

Binary quadratic forms

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1. EQUIVALENCE

Definition 1.1. Two forms are called *equivalent* if they are in a same orbit with respect to $\mathrm{GL}_2(\mathbb{Z})$ -action.

Definition 1.2. Two forms are called *properly equivalent* if they are in a same orbit with respect to $\mathrm{SL}_2(\mathbb{Z})$ -action.

2. DEFINITE FORMS

Proposition 2.1. *The $\mathrm{SL}_2(\mathbb{Z})$ -action on the definite forms is not faithful, i.e. the kernel is given by a nontrivial group $\{\pm I\}$.*

Proposition 2.2. *The $\mathrm{PSL}_2(\mathbb{Z})$ -action on the definite forms is faithful.*

2.1. Positive definite forms.

Proposition 2.3. *The set of positive definite forms admits the $\mathrm{SL}_2(\mathbb{Z})$ -action (also $\mathrm{PSL}_2(\mathbb{Z})$ -action).*

Proposition 2.4. *The $\mathrm{PSL}_2(\mathbb{Z})$ -actions on positive definite forms and negative definite forms are isomorphic.*

3. INDEFINITE FORMS

4. CLASS GROUP