Category Theory for Homological Algerba

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1. Additive category

There are three main concepts we need to catch:

(1) zero morphisms and zero object;
(2) biproduct;
(3) additive functors and enriched functors.

1.1. Zeros.

We get started from definitions.

Definition 1.1. A *zero object* is an object which is initial and terminal.

Definition 1.2. A category is said to have *zero morphisms* if every hom-set contains a specified morphism denoted by 0 that satisfies $0 \circ f = 0$ and $f \circ 0 = 0$ for all morphisms f.

In other words, we can say that the existence of zero morphisms is equivalent to $\mathbf{Set}_*$-enrichment.

1.2. Biproducts.

Simply saying, biproducts is something that is both product and coproduct. To define biproduct, we need zero morphisms, so the $\mathbf{Set}_*$-enrichment will be assumed.

Definition 1.3. aaa

1.3. Categories.

Here are definitions of categories in which we are interested.

Definition 1.4. A *pointed category* is a cateogry with a zero object.

Definition 1.5. A *semiadditive category* is a **CMon**-enriched category with a zero object.

Definition 1.6. A *additive category* is a **Ab**-enriched category with a zero object.

Note that we have of course several possible ways to give some equivalent definitions. For example, the following proposition is well-admitted as the definition of semiadditive category.

Theorem 1.1. *A* $\mathbf{Set}_*$*-enriched category is semiadditive if and only if it has all finite biproducts.*

A *preadditive category* is another name of **Ab**-enriched category.

$$
\begin{tikzcd}
\text{Additive category} \arrow[r] \arrow[d] & \text{Semiadditive category} \arrow[r] \arrow[d] & \text{Pointed category} \arrow[d] \\
\textbf{Ab}\text{-enriched category} \arrow[r] \arrow[u, dashed, bend left, "\text{zero object}"] & \textbf{CMon}\text{-enriched category} \arrow[r] \arrow[u, dashed, bend left, "\text{zero object}"] & \textbf{Set}_*\text{-enriched category} \arrow[u, dashed, bend left, "\text{zero object}"]
\end{tikzcd}
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