

Title of the Article: Curved folding and planar cutting of simple closed curve on a conical origami

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Comments to the Author(s)

The paper states an interesting result along the lines of the “fold-and-cut” theorem. Demaine and others have shown that by flat-folding a sheet of paper and then cutting along a straight line, any polygonal domain can be obtained on the unwrapped paper. The submitted paper deals with conical folding, and shows that every star shaped domain (under suitable assumptions) can be obtained by a planar cut.

The paper is well written and mathematically correct. I recommend it for publication. I would like to point out some points which should be made clear before publication.

1. the concept of “piecewise- C^1 ” map is not uniquely defined in literature. The author should make a clear definition of what he means (I think he is using the one defined in [2] but this should be stated and anyway the definition should be repeated);
2. the paper defines the concept of “conical origami” as a map whose image is contained in a cone. However I think that in Proposition 2.1 the author is actually using a stronger assumption which is that the map u should be 1-homogeneous. In fact if one makes some flat folding of a sheet of paper and then makes a cone with the already folded paper, then the resulting map has image in a cone (so it would satisfy the hypothesis of Proposition 2.1). However the half lines lying on the cone are not guaranteed to be fully contained in the image of u . This property is used in the proof and I believe that actually the statement could be false. In fact there exists a domain which is not star shaped but which can be flat folded to a star-shaped domain so that the boundary goes to the boundary. So if the star shaped domain can be cut from a cone (as the submitted paper shows) then also the original domain (which is not star shaped) can be obtained.
3. in the main Theorem (page 2) a natural question is whether condition (1) is always satisfied. If it is not, an example would be given.
4. The definition of the function A_z (first equation of page 7) should be given before the main statement.

Reading the paper I can also suggest some improving of the language.

5. replace “graph with straight sides” with “polygon” or “polygon with straight edges”;
6. In the fourth line of the introduction the phrasing suggests that the result proven is a generalization of the flat fold-and-cut theorem, which is not;
7. In page 1 line -5. Replace “whose image is a cone by the conical origami” with “whose image is a cone a conical origami”.
8. In page 2 line 10. Replace “Exactly the kernel” with “More precisely, the kernel”.
9. In page 2 line 27. Replace “we call the curve cut on u ” with “we call the curve a cut on u ”.
10. Page 2 line -10. Replace “simple closed curve to be cut” with “simple closed curve to be a cut”
11. Page 2 line -6. Replace “be the preimage of its vertex” with “as the preimage of its vertex”.

12. Page 4 line -11. Replace “it is contradiction” with “it is a contradiction”.
13. Page 4 line -3. Replace “half-line intersects” with “half-line which intersects”.
14. Page 7 line 13. Replace “increasing for ψ ” with “increasing in ψ ”. Line 14: replace “the inverse function” with “an inverse function”.
15. Page 7 line -7. Replace “u is the conical origami” with “u is a conical origami”.
16. Page 8 line -9. Replace “u belongs to between” with “u is between”.
17. Page 10 line 13. Replace “linear for ρ ” with “linear in ρ ”. Line -2: replace “such that:” with “by:”.