Physics

-Problem set-

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Contents

1 Classical mechanics

In Hamiltonian mechanics, the phase space M is defined to be cotangent bundle of a configuration manifold. According to Newton's principle of determinacy, a particle at a specific time corresponds to a point in M, and the point contains all informations of a particle. A function on M is a physical quantity, such as position, momentum, angular momentum, etc. Especially, positions and momenta with respect to each dimension provide with canonical coordinate functions on M. Therefore, every function on M can be realized by a function of positions and momenta.

A *Hamiltonian function* H is also just a function on M. In physics, if a Hamiltonian function is given, the equation of motion is generated. In other words, Hamiltonian function defines a physical problem.

Definition (Hamilton's equations of motion)**.** For a Hamiltonian function H, Hamilton's equations of motion are given by

$$\dot{x} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x}.$$

Using the Poisson bracket, the equations can be represented by

$$\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}.$$

Example 1.1 (Harmonic oscillator)**.** Let $M = T^*\mathbb{R}$ and

$$H(x,p) = \frac{p^2}{2m} + \frac{1}{2}kx^2.$$

This Hamiltonian function defines a problem of 1-dimensional harmonic oscillator. The equations of motion are

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}, \qquad \dot{p} = -\frac{\partial H}{\partial x} = -kx.$$

Therefore, we get the familiar equation for a harmonic oscillator

If H has a symmetry under transformations in time, namely, H does not depend on t explicitly, then

A problem that H is explicitly independent on p is difficult to occur physically.

2 Quantum mechanics

2.1. *Hydrogen atom.*

Hydrogen

(a) hydrogen

3 Relativity theory

4 Particle physics

4.1. *Yukawa potential.*

The wave equation for a massive field is the Klein-Gordon equation

$$(\Box + m^2)U = 0.$$

(a) deriving the Yukawa potential as a Green function by assuming static case.

(b) relation with Coulomb potential.

(c) the Fourier transformation.

(d) range of (strong) nuclear force and mass of pion

4.2. *Negative energy solution and antiparticles.*

Dirac's interpretation.

(a) negative energy solution of the Klein-Gordon equation

(b) time reversal?

4.3. *Polarization of photon field.*

This is an empty problem. A quadratic function is defined by

$$f(x) = ax^2 + bx + c$$

where a, b, c are real numbers.

(a) Show that the equation $f(x) = 0$ has a real solution if and only if $b^2 - 4ac \geq 0$.

(b) Find the formula of solutions.

4.4. *Aharonov-Bohm effect.*

This is an empty problem. A quadratic function is defined by

$$f(x) = ax^2 + bx + c$$

where a, b, c are real numbers.

(a) Show that the equation $f(x) = 0$ has a real solution if and only if $b^2 - 4ac \geq 0$.

(b) Find the formula of solutions.