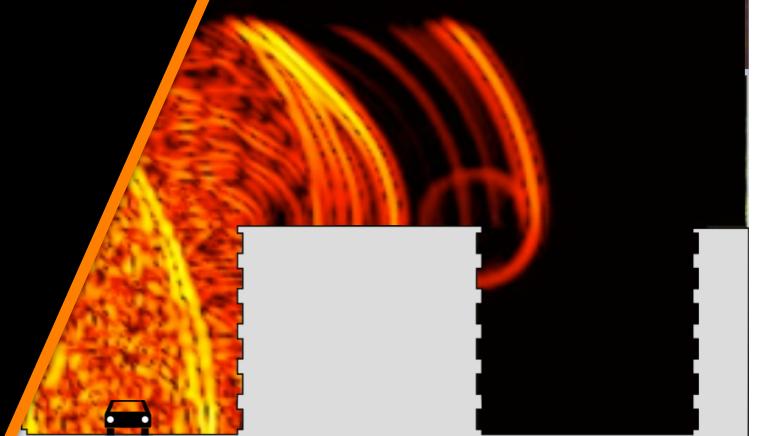


Architectural Acoustics

Week 4 Environmental Acoustics
Lecture E.1-E.2

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EINDHOVEN
UNIVERSITY OF
TECHNOLOGY

Contents

Week 1

- Wave function (Harmonic motion)
- Complex numbers
- Impedance
- Resonance
- Fourier Transform
- Impulse response and Transfer function

Week 2

- Wave equation in fluids
- Harmonic waves
- Intensity
- Sound pressure level
- Wave equation in solids
- Quiz!

Contents

Week 3

- Attenuation of sound
- Spherical waves
- Radiation of sound in air

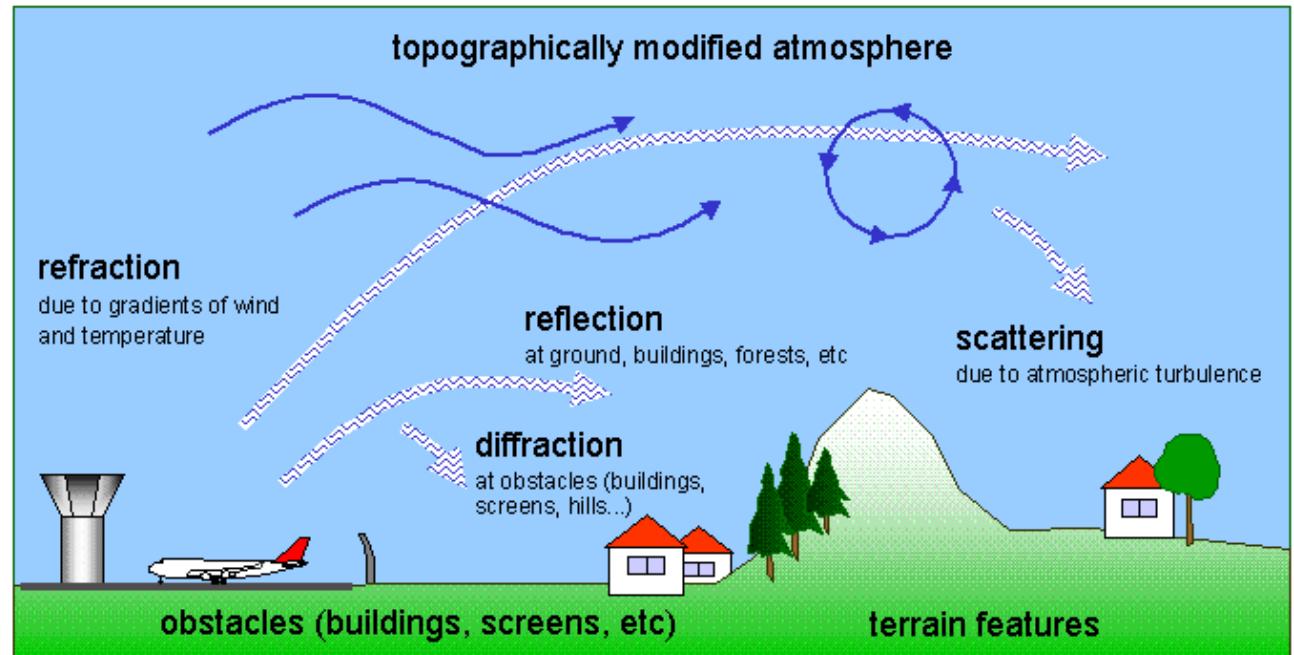
Contents Environmental Acoustics

Lecture E.1

- Reflection
- Refraction

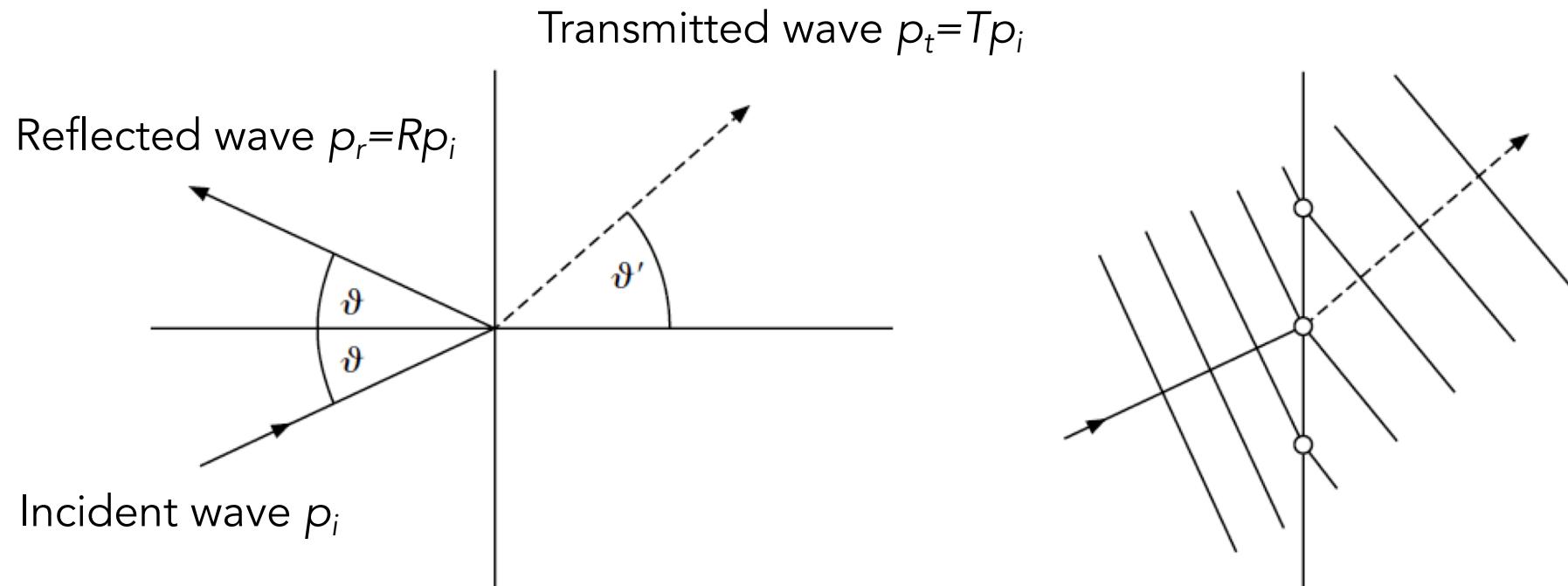
Lecture E.2

- Diffraction
- Scattering
- Quiz



Reflection

Plane wave reflection coefficient



Reflection

Plane wave reflection coefficient

$$R = \frac{Z'_0 \cos \theta - Z_0 \cos \theta'}{Z'_0 \cos \theta + Z_0 \cos \theta'} \quad \frac{c}{\sin \theta} = \frac{c'}{\sin \theta'}$$

$$\alpha = 1 - |R|^2$$

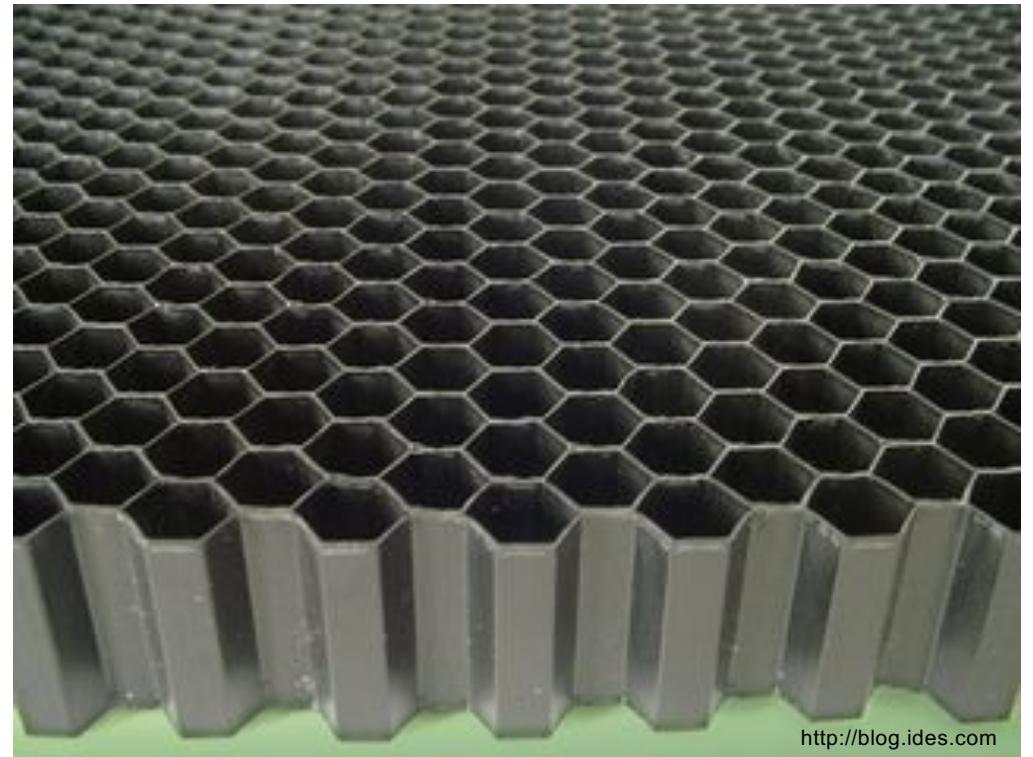
Some examples, for normal incident sound ($\theta = 0$)

$Z'_0 = \infty,$	$R = 1, \quad \alpha = 0$	second medium is hard (acoustically rigid)
$Z'_0 / Z_0 = 1,$	$R = 0, \quad \alpha = 1$	second medium is equal to first medium
$Z_0 = 0$	$R = -1, \quad \alpha = 0$	second medium is soft

