“Level Up”: Leveraging Skill and Engagement to Maximize Player Retention in Online Video Games

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In this study, we propose a novel two-stage data analytic modeling approach, combining statistical methodologies with optimization techniques to model player engagement as a function of game-play motivation to maximize customer retention via matching in the large and growing online video game industry. In the first stage, we build a Hidden Markov Model (HMM) to capture the evolution of gamers’ latent engagement state and the state-dependent participation behavior. We then calibrate the HMM using a longitudinal dataset obtained from a major international video gaming company, which contains detailed information on 1,309 randomly sampled gamers’ playing history over a period of 29 months playing more than 700,000 unique game rounds. We find high-, medium- and low-engagement-state gamers respond differentially to motivations such as achievement and challenge. In the second stage, we use the results from the first stage to develop a matching algorithm that learns the gamer’s current engagement state and exploits that learning to match the gamer to a round to maximize retention. Our (real-time) algorithm increases gamer retention by 4-8% conservatively, leading to economically significant revenue gains for the company.

Key words: Online Video Games, Gamer Behavior, Customer Engagement, Hidden Markov Models, Optimization

1. Introduction

The video game industry is a large and important part of the overall entertainment industry, with 2016 global revenues at $101.1 billion (Newzoo 2017). In this industry, the fastest growth in revenues is from online games, projected to reach $35 billion by 2017, up from $19 billion in 2011 (Gaudiosi 2012). Retaining players and keeping them engaged is a key challenge faced by the video game companies, because player engagement and retention is strongly correlated with a video game’s financial success (GameAnalytics 2015). Apart from the content and mechanism of the video game itself, an important factor that plays a significant role in influencing player engagement and retention, especially in multiplayer video games, is how players are assigned to available game rounds. The assignment algorithm determines whom a player plays with (or against) in each round, which directly influences her game-play experience/outcome in the current round, and indirectly affects her future participation. Traditional matching algorithms used in most online video games do not utilize the detailed gamer playing history from multiplayer games – they either match players randomly, or use simple assignment rules such as matching players with similar skill levels into the same round. In a data rich environment such as online video games, existing parsimonious assignment rules are likely to be suboptimal. Video game companies start to realize the room for improvement; however, there has not been systematic research on video game player behavior that could guide improvements of player matching.

In this study, we propose a novel data analytic strategy that uses motivation-based correlates of an unobservable, *player engagement*, and evaluate its impact on an observable, *player retention*. In particular, we first identify factors in players' past game-play experiences that affect their engagement state and its evolution over time as well as future participation. Based on the impact of these factors, we propose a new matching algorithm that maximizes player retention over a specified time period. We do this using a two-stage modeling approach that combines statistical methodology with optimization techniques. In the first stage, we build a Hidden Markov Model, or HMM (Netzer et al. 2008, Singh et al. 2011, Sahoo et al. 2012, Yan and Tan 2014, Abhishek et al. 2015, Ayabakan et al. 2016, Chen et al. 2017, etc.) to capture the transition of players' latent engagement state and the state-dependent participation behavior. In the second stage, we use the estimates from the first stage to develop and calibrate a dynamic matching algorithm that incorporates non-stationary behavior of the player in a finite-horizon non-discounted setting.

We apply the HMM to a longitudinal dataset made available to us by a major international video gaming company for a first-person shooter game.[1] The dataset contains detailed information on the playing history of 1,309 randomly sampled gamers over a period of 29 months and spans more than 700,000 unique game rounds. The first-stage estimation results suggest that players can be classified into high, medium and low engagement states with different motivations impacting each state. For example, *achievement* (measured via total score) has a positive effect on players in a low or a high engagement state but not on players in a medium engagement state. *Challenge* (measured by within-period score variation) positively affects the engagement level of players who are currently in either a low or a medium engagement state, but negatively affects the propensity of players in a high engagement state to stay highly engaged. Finally, *curiosity* (or the lack thereof, proxied by the time

[1] "First-person Shooter" is a video game genre centered on gun and projectile weapon-based combat through a first-person perspective i.e., the player experiences the action through the eyes of the protagonist. Early representatives are *Doom* and *Half-Life*; the largest current franchise is *Call of Duty.* First person shooter games represent the largest genre of video games in terms of units sold in 2015 (see http://www.theesa.com/wp-content/uploads/2016/04/Essential-Facts-2016.pdf).

