



PERFORMANCE COMPARISON OF MVDR, MUSIC, AND ESPRIT ALGORITHMS IN SIGNAL CLASSIFICATION

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ABSTRACT

In this study, the performance of three well-known algorithms for classifying/localizing arriving signals at an array of antennas is presented. These three algorithms are (1) Minimum Variance Distortion less Response (MVDR), (2) Multiple Signal Classification (MUSIC), and (3) Estimation of Signal Parameter via Rotational Invariance (ESPRIT). Exhaustive simulations are done to evaluate and analyze these three localization algorithms where the array SNR, separation and relative power of sources, and number of samples varied to determine how each algorithm performs. Simulation results show that MVDR relies on the received signal's relative power, while both MUSIC and ESPRIT can still detect signals that are closely spaced.

Keywords: MVDR, MUSIC, ESPRIT.

1. INTRODUCTION

Signal classification has a variety of temporal and spatial applications. In temporal signal processing, these applications encompass speaker recognition [1] [2] and source separation [3] to name a few. In spatial signal processing, the estimation of the direction of arrival (DOA) of incoming signals is significant. The effectiveness of DOA estimation algorithms is evaluated in the presence of multiple signals and multiple smart antennas or antenna arrays. Accurate signal localization has applications related to natural catastrophes (earthquakes), radar and sonar [4].

Multiple Signal Classification [5] estimates the signal by exploiting the fact that the noise subspace is orthogonal to the signal subspace while Minimum Variance Technique relies on the covariance matrix of the received signal. The following expressions (1) and (2) are used in estimating the incident signal parameters especially the Angle of Arrival or Direction of Arrival (AOA or DOA).

$$P_{MUSIC}(\theta) = \frac{1}{a(\theta) E_N E_N^* a(\theta)} \quad (1)$$

$$P_{MVDR}(\theta) = \frac{1}{a^H(\theta) R^{-1} a(\theta)} \quad (2)$$

where E_N is the noise eigenvector, $a(\theta)$ is the mode vector and R is the covariance matrix of the received signal. ESPRIT [6], on the other hand, exploits the sensor array invariance. The key expression in estimating the DOA is given in (3).

$$(\Gamma_1 U_s) \Phi = \Gamma_2 U_s \quad (3)$$

where

$$\Gamma_1 = [I_{M-1 \times M-1} \mid 0_{M-1 \times 1}]$$

$$\Gamma_2 = [0_{M-1 \times 1} \mid I_{M-1 \times M-1}]$$

The eigenvalues of Φ contains the information on the DOA of the incident signals and is solved by (4).

$$\Phi = [(\Gamma_1 U_s)^H \Gamma_2 U_s]^{-1} (\Gamma_1 U_s)^H \Gamma_2 U_s \quad (4)$$

(1), (2), and (4) are the core equations implemented in evaluating MVDR [7], MUSIC, and ESPRIT.

In [8], the DOA estimation was done given a reactance-based uniform circular arrays based on the MUSIC algorithm. Their results showed that the reactance of the active ports can estimate the signal's DOA and had a good performance in terms of error and resolution estimation. Another work in [9] employed MUSIC for constructing the noise subspace given a set of sensor outputs that are randomly chosen from its sensor arrays. They exploited the Nystrom method then developed a low-complexity MUSIC algorithm to utilized in far-field source localization. In [10], the MVDR-LASSO was proposed. It combined the benefits of the virtual array concept and the compressive sensing technique. The MVDR algorithm was used as noise and reverberation suppressors in [11], while in [12], MVDR was considered for robust automatic speech recognition application.

A low-complexity joint discrete Fourier transform and ESPRIT was proposed in [13] for time-of-arrival (TOA) and DOA estimation used in vehicle frequency-modulated continuous wave radars. Finally, in [14], the unitary ESPRIT was adopted to obtain DOA estimations from transmit and receive arrays. Unique



DOA is achieved by finding the results that coincide from these arrays based on co-primeness.

In this work, we present exhaustive simulation results that will evaluate the performance of the three stated source localization algorithms. By considering and varying all possible parameters that may affect how a source signal can be classified, this research provides how one estimation algorithm is better when compared to the rest.

This paper is organized as follows: Section II defines the problem and source signal model. Section III presents and evaluates the simulation results derived from using the three algorithms. Section IV summarizes and concludes this study.

2. PROBLEM DEFINITION AND VARIABLE SET-UP

The source signal data model used is shown in (5).

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} a_1(\theta) & a_2(\theta) & \cdots & a_N(\theta) \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_D \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix} \quad (5)$$

or $X = AF + W$, where X is the received signal, A is the array manifold with each column being equal to (6). F is the incident signal array and W is the noise vector.

$$a_i(\theta) = \begin{bmatrix} 1 & e^{j\frac{2\pi}{\lambda}d\cos\theta} & \cdots & e^{j(N-1)\frac{2\pi}{\lambda}d\cos\theta} \end{bmatrix}^T \quad (6)$$

We define two uncorrelated complex signals used as incident signals, F . A signal Signal1 is arriving at an angle 110deg and has a signal power of 50 W, while a signal Signal2 is coming from 135deg with a 60-W signal power. Additive noise is also complex and has an SNR of 0 dB relative to the weakest signal.