Reflection

Locally reacting medium

$$R = \frac{Z \cos \theta - Z_0}{Z \cos \theta + Z_0}$$



Reflection

Absorption
coefficient

$$\alpha = 1 - |R|^2$$

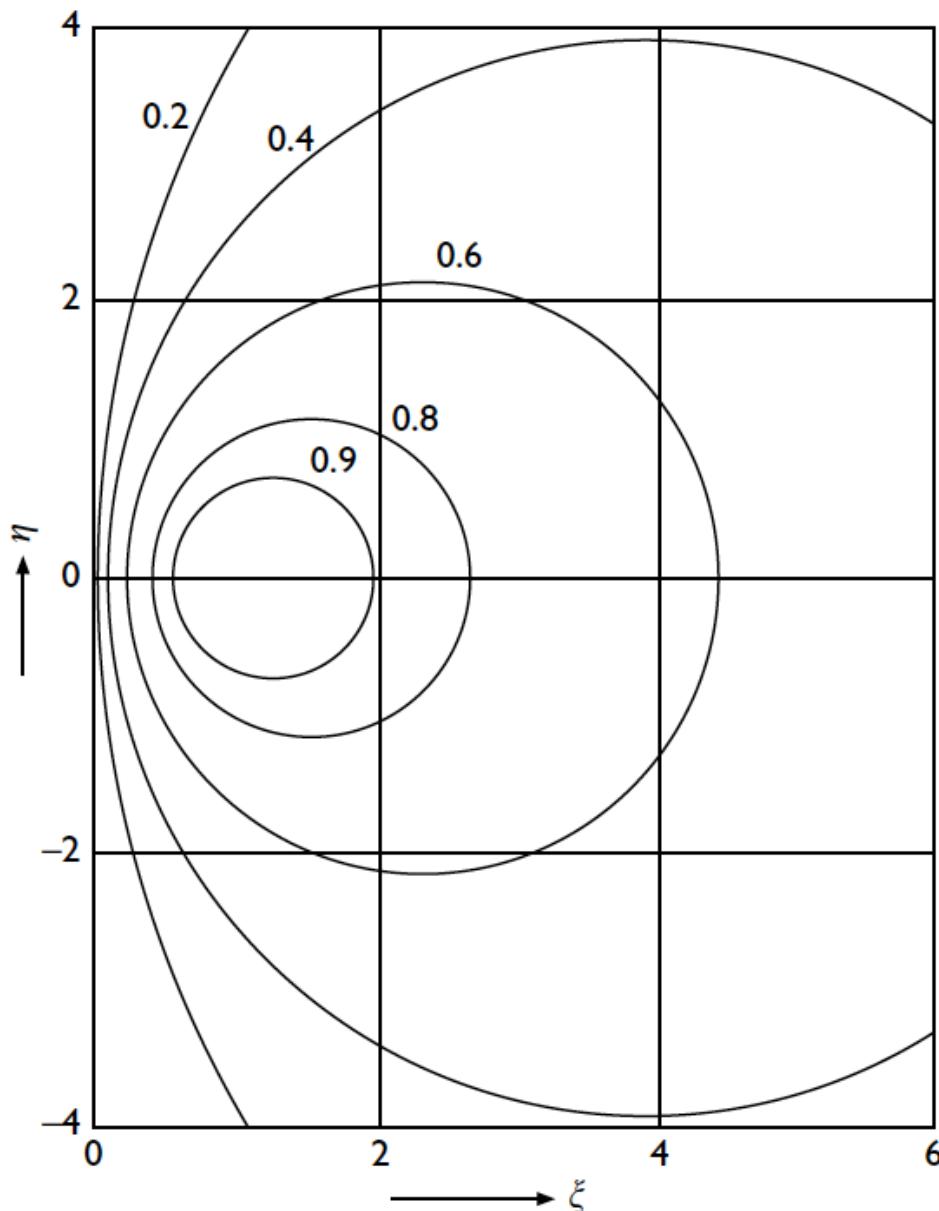


Figure 6.5 Circles of equal absorption coefficient in the complex plane of the specific wall impedance.

Reflection

Important for a porous material:

- Flow resistivity
- Thickness
- Porosity



Wavenumber and specific impedance of a simplified porous material
(according to book)

$$k' = \frac{\omega}{c} \sqrt{1 - \frac{j\sigma\Xi}{\rho_0\omega}}$$

$$\underline{Z'_0} = Z_0 \sqrt{1 - \frac{j\sigma\Xi}{\rho_0\omega}}$$

Reflection

Flow resistivities of outdoor ground surfaces in
kPa•s/m²

Description	Type	(kPa•s/m ²)
Very soft (snow or moss-like)	A	12.5
Soft forest floor (short, dense heather-like or thick moss)	B	31.5
Uncompacted, loose ground (turf, grass, loose soil)	C	80
Normal uncompacted ground (forest floors, pasture field)	D	200
Compacted field and gravel (compacted lawns, park area)	E	500
Compacted dense ground (gravel road, car park)	F	2000
Hard surfaces (most normal asphalt, concrete)	G	20 000
Very hard and dense surfaces (dense asphalt, concrete, water)	H	200 000

Reflection

Impedance model for outdoor surfaces (high frequency approach):
one parameter model Delany and Bazley

$$Z_{DB} = 1 + 9.08 \left(\frac{1000f}{\Xi_{\text{eff}}} \right)^{-0.75} - j11.9 \left(\frac{1000f}{\Xi_{\text{eff}}} \right)^{-0.73}$$
$$k_{DB} = 1 + 0.0978 \left(\frac{f}{\Xi_{\text{eff}}} \right)^{-0.700} - j0.189 \left(\frac{f}{\Xi_{\text{eff}}} \right)^{-0.595}$$

More sophisticated models for porous materials (may) include:
(variable) porosity, tortuosity, pore shape factor, thermal characteristic length, viscous characteristic length

