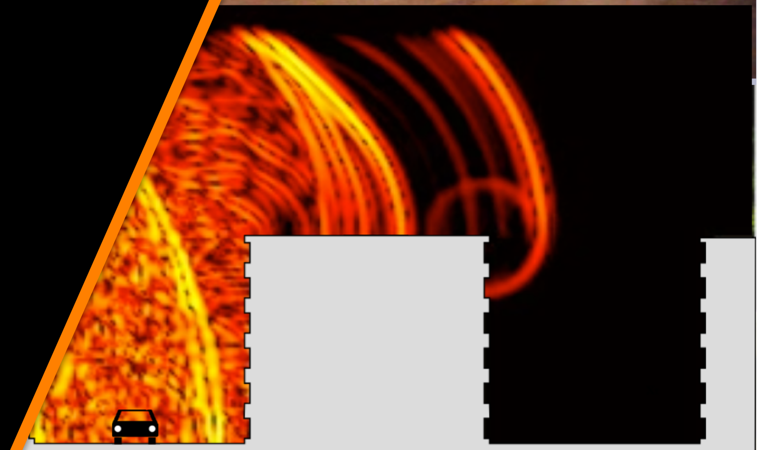
Architectural Acoustics

Week 3: Fundamentals of Acoustics
Lecture F.5-F.6

prof.dr.ir. Maarten Hornikx



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Contents

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- Wave function (Harmonic motion)
- Complex numbers
- Impedance
- Resonance

- Fourier Transform
- Impulse response and Transfer function

Week 2
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- Harmonic waves
- Intensity

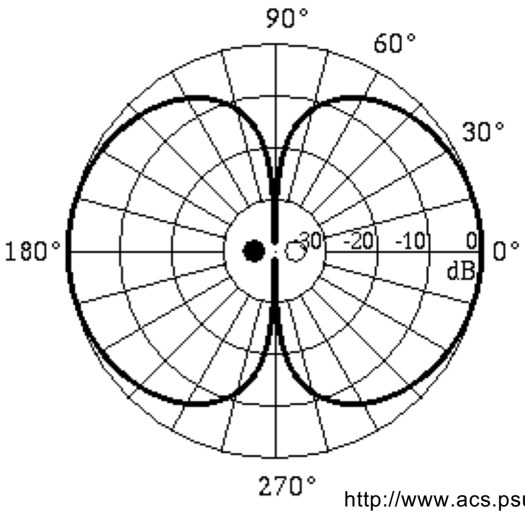
- Sound pressure level
- Wave equation in solids
- Quiz!

Contents week 3

- Attenuation of sound
- Spherical waves

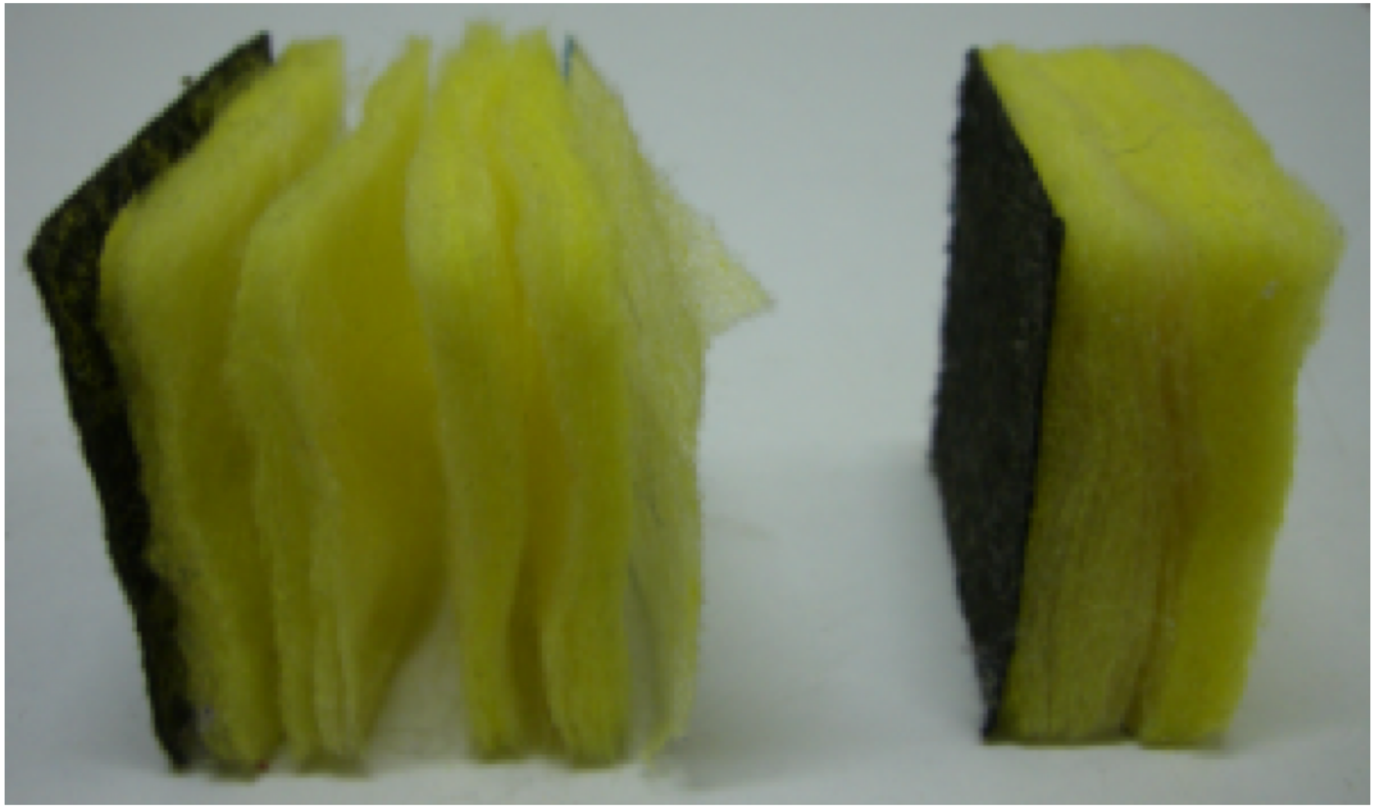
- Radiation of sound in air
- Quiz!

