Architectural Acoustics

Exercises week 1 and 2: answers

2020-02-16

Question 1

a) $\hat{p} = \sqrt{Re\{p\}^2 + Im\{p\}^2} = 0.22$ Pa

b) Assume $p(x,t) = \hat{p}e^{j(\omega t - kx + \varphi)}$, then $p(x = 0, t = 0) = \hat{p}e^{j\varphi}$, and $\varphi = atan\left(\frac{Im\{p\}}{Re\{p\}}\right)$=1.11 rad

Question 2

a) The frequency is computed from $\omega = 628{,}31 = 2\pi f$. Than, *f = 100* Hz.

b) We make use of the relation *Z = F/v*, from which we get

$$v(t) = \frac{5e^{j628{,}31t}}{1.10^4\ e^{\frac{j\omega}{800}}} = 5.10^{-4}e^{j(628{,}31t - \pi/4)}$$

c) No, they differ by a phase of $\pi/4$ rad.

d) power injected can be computed from

$P_a\ = \frac{1}{2}\hat{v}^2 Re\{Z\}$=$\frac{1}{2}0{,}25.10^{-6} * 1.10^4 \cos(\pi/4\ )$ W

e) The resonance frequency is computed from $f\ = \frac{1}{2\pi}\sqrt{\frac{1}{mn}}$ ≈50 Hz.

Question 3

The spectrum of the signal can be computed as

$$P(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p(t)e^{-j\omega t}dt$$

The argument of the transform $p(t)e^{-j\omega t}$ is equal to $p(t)$ for this specific case. This means:

$$P(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p(t)dt$$

It is obvious that $P(\omega)$ does not depend on the frequency since the frequency does not appear at the right side of the last equation.

Question 4

a) $L = 20log_{10}\ \left(\frac{\tilde{p}}{p_b}\right) = 20log_{10}\ \left(\frac{0.045}{2.10^{-5}}\right) = 67.1$ dB

b) $L(\omega) = 20log_{10}\ \left(\frac{\tilde{p}(\omega)G(\omega)}{p_b}\right)$. For 1000 Hz, we find *L* = 71.2 dB.

Question 5

We only have longitudinal sound waves in air.

Question 6

a) First, k = $\frac{\omega}{c} = \frac{2\pi f}{c}$, then we find $p(1{,}1) = \hat{p}e^{-j626{,}47}$ = -0.2742 + 0.9617i Pa.

b)

$$-\frac{\partial p}{\partial x} = \rho_0 \frac{\partial v_x}{\partial t}$$

Then

$-\frac{\partial p}{\partial x} = -jkp$ and $\frac{\partial v_x}{\partial t} = -j\omega v_x$, so $v_x = \frac{jk}{j\omega\rho_0} p = \frac{p}{\rho_0 c}$