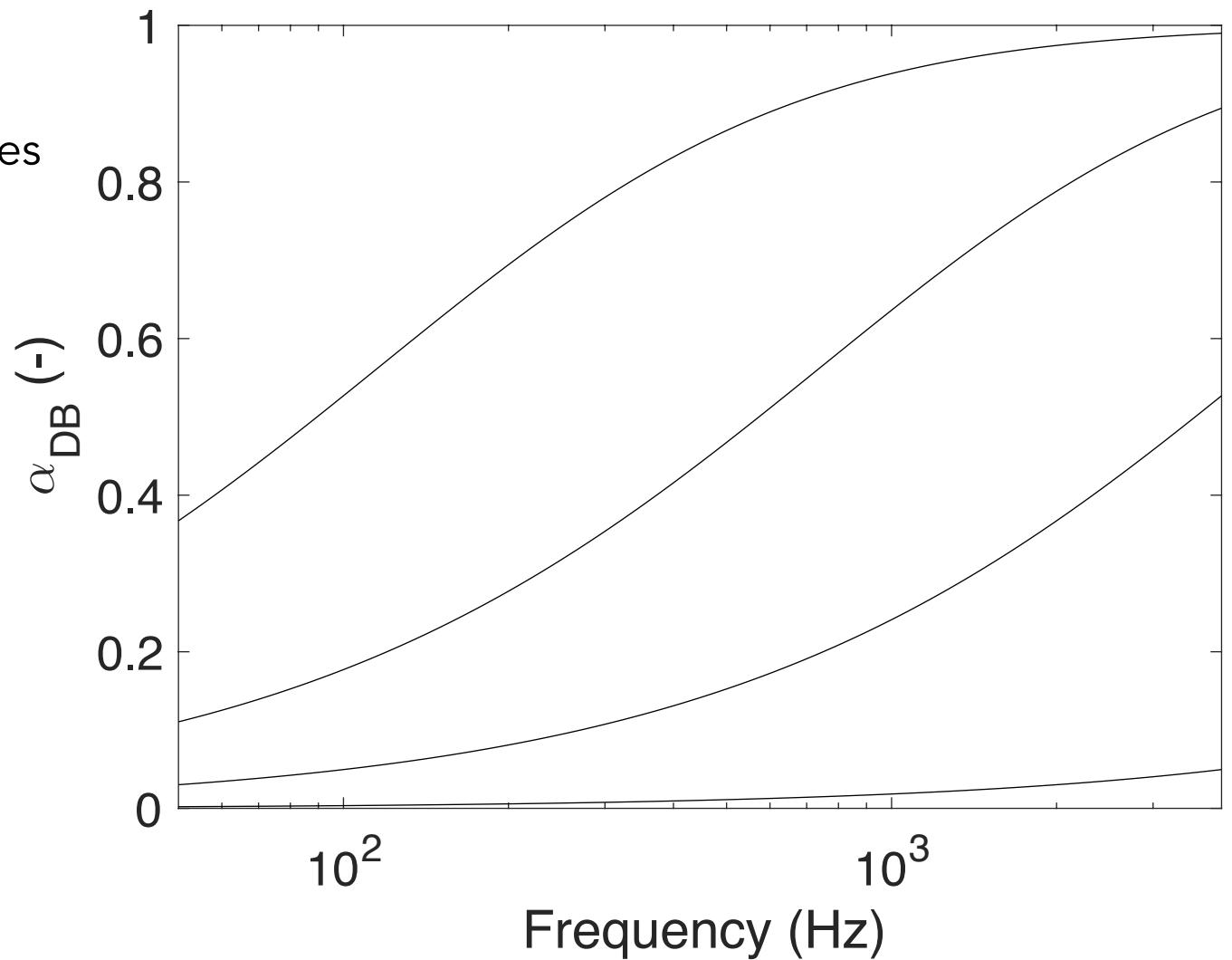
Reflection

Absorption coefficient for
normal incident sound waves

DB impedance model

Flow resistivity classes

A, C, E, G



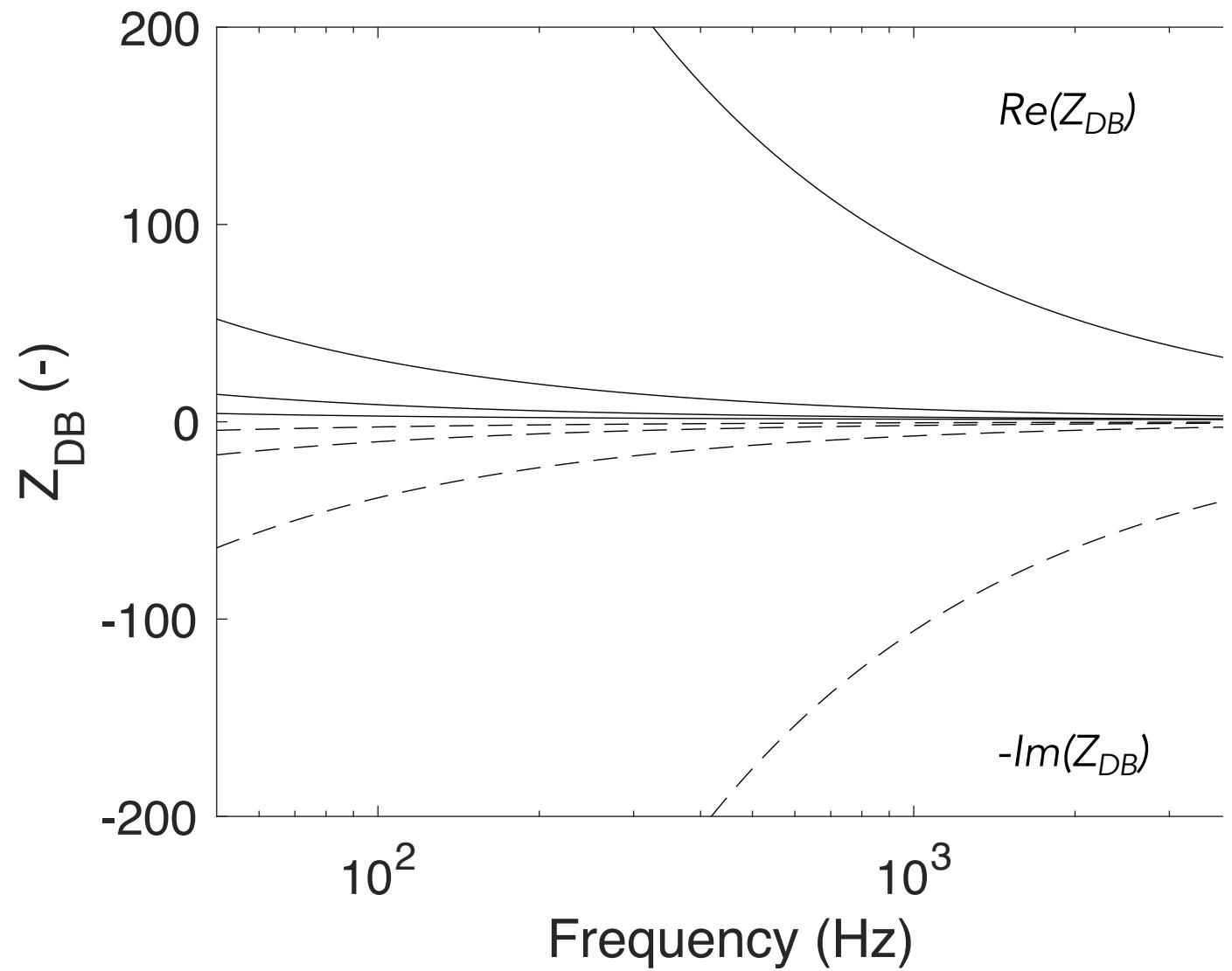
Own work

Reflection

Normalized surface
impedance for normal
incident sound waves

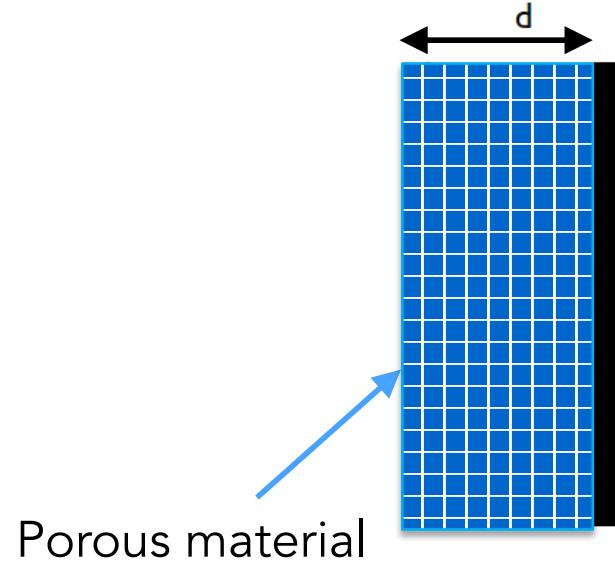
DB impedance model

Flow resistivity classes
A, C, E, G



Own work

Reflection



$$Z = -j \underline{Z}'_0 \cot(\underline{k}'d)$$

Impedance of porous layer with rigid backing at distance d

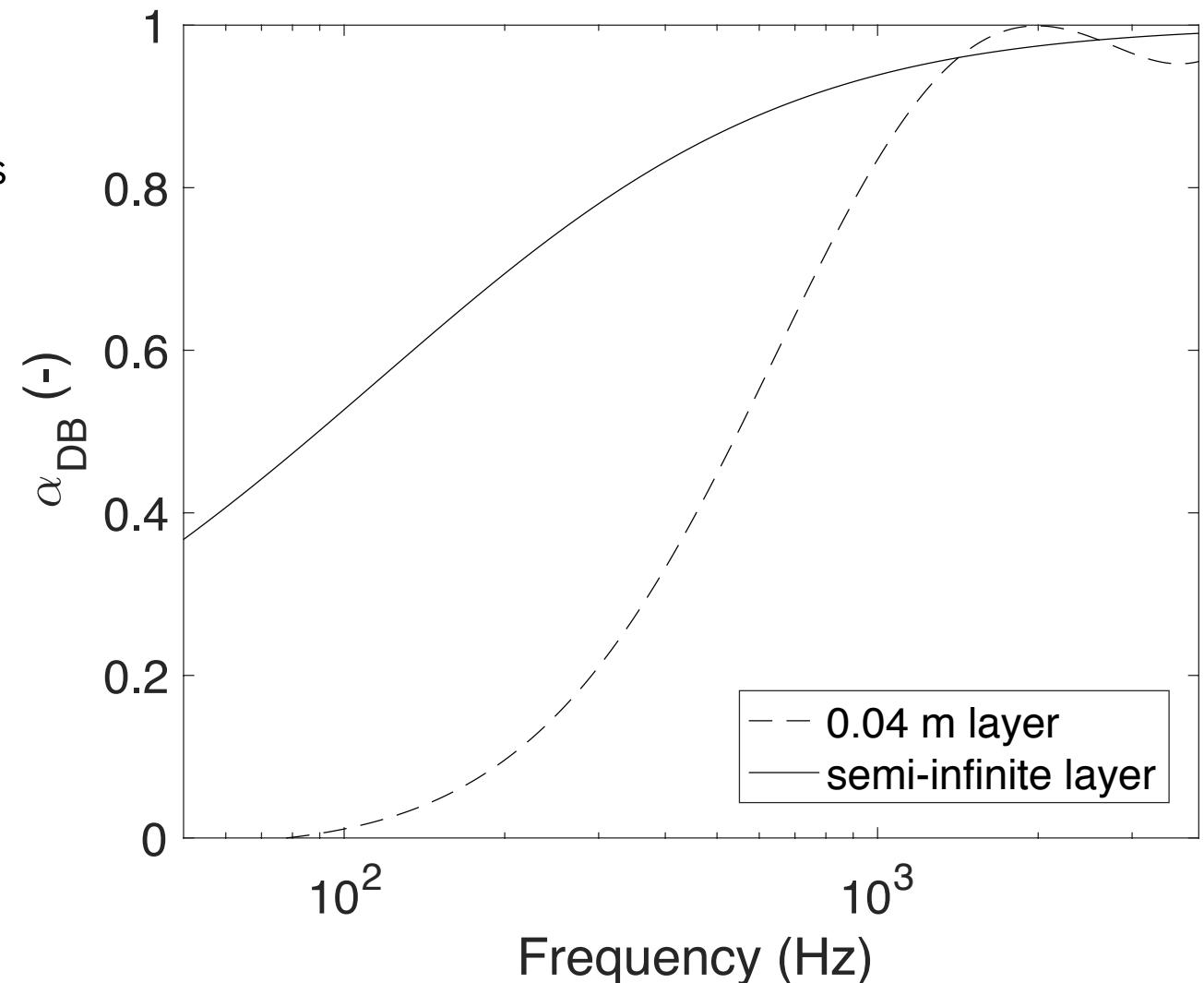
Reflection

Absorption coefficient for
normal incident sound waves

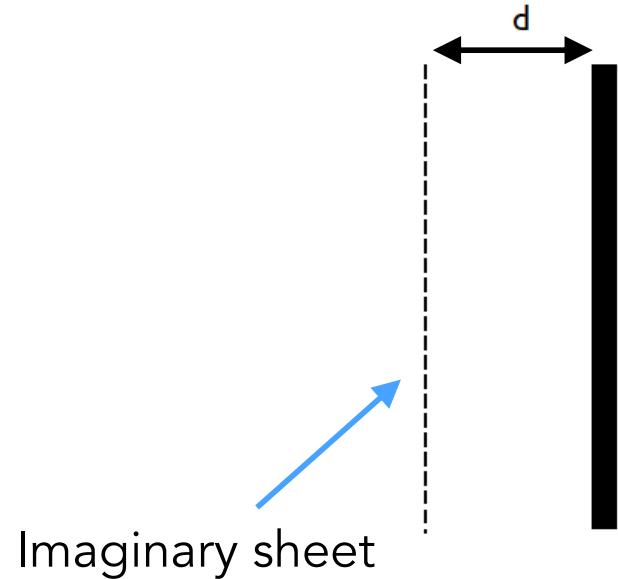
DB impedance model

Flow resistivity class A

Layer thickness:
0.04 m versus semi-infinite



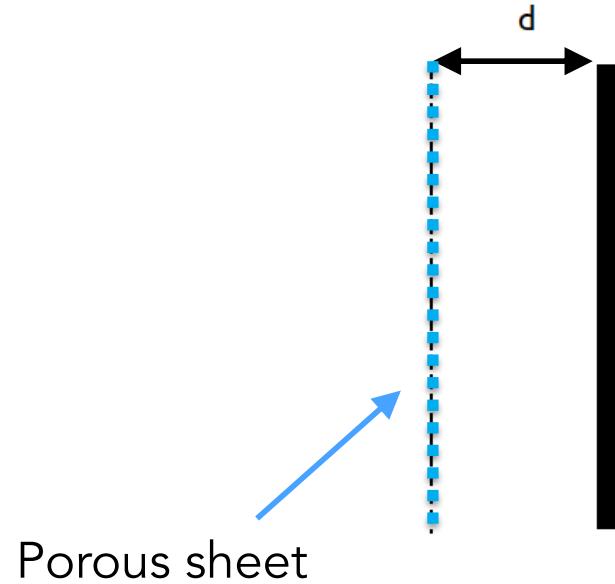
Reflection



$$Z = Z_0 \frac{1 + e^{-j2kd}}{1 - e^{-j2kd}} = -jZ_0 \cot(kd)$$

Impedance of air layer with rigid backing at distance d

Reflection



$$Z = r_s - jZ_0 \cot(kd)$$

Impedance of a thin porous sheet on an air layer
with rigid backing at distance d

Reflection

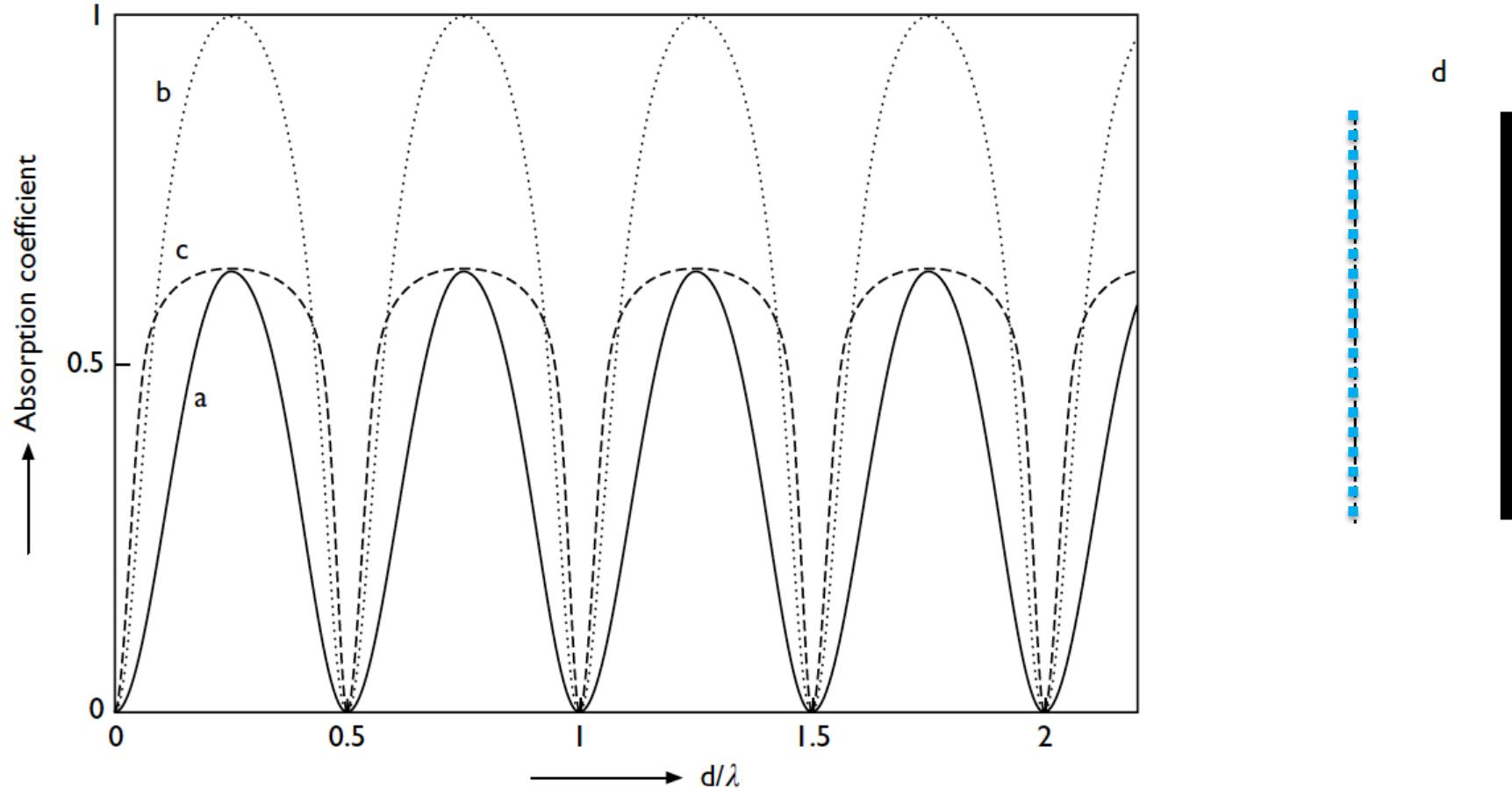
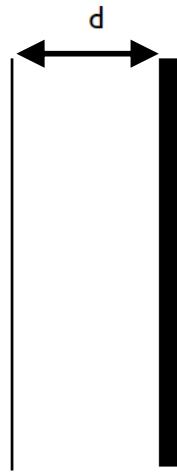


Figure 6.11 Absorption coefficient of a porous sheet (see Fig. 6.10b). Curve a: $r_s = Z_0/4$, curve b: $r_s = Z_0$, curve c: $r_s = 4Z_0$.

Kuttruff, H. (2007). *Acoustics: an introduction*. CRC Press.