since sign-up) decreases the engagement level over time with the rate of decrease being higher for less engaged players. The second stage uses the estimated HMM parameters to generate predictions vis–à–vis gamer behavior and then matches each gamer to a game round that is expected to yield the most collectively desirable outcome. Specifically, we develop a novel matching algorithm that first "learns" the current engagement state of a player using her past game-play history, and, based on this information, assigns the player to a new round to maximize future engagement. We compare our approach with approaches commonly used in the industry (matching players randomly and matching players based on skill level) on customer retention (measured as the frequency of playing and the average number of rounds played). We show that our proposed matching algorithm provides real-time matches and improves retention (conservatively) between 4.04% and 7.79%. We also show that this improvement translates to economically significant outcomes for the firm.

In summary, our paper makes three main contributions. First, it is one of the first studies that uses real-world data to examine the behavioral dynamics of customers (gamers) in the fast-growing online video game market, where engagement and retention play essential roles in the success of a product (i.e., an online video game). Specifically, we build an HMM to capture the drivers of customer engagement and retention and their impact on relevant firm outcomes. Second, from a methodological point of view, our two-stage data analytic modeling approach combines statistical methodology (to understand gamer behavior) and dynamic optimization (to derive firm policy). In doing so, we demonstrate how statistical modeling and optimization techniques can be integrated seamlessly to generate business insights and assist decision making. Finally, our proposed algorithm provides real-time player matching resulting in economically significant improvements for the firm. Our algorithm is general enough to be used in experiential settings where engagement states vary across customers and evolve in a dynamic manner.

The rest of the paper is organized as follows. In §2, we review the relevant literature. §3 describes the institutional setting, and provides an overview of the data and along with some "model free" data patterns. We discuss the HMM in §4 and the associated results in §5. §6 describes the proposed matching algorithm and its performance. We conclude in §7.

2. Literature Review

Our paper is related to four streams of literature. First, it is related to the emerging literature on the video game industry. There have been several studies on the structure of the video game markets (Shankar and Bayus 2003, Nair 2007, Liu 2010, Dube et al. 2010, Zhu 2012, Anderson Jr. 2014, Zhou 2016), in which the market for video games is typically characterized by an oligopolistic structure with both direct and indirect network effects. A number of papers examine other important aspects of video games, including the business models for video games (Roquilly

2011), pre-ordering strategy (Chu and Zhang 2011), "freemium" pricing strategy (Wu et al. 2013, Niculescu and Wu 2014), word-of-mouth effects (Zhu 2010, Hao et al. 2011, Dou et al. 2013) and piracy issues (Lahiri and Dey 2013). Most of the existing studies focus on customers' purchase decision in video game markets; to our best knowledge, with the exception of Kwon et al. (2016), which studies the consumption pattern of mobile social apps (social games are among one of the categories studied), there has been little research that utilizes individual-level behavioral data to examine the engagement evolution and retention of video game players.

Second, since video games are hedonic products (Voss et al. 2003), consumers' consumption behavior in this market can be quite different from those in non-hedonic product markets, which have been studied extensively. Researchers from media psychology, communications, and computer science have investigated motivations for playing computer games (including online and offline games), mostly using experimental and qualitative approaches. Motives for playing games provide an alternative perspective on engagement which involve appraisals of feelings experienced while playing games (Boyle et al. 2012). Malone (1981) applies intrinsic motivation concepts to computer games and identifies three basic motivational categories: fantasy, challenge, and curiosity. Later studies find that other psychological constructs, such as in-game autonomy and competence (Ryan et al. 2006) and self-efficacy (Klimmt and Hartmann 2006), are also important motivating factors for video game consumption. It has also been shown in the literature that rewards (such as experience points, levels, and wealth) help motivate players, create loyalty, and signal social status (Salen and Zimmerman 2004, Bartle 2004, Amabile and Kramer 2011, Deterding 2012, Hofacker et al. 2015). In this study, we do not aim to directly alter any of the design elements of a video game (e.g., rewards, incentive structures, and game levels). Instead, we treat the design of the game as given, and seek to improve the matching algorithm that affects players' game-play experience and the realization of the rewards they receive, with the goal of maximizing player retention. The empirical model we construct is informed by the literature on the player motivation and engagement. This enables us to use real-world player data to empirically examine how different motivational factors affect players' engagement and retention, and then to apply our empirical findings towards improving the player matching algorithm.

