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CUTTING

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ABSTRACT. abstrsct

1. Introduction

2. Definition

Definition. Let O be the pole of the plane U that has polar coordinate (r, θ) and M be a surface that is parameterized by an isometry $\phi : U \backslash \{O\} \to U \times \mathbb{R}$ such that $\phi(P) \to (0, \theta, z)$ as $P \to O$ for a positive real number z. Then M is developable surface that forms generalized cone from Lemma *. Suppose that γ is Jordan curve lying on $U \backslash \{O\}$. If z-coordinate of any point of the image $\phi(\gamma)$ vanishes, the curve γ is called *cutting* with *folding* ϕ.

Theorem 2.1. *If a Jordan curve γ lying on $U \backslash \{O\}$ is cutting, then the pole O belongs to interior of γ. Moreover, there exists a bijective function between the points on γ and angular coordinates of each point.*

Proof. □

Definition. Let a mapping $\tau : U \to U$ be *folding transformation* of folding ϕ if $\tau \circ \phi^{-1}$ is a projection such that $\tau \circ \phi^{-1}(r, \theta, z) = (r, \theta)$.

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Theorem 3.1. *For certain positive real number Z, if a function $q(z, \theta)$ satisfies the followings:*

(1) *$q(z, \theta)$ is continuous and bounded for z and θ;*
(2) *$q(z, \theta)$ is strictly increasing for z;*
(3) *$q(z, \theta)$ is periodic with period 2π for θ;*
(4) *there is an interval $[\alpha, \beta) \subset [0, 2\pi)$ s.t. $q(z, \theta)$ is monotonic over $\theta \in [\alpha, \beta)$;*
(5) *$q(0, \theta) = 1$ for all θ in the domain*

for all real number z in $(0, Z)$ and θ, then there exists a real number $z_0 \in (0, Z)$ such that for all $z \in (0, z_0)$ there is a function $p(z, \theta)$ such that:

(6) *$|p(z, \theta)| = q(z, \theta)$;*
(7) *$p(z, \theta)$ is piecewise continuous for θ;*
(8) *$p(z, \theta)$ has simple folding curve for θ;*

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(9) $\int_0^{2\pi} p(z,\theta)\,d\theta = 2\pi$

Proof. Since $q = 1$ where $z = 0$ and $q(z,\theta)$ is strictly increasing for z, a real number ϕ is positive which is defined such that

$$\phi = \int_0^{2\pi} q(Z,\theta)\,d\theta - 2\pi > 0.$$

Let $Q(z;a,b)$ be a function such that

$$Q(z;a,b) = \int_0^{2\pi} q(z,\theta)\,d\theta - 2\int_{\frac{2a+b}{3}}^{\frac{a+2b}{3}} q(Z,\theta)\,d\theta.$$

That $q(Z,\theta)$ is bounded implies

$$\int_{\frac{2a+b}{3}}^{\frac{a+2b}{3}} q(Z,\theta)\,d\theta \leq \frac{b-a}{3}\sup\{q(Z,\theta) : \theta \in \mathbb{R}\}$$

for all interval $[a,b)$, therefore if we take an interval $[a_0,b_0) \subset [\alpha,\beta)$ whose length is less than $3\phi/2\sup\{q(Z,\theta)\}$, we have

$$Q(Z;a_0,b_0) > 2\pi. \tag{1}$$

$Q(0;a_0,b_0) = 2\pi - \frac{2}{3}(b-a) < 2\pi$ and the inequality (1) imply that there exists $z_0 \in (0,Z)$ which makes $Q(z_0;a,b)$ be supposed to be 2π according to IVT. It means that there is an interval $[a_0,b_0)$ such that $q(z,\theta)$ is monotonic over $\theta \in [a_0,b_0)$ and there exists $z_0 \in (0,Z)$ such that $Q(z_0;a,b) = 2\pi$

To prove the proposition for all $z < z_0$, consider a positive real number t less than $(b-a)/2$. $Q(z;a_0+t,b_0-t)$ is continuous for t and

$$Q(z;a_0,b_0) < Q(z_0;a_0,b_0) = 2\pi$$
$$Q(z;a_0+t,b_0-t) = 0 < 2\pi$$

imply there exists $t \in (0,(b-a)/2)$ such that $Q(z;a,b) = 2\pi$ letting $[a,b) = [a_0+t,b_0-t)$. $q(z,\theta)$ is obviously monotonic over $\theta \in [a,b)$.

Let $p(z,\theta)$ be a function such that

$$p(z,\theta) = \begin{cases} -q(z,\theta), & \text{if } \theta \in [a,b) \\ q(z,\theta), & \text{if } \theta \notin [a,b) \end{cases}$$

where a,b is the numbers determined above. Then we can prove the function $p(z,\theta)$ satisfy the conditions. □

In Theorem 3.1, let trivial supremum of z defined the supremum of Z, non-trivial supremum of z defined the supremum of z_0.

References

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