Reflection



$$Z = j\omega m' - jZ_0 \cot(kd)$$

Impedance of a non-porous sheet on an air layer with rigid backing at distance d

Reflection

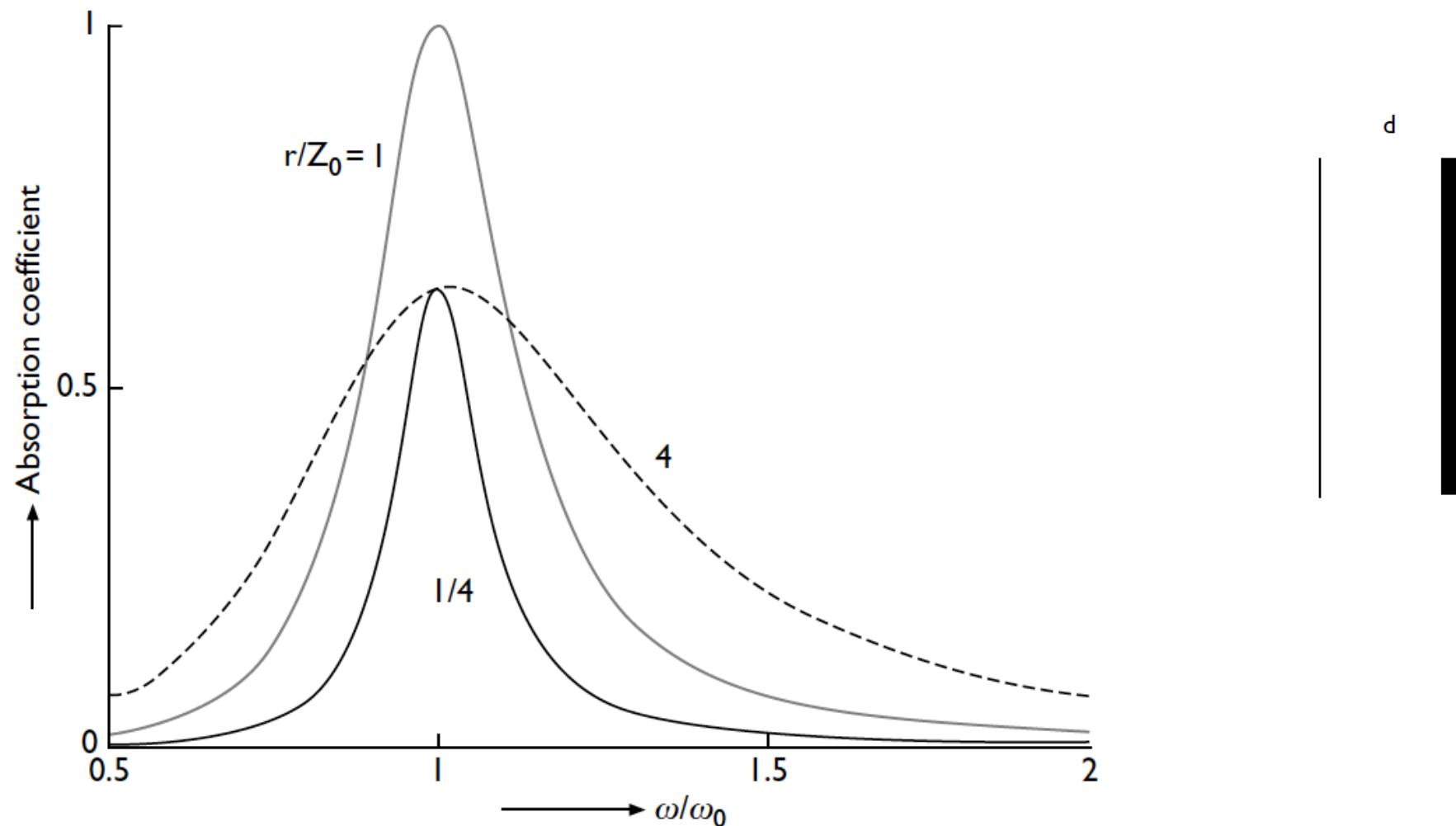


Figure 6.12 Absorption coefficient of a panel in front of a rigid wall (see Fig. 6.10b) (panel resonator) for $m'\omega_0 = 4Z_0$. Parameter: r/Z_0 .

Kuttruff, H. (2007). *Acoustics: an introduction*. CRC Press.

Reflection

Point source over ground surface

$$p(x_r, z_r, t) = A \left(\frac{e^{-jkr_1}}{r_1} + R_{\text{sph}} \frac{e^{-jkr_2}}{r_2} \right) e^{j\omega t}$$

$$r_1 = \sqrt{(x_r - x_s)^2 + (z_r - z_s)^2}$$

$$r_2 = \sqrt{(x_r - x_s)^2 + (z_r + z_s)^2}$$

x_s, z_s



x_r, z_r



Ground reflection

Point source over ground surface

$$p(x_r, z_r, t) = A \left(\frac{e^{-jkr_1}}{r_1} + R_{\text{sph}} \frac{e^{-jkr_2}}{r_2} \right) e^{j\omega t}$$

$$r_1 = \sqrt{(x_r - x_s)^2 + (z_r - z_s)^2}$$

$$r_2 = \sqrt{(x_r - x_s)^2 + (z_r + z_s)^2}$$

$$R_{\text{sph}} = R + (1 - R)F(w) \quad \text{sphe}$$

$$F(w) = 1 - j\sqrt{\pi}e^{-w^2} \operatorname{erfc}(jw) \quad 1$$

$$w \approx \frac{1}{2}(1 - j)\sqrt{kr_2}(\cos(\theta) + \frac{1}{Z})$$

$$R \approx R_{\text{sph}} \quad \text{if} \quad |w| > 4$$

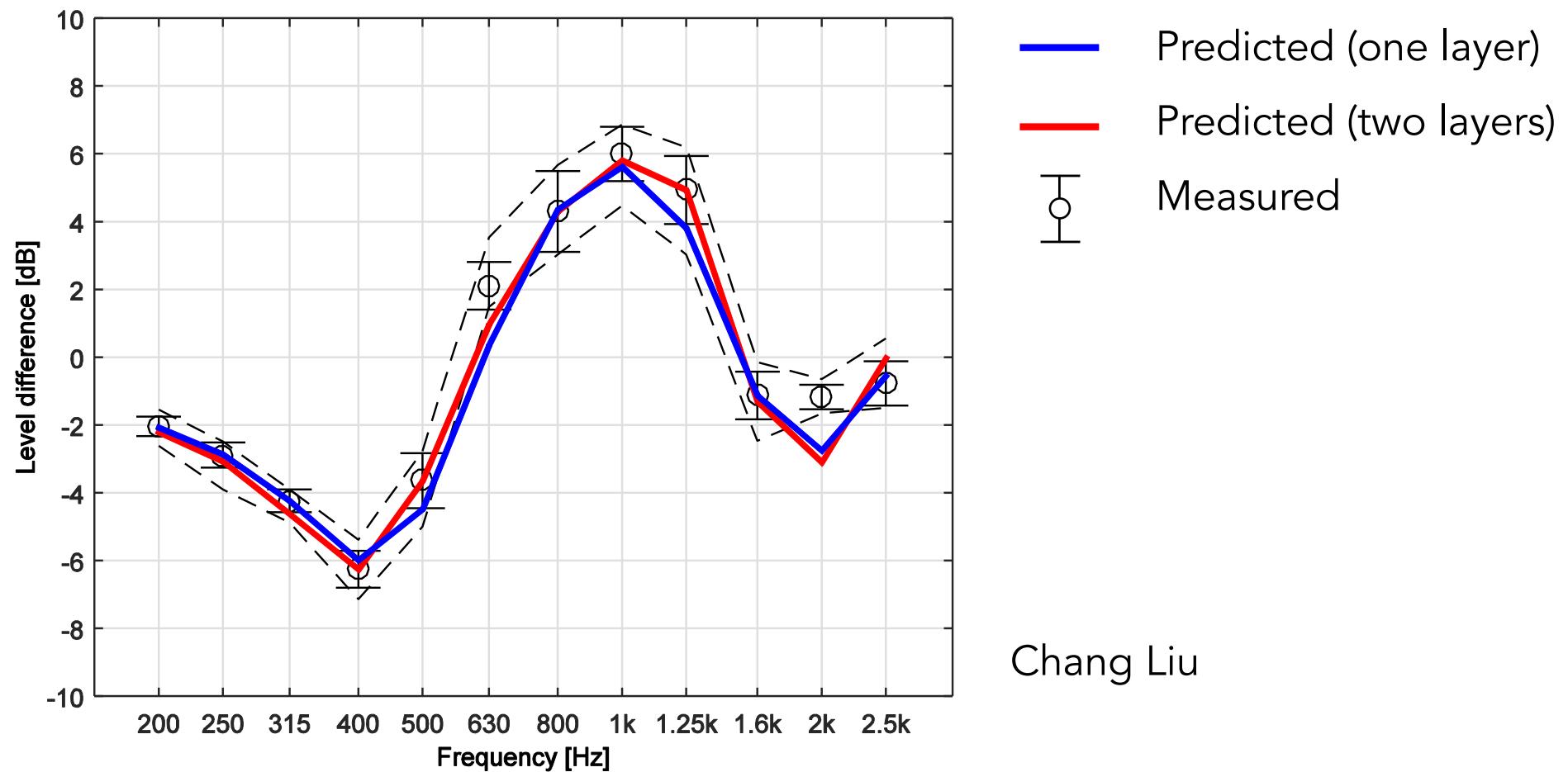
Ground reflection

Measurement at Cascade building (Chang Liu)



Ground reflection

Prediction using slit-pore model (porosity and flow resistivity as parameters)



Chang Liu

Ground reflection

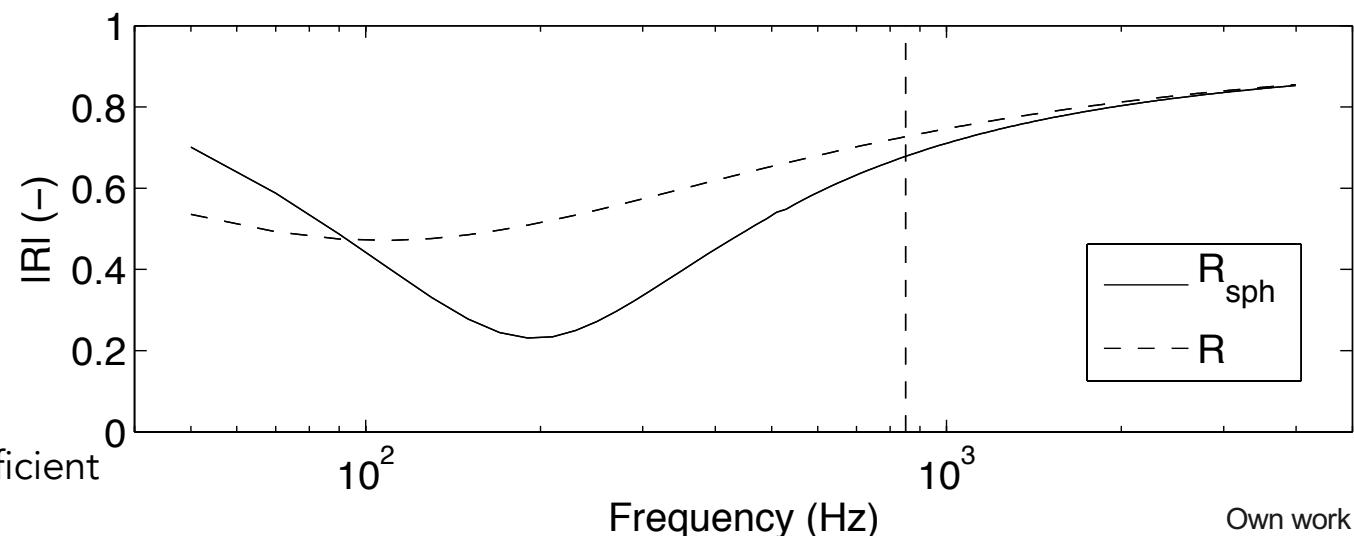
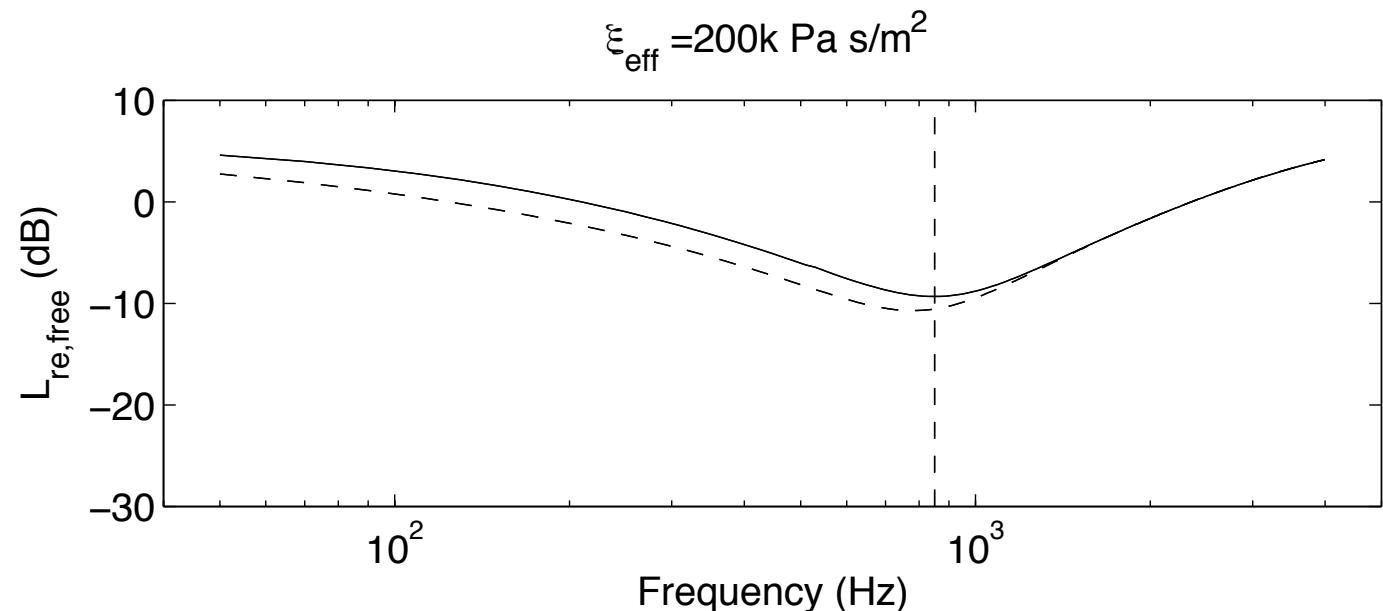
Example:

