonance frequency of 440 Hz and a Q of 30. What is the linewidth of its resonance curve?
2. Sketch a waveform that represents the displacement of the mass in Fig. 4.2 as a function of time. Then carefully sketch a second wave one-fourth cycle in advance of the first to represent the driving force at resonance. Label each curve correctly.
3. Determine the frequencies of the fundamental and first overtone (second partial) for the following. Neglect end corrections.
   (a) A 16-ft open organ pipe
   (b) a 16-ft stopped organ pipe (one open end, one closed end)
4. Extend Figs. 4.7 and 4.8 to include two more modes each.
5. Find the difference in the fundamental frequency, calculated with and without the end correction, of an open organ pipe 2 m long and 10 cm in diameter.
6. A nylon guitar string 65 cm long has a mass of 8.3×10^{-4} kg/m and the tension is 56 N. Find the frequencies of the first four partials.
7. A steel bar 1 m long is held at the center and tapped on one end. Because its ends are free to move, its modes of longitudinal vibration will be similar to those of the air in a pipe open at both ends. Using the speed of sound given in Table 3.1, calculate the frequencies of the first three longitudinal modes.
8. Determine the frequencies of the pipes in Problem 3 if helium is substituted for air. (The speed of sound in helium is given in Table 3.1.)

EXPERIMENTS FOR HOME, LABORATORY, AND CLASSROOM DEMONSTRATION

Home and Classroom Demonstration

1. *Resonance of hand-held oscillator* It is easy to demonstrate resonance by moving the top of a spring, to which a mass is attached, up and down at the correct frequency. When the hand is moved up and down slowly, the mass is seen to move in phase with the hand; when the hand is moved quite rapidly, it can be seen that the mass and the hand move in opposite phase, as shown in Fig. 4.3.

2. *Resonance of a driven oscillator* Quantitative data require a sinusoidal drive with variable frequency. Although it is easy to demonstrate that the amplitude of a mass-spring oscillator has maximum value at the resonance frequency (Fig. 4.2), it is more difficult to show the important phase change at resonance (Fig. 4.3). Attaching markers as shown at the right is fairly effective. In the Pasco ME9210A harmonic motion analyzer (now discontinued), a flashing LED showed the phase relationship between the driver and the oscillating mass.

3. *Resonance of a wire string* The resonances of a thin wire string should be demonstrated, preferably both by electromagnetic excitation and by bowing with a violin bow (see Fig. 4.6).

4. *Resonance of a closed tube* A tuning fork is held above one end of a glass tube whose other end is immersed in a large reservoir of water. The tube is raised or lowered in the water until resonance occurs.

Another way to change the length of a closed-end tube is to connect it to a reservoir by means of a hose. The water height in the tube is adjusted by raising or lowering the reservoir.

5. *Resonance of a tube by a loudspeaker* A loudspeaker L driven by an audio generator sets up standing waves in a large cardboard or Plexiglas tube. A small microphone M can be moved up and down to locate the pressure maxima for each resonance.

6. *Tubing fork resonator* The Ames tube, sold by Riverbank Laboratories, is a tuning fork and open-end resonance tube combined. Their Ames tube kit includes materials for several interesting demonstrations. Choirchimes, made by Malmark, Inc. (a handbell manufacturer), similarly combine a tuning fork with a closed-end resonator. Choirchimes, which are popular with handbell choirs in churches and schools, include a clapper with which to set the tuning fork into vibration.

7. *Smoke alarm vibrator* A vibrator of the type used in smoke alarms is attached to one end of a tube, which is adjustable in length. When powered by a battery, the vibrator generates a tone with several harmonics, and the tube can be adjusted to resonate with individual harmonics.

8. *Kundt's tube* In the classical Kundt's tube, a brass rod is stroked with a rosin-soaked cloth so that it vibrates (longitudinally) at its resonance frequency. A plunger is adjusted until standing waves appear in cork dust in the glass tube. Note that two resonances are being demonstrated: one in the brass rod and one in the air column.

9. *Electrified Kundt's tube* In the electrified Kundt's tube, a horn driver or small loudspeaker drives the tube. In this case, it is easy to obtain striations in the cork dust, which fascinated Lord Rayleigh (*Theory of Sound*, Section 253b) and others.

10. *Impedance of an air column* If the loudspeaker in Experiment 9 is driven at a nearly constant current (by adding a resistor of 200 Ω or more in series), the voltage across the speaker (and hence the electrical impedance) can be shown to rise at each resonance. (If the voltage and current are displayed on an oscilloscope, a shift in phase can also be shown.)