http://www.ohio.edu/people/schneidw/audio/lx521.html



http://www.acs.psu.edu

Acoustic attenuation



www.isover.com

Acoustic attenuation:
Conversion of acoustic energy into heat

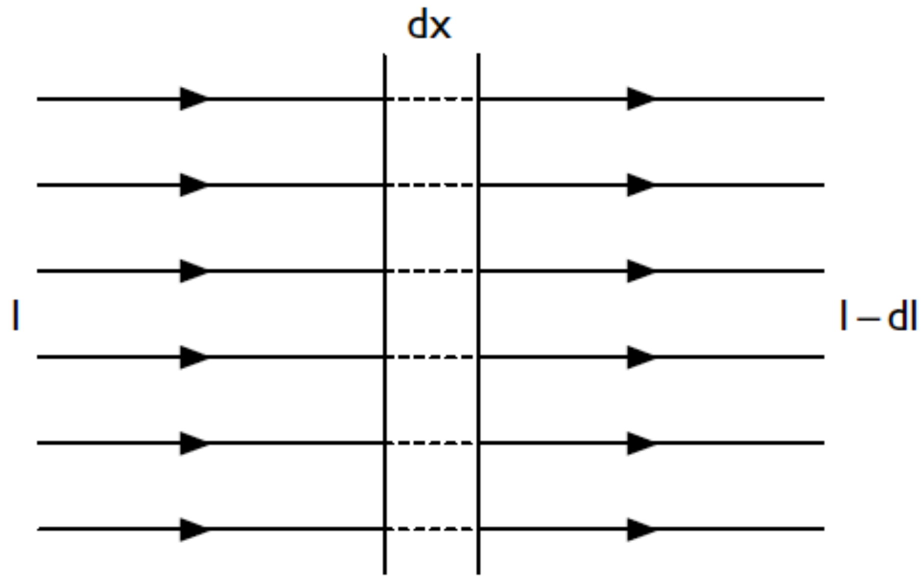


Figure 4.6 Attenuation of a plane wave.

$$-\mathrm{d}I = mI\mathrm{d}x$$

$$I(x) = I_0 e^{-mx}$$

Acoustic attenuation

$$p(x,t) = \hat{p}e^{-mx/2}e^{j(\omega t - kx)}$$

$$\underline{k} = k - j\frac{m}{2}$$

$$p(x,t) = \hat{p}e^{j(\omega t - \underline{k}x)}$$

m Attenuation constant

Acoustic attenuation: 2 types

1) Classic attenuation

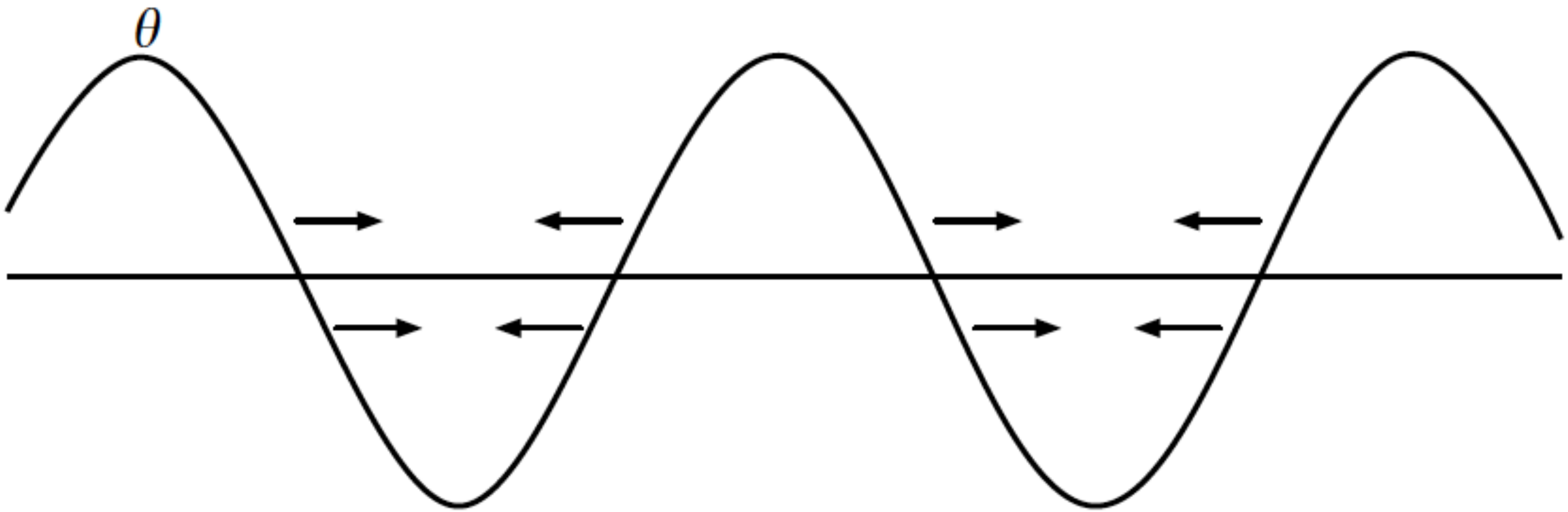


Figure 4.7 Heat flux in a sound wave.

Kuttruff, H. (2007). *Acoustics: an introduction*. CRC Press.