Methodologically, our research adds to the growing literature that uses HMM to analyze business and management related questions. Netzer et al. (2008) develop an HMM to examine the dynamics of the customer relationships and the effect of customer-firm interactions on the evolution of customer relationships and consumer behavior; Singh et al. (2011) use an HMM to study developers' learning dynamics in open source software projects; Sahoo et al. (2012) propose a performance-improving HMM-based collaborative filtering algorithm, which allows tracking unobserved user preference changes; Abhishek et al. (2015) develop an HMM of an individual consumer's behavior based on the concept of a conversion funnel to address the problem of attribution in multi-channel advertising; Yan and Tan (2014) and Ayabakan et al. (2016) use the HMM framwork to capture the evolution of patient latent health condition in their respective papers, with the former focusing on the effect of social support offered by online healthcare communities and the latter focusing on the impact of telehealth on patient health; Chen et al. (2017) examine the dynamics of user voluntary contributions in online communities using an HMM with latent motivation states under the public goods framework. In this paper, we develop an HMM of video game players' behavioral dynamics. We believe that the HMM framework is well suited for our context, because one of the goals of this research is to capture an important but hard to measure latent construct – *engagement*, and evaluate its impact on an observable outcome variable – *customer retention*. We incorporate covariates into the transition matrices of the HMM, which allows a rich description of the evolution of a player's game-play behavior over time. The HMM we propose also provides interpretability: it is a quantitative model that corresponds to theories of video-game design and is based on findings from behavioral and qualitative research on gamer behavior.

The optimization part of this paper is also related to a large body of literature on dynamic optimization with partially observable states, especially that of Partially Observable Markov Decision Process (POMDP) (see Krishnamurty (2016) for an overview). However, existing approaches to solve POMDP are not easily applicable to our problem for at least two reasons. First, most existing work that develops efficient algorithms for POMDP assumes a stationary transition matrix and infinite-horizon discounted setting. In contrast, in our work, the corresponding probability of transitioning from an engagement state to another engagement state also depends on how long the player has stayed in the system, i.e., the transitions are nonstationary. Second, unlike in a typical POMDP setting where the decision maker only needs to choose one action per period, in our problem, a player may play multiple rounds in one period and the game designer needs to choose which candidate game to match to the player for each round after observing the outcomes in the previous round. Thus, we are essentially dealing with a variant of classical POMDP in which the decision maker is faced with a sequential decision making within each period, and the state at the beginning of the next period is determined by the collective outputs of the sequential decisions made in the current period. This significantly increases the complexity of the problem beyond the typical POMDP and we are not aware of extant literature that addresses this setting. Since efficiency and speed of the matching algorithm is of utmost important in our application context, in this paper, we depart from standard POMDP techniques and develop an algorithm that combines approximate learning (or belief update) with approximate matching.

3. Research Context, Data and Descriptive Evidence

As noted earlier, the data we use was provided by a major international video gaming company for a first-person shooter game.[2] In the game, players take on the persona of various military roles, such as assault, reconnaissance, and engineer, and play on maps of urban streets, metropolitan areas, and open landscapes. The game can be played on a dedicated game console (e.g., Xbox 360) or on a regular personal computer (PC). Players typically play in teams and the objective may be to capture a flag, kill the rival teams a certain number of times, or attack/defend stations, depending on their chosen game mode. Players earn score (points) individually in each round they play, by injuring enemies, killing enemies, killing a certain number of enemies in a row without a death, completing tasks such as capturing the flag, or completing a role-based task, such as healing other players as a medic.[3] The cumulative score players collect determines their "rank." Each rank has a set cumulative score that must be earned to "level up", i.e., reach the next rank level.[4] After leveling up, players can unlock the new rank's specializations or weapons. To join a game round, players can choose either "Quick Match" or "Server Browser." If Quick Match is selected, players simply choose a game mode and one of the associated maps after which they will be matched by the company into a currently available game round. If Server Browser is selected, players can browse servers, looking for a specific game round to join; the matching choice in this case is made by the gamer, not the company. Although the video game company does not record which method, Quick Match or Server Browser, a player chooses to join a round, they have some internal data suggesting that the proportion of players who choose Quick Match is much higher for console players than for PC players.