The received signal X , generated by (5) and (6), will be processed to determine the DOA using the algorithms MVDR, MUSIC, and ESPRIT.

For the three algorithms, a uniform and linear array of five antennas will be used, except for ESPRIT which is assumed to have doublets. This antenna array has inter-element separation of $\lambda/2$, i.e., each mode vector is expressed as:

$$a_i(\theta) = \begin{bmatrix} 1 & e^{j\pi\cos\theta} & \cdots & e^{j(N-1)\pi\cos\theta} \end{bmatrix}^T \quad (7)$$

3. SIMULATION RESULTS AND ANALYSIS

Simulation is carried out using the generated received signal and the autocorrelation matrix which can be easily derived after. Exhaustive simulations have been run for

performance evaluation of the three algorithms. These simulation scenarios are:

- Effect of the number of samples in estimation
- Effect of the relative power of the incident signals
- Value of array SNR
- Effect of closely spaced signals
- Effect of the correlation level of the incident signals

3.1 Effect of Number of Samples in Estimation

In this section, we investigate the effect of the number of samples processed in the estimation of the DOA. We vary the number of samples from 8000, 1000, and 100 samples. Array SNR is zero and the incident signal powers are 50W and 60W. Figure-1 shows the changes in the estimate of the signal DOA for MUSIC and MVDR, respectively. For ESPRIT, the effect of the number of samples in the estimation is shown in Table-1.

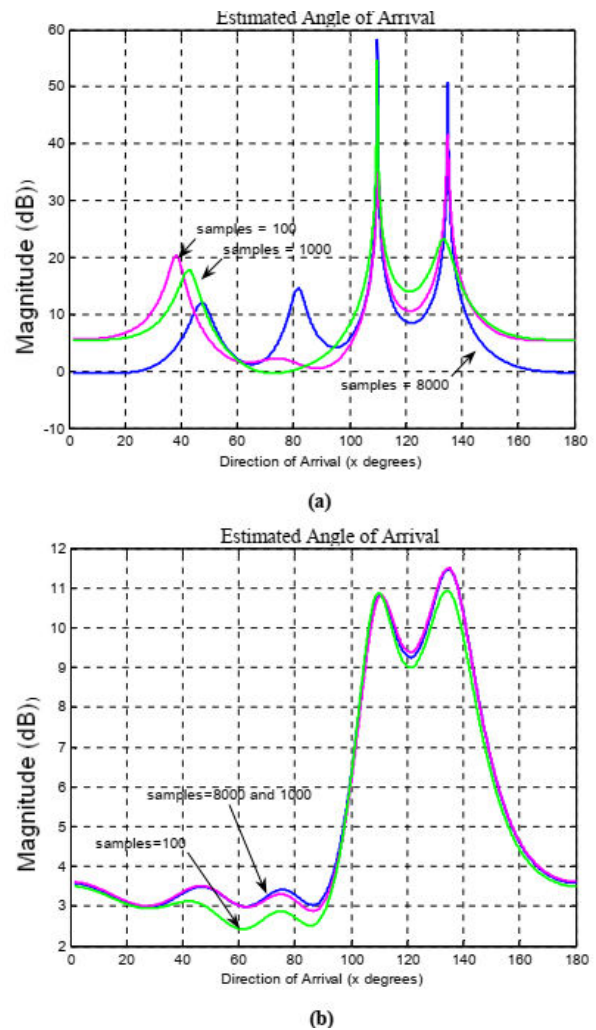


Figure-1. (a) MUSIC and (b) MVDR estimated DOA for received signals coming from 135° and 110°.

Figure-1(a) shows that as the number of samples is increased, peaks at the estimated DOA becomes higher signifying that as information is added in the estimation,



the confidence on the estimate becomes higher (confidence in this paper is associated with magnitude of the estimate). It can also be noticed that at lower number of samples, 'uncertainties' in the estimates, which is manifested by the lower peaks, are higher. It can be observed that the MUSIC estimate is dependent on the number of samples. More samples make the estimate more certain. Figure-1(b) shows the DOA estimation using the MVDR algorithm. In this plot, 8000 samples and 1000 samples have no significant effect on the estimation of the DOA. Though at 100 samples, the estimate varies a little. This small change in amplitude in Signal 2 estimate is small compared to the big change in the number of samples from 8000 to 100. From the plot, it can be said that changing the number of samples will not significantly affect the estimation performance of the MVDR. Table-1 gives the estimated DOA using ESPRIT with different number of samples. From the estimate, certainty cannot be evaluated since the algorithm gives no estimate of other angles unlike with the MUSIC and MVDR where it weeps from 0 to 360 degrees and the decision where the DOA is suggested by its peaks.

Table-1. ESPRIT estimated DOA for different number of samples. Signals are coming from 135^0 and 110^0 .

