Physics I : Classical theory

Written by Ikhan Choi
Solved by Ikhan Choi

Contents

CHAPTER 1

Classical mechanics

1. Newtonian mechanics

Uniformly accelerated motion. For rectilinear motion with constant velocity v,

$$x(t) = vt + x_0.$$

For uniformly accelerated motion with constant acceleration a and the initial velocity v_0, we have

$$v(t) = at + v_0,$$

and by integrating

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0.$$

Problems.

PROBLEM 1.1. Take x-axis to be the horizontal earth and y-axis to be vertical. A boy is on the earth, and let the position of the boy be $(0, 0)$. A bird in the sky (l, h) get a shot at time $t = 0$ and falls freely. Almost simultaneously, The boy throw a small rock with the initial velocity v_0 and the angle θ from the earth. Let g be the gravitational acceleration.

(1) Find the time t at which the rock reaches a vertical line $x = l$.
(2) At this time, find the y-coordinate of the rock and the bird y_r, y_{bird}.
(3) They collided. Find θ.
(4) Find the condition for v_0 to make them collide each other.
(5) Show that if you are a bird, a rock is in a uniform rectilinear motion. Find the time for collision after throwing.

SOLUTION.

(1) The horizontal component of the velocity is $v_0 \cos\theta$. Therefore, the time traveling the length l is

$$t = \boxed{\frac{l}{v_0 \cos\theta}}.$$

(2) Let t be the answer of the previous problem. Their accelerations are $-g$. Therefore,

$$y_{rock} = \frac{1}{2}(-g)t^2 + (v_0 \sin\theta)t + (0) =$$

and

$$y_{bird} = \frac{1}{2}(-g)t^2 + (0)t + (h) = .$$

(3)

□

PROBLEM 1.2. Let m, b, and k be a positive constant such that $b^2 - 4mk < 0$. Consider an underdamped oscillatior with sinusoidal driving force

$$m\ddot{x} + b\dot{x} + kx = A \sin \omega t, \qquad x(0) = x_0,\ \dot{x}(0) = 0.$$

For convenience, define positive constants $\beta := \frac{b}{2m}$, $\omega_0 := \sqrt{\frac{k}{m}}$, and $\omega_1 := \sqrt{\omega_0^2 - \beta^2}$.

The solution of this equation when $A = 0$ is called the complementary solution and is given by

$$x_c(t) = x_0 e^{-\beta t} \cos \omega_1 t.$$

(1)

(2) Find the value of ω such that the amplitude of particular solution is maximized.

In Hamiltonian mechanics, the phase space M is defined to be cotangent bundle of a configuration manifold. According to Newton's principle of determinacy, a particle at a specific time corresponds to a point in M, and the point contains all informations of a particle. A function on M is a physical quantity, such as position, momentum, angular momentum, etc. Especially, positions and momenta with respect to each dimension provide with canonical coordinate functions on M. Therefore, every function on M can be realized by a function of positions and momenta.

A *Hamiltonian function* H is also just a function on M. In physics, if a Hamiltonian function is given, the equation of motion is generated. In other words, Hamiltonian function defines a physical problem.

DEFINITION 1.1 (Hamilton's equations of motion). For a Hamiltonian function H, Hamilton's equations of motion are given by

$$\dot{x} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x}.$$

Using the Poisson bracket, the equations can be represented by

$$\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}.$$

PROBLEM 1.3 (Harmonic oscillator). Let $M = T^*\mathbb{R}$ and

$$H(x, p) = \frac{p^2}{2m} + \frac{1}{2}kx^2.$$

This Hamiltonian function defines a problem of 1-dimensional harmonic oscillator. The equations of motion are

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}, \qquad \dot{p} = -\frac{\partial H}{\partial x} = -kx.$$

Therefore, we get the familiar equation for a harmonic oscillator

SOLUTION.

(1) Here is the proof.
(2) Here is a proof.

$\square$

If H has a symmetry under transformations in time, namely, H does not depend on t explicitly, then

A problem that H is explicitly independent on p is difficult to occur physically.

CHAPTER 2

Electromagnetism

CHAPTER 3

Relativity theory

CHAPTER 4

Statistical mechanics

CHAPTER 5

Fluid dynamics

1. Quantum mechanics

PROBLEM 1.1 (Hydrogen atom). Hydrogen hydrogen

2. Particle physics

Problem 2.1 (Yukawa potential). The wave equation for a massive field is given by the Klein-Gordon equation

$$(\Box + m^2)u(t,x) = 0,$$

where m is mass.

(1) Derive the Yukawa potential

$$u(x) = k\frac{e^{-\frac{r}{m}}}{r}$$

where $r = |x|$, as a Green function by assuming static case.

(2) Letting $m = 0$, discuss the relation with Coulomb potential.

(3) By taking Fourier transform.

(4) Find an approximate range of strong nuclear force and mass of pion.

Problem 2.2 (Negative energy solution and antiparticles). Dirac's interpretation. negative energy solution of the Klein-Gordon equation
time reversal?

Problem 2.3 (Polarization of photon field).

Problem 2.4 (Aharonov-Bohm effect).