The data set includes the complete game-play history of 1,309 players (focal players) from October 2011 (product launch) to March 2014. These players play 710,212 unique game rounds in this period with the total number of players playing these rounds at 9.5 million. In the data, we observe results of each round played by each focal player. The round-level information includes map, game mode, the date on which the round began, the duration of the round, the number of unique players in the round, the maximum number of players in the round at any given time, and the maximum number of players allowed in the round at any given time. For each player in each record, we observe her rank at the start and the end of the round, the number of seconds the player spent

[2] We are unable to identify the company or the game due to a non-disclosure agreement.

[3] As the data contain limited information on team membership, we do not consider team dynamics directly in our approach. Note that we show later in this section that the company assigns players to teams randomly implying that there is no systematic difference in member profiles between competing teams in a game round. Thus, we do not expect team membership/composition to have a meaningful impact on a player's game-play outcome in a round.

[4] The cumulative score is non-decreasing. Hence, players can only move from a lower rank to a higher rank, but not the other way around.

in the round, and the total score and combat score she earned in the round. We are only given the user ID's for the 1,309 sampled focal players. For non-focal players, we observe neither their user ID's nor their complete playing history. In addition, there is little overlap in the game rounds played by different focal players; 0.2% of the rounds in the sample have more than one focal player playing in them. For these reasons, we do not observe the "game-play network" among players (whether a player is playing with the same set of players again and again, or playing with different sets of players in different rounds) and thus cannot model it explicitly. However, for each round one or more focal players played in, we observe the ranks of all players in that round. We can use these observed ranks to construct variables that describe with whom the focal player played in that round, which can impact focal players' future game-play behavior indirectly as the player composition for a round can affect game-play outcomes.

Summary statistics of the round-level characteristics are provided in Table 1. Player-round-level variables for focal players are summarized in Table 2. Variables reported in Table 2 are defined as follows:

- Rank at Start/End: the focal player's rank at the start/end of the current round.
- Player Seconds: the number of seconds the player spent in the round.
- Total Score: the sum of all points earned from the round.
- Combat Score: points earned from killing other players.

In addition to the round level information, we also observe a set of time-invariant player characteristics, such as country, age, gender (these three variables are self-reported by the users), and the primary platform. The summary statistics of player characteristics, as well as the total number of rounds players played, and their rank at the end of the time interval spanned by the data (Ending Rank) are reported in Table 3.

Table 1 Round-Level Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.
Duration (Seconds)	1522.07	4065.60	10.33	99300
Number of Unique Players	34.95	18.51	2	199
Max Simultaneous Players	20.60	9.80	1	64
Max Simultaneous Players Allowed	43.87	0.724	2	100
Average Player Rank at Start	44.87	17.84	1	145
Std. Dev. of Player Rank at Start	24.91	10.72	0	87.68

Note: N_r=710,212; Rounds with duration greater than 100,000 seconds (1.16 days) dropped.

The most direct way to measure the effect of the game-play outcome on players' future game-play behavior, is to study how the game-play outcome in a single round affects a player's subsequent game-play behavior (e.g., how soon the player returns to play the next round). However, in our

Table 2 Focal Players' Round-Level Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.
Rank at Start	51.86	34.46	0	145
Rank at End	51.91	34.43	0	145
Player Seconds	798.64	703.30	1.03	9928
Total Score	4883.17	7020.62	0	455600
Combat Score	2510.68	2935.76	0	256800

Note: N_r=710,212.