Samples	Via ESPRIT Technique	
8000	134.949	109.910
1000	134.990	109.979
100	134.885	108.979

The effect of the number of processed samples is seen in the 'certainty' of the estimate. MUSIC shows dependence on the number of samples while MVDR and ESPRIT show otherwise. MUSIC behavior in dependence on number of samples can be explained by the noise eigenvalues, λ_{\min} , spread with the number of samples. As the number of samples is increased, the spread of this eigenvalues decreases [2], therefore, noise variance is larger at lower number of samples, and certainty becomes lower. In contrary to this, MVDR estimation relies only in the covariance matrix, which gives values normalized by the number of samples, i.e., $E[rx]$. Being normalized by the number of samples, the effect of the said variable will not significantly affect the estimation of DOA. ESPRIT, likewise, shows no significant effect on the variation of the number of samples since the algorithm spits out estimated DOA's without giving any information on the confidence of estimate with respect to the other angles.

3.2 Effect of Number of Samples in Estimation

In this section, we vary the relative power of the incident signals and observe its effect(s). Initially, the power of the *Signal 2* is 60W while the power of *Signal 1* is 50W. Keeping the other variables and the power of the weaker signal constant, (number of samples = 8000),

power level of *Signal 2* is changed from 60W to 100W and to 500W, that is, the relative power ratio.

Case1: Signal 2: Signal 1 = 6:5

Case2: Signal 2: Signal 1 = 2:1

Case3: Signal 2: Signal 1 = 10:1

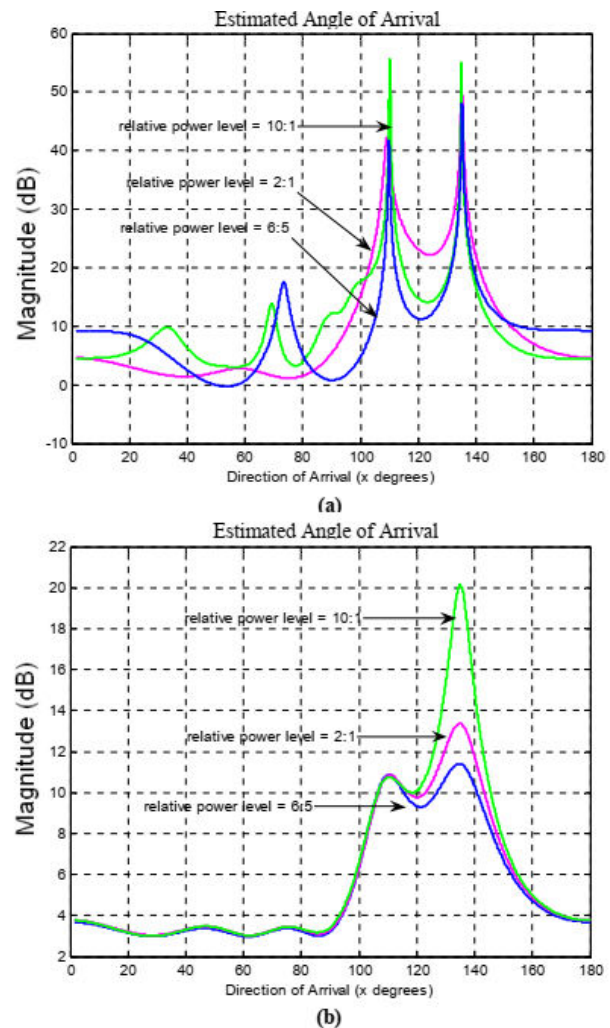


Figure-2. (a) MUSIC and (b) MVDR estimated DOA while relative power of signal from 135^0 and 110^0 is varied.

In Figure-2(a), it can be seen that large difference in relative power of the two incident signals gives no clear relation with regard to the 'pseudospectrum' of the DOA. For example, the relative power 10:1, MUSIC gives almost the same height (~55dB) for DOA of *Signal1* and *Signal 2* estimates whereas the relative power of 6:5 gives difference of around 6dB in the signals' DOA estimate. In the previous observation, the magnitude in the 'DOA pseudo spectrum' was associated with the confidence of the estimate, i.e., in MUSIC, a power of some "signal 2" that is 10 times the power of some "signal 1" will not necessary yield a more confident estimate of "signal 2" as compared to the estimate of "signal 1".



Figure-2(b) shows the MVDR DOA estimate which, apparently, correlates the relative power of the signals to its estimation. In relative power level 6:5, the plot shows that *Signal 2* is quite higher in magnitude. *Signal 2*, when power is increased, such that its power is twice the power of *Signal 1*, also exhibits increased in magnitude in the estimate. When further increased, such that the power level ratio is now 10:1, estimate magnitude is also increased. The *Signal 1*, having no change in signal power, shows no change in the amplitude of its estimate. That is, for MVDR, the relative power levels of signal sources manifest in the algorithm's estimates. On the other hand, the relative power of the signals is not apparent in the ESPRIT (as shown in Table-2) estimate since the estimator only outputs the estimated DOA of the signal. If the weaker signal can be estimated successfully by ESPRIT, then estimation of the stronger signal should not be a problem.

Table-2. Estimated DOA via ESPRIT when relative signal power of source from 135 and from 110 is varied.

