Diachrony of spectrum

Ikhan Choi

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Definition

Let R be a commutative ring. The *spectrum* of R is the set of prime ideals of R.

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Question: Why is it defined like this?

Hydrogen spectral series

Hilbert spaces in quantum mechanics

Rydberg's formula : Bohr model

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$$mvr = n\hbar.$$

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Bohr's postulates:

- ▶ The electrons are on certian stable orbits.
- ▶ The stationary orbits are computed by the old quantization assumption for angular momenta:

$$mvr = n\hbar.$$

- ▶ An electron absorbs or emits light frequency f when they jump from an orbit to another, satisfying

$$\Delta E = hf.$$

Rydberg's formula : Bohr model

From the three relations

$$mvr = n\hbar, \quad \frac{mv^2}{r} = k\frac{e^2}{r^2}, \quad E = K + V = \frac{1}{2}mv^2 - k\frac{e^2}{r},$$

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$$mvr = n\hbar, \quad \frac{mv^2}{r} = k\frac{e^2}{r^2}, \quad E = K + V = \frac{1}{2}mv^2 - k\frac{e^2}{r},$$

we deduce

$$E = -\frac{k^2 e^4 m}{2\hbar^2}\frac{1}{n^2}.$$

Rydberg's formula : Schrödinger equation

Separation of variables

Spectral theorem of normal matrices

Spectral theorem of compact operators

Spectral theorem of elliptic operators

Banach algebras and C^*-algebras

Example 1 : Bounded operators

Example 2 : Continuous functions

Spectra, multiplicative homomorphisms, maximal ideals

Gelfand-Naimark theorem

Algebraic variety

Coordinate ring

Maximal ideal is a point

Problem of unified codomains

Functoriality