Table 3 Player-Level Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.
Age	29.32	9.22	15	80
Total Rounds Played	559.52	778.84	1	8368
Ending Rank	43.32	29.57	1	145
Country	US: 475	GB: 125	DE: 105	Other: 604
Gender	M: 48	F: 2	Unknown: 1259	
Primary Platform	PCWIN: 319	PS3: 485	XBOX360: 505	

Note: N=1,309.

data set, we observe only the date on which each round was played; the time of day when a round was played is unavailable to us. As a result, we do not know the exact order of the rounds played by the same individual within the same day, or the time gap between different rounds. An alternative modeling approach is to aggregate players game-play behavior and the game-play outcome to the daily or the weekly level, then study how a player's daily/weekly game-play outcome affects her game-play activities in future periods. This approach has multiple advantages – it not only resolves the data issue described above, but also simplifies the analysis, because players' game-play process is modeled as a discrete-time process instead of a continuous-time process. It also allows us to examine how variables characterizing player game-play outcome within a certain period of time (e.g., the standard deviation of the scores they receive in a period) affect their future game-play activities. We adopt this "aggregate approach" and define a period as one week. We choose to aggregate the player game-play information to the weekly level, rather than the daily level, because the mean and median of the probability of playing on a given day across all 1,309 players are 0.098 and 0.063 respectively, resulting in too many zeros in the daily-level data. This can adversely affect the identification of the HMM parameters (see §4). The summary statistics of the weekly game-play activities and outcome variables are provided in Table 4. For each player, we consider only the weeks after the player registered for the game and before the player upgraded to the next version of the same game. This is because it is evident from the data that once players upgrade to the new version of the game, they rarely return to the older version of the game. Therefore, as soon as a player upgrades to the newer version of the game, she exits our analysis.

Table 4 Week-Player-Level Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.
Number of Rounds Played	4.61	14.27	0	381
Weekly Player Seconds	3682.34	11618.09	0	279418
Weekly Total Score	22517.81	88919.16	0	4061000
Weekly Combat Score	11576.70	43736.48	0	2101650

Note: N=1,309; T_i=113-127.

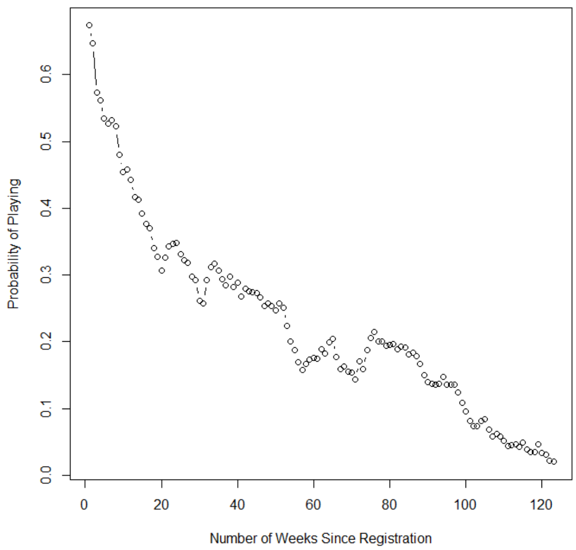
We find that players' weekly game-play activity is not stable over time – there is a natural decline in their playing patterns. As shown in Figure 1, both the probability of a player playing in a given week and the number of rounds she plays decrease over time. This pattern could be partly explained by the motivation factor of *curiosity* – as an individual plays the game for a longer period of time, there are fewer and fewer surprises, and the player may gradually become bored and less engaged. We also use available data to construct measures or proxies for gamer motivations that lead to higher engagement. First, we use the total score obtained by a player during one period to capture *competence* and *self-efficacy* (Ryan et al. 2006, Klimmt and Hartmann 2006).[5] Second, we use the standard deviation of the scores obtained by a player in different rounds played within a period to capture *challenge*. After controlling for the total score a player receives in a week, the larger the variation in scores she receives weekly from different rounds, the more she is challenged in that week (Sherry et al. 2006). Third, we use the number of weeks in which a player has played the game since her last rank increase to capture any *achievement/progress* or lack thereof (Sherry et al. 2006).

We first estimate two descriptive (OLS) models to explore how the game-play outcome variables and the number of weeks since a player signed up for the game are related to the probability of a player's playing in a given week (d_{it}) and the number of rounds she plays (r_{it}). The estimation results, reported in Table 5, suggest that all these variables seem to affect players future (at least near-term future) game-play behavior.