Relative Power Level	Via ESPRIT Technique	
60:50	134.979	109.877
100:50	134.943	110.189
500:50	135.092	109.911

As the relative power level is varied, MUSIC estimate shows no direct relation with the relative power of the signals unlike the MVDR which exhibits high correlation with its estimates and the relative signal power. This high correlation with the relative power of the signal and the MVDR estimate comes from the fact that the MVDR estimation depends on the covariance matrix R , which also contains the received signal power levels. Unlike MVDR, MUSIC depends only on the noise eigenvalues in estimating the signal parameters.

3.3 Effect of the Array SNR

The effect of the array SNR with respect to the weakest signal is observed next. In this simulation, the signal power is returned to 50W (for *Signal 1*) and 60W (for *Signal 2*). The number of samples is still 8000 while keeping the other variables unchanged. Array SNR is the variable here and will be changed with respect to the weakest signal, i.e. *Signal 1*. SNRs to be used are 0dB, -10dB and 10dB. Figure-3(a) shows the performance of MUSIC algorithm to array SNR. Major assumption is made in this part that affects greatly the desirable performance of the algorithm, i.e., the a-priori knowledge of the number of incident signals. MUSIC, after Eigen decomposition, must decide how many signals are present by looking on the peaks that popped out of the noise eigenvalues. If the signal is deeply buried in noise, the decision block of the algorithm might not notice the signal because its level is just above the noise floor. With a-priori

knowledge of the number of signals, this buried signal will still be considered as belonging to the signal subspace; thus, the noise subspace will still be an accurate noise subspace. From Figure-3(a), it is shown that MUSIC can still estimate the DOA of the incident signals even if it is buried in noise if the decision block correctly identifies the number of signals present. Misjudging on the number of signals present will greatly affect the estimation of the signals since the noise subspace will contain the signal eigenvector.

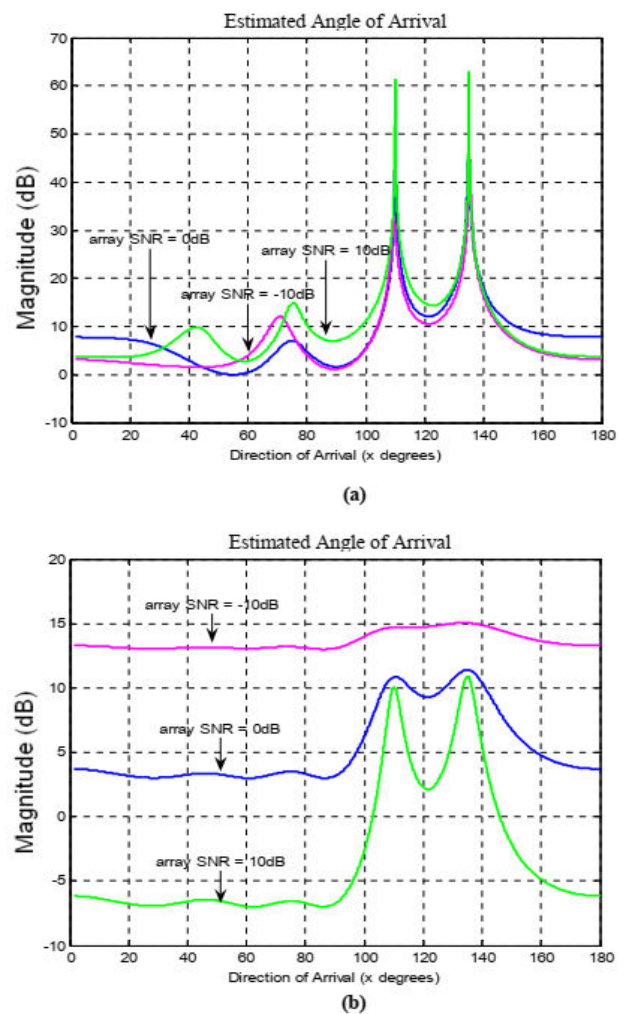


Figure-3. (a) MUSIC and (b) MVDR estimated DOA with array SNR is changed with respect to the weakest signal.

MVDR performance with respect to the array SNR is shown in Figure-3(b). MVDR, from previous sections, shows direct relation with the power of the received signal. Being dependent on the autocorrelation of the received signal, it is also expected that MVDR will change as SNR is changed. In Figure-3(b), at SNR = 10dB, the MVDR successfully estimated the signal DOA (1100 and 1350). At SNR = 0dB, the estimate becomes hazy, sharp edges for the peaks are smoothen and the dent becomes shallower, noise level here is 10 times stronger than the previous case. When SNR = -10dB, the estimate



fails because peaks are gone, and the levels are the same from 1000 to 1400. At this state, *Signal 1* is buried in noise as well as *Signal 2* because they both have comparable powers.

Table-3 shows the signals DOA estimate via ESPRIT. Since ESPRIT exploits the same data model as MUSIC, it is expected that it will not have much problem in estimating the signal parameters provided that the number of incident signals is known.

Table-3. ESPRIT estimate with array SNR varied with respect to the weakest signal.

Array SNR	Via ESPRIT Technique	
0	135.101	110.058
10	134.869	109.780
-10	135.000	109.989

3.4 Effect of the Angle of Separation of the Sources

In this part of the experiment, signal separation is gradually decreased until the separation is less than the beamwidth of the antenna array. In this simulation, all variables are fixed (number of samples = 8000, SNR = 0dB, relative power = 6:5) including signal 1's DOA is fixed to 110° . Signal 2 AOA is varied and set to $110 + 5$, $+3$ and $+1$ degrees.