Acoustic attenuation: 2 types

2) Molecular attenuation

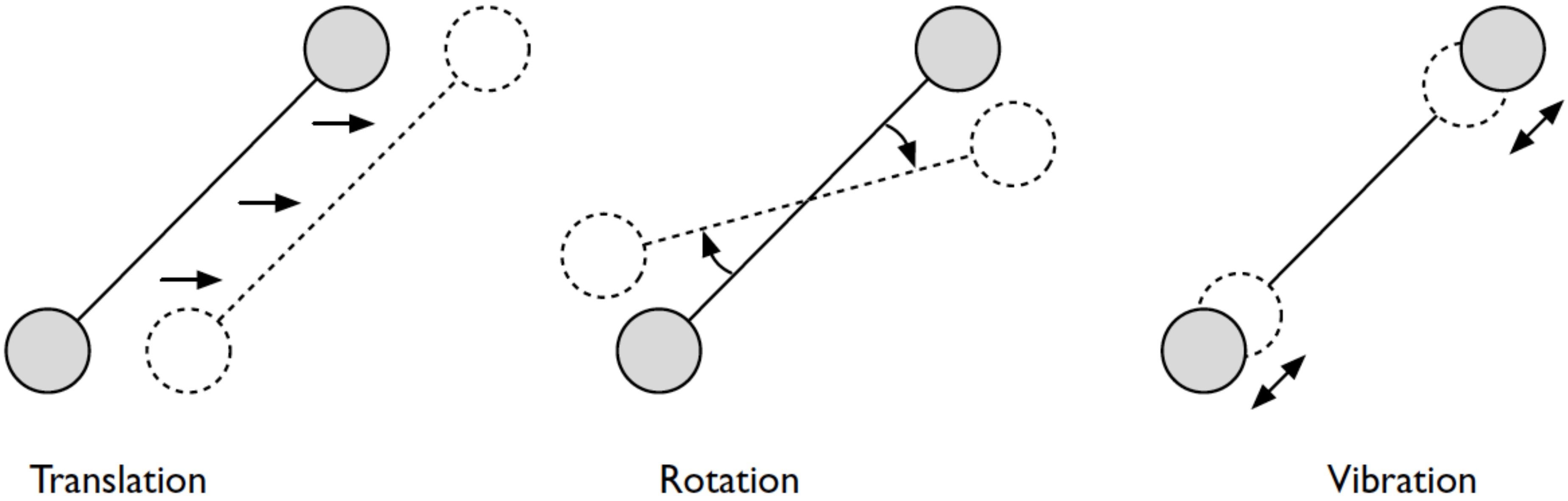


Figure 4.9 Possible motions of a polyatomic molecule. (By courtesy of S. Hirzel Verlag, Stuttgart.)

Kuttruff, H. (2007). *Acoustics: an introduction*. CRC Press.

Acoustic attenuation: classical attenuation

- Due to heat conductivity and viscosity
- We assumed no heat conduction in the derivation to the wave equation, but there is a small amount of conduction leading to acoustic attenuation

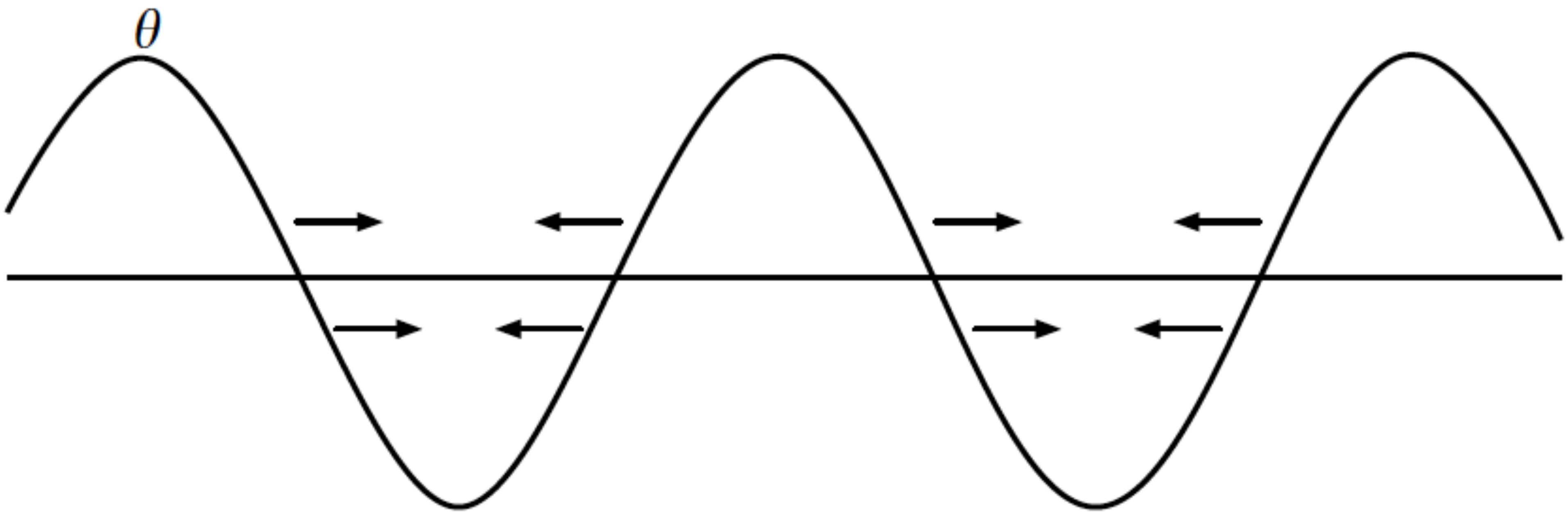


Figure 4.7 Heat flux in a sound wave.

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Acoustic attenuation: classical attenuation

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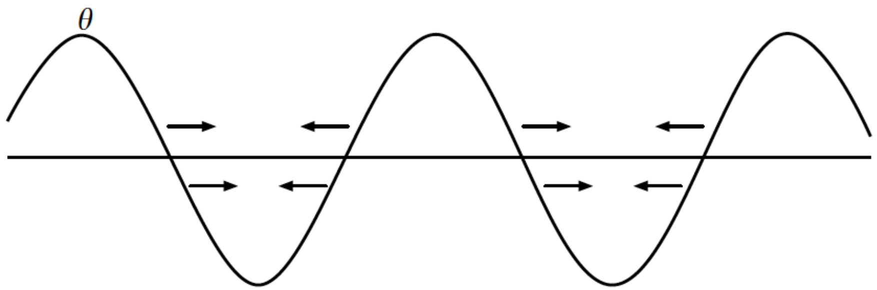
$$\theta = T_0 \frac{\kappa - 1}{\kappa} \frac{p}{p_0}$$



θ	Sound temperature
κ	Adiabatic heat constant

Acoustic attenuation: classical attenuation

- Thermal attenuation constant

$$m_{th} = \frac{\kappa - 1}{\kappa} \frac{\nu\omega^2}{\rho_0 c C_v c^3}$$

ν Heat conductivity of propagation medium

- Viscous attenuation constant

$$m_{vis} = \frac{4\eta\omega^2}{3\rho_0 c^3}$$

η Viscosity of propagation medium

Acoustic attenuation: molecular attenuation

- If a gas is in equilibrium, the thermal energy stored in translation, rotation and vibration
- When a sound wave passes by, there is will be a new equilibrium state of energy storage, and it has be redistributed among the three kinds of motion (with translation first)

