

**1.Solution:**

Let  $A$  be the  $|V| \times |V|$  adjacency matrix of  $G$ .

**Pseudocode:**

```

1: for i ← 1 to |V|
2:   for j ← 1 to |V|
3:      $G^2[i][j] \leftarrow 0$ 
4:     for t ← 1 to |V|
5:        $G^2[i][j] \leftarrow \max(G^2[i][j], a[i][t]*a[t][j])$ 
6: return  $G^2$ 

```

**Time Complexity:**

In this pseudocode, there are three loops. For the third loop, the operation only costs  $O(1)$  time. Thus the time complexity is  $O(|V|^3)$ .

**2. Solution:**

Discovery/finishing times from performing the DFS traversal of the graph are as follows:

Vertex	Discover time	Finish time
m	1	20
q	2	5
t	3	4
r	6	19
u	7	8
y	9	18
v	10	17
w	11	14
z	12	13
x	15	16
n	21	26
o	22	25
s	23	24
p	27	28

The sequence is obtained by reading off the entries in decreasing order of finish time:  $p(28)$ ,  $n(26)$ ,  $o(25)$ ,  $s(24)$ ,  $m(20)$ ,  $r(19)$ ,  $y(18)$ ,  $v(17)$ ,  $x(16)$ ,  $w(14)$ ,  $z(13)$ ,  $u(8)$ ,  $q(5)$ ,  $t(4)$ .

**3. Solution:**

Suppose that the form of the graph is the adjacency list form.

**Pseudocode:**

```

1: for each vertex  $u \in V$  do
2:    $\text{in-degree}[u] \leftarrow 0$  # take time  $O(|V|)$ 
3: for each vertex  $u \in V$  do
4:   for each  $v \in \text{Adj}[u]$ 
5:      $\text{in-degree}[v] \leftarrow \text{in-degree}[v] + 1$  # computes the in-degree for all vertices,
which take time  $O(|V|+|E|)$ 
6:  $Q \leftarrow \emptyset$  #  $Q$  is a queue

```

```

7: for each vertex  $u \in V$  do
8:   if in-degree[u] = 0 then
9:     Enqueue (Q,u) # collects a list with all vertices of in-degree 0, which take
        time  $O(|V|)$ 
# If the in-degrees of some vertex decreases to 0, then it's added to the list
10: while  $Q \neq \emptyset$  do
11:    $u \leftarrow$  Dequeue (Q)
12:   output u
13:   for each  $v \in \text{Adj}[u]$  do
14:     in-degree[v]  $\leftarrow$  in-degree[v] - 1
15:     if in-degree[v] = 0 then
16:       Enqueue (Q,v) # take time  $O(|V|+|E|)$ 
# The sorted result is given by the order in which they are extracted from the list.
Time Complexity:  $O(|V|+|E|)$ .

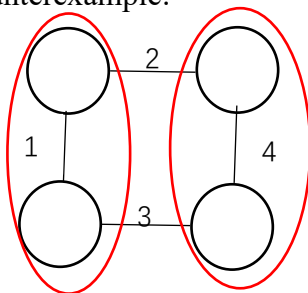
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**Discussion:** If the input graph is cyclic, then the list becomes empty before finishing all the vertices, and it's impossible for this algorithm to complete the sort.

#### 4. Solution:

The given algorithm will fail.

Counterexample:



According to the given algorithm, we can get a tree of total cost  $1+4+2=7$ . But obviously, the weight of the minimum-spanning-tree should be  $1+2+3=6$ . So the given algorithm fails to obtain a minimum spanning tree.