Problem Set : Mathematical Physics

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CHAPTER 1

Classical mechanics

In Hamiltonian mechanics, the phase space M is defined to be cotangent bundle of a configuration manifold. According to Newton's principle of determinacy, a particle at a specific time corresponds to a point in M, and the point contains all informations of a particle. A function on M is a physical quantity, such as position, momentum, angular momentum, etc. Especially, positions and momenta with respect to each dimension provide with canonical coordinate functions on M. Therefore, every function on M can be realized by a function of positions and momenta.

A *Hamiltonian function* H is also just a function on M. In physics, if a Hamiltonian function is given, the equation of motion is generated. In other words, Hamiltonian function defines a physical problem.

DEFINITION 0.1 (Hamilton's equations of motion). For a Hamiltonian function H, Hamilton's equations of motion are given by

$$\dot{x} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x}.$$

Using the Poisson bracket, the equations can be represented by

$$\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}.$$

PROBLEM 0.1 (Harmonic oscillator). Let $M = T^*\mathbb{R}$ and

$$H(x,p) = \frac{p^2}{2m} + \frac{1}{2}kx^2.$$

This Hamiltonian function defines a problem of 1-dimensional harmonic oscillator. The equations of motion are

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}, \qquad \dot{p} = -\frac{\partial H}{\partial x} = -kx.$$

Therefore, we get the familiar equation for a harmonic oscillator

SOLUTION.

(1) Here is the proof.
(2) Here is a proof.

$\square$

If H has a symmetry under transformations in time, namely, H does not depend on t explicitly, then

A problem that H is explicitly independent on p is difficult to occur physically.

CHAPTER 2

Classical field theory

CHAPTER 3

Relativity theory

CHAPTER 4

Quantum mechanics

PROBLEM 0.2 (Hydrogen atom). Hydrogen
hydrogen

1. Particle physics

PROBLEM 1.1 (Yukawa potential). The wave equation for a massive field is given by the Klein-Gordon equation

$$(\Box + m^2)u(t, x) = 0,$$

where m is mass.

(1) Derive the Yukawa potential

$$u(x) = k\frac{e^{-\frac{r}{m}}}{r}$$

where $r = |x|$, as a Green function by assuming static case.

(2) Letting $m = 0$, discuss the relation with Coulomb potential.

(3) By taking Fourier transform.

(4) Find an approximate range of strong nuclear force and mass of pion.

PROBLEM 1.2 (Negative energy solution and antiparticles). Dirac's interpretation. negative energy solution of the Klein-Gordon equation
time reversal?

PROBLEM 1.3 (Polarization of photon field).

PROBLEM 1.4 (Aharonov-Bohm effect).

CHAPTER 5

Quantum field theory