Figure-4(a) shows the MUSIC performance when the signal sources are closely spaced together. In the plot, MUSIC fails to differentiate *Signal 1* and *Signal 2* when these are separated by 3 degrees or less. However, at around 5 degrees, MUSIC starts to distinguish the two signals from the other.

In Figure-4(b), same characteristics as the previous algorithm are exhibited by MVDR in closely spaced incident signals. However, MVDR in this aspect is inferior as compared with MUSIC because existence of two signals is not noticed by MVDR at 5 deg. ESPRIT, on the other hand, gives no issue as to how many peaks are obtained and where these peaks are located since there are always two parameters obtained every time, provided that the number of signals is known. The accuracy of the ESPRIT estimates however is important. In Table-4, though ESPRIT gave two estimates, the accuracy of the estimate is quite far from the actual DOA.

Table-4. ESPRIT estimated DOA when signal sources are closely spaced.

Spectral Separation	Via ESPRIT Technique			
	Estimate	Actual	Estimate	Actual
1 deg	110.491	111	108.810	110
3 deg	114.828	112	110.889	110
5 deg	14.571	115	109.299	110

MUSIC and ESPRIT suggest that both algorithms are useful in estimating the DOA of two closely spaced signals. However, MUSIC can only distinguish signals up to 5 degrees from this simulation but have poor accuracy while ESPRIT incurred error of up to 3 degrees. More iteration should be made for more reliable statistics. Meanwhile, MVDR unlike MUSIC and ESPRIT, as shown in Figure-4(b), suggests that the algorithm is not effective for closely spaced signals.

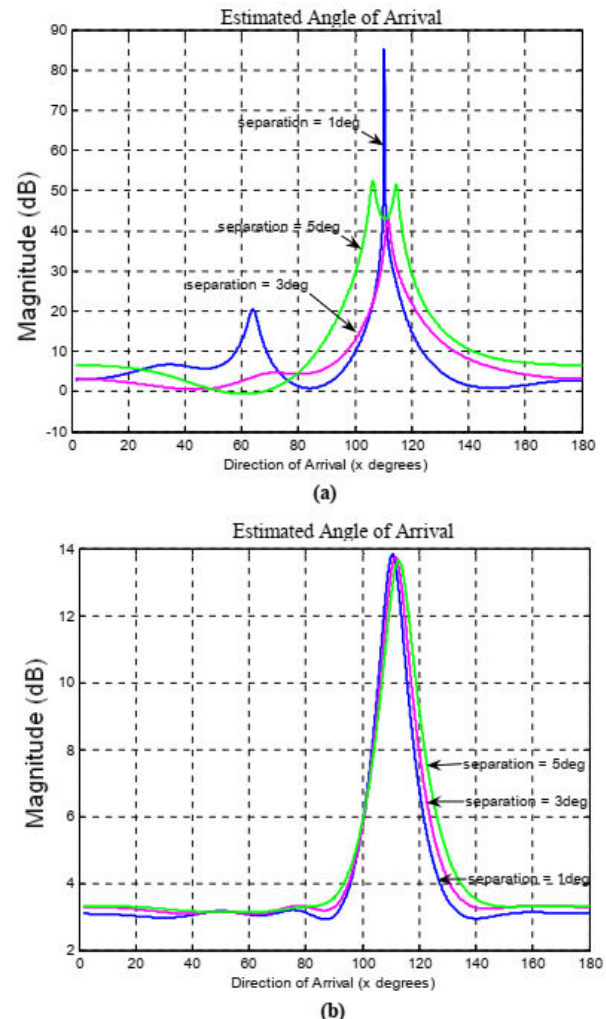


Figure-4. (a) MUSIC and (b) MVDR estimated DOA when the incident signals are closely spaced.

3.5 Effect of the Correlation of Incident Signals

In this part of simulation, *Signal 1* (DOA = 110°) and *Signal 2* (DOA = 135°) are generated with some correlation level. Equations (8) and (9) are used to generate correlated signals 1 and 2:

$$\rho_{S1,S2} = \frac{\text{Cov}(S1, S2)}{\sigma_{S1}\sigma_{S2}} \quad (8)$$



$$S2 = S1 + \sqrt{N_0} \text{noise} \quad (9)$$

where $\rho_{S1, S2}$ is the correlation level, $Cov()$ is the covariance, σ is the standard deviation of the signal, N_0 is the noise power, and 'noise' is white noise. *Signal2* (*s2*) is generated using the second equation. Correlation levels are set to 10%, 50% and 90%.

The variables are set to their initial condition (shown again below for convenience) to form the base condition of the signals.

Source 1: DOA: 110deg

Signal Power: 50 W

Source 2: DOA: 135deg

Signal Power: 60 W

SNR: 0dB wrt the signal with lowest power

Number of samples, SNR wrt the weakest signal, and spacing of the signal sources are the variables in addition to the correlation level.