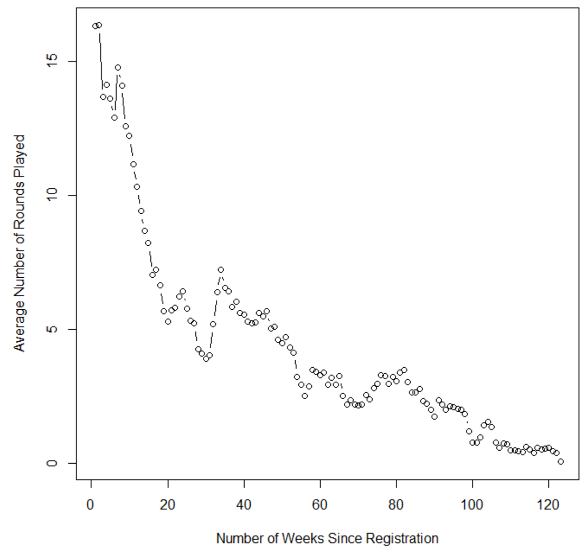
Since one of the goals of this paper is to design a better player-matching algorithm, we also need to ensure that we understand the matching pattern in our data. In Figure 2, we plot the histogram of the range of the ranks of players playing in each round. The figure shows that the rank range in the majority of the game rounds observed in our data is quite large (≥ 50), indicating that the current player assignment is quite random. The randomness in matching in the data generating process helps us rule out issues of selection (or broadly speaking, endogeneity).

[5] The other two observed outcome variables, player seconds and combat score, are highly correlated with the total score and thus we exclude them from the model. Therefore, we use only the total score to measure *competence* and *self-efficacy*.

Figure 1 Weekly Player Activity



(a) Average Probability of Playing



(b) Average Number of Rounds Played

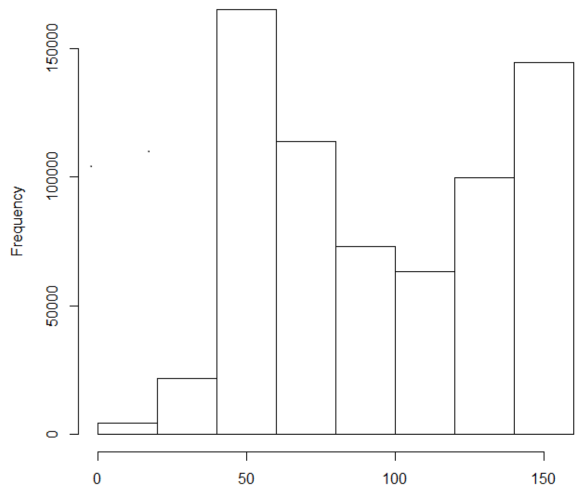
Table 5 Regression Results

	Dependent variable:	
	$\mathrm{I}(d_{it} = 1)$	$\ln(r_{it})\|d_{it} = 1$
$\ln(TotalScore_{it-1})$	0.537^{***}	0.265^{***}
	(0.017)	(0.007)
$\ln(StdScore_{it-1})$	-0.370^{***}	-0.284^{***}
	(0.023)	(0.010)
$\ln(WeekSameRank_t)$	0.139^{***}	-0.039^{***}
	(0.023)	(0.011)
$\ln(WeekSinceRegistration_t)$	-0.707^{***}	-0.152^{***}
	(0.011)	(0.007)
Individual Fixed Effects	Yes	Yes
Observations	116,621	24,785
R^2	0.217	0.368
Max. Possible R^2/Adjusted R^2	0.520	0.332
Log Likelihood/F Statistic	−28,507.22	10.47
Note:	*p<0.1; **p<0.05; ***p<0.01	

4. Empirical Model

The main focus in the first stage of our two-stage data analytic approach is to construct an empirical model, more specifically, an HMM, to capture time-varying player engagement and then use the real player game-play history to calibrate the model. An HMM is a model of a Markov stochastic process that is not directly observed, but can be inferred through another set of stochastic processes that produces a sequence of observations.