$$(x_r, z_r) = (50 \text{ m}, 0.5 \text{ m}),$$

$$(x_s, z_s) = (0 \text{ m}, 1.5 \text{ m})$$

$$Z = Z_{\text{DB}}, \text{ with}$$

$$\Xi = 2 \times 10^5 \text{ Pa s/m}^2$$



R_{sph} = spherical wave reflection coefficient

R = plane wave reflection coefficient

Own work

Ground reflection

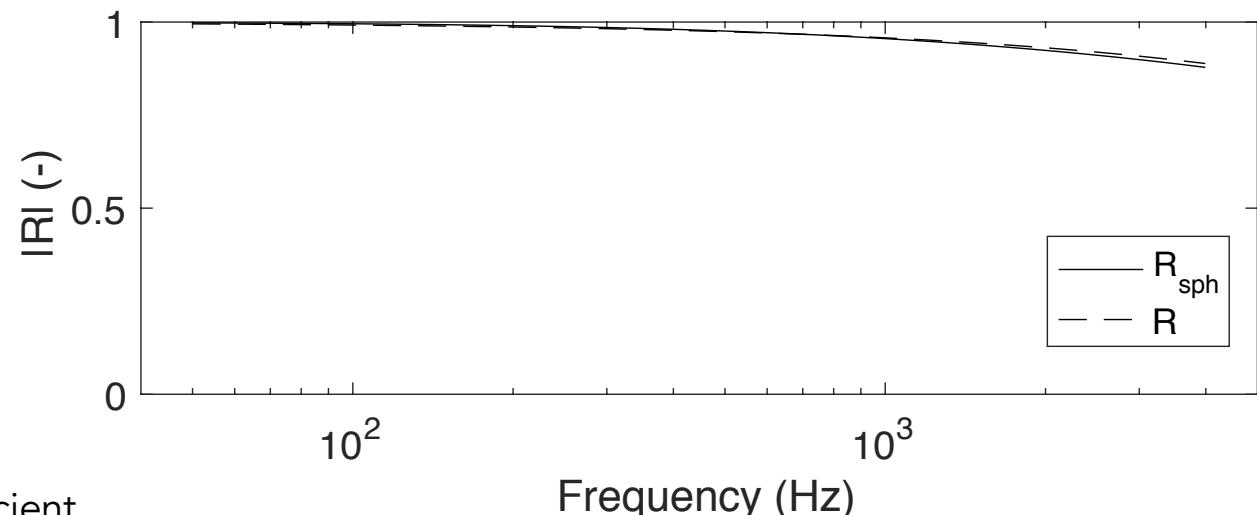
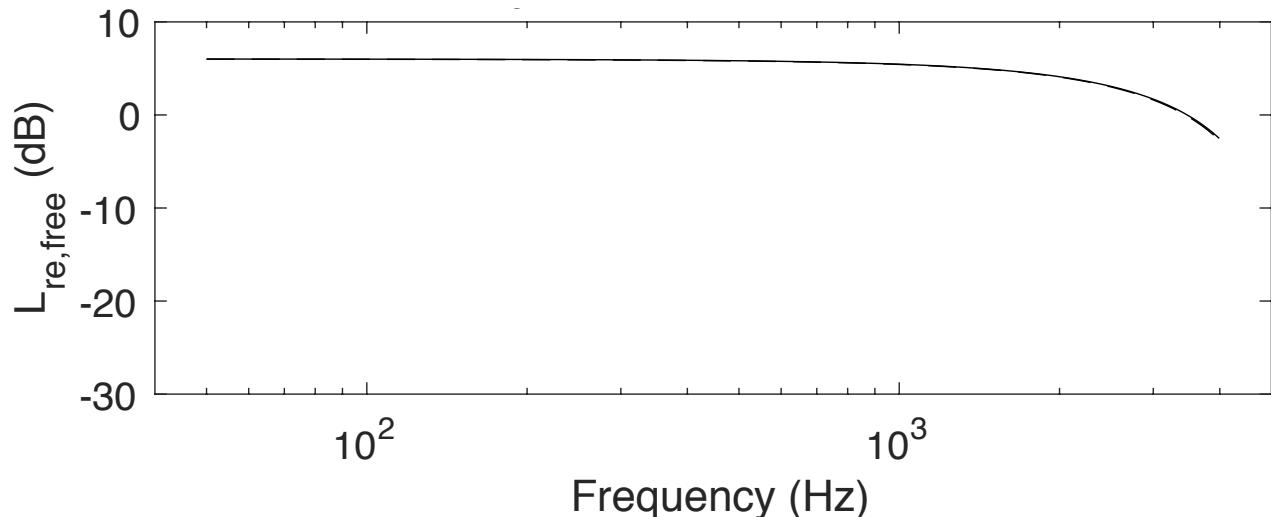
Example:

$$(x_r, z_r) = (50 \text{ m}, 0.5 \text{ m}),$$

$$(x_s, z_s) = (0 \text{ m}, 1.5 \text{ m})$$

$$Z = Z_{\text{DB}}, \text{ with}$$

$$[\Sigma] = 2 * 10^8 \text{ Pa s/m}^2$$



R_{sph} = spherical wave reflection coefficient

R = plane wave reflection coefficient

Own work

Ground reflection

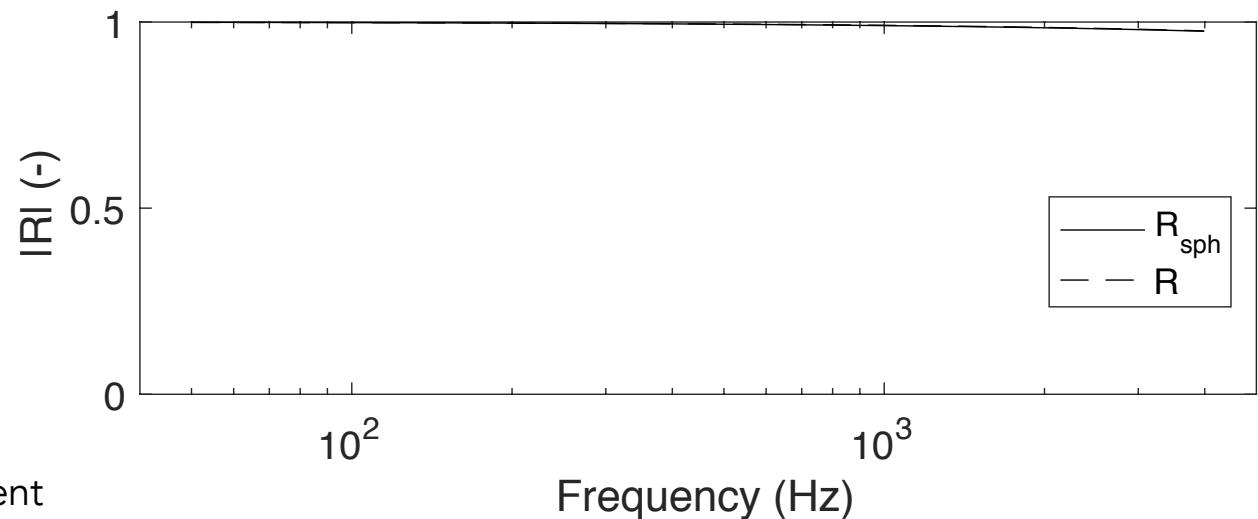
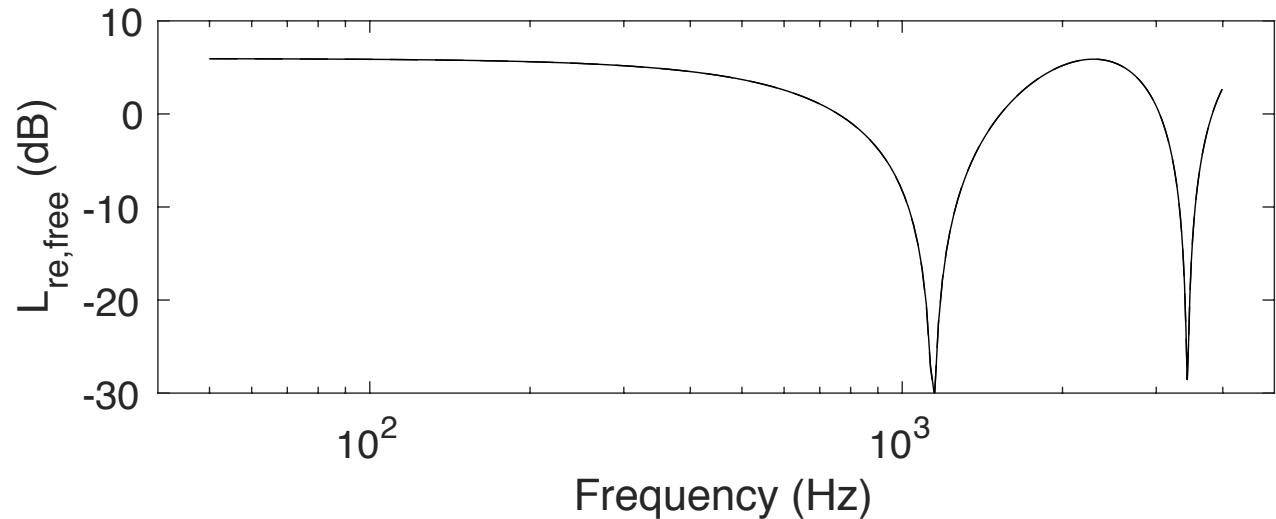
Example:

