Problem Set : Differential Equations

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Contents

CHAPTER 1

Chapter

1. Section

PROBLEM 1.1. Let m, b, k, A, and ω be positive real constants. Consider an underdamped oscillatior with sinusoidal driving force, which is descibed by the following second order ordinary differential equation.

$$m\ddot{x} + b\dot{x} + kx = A\sin\omega t, \qquad x(0) = x_0,\ \dot{x}(0) = 0.$$

For convenience, define $\omega_0 := \sqrt{\frac{k}{m}}$.

(1) The solution of this equation when given the free oscillation condition $A = 0$ has the form

$$x_c(t) = x_0 e^{-\beta t}\cos\omega_1 t.$$

Compute the constants β and ω_1. This solution is called the *complementary solution.*

(2) Show that the contemporary solution is asymptotically stable, i.e.

$$\lim_{t\to\infty} x_c(t) = 0.$$

(3) Show that the solution decays most fastly at $\beta = \omega_0$.

(4) Find the value of ω such that the amplitude of particular solution is maximized.

SOLUTION.

(1) $\beta = \frac{b}{2m}$, $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$.

(2)

□

PROBLEM 1.2. Let Ω be a bounded Lipschitz domain. Consider an eigenvalue problem

$$\begin{cases} -\Delta u = \lambda u, & \text{in } x \in \Omega, \\ u = 0, & \text{on } x \in \partial\Omega. \end{cases}$$