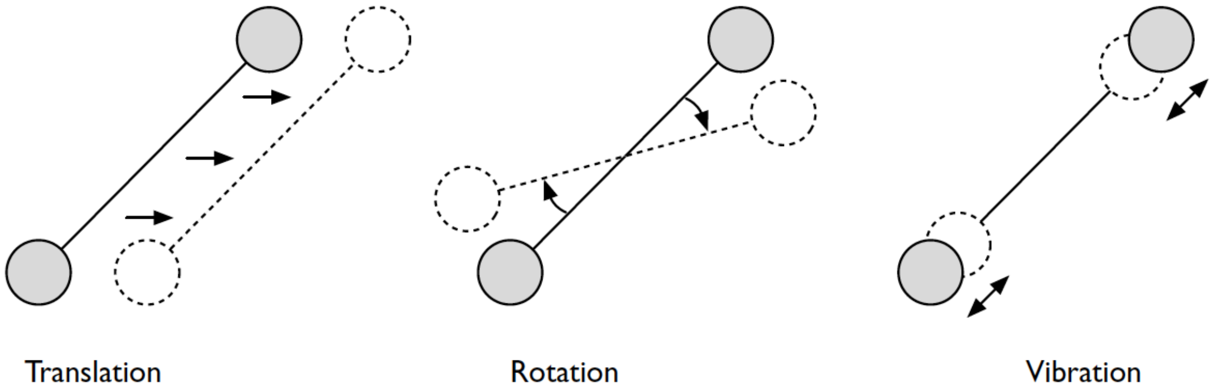


Figure 4.9 Possible motions of a polyatomic molecule. (By courtesy of S. Hirzel Verlag, Stuttgart.)

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Acoustic attenuation: molecular attenuation

- Molecular attenuation

$$m_{rel} \sim \frac{\omega\tau}{1+(\omega\tau)^2}$$

τ Relaxation time of propagation medium

- Speed of sound slightly dependent on frequency, dispersion!

Acoustic attenuation

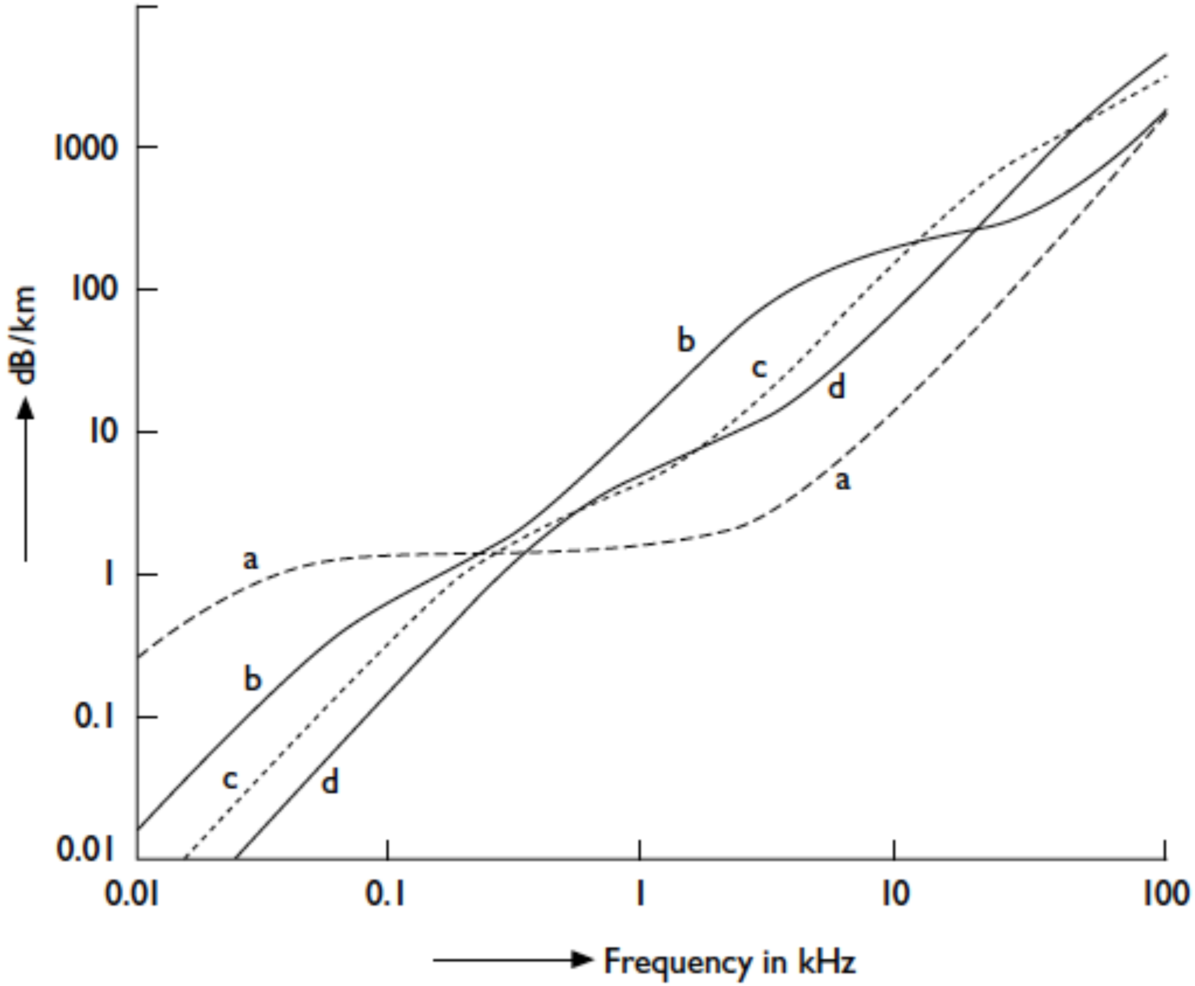


Figure 4.12 Attenuation of air (in dB/km) at 20°C and normal pressure. Parameter: relative humidity: (a) 0%, (b) 10%, (c) 40%, (d) 100%.

Kuttruff, H. (2007). *Acoustics: an introduction*. CRC Press.