For result of varying the number of samples, Figures 5(a-c) illustrate the MUSIC estimate of the DOA with the correlation levels 10%, 50% and 90%. Figures 5(a-c) show that the number of samples affects the estimation (from previous results). The effect of correlation levels for 8000 and 1000 are not seen in the estimate of MUSIC while its effect is very apparent when the signal samples is 100. This result can be explained that signals are generated by function (randn) that has variance

equal to 1. Statistics about the produced vector of the random generator depends on the length of the vector. That is, the longer the vector the closer the statistics to its desired values. For signals with 100 samples, correlation coefficient may have been greater than of the desired level (i.e., 90%) such that ambiguities occur. Ambiguities for highly correlated signals are experienced by the algorithm MUSIC since it relies on the signal and noise eigenvalues of the received signals. If the incident signals are highly correlated (say 90% up) Eigen decomposition will fail because their eigenvalues will not be orthogonal (uncorrelated). Eigen values in the decomposition should be orthogonal.

For MVDR, correlation level does not generally affect the performance of the algorithm mainly because it does not depend on identification of the signal and noise subspaces. Determination of signal and noise subspaces requires orthogonality of the signals to determine noise subspace. Therefore, MVDR performances in estimating the DOA will still be the same even if the signals are highly correlated.

In Table-5 for ESPRIT estimates, 8000 and 1000 input samples processed by ESPRIT show good estimation of the DOA with maximum error of around 0.2 degrees even if the signals are 50% and 90% correlated. However, at 100samples 50% correlation and 90% correlation, error rises to 0.96 degrees and 1.8 degrees respectively. High dependency on signal orthogonality of the algorithm, like MUSIC's, results in such behavior.

Table-5. ESPRIT estimated DOA for correlated signals with varying correlation levels and number of samples.

Samples	Via ESPRIT Technique 10% Correlation		Via ESPRIT Technique 50% Correlation		Via ESPRIT Technique 90% Correlation	
8000	135.1648	109.9428	134.9919	109.9804	134.8854	110.1462
1000	134.9929	109.9024	135.1125	110.9804	135.3884	109.8866
100	135.0428	109.9882	135.3794	110.9610	135.4403	108.8251

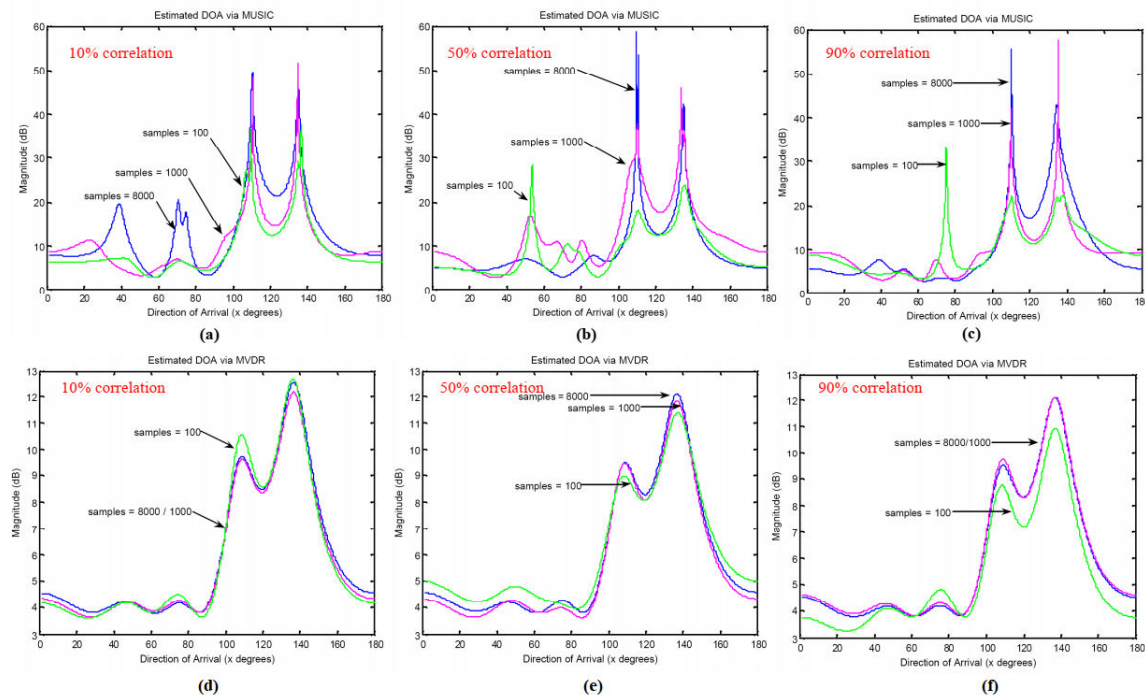


Figure-5. (a-c) Estimated DOA of MUSIC and (d-f) MVDR for correlated signals with varying correlation levels and number of samples.

Figure-6 presents the results of varying SNR while the signals are also correlated. Figures 6(a-c) belong to MUSIC algorithm with 10%, 50%, and 90% correlation levels.

