1. [20 pts] (4.3 in the textbook) Consider the use of cross-validation (CV) to optimize the broadness of a theoretical correlogram (e.g., the value of δ in eq(4.54) (page 245)) for ordinary kriging.
(a) Create an R object for optimizing δ in eq(4.54). The cross-validation is carried out by deleting data one by one, except those located at the two ends of the data region, and estimating the prediction error by comparing the estimated values with deleted data.
(b) Using the data shown in Problem 2.3 in Chapter 2, optimize δ by the use of the object created in (a). The data in Problem 2.3 is following: 9.6, 12.8, 14.6, 15.6, 15.5, 15.1, 15.6, 13.8, 13.9, 16.1, 17.3, 18, 19.9, 20, 19.9, 18.2, 15.8, 11.2, 9.6, 15.8, 16.7, 17.5, 13.7, 15.7, 20.6, 21.2, 16.7, 16, 20.7, 17.6.

2. [10 pts] (4.5 in the textbook) Construct an R object for calculating and charting the elements of a hat matrix for smoothing based on simple kriging (eq(4.59) (page 248) and eq(4.60)). Eq(4.65) (page 248) gives the definition of the hat matrix.

3. [10 pts] (4.7 in the textbook) Use universal kriging with one predictor to carry out tasks similar to those performed in Q1; the degree of the polynomial equation is also optimized. Here a rough trend is modeled by a polynomial equation of one predictor.

4. [40 pts] (4.9 in the textbook) Consider methods for obtaining the regression equation defined in eq(4.116)(page 263) without using an iterative procedure; solve a normal equation by a noniterative method.
(a) Derive a normal equation to obtain eq(4.116); use the matrix below as a design matrix.

$$\begin{pmatrix} 1 & X_{11} & X_{11}^2 & \cdots & X_{11}^p & X_{12} & X_{12}^2 & \cdots & X_{12}^q \\ 1 & X_{21} & X_{21}^2 & \cdots & X_{21}^p & X_{22} & X_{22}^2 & \cdots & X_{22}^q \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n1} & X_{n1}^2 & \cdots & X_{n1}^p & X_{n2} & X_{n2}^2 & \cdots & X_{n2}^q \end{pmatrix}$$

(b) Create an R object for computing regression coefficients using the normal equation obtained in (a). Moreover, produce an R object for calculating corresponding hat matrix.
(c) Construct an R object (using `poly()`) for deriving a regression equation in the form of eq(4.116) using the design matrix below.

$$\begin{pmatrix} r_0(X_{11}) & r_1(X_{11}) & r_2(X_{11}) & \cdots & r_p(X_{11}) & s_1(X_{12}) & \cdots & s_q(X_{12}) \\ r_0(X_{21}) & r_1(X_{21}) & r_2(X_{21}) & \cdots & r_p(X_{21}) & s_1(X_{22}) & \cdots & s_q(X_{22}) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r_0(X_{n1}) & r_1(X_{n1}) & r_2(X_{n1}) & \cdots & r_p(X_{n1}) & s_1(X_{n2}) & \cdots & s_q(X_{n2}) \end{pmatrix},$$

where $\{r_1(\cdot), r_2(\cdot), \cdots, r_p(\cdot)\}$ and $\{s_1(\cdot), s_2(\cdot), \cdots, s_q(\cdot)\}$ are orthonormal polynomials. Furthermore, $r_0(\cdot) \equiv \frac{1}{\sqrt{n}}$.
(d) Confirm that the hat matrix given by the object created in (b) is identical to that given by the object constructed in (c) using the simulated data.

5. [10 pts] (4.16 in the textbook) Modify the object `aceit1()` presented in (M) to be able to use the smoothing spline as a smoother for each variable. Provide an example with a plot using the constructed object.