$$(x_r, z_r) = (10 \text{ m}, 0.5 \text{ m}),$$

$$(x_s, z_s) = (0 \text{ m}, 1.5 \text{ m})$$

$$Z = Z_{\text{DB}}$$
, with

$$[\mathbf{H}] = 2 * 10^8 \text{ Pa s/m}^2$$

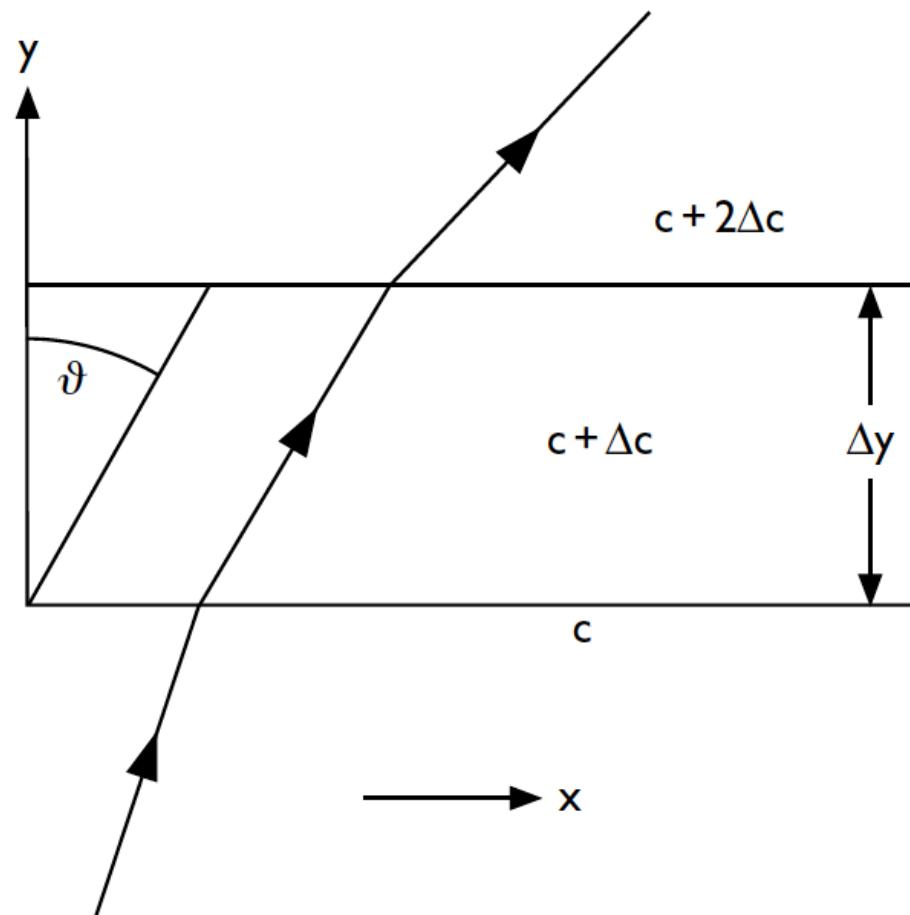


R_{sph} = spherical wave reflection coefficient

R = plane wave reflection coefficient

Own work

Refraction



Refraction

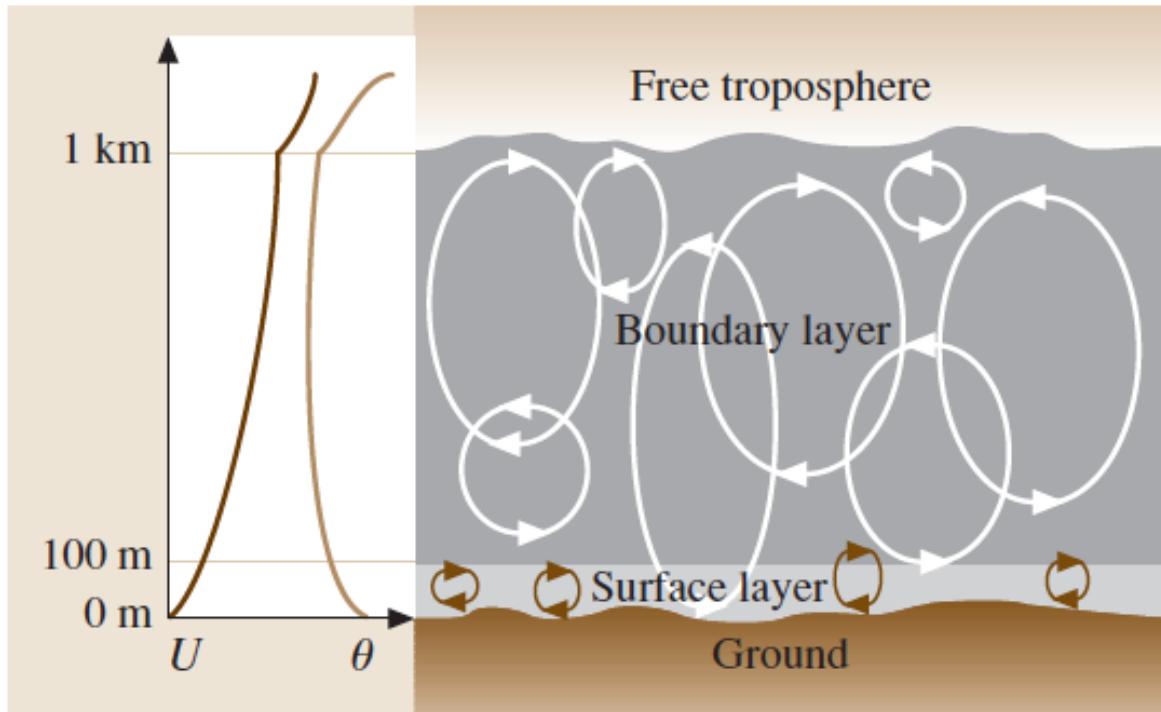
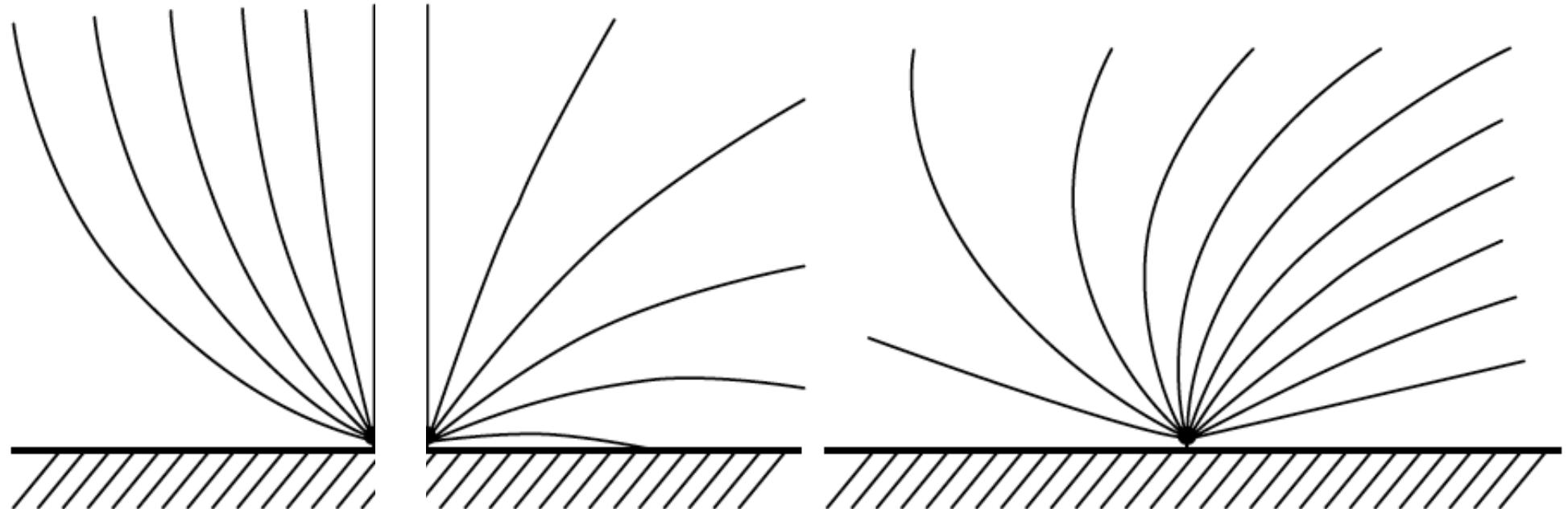


Fig. 4.14 Schematic representation of the daytime atmospheric boundary layer and turbulent eddy structures. The curve on the *left* shows the mean wind speed (U) and the potential temperature profiles ($\theta = T + \gamma_d z$, where $\gamma_d = 0.098 \text{ } ^\circ\text{C/km}$ is the dry adiabatic lapse rate, T is the temperature and z is the height)

Refraction



Temperature gradient

Left part: temperature decreasing with height
Right part: temperature increasing with height

Wind velocity increasing with height, blowing
Towards right direction

Refraction

Ray approach:

$$v_{0,x} = b \ln \left(\frac{z}{z_0} + 1 \right) \quad \text{logarithmic wind speed profile}$$

$$h_n \approx \frac{|x_s - x_r|}{n} \sqrt{\frac{b}{2\pi c}}$$

maximum height of n^{th} sound ray in logarithmic wind speed profile

$$\Delta L_{wind} \approx 10 \log_{10} (N_{rays}) = 10 \log_{10} \left(\frac{8h_1}{z_s + z_r} \right)$$

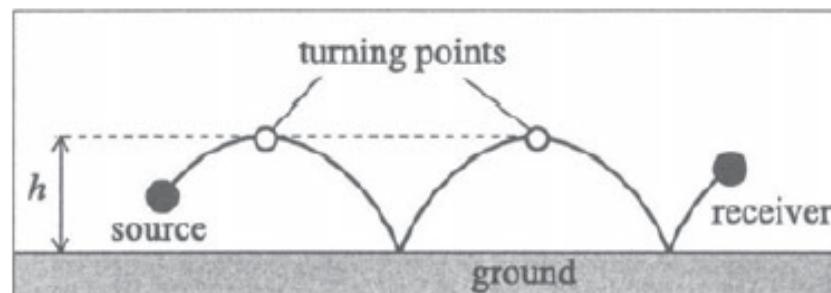
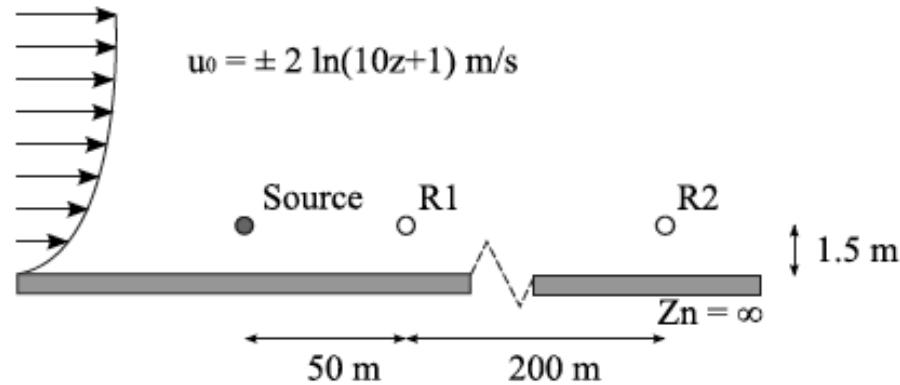
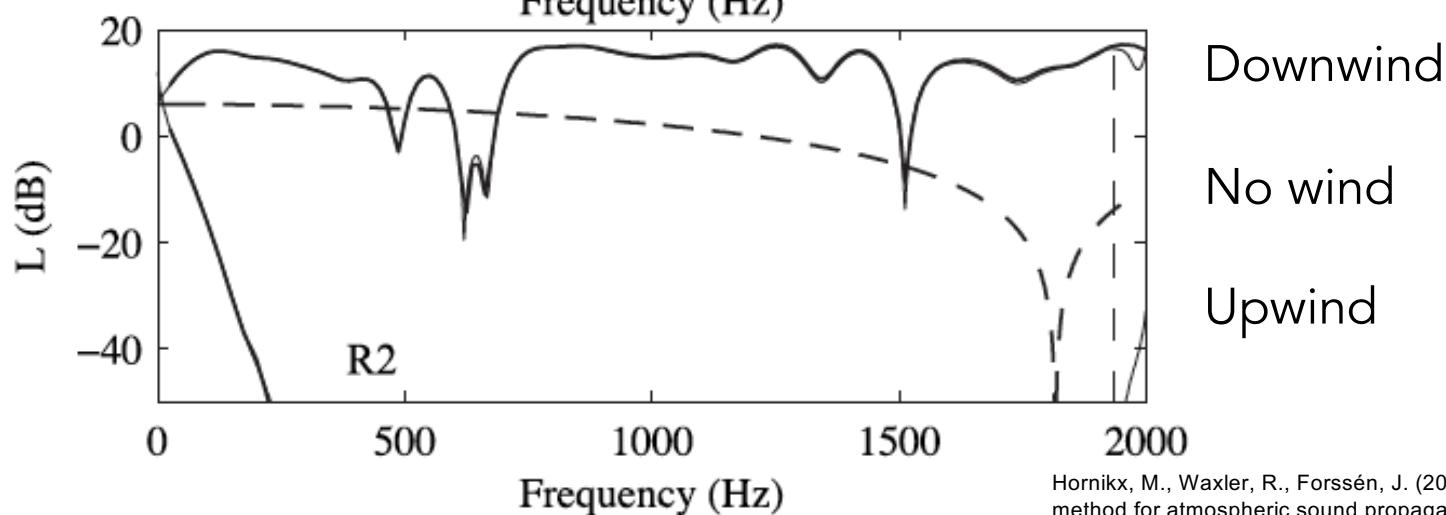
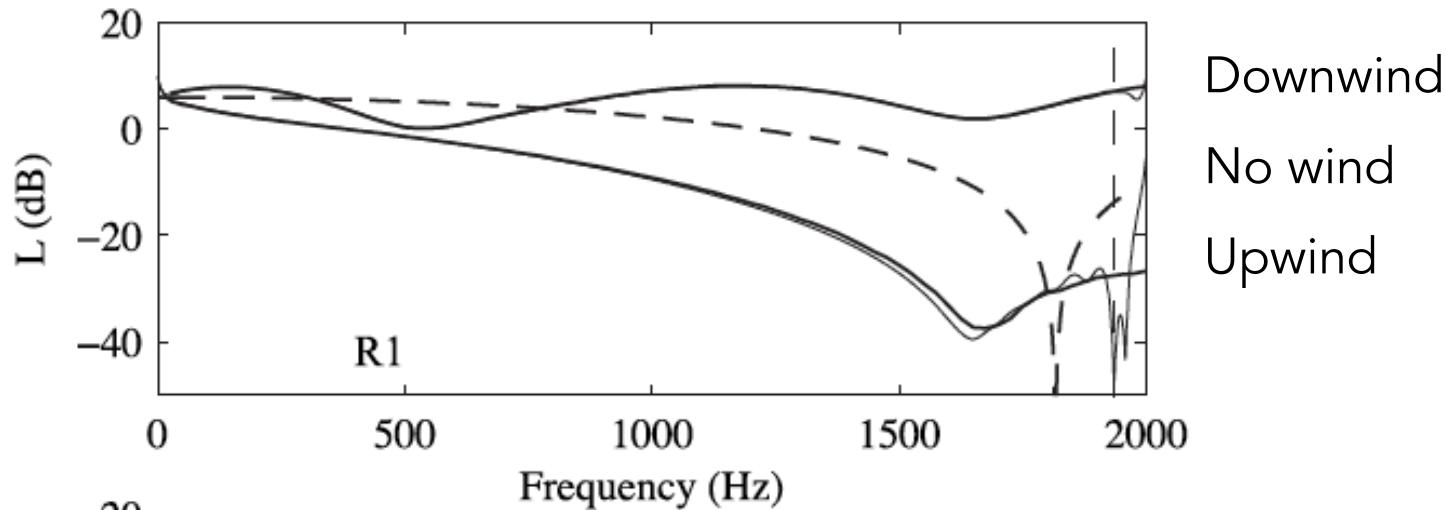