Like the previous results, varying the SNR results in possible peaks that may pop out of the noise floor. A-prior knowledge of the number of incident signals is necessary for better estimation of the DOA especially if the SNR is too low. At 10% correlation, the algorithm's estimate still yields well except for SNR = -10dB which gives around 108 degrees for DOA of 110. At 50% correlation, ambiguity starts to rise since the signal subspace was not successfully described by the signal eigenvectors. Estimate though is successful since 110 degrees and 135 degrees are obvious in the plots.

MVDR, in Figures 6(d-f) consistently show independence on correlation of signals as observed in its simulation results. In fact, Figures 6(d-f) is comparable to Figure-3(b) where the signals are uncorrelated.

ESPRIT estimates (Table-6), moreover, gives no large effect in changing the correlation levels, of course

with the addition of prior knowledge on the number of signals. However, at 50% correlation, signals estimate error increases from 0.2 maximum error to 1.5 degrees maximum error.

When signals are closely spaced as in Figure-7(a-c), MUSIC produces peaks due to the ambiguity in the signal eigenvectors. If signals are closely spaced, the tendency of the MUSIC is to decide two peaks (for two incident signals) to establish the noise subspace. If the signals are correlated, the second eigenvector that MUSIC might classified as signal eigenvector belongs to the noise subspaces, i.e. that noise will have its pick at the 'pseudo spectrum' of the DOA. Thus, Figures 7(a-c) with 5 degree separation.

MVDR, again, is consistent of its independence from the correlation of the signals as it gives no difference in its performance when the signal is uncorrelated, shown in Figures 7(d-f) and Figure-3(b). Erratic estimate occurs in ESPRIT (Table-7) as the algorithms data model uses the data model of the MUSIC. 2nd peak might be mistakenly decided by the algorithm as the DOA of the second signal.

Table-6. ESPRIT estimate with array SNR varied with respect to the weakest signal.

Array SNR	Via ESPRIT Technique, Signals 10% Correlated		Via ESPRIT Technique, Signals 50% Correlated		Via ESPRIT Technique, Signals 90% Correlated	
1	99.6482	110.6193	56.9323	110.6347	53.9429	110.5921
3	105.7015	112.1801	114.6089	111.5237	103.9715	111.8602
5	108.7909	114.8711	117.1551	112.0689	116.0627	111.2545



In summary, correlation of the signal affects the performance of the eigenvector-based algorithms which depends highly on the orthogonality of the signals and the noise subspaces. If the signal subspace is not completely defined, estimation will be affected.

4. CONCLUSIONS

In this work, the MVDR, MUSIC and ESPRIT DOA estimation algorithms have been evaluated. MVDR shows dependency on the relative power of the received signals while MUSIC and ESPRIT show a little to none at all. MUSIC, on the other hand, shows dependency on the number of processed signals. Longer sample length is desirable as compared to shorter samples of the received signal. Both MUSIC and ESPRIT give desirable response in detecting closely spaced signals in space in contrast to MVDR. Furthermore, correlation of the incident signals affects the performance of the eigenvector-based estimators, i.e., MUSIC and ESPRIT. The negative effect is apparent especially if the signals to be processed are short, have low SNR or closely spaced.

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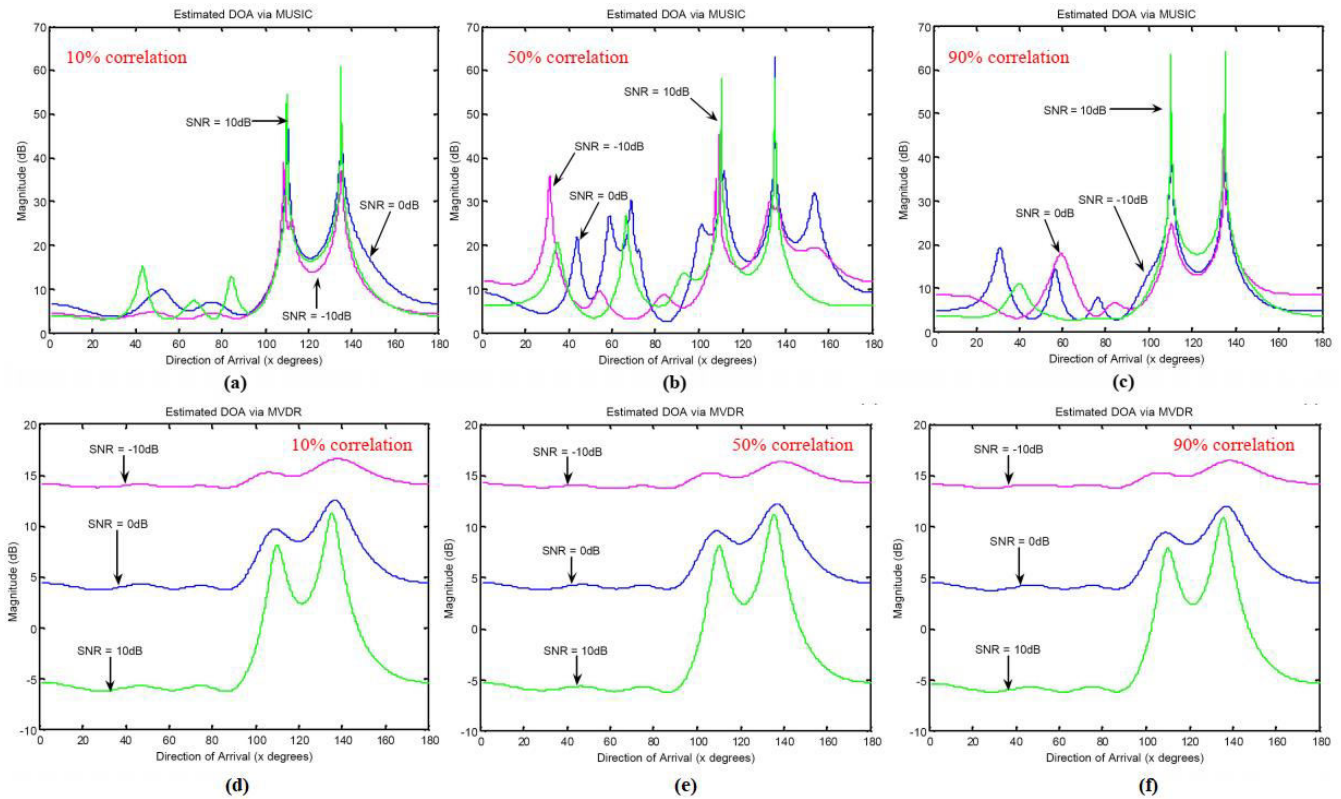