Spherical waves

- Wave equation in polar coordinates

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

- Recall the 1D wave equation

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

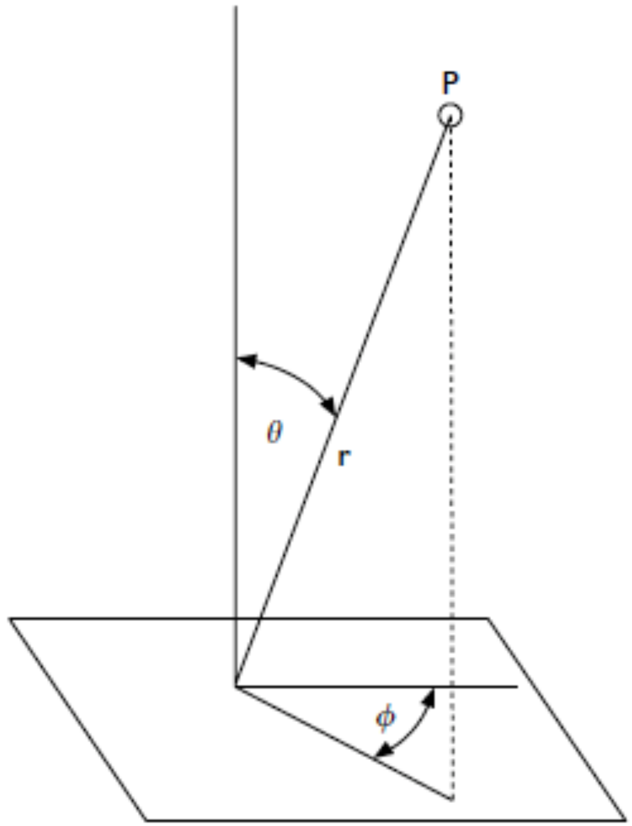


Figure 5.2 Spherical polar coordinates.

Spherical waves

- Pressure solution to wave equation in polar coordinates (point source solution)

$$p(r,t) \quad = \quad \frac{A}{r} e^{j(\omega t - kr)}$$

- Velocity solution ...

$$v_r \quad = \quad \frac{p}{\rho_0 c} \cdot \left(1 + \frac{1}{jkr}\right)$$

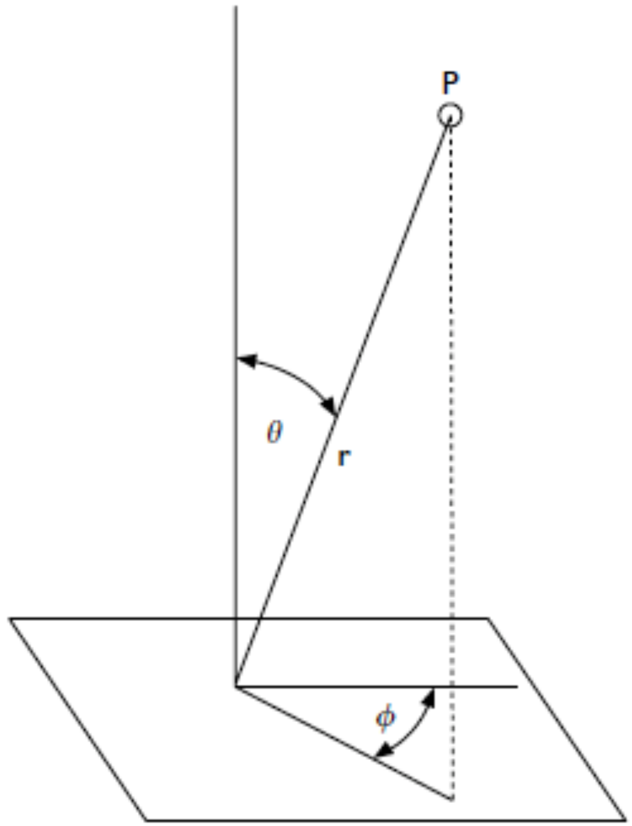
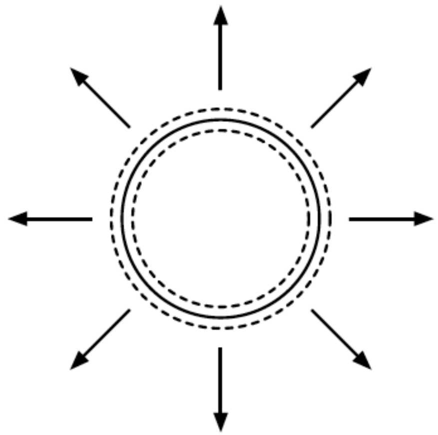


Figure 5.2 Spherical polar coordinates.

Sound radiation

- Radiated sound from a source is described by the radiation impedance Z_r
- For breathing sphere