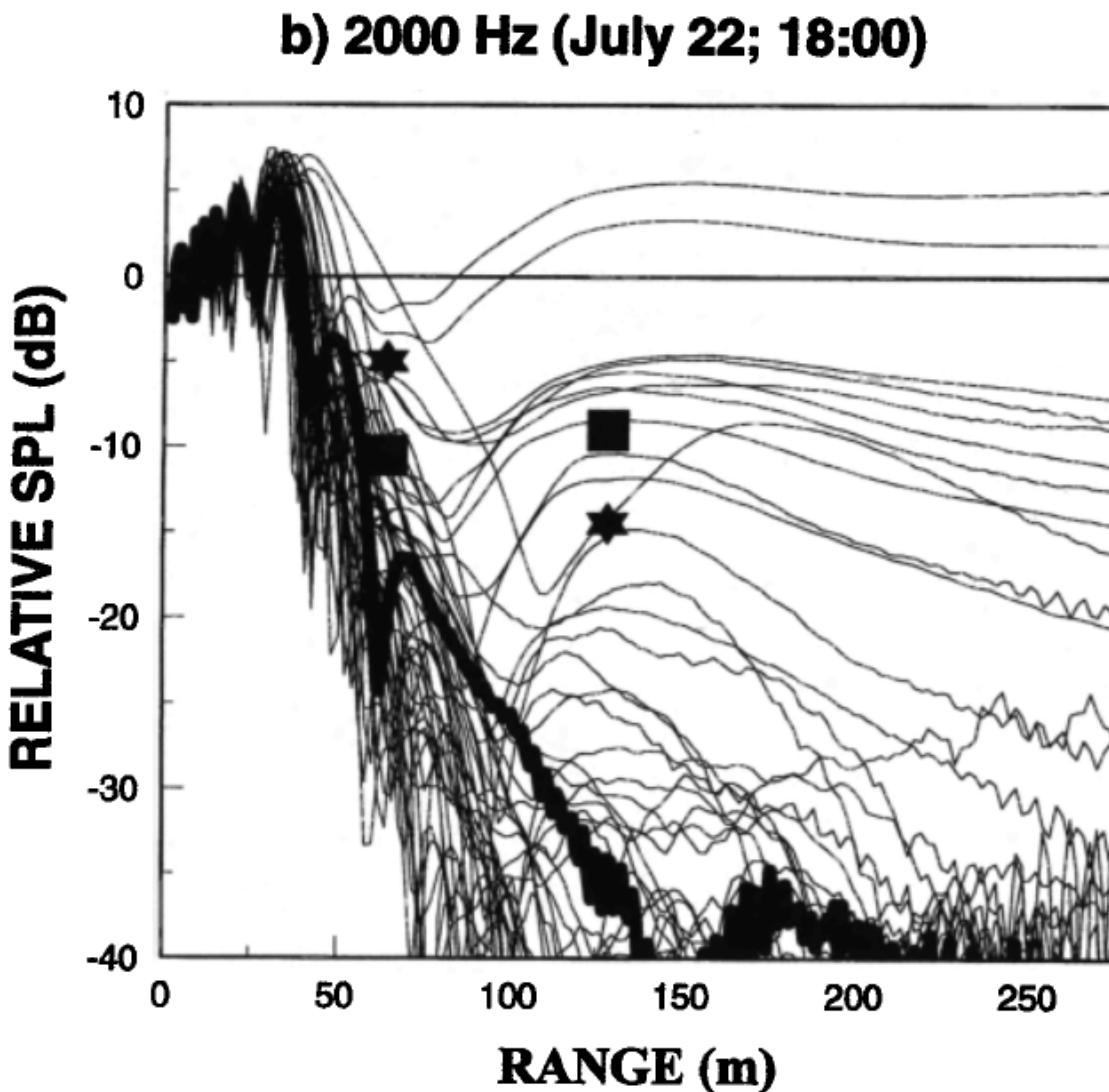
Figure 4.9. Example of a sound ray with two turning points ($n = 2$). The maximum height of the ray, denoted as h , is indicated.



- Level relative to free field
(without ground)



Refraction



Instantaneous sound pressure levels
relative to free field

Source and receiver height = 2m

Upward refraction

Refraction

Coupled linear acoustic equations, non-moving propagation medium

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}$$

$$\frac{\partial v_y}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y}$$

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial z}$$

$$\frac{\partial p}{\partial t} = -\rho_0 c^2 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

Refraction

The linearized Euler equations (LEE), acoustic equations for moving propagation medium

$$\begin{aligned}\frac{\partial v_x}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial x} - \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) v_{0,x} - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) v_x \\ \frac{\partial v_y}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial y} - \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) v_{0,x} - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) v_y \\ \frac{\partial v_z}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial z} - \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) v_{0,x} - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) v_z \\ \frac{\partial p}{\partial t} &= -\rho_0 c^2 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) p\end{aligned}$$

$v_{0,x}, v_{0,y}, v_{0,z}$ = mean wind velocity components

Refraction

Effective sound speed approach, valid for

- Sound propagation direction similar to direction of horizontal wind velocity component

$$c_{\text{eff}}(z) = c(0) \sqrt{\frac{T(z) + 273.15}{273.15}} + v_{0,x}(z)$$

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}$$

$$\frac{\partial v_y}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y}$$

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial z}$$

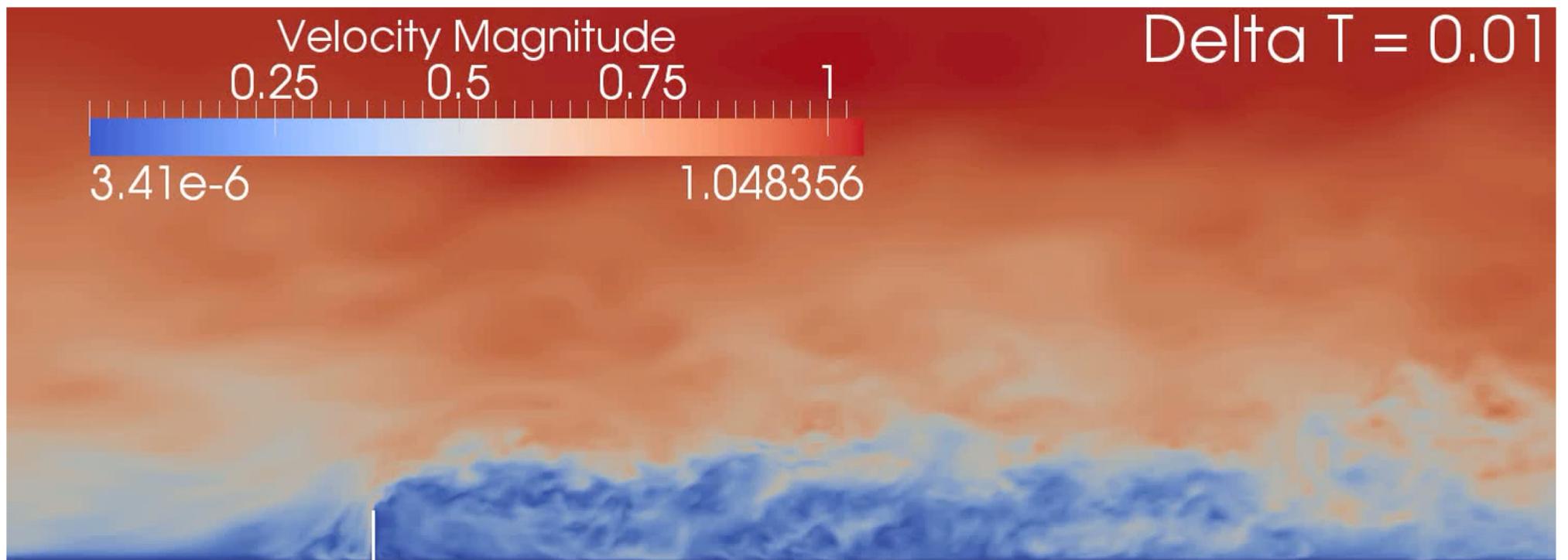
$$\frac{\partial p}{\partial t} = -\rho_0 c^2 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$



$$c = c_{\text{eff}}$$

Refraction

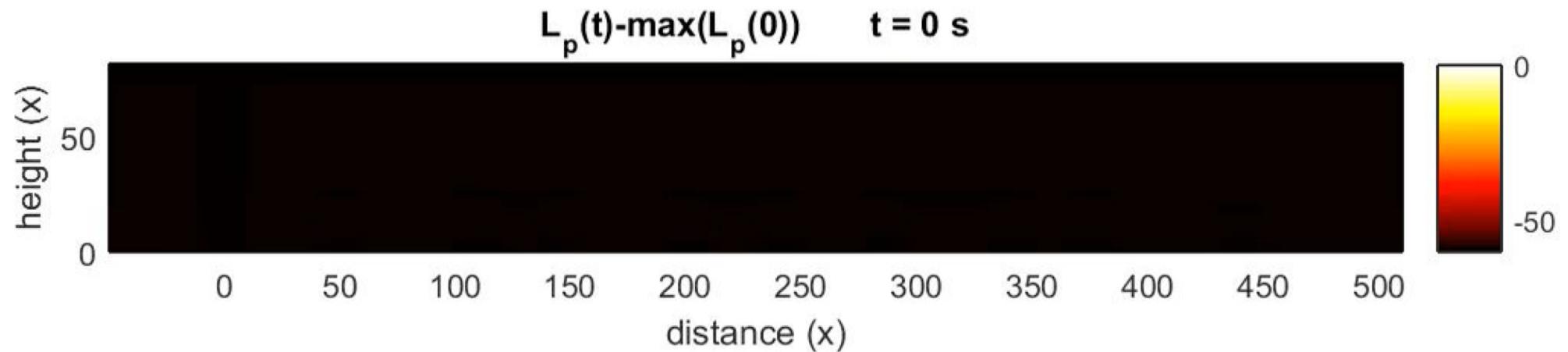
Atmospheric turbulence: Fluctuation on mean wind velocity component