11. *Resonances of a tin whistle* Some tin whistles (or plastic whistles) will sound several notes with the end closed as well as open. With a closed end the fundamental will be an octave lower and only the odd-numbered harmonics will sound (f, $3f$, $5f$, etc.). With a closed end, all the harmonics of the higher note will sound ($2f$, $4f$, $6f$, etc.). Thus by alternately opening and closing the end, a complete harmonic series is obtained and bugle calls can be played.

12. *Open and closed organ pipes* Many cylindrical organ pipes will sound several notes with the end closed as well as open. With a closed end, the fundamental will be an octave lower and only the odd-numbered harmonics will sound. With both ends open, both odd and even harmonics will sound.

13. *Flutes and clarinets* Flutes and recorders behave like open pipes and thus sound both odd and even harmonics of the fundamental (Fig. 4.7). Clarinets behave like closed-end pipes and thus play only the odd-numbered harmonics (Fig. 4.8).

14. *Rubens flame tube* Flame tubes with regularly spaced holes (Meiners 1985, 19–3.5) are widely used to demonstrate standing waves, but the explanation of why variations in average gas pressure occur requires a little thought (see T. D. Rossing, "Average pressure in standing waves," Phys. Teach. *15*, 260 (1977)).

15. *Pipe excited with a Meeker burner* A pipe about 1 to 2 m in length will resonate when lowered to optimum position over a Meeker (Fisher) grid-top burner. Large-diameter tubes are easiest to excite. Adjust the burner until blue tips are formed about 1 mm above the grid.

16. *Rijke tube* The Rijke tube has one or more layers of wire gauze at about one-fourth of its length to produce turbulence. If you heat the gauze cherry red, the tube sings when removed from the flame (as long as it is near vertical). To amuse the class, sound can be "poured out" into a bucket and then "poured back" into the tube.

17. *Singing corrugated hose* Plastic corrugated hose emits tones when held near one end and twirled in a circle. Sump pump hose (1 m long × 2.5–3.5 cm in diameter) from building supply stores works well, as do "hummers," sold in toy stores. Some spiral-wound vacuum cleaner hose will not work, nor will hose with closely spaced corrugations.

By measuring the frequencies of several harmonics, the speed of sound in the hose can be determined. It is up to 10% slower than the speed of sound in air, due to the periodic loading effect of the corrugations.

18. *Soda-straw reed instrument* Flatten one end of a soda straw and cut an arrow-shaped end with scissors. Place well into your mouth and blow hard. You should get a tone as the end vibrates in the manner of an oboe or bassoon reed. While blowing, shorten the straw in small steps with the scissors to hear the frequency rise. Alternatively, you can cut some tone holes in the straw that can be covered or uncovered with your fingers. Or, you can add a second straw of slightly greater or lesser diameter, so that the tube can be lengthened by sliding the outer (or inner straw) to create a sort of "kazoo."

19. *Longitudinal resonances in a singing rod* An aluminum rod about 1.3 cm (0.5 in.) in diameter and 2 m long can be held at the center and stroked longitudinally to give a rather loud sound. When held at the center, the rod vibrates at a frequency such that it is one-half a wavelength. Holding the rod at one-fourth of its length causes it to vibrate in its second harmonic. The rod can be sprayed with a tacky material (such as Firm Grip, sold in sporting goods stores) or the fingers can be coated with a little rosin to enhance the stick-slip action. Some demonstrators prefer to stroke the rod with a paper towel soaked in rubbing alcohol (Carpenter and Minnix 1993).

20. *Longitudinal and transverse vibrations of a rod* An aluminum rod about 0.6 cm (0.25 in.) in diameter and 2 m long can be used to demonstrate a great difference in speed between longitudinal and transverse waves. Hold the rod at the center and stroke it (as in Experiment 19) or tap it on the floor to excite the lowest longitudinal resonance. Then hold it at about one-fourth of its length and strike the center with the other hand to excite the lowest transverse resonance. The vibrations are slow enough to be readily visible. Formulas for transverse and longitudinal wave speeds are given in Table 13.1.

21. *Breaking a beaker (or wineglass)* Tap a beaker (or wineglass) to determine the frequency of its fundamental

mode. Supply a signal of that frequency to an audio amplifier and drive a loudspeaker positioned close to the beaker (a compression horn driver works well, because the sound output is concentrated in a small area). The beaker must have a high Q, and the sound source must have ample power in order for the beaker to break.

Laboratory Experiments

Resonance (Experiment 7, *Acoustics Laboratory Experiments*)

Vibrating strings (Experiment 4, *Acoustics Laboratory Experiments*)

Vibrations of rods: longitudinal and transverse (Experiment 45, *Acoustics Laboratory Experiments*)