$$Z_r = \frac{F}{v} = \frac{pS}{v}$$

$$= \frac{S\rho_0 c}{1 + 1/jka}$$

Breathing sphere impedance

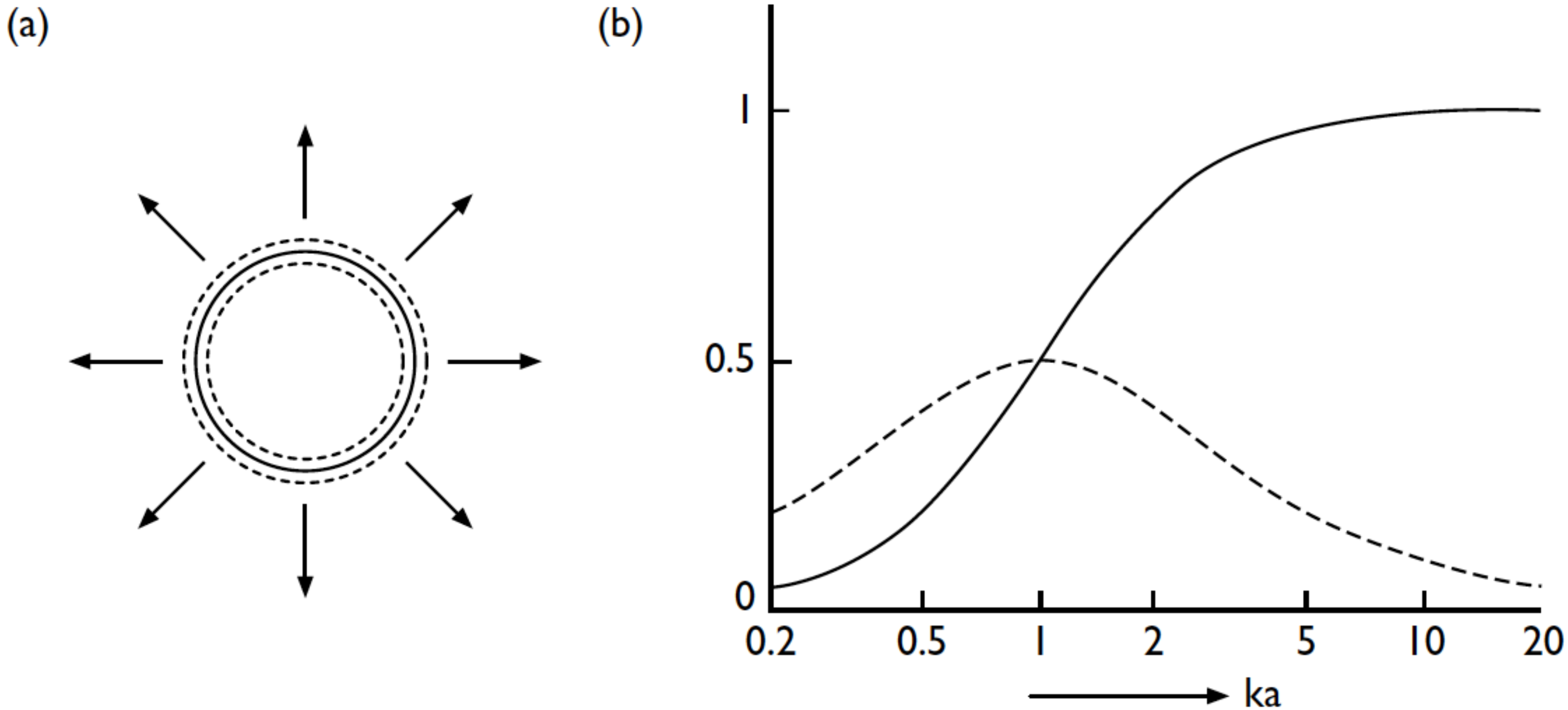


Figure 5.10 Breathing sphere: (a) schematically, (b) normalized radiation impedance (solid line: real part R_r/SZ_0, broken line: imaginary part X_r/SZ_0).

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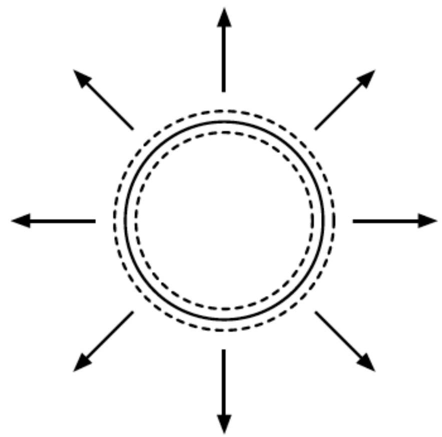

$$Z_r = \frac{F}{v} = \frac{pS}{v}$$

$$= \frac{S\rho_0 c}{1 + 1/jka}$$

- Radiated power

$$R_r = S\rho_0 c \frac{(ka)^2}{1 + (ka)^2} .$$

Directivity

- All sound sources can be modelled by a summation of point sources

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- The resulting sound pressure, far away from the source

$$p(r, \theta, \phi, t) \quad = \quad \frac{A}{r} R(\theta, \phi) e^{j(\omega t - kr)}$$

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- The resulting sound pressure, far away from the source

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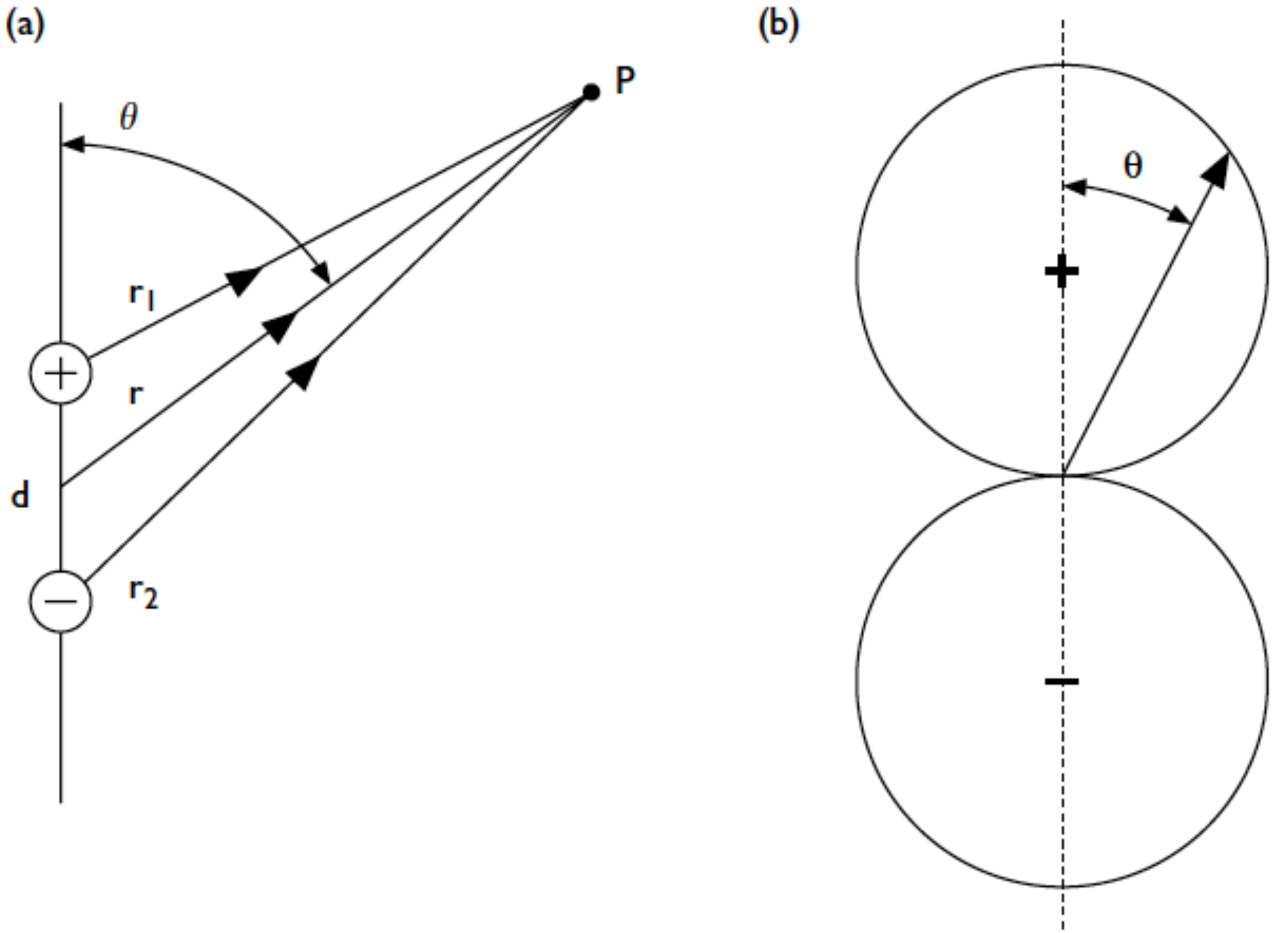
Dipole