Jasper Thomas

Refraction

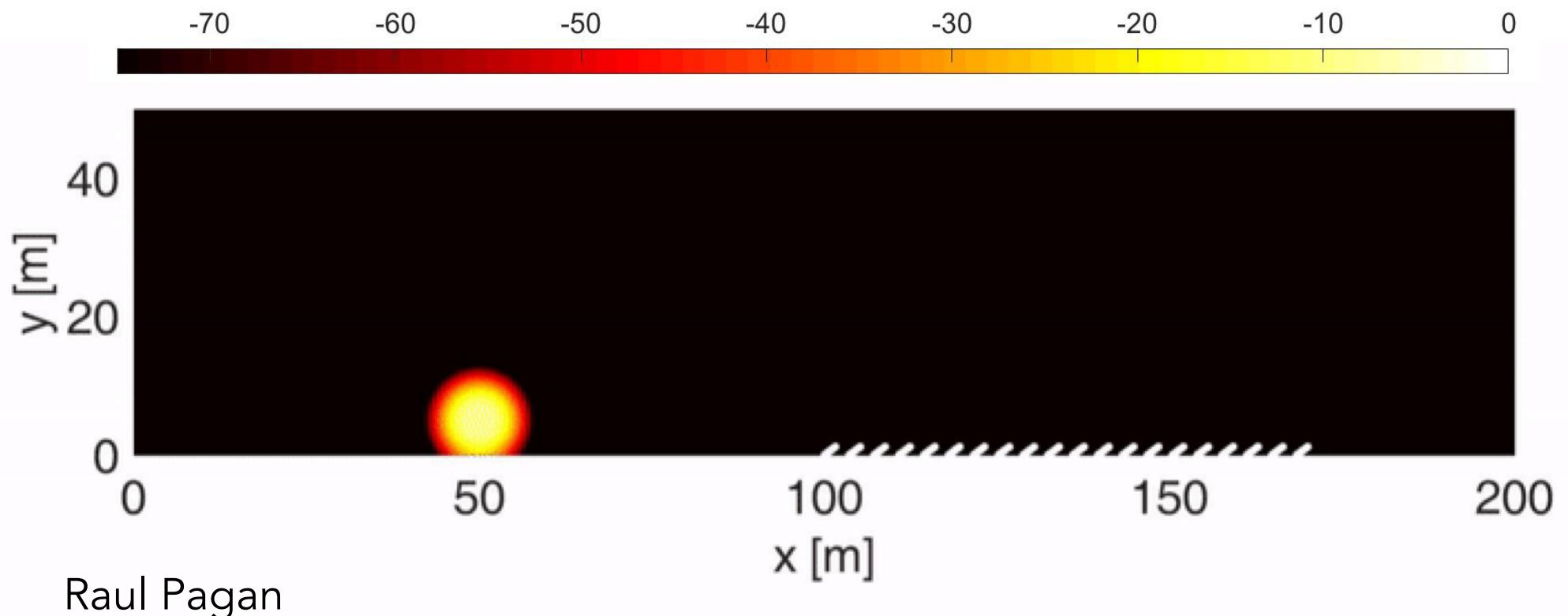
Sound propagation over building roofs



Maud Dohmen

Refraction

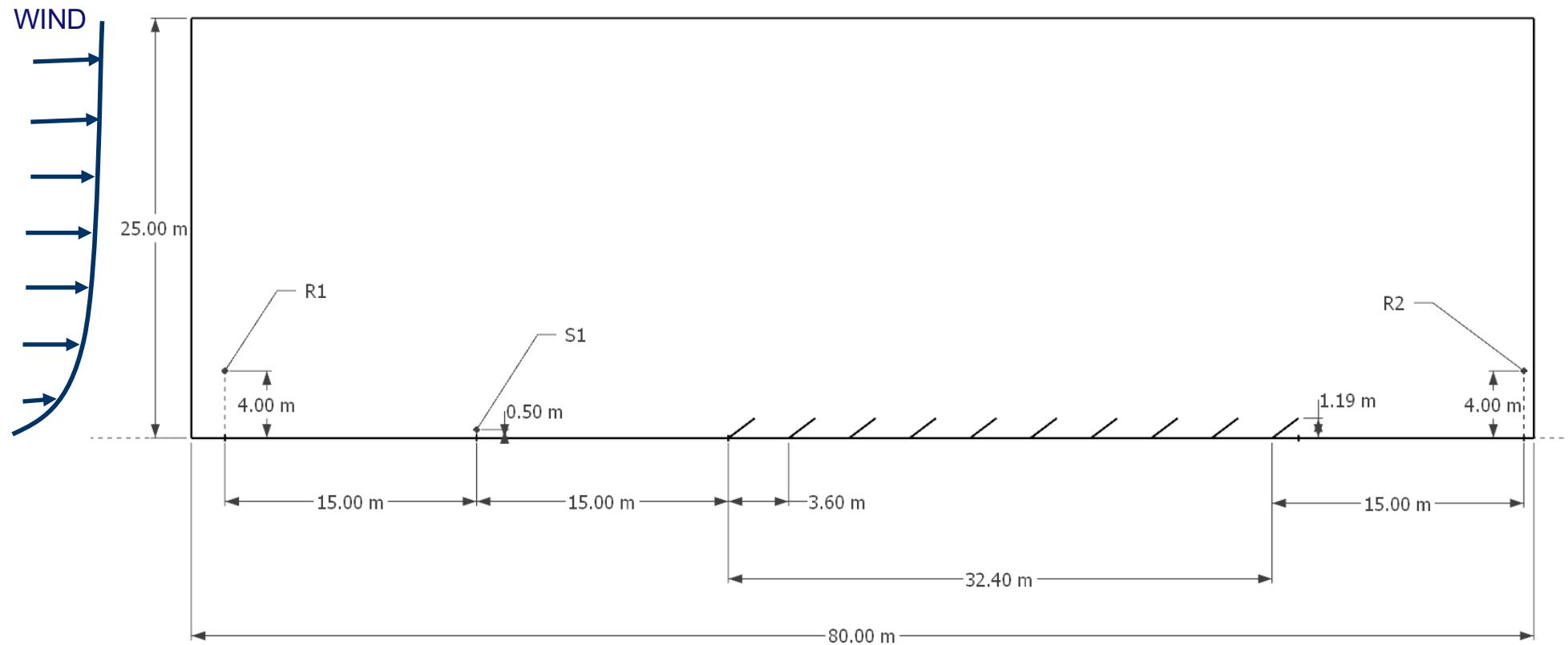
Solar panel fields



Raul Pagan

Refraction

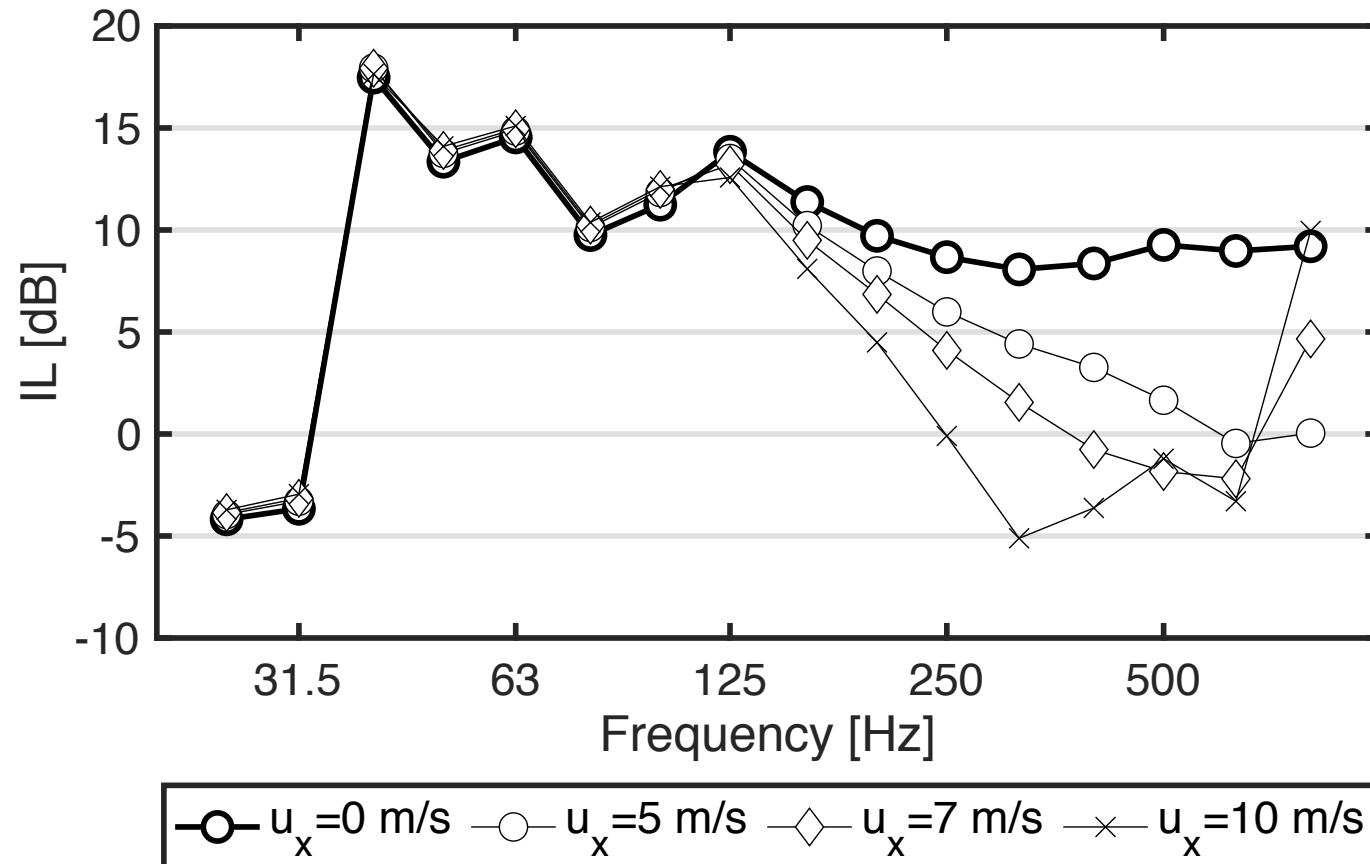
Solar panel fields



Raul Pagan

Refraction

Solar panels fields (37° on ground)



Raul Pagan

Formulas week 4 part a

$$\frac{c}{\sin \theta} = \frac{c'}{\sin \theta'} \quad \text{Refraction between two media}$$

$$\theta_t = \arcsin \left(\frac{c}{c'} \right) \quad \text{Critical angle of total reflection}$$

$$R = \frac{Z \cos \theta - Z_0}{Z \cos \theta + Z_0} = \frac{\zeta \cos \theta - 1}{\zeta \cos \theta + 1} \quad \text{Reflection from wall} \quad (17)$$

$$R = \frac{Z'_0 \cos \theta - Z_0 \cos \theta'}{Z'_0 \cos \theta + Z_0 \cos \theta'} \quad \text{Reflection from different medium}$$

$$\alpha = 1 - |R|^2 \quad (18)$$

$$Z = Z_0 \frac{1 + e^{-j2kd}}{1 - e^{-j2kd}} = -jZ_0 \cot(kd)$$

Impedance of air layer with rigid backing at distance d

$$Z = -jZ'_0 \cot(k'd) \quad (19)$$

Impedance of porous layer with rigid backing at distance d

$$k' = \frac{\omega}{c} \sqrt{1 - \frac{j\sigma\Xi}{\rho_0\omega}} \quad \text{Wavenumber of porous layer}$$

$$Z'_0 = Z_0 \sqrt{1 - \frac{j\sigma\Xi}{\rho_0\omega}} \quad \text{Impedance of porous layer}$$

$$Z = Z_0 + r_s \quad \text{Impedance of a porous sheet}$$

$$Z = r_s - jZ_0 \cot(kd) \quad (20)$$

Impedance of a thin porous sheet on an air layer with rigid backing at distance d

$$Z = j\omega m' - jZ_0 \cot(kd)$$

Impedance of a non-porous sheet on an air layer with rigid backing at distance d

Formulas week 4 part a

Reflection

$$Z_{DB} = 1 + 9.08 \left(\frac{1000f}{\Xi_{\text{eff}}} \right)^{-0.75} - j11.9 \left(\frac{1000f}{\Xi_{\text{eff}}} \right)^{-0.73} \quad (39)$$

Delany and Bazley ground impedance

$$k_{DB} = \frac{\omega}{c} \left(1 + 0.0978 \left(\frac{f}{\Xi_{\text{eff}}} \right)^{-0.700} - j0.189 \left(\frac{f}{\Xi_{\text{eff}}} \right)^{-0.595} \right)$$

$$p(x_r, z_r, t) = A \left(\frac{e^{-jkr_1}}{r_1} + R_{\text{sph}} \frac{e^{-jkr_2}}{r_2} \right) e^{j\omega t} \quad \text{point source solution above ground surface} \quad (40)$$

$$r_1 = \sqrt{(x_r - x_s)^2 + (z_r - z_s)^2}$$

$$r_2 = \sqrt{(x_r - x_s)^2 + (z_r + z_s)^2}$$

$$R_{\text{sph}} = R + (1 - R)F(w) \quad \text{spherical wave reflection coefficient}$$

$$F(w) = 1 - j\sqrt{\pi}e^{-w^2} \operatorname{erfc}(jw) \quad \text{boundary loss factor}$$

$$w \approx \frac{1}{2}(1 - j)\sqrt{kr_2}(\cos(\theta) + \frac{1}{Z}) \quad \text{numerical distance}$$

$$R \approx R_{\text{sph}} \quad \text{if } |w| > 4$$

Formulas week 4 part a

Refraction

$$\begin{aligned}
 c_{\text{eff}}(z) &= c(0) \sqrt{\frac{T(z) + 273.15}{273.15}} + v_{0,x}(z) \\
 \frac{\partial v_x}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial x} - \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) v_{0,x} - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) v_x \\
 \frac{\partial v_y}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial y} - \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) v_{0,x} - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) v_y \\
 \frac{\partial v_z}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial p}{\partial z} - \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) v_{0,x} - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) v_z \\
 \frac{\partial p}{\partial t} &= -\rho_0 c^2 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) - \left(v_{0,x} \frac{\partial}{\partial x} + v_{0,y} \frac{\partial}{\partial y} + v_{0,z} \frac{\partial}{\partial z} \right) p
 \end{aligned} \tag{41}$$

Linearized Euler equations (4 equations above)

$$\begin{aligned}
 v_{0,x} &= b \ln \left(\frac{z}{z_0} + 1 \right) \quad \text{logarithmic wind speed up} \\
 h_n &\approx \frac{|x_s - x_r|}{n} \sqrt{\frac{b}{2\pi c}} \\
 &\quad \text{maximum height of } n^{\text{th}} \text{ sound ray in logarithmic wind speed profile} \\
 \Delta L_{\text{wind}} &\approx 10 \log_{10} \left(\frac{8h_1}{z_s + z_r} \right)
 \end{aligned} \tag{42}$$

Sound pressure level increase due to log wind speed profile