Figure-6. (a-c) Estimated DOA of MUSIC and (d-f) MVDR for correlated signals with varying correlation levels and SNR.

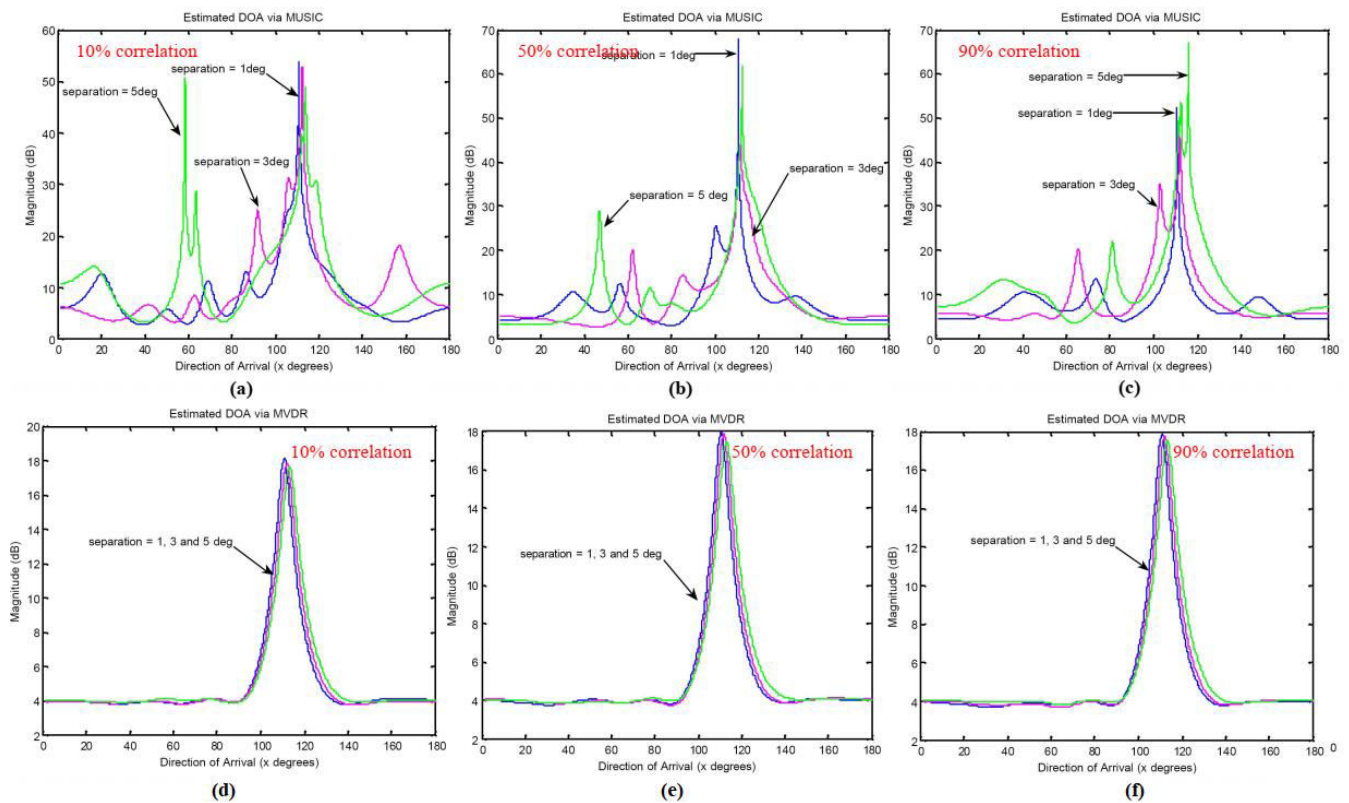


Figure-7. (a-c) Estimated DOA of MUSIC and (d-f) Estimated DOA of MVDR for correlated signals with varying correlation levels and source signal separation.