http://www.ohio.edu/people/schneidw/audio/lx521.html



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Dipole

$$p(r,t) = p_1 + p_2 = \frac{A}{r_1} e^{j(\omega t - kr_1)} - \frac{A}{r_2} e^{j(\omega t - kr_2)}$$

$$r_{1,2} = r \pm \frac{\bar{d}}{2} \cos\theta \qquad \text{for } kd << 1$$

$$p(r,t) = \frac{jA\omega d}{rc} \cos\theta e^{j(\omega t - kr)}$$

Linear array

http://witstage.en.alibaba.com/productshowimg/653563332-214406893/line_array_loudspeaker.html

Linear array

$$p(r, \alpha, t) = \frac{A}{r} \sum_{n}^{N-1} e^{jkdn \sin\alpha} e^{j(\omega t - kr)}$$

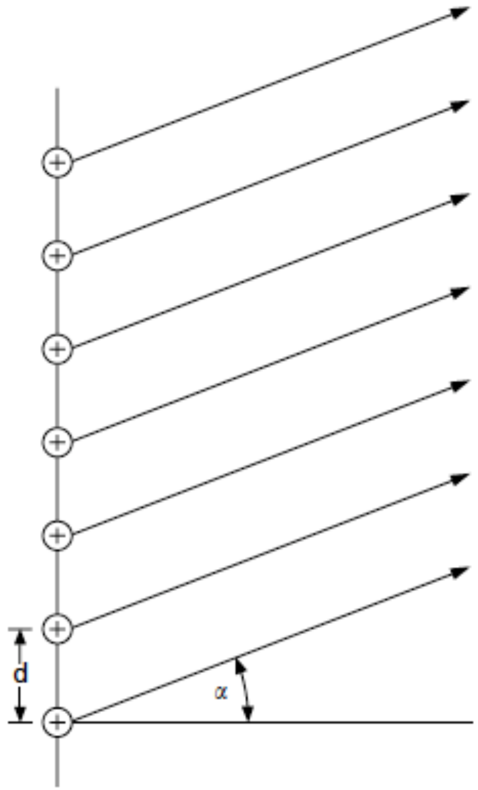

Figure 5.7 Linear array of point sources.

Loudspeaker array

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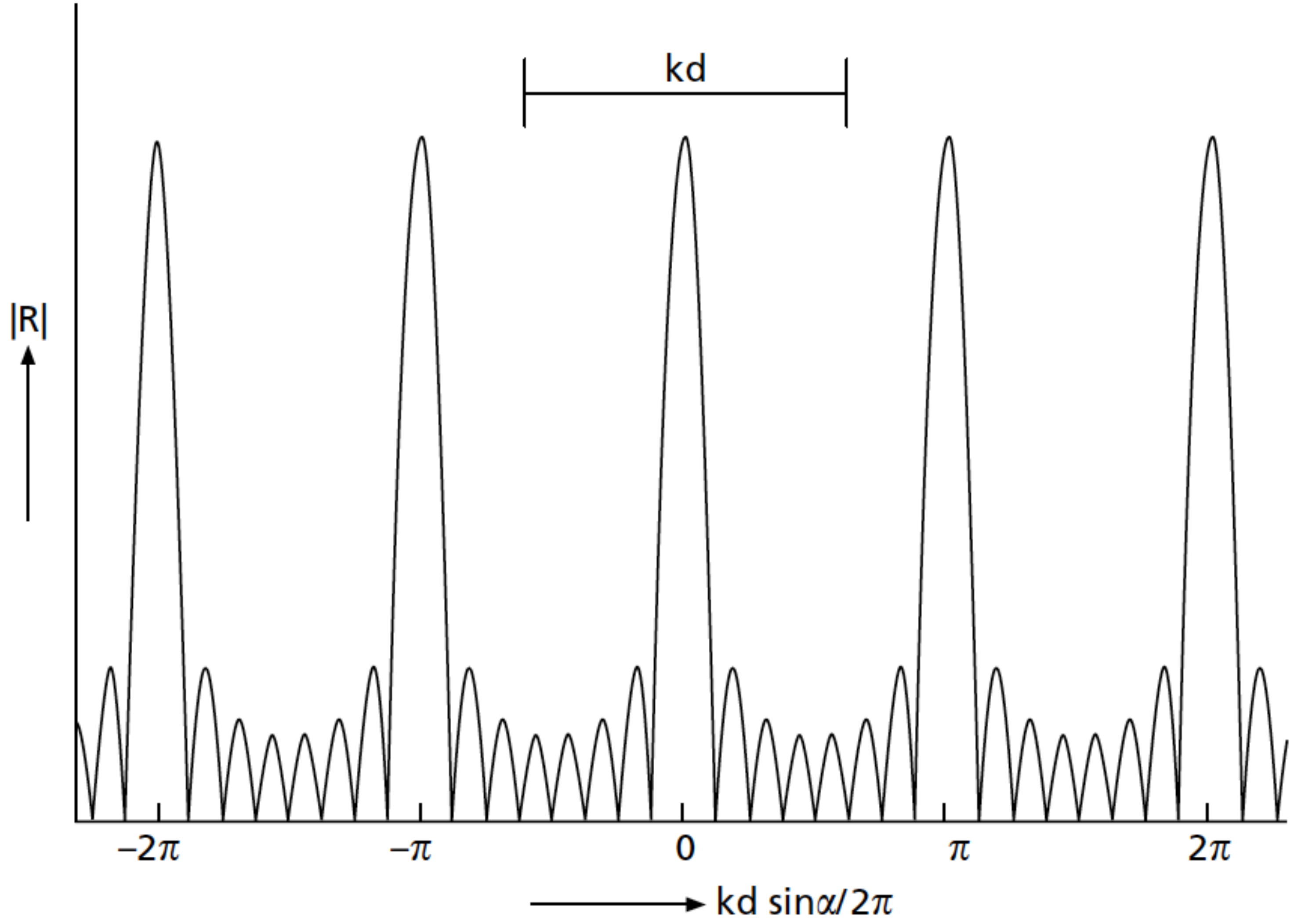
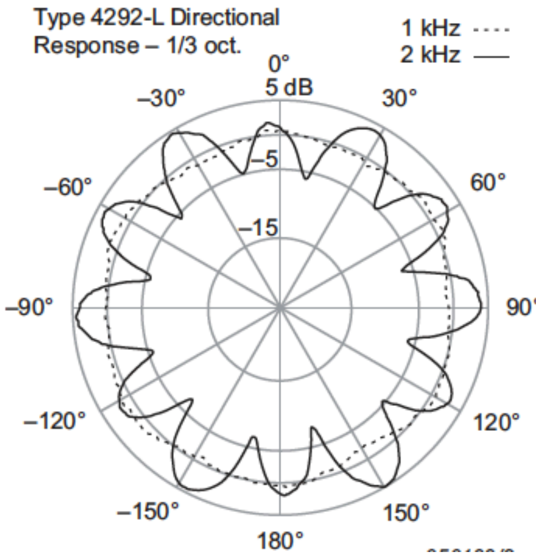
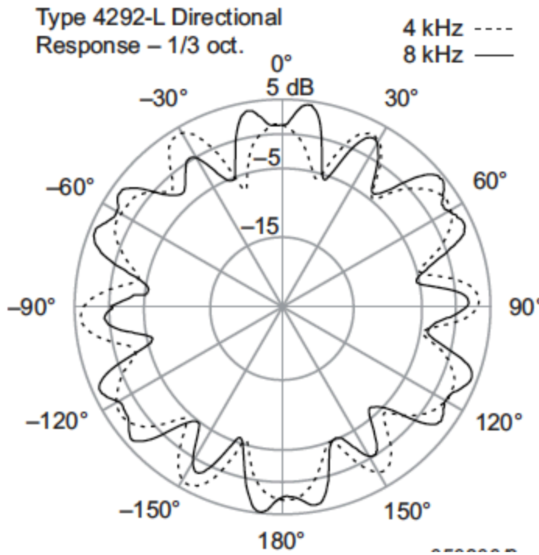


Figure 5.8 Directional factor (magnitude) of a linear array with eight elements.

Piston



Type 4292-L Directional
Response – 1/3 oct.
1 kHz
2 kHz
0°
5 dB
–30°
30°
–5
–60°
60°
–15
–90°
90°
–120°
120°
–150°
150°
180°



Type 4292-L Directional
Response – 1/3 oct.
4 kHz
8 kHz
0°
5 dB
–30°
30°
–5
–60°
60°
–15
–90°
90°
–120°
120°
–150°
150°
180°

Formulas week 3

$$\begin{aligned}
p(x,t) &= \hat{p}e^{-mx/2}e^{j(\omega t - kx)} = \hat{p}e^{j(\omega t - \underline{k}x)} \qquad (13)\\
\underline{k} &= k - j\frac{m}{2} = \frac{\omega}{c} - j\frac{m}{2}\\
m_{th} &= \frac{\kappa - 1}{\kappa}\frac{\nu\omega^2}{\rho_0 c C_v c^3} \qquad \text{Thermal attenuation constant}\\
m_{vis} &= \frac{4\eta\omega^2}{3\rho_0 c^3} \qquad \text{Viscous attenuation constant}\\
m_{rel}\cdot\lambda &= 2\pi\varepsilon\frac{\omega\tau}{1+(\omega\tau)^2}\left(\frac{c}{c_0}\right)^2 \qquad \text{Relaxation attenuation constant}\\
m_{rel} &\sim \frac{\omega\tau}{1+(\omega\tau)^2}\\
\theta &= T_0\frac{\kappa - 1}{\kappa}\frac{p}{p_0}
\end{aligned}$$

Formulas week 3

Spherical waves and sound radiation (chapter 5)

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \quad \text{Wave equation in polar coordinates}$$

$$p(r,t) = \frac{j\omega\rho_0\hat{Q}}{4\pi r}e^{j(\omega t-kr)} \quad \text{Point source solution} \tag{14}$$

$$v_r = \frac{p}{\rho_0 c}\cdot\left(1+\frac{1}{jkr}\right) \quad \text{Radial particle velocity}$$

$$P_r = 4\pi r^2 I = \frac{\rho_0\omega^2\hat{Q}^2}{8\pi c} \quad \text{Acoustic power from point source}$$

$$p(r,\theta,\phi,t) = \frac{A}{r}R(\theta,\phi)e^{j(\omega t-kr)} \quad \text{Sound pressure with directivity function} \tag{15}$$

$$L_{\mathrm{P}} = 10\log_{10}\left(\frac{P_r}{P_0}\right) dB$$

$$p(r,t) = \frac{j\omega\rho_0\hat{Q}}{4\pi}\left(\frac{e^{-jkr_1}}{r_1}-\frac{e^{-jkr_2}}{r_2}\right)e^{j\omega t} \quad \text{Dipole source solution} \tag{16}$$

$$p(r,\alpha,t) = \frac{A}{r}\sum_{n}^{N-1} e^{jkdn\sin\alpha}e^{j(\omega t-kr)} \quad \text{Linear array}$$

$$Z_r = \frac{S\rho_0 c}{1+1/jka} \quad \text{Radiation impedance of breathing sphere}$$

$$R_r = S\rho_0 c\frac{(ka)^2}{1+(ka)^2} \rightarrow \frac{\rho_0 S^2}{4\pi c}\cdot\omega^2 \quad \text{for } ka \ll 1$$

Radiation resistance of breathing sphere