Figure 2 Histogram of Rank Range in Each Round



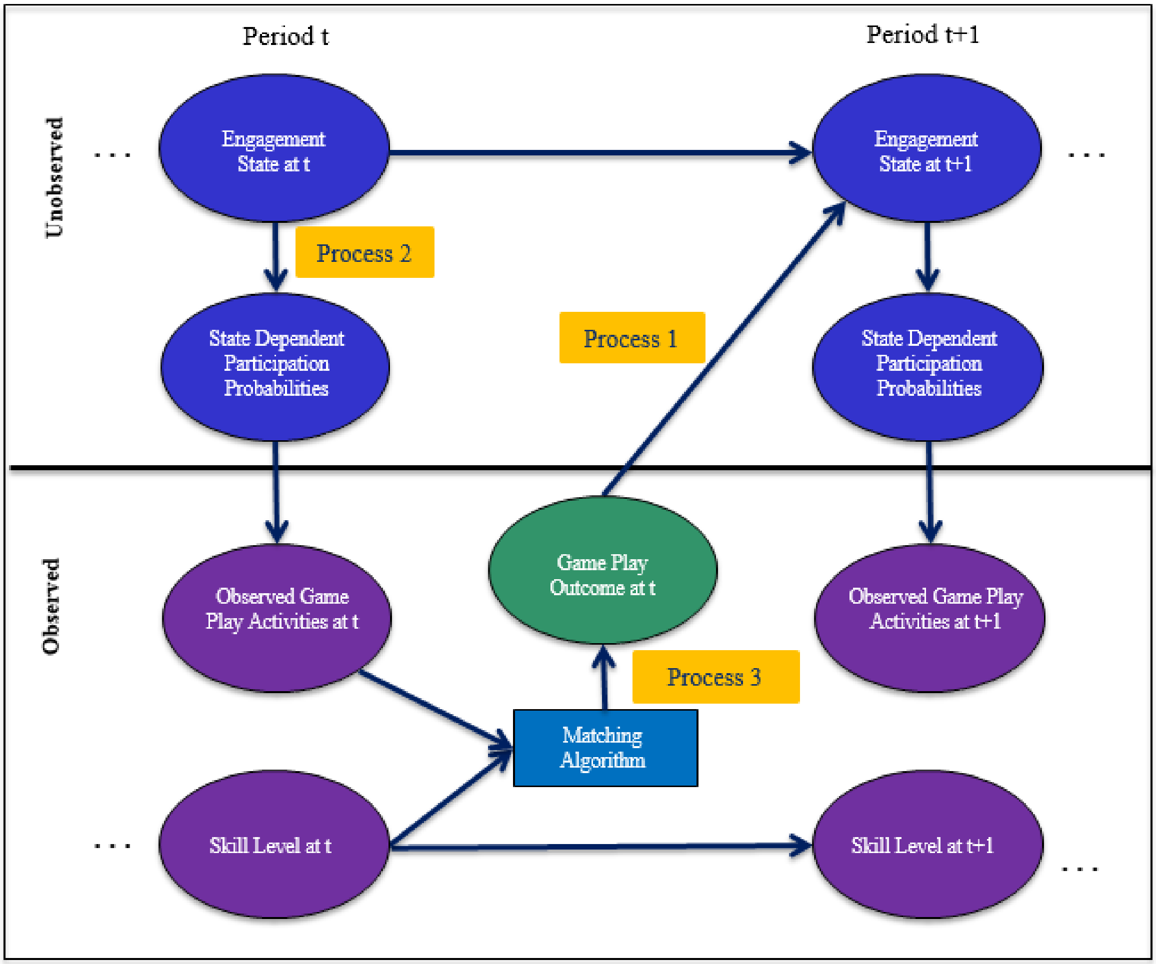
In our context, players' "skill" level is measured by their rank (determined by players' cumulative score, as discussed in §3), the evolution of which is observed and deterministic. Players' "engagement" level is the latent state variable (impacted by the motivations described earlier), and the latent engagement states stochastically determine their game-play behavior, which are observable to researchers. The HMM of players' participation behavior can be visualized as in Figure 3. The proposed HMM consists of three major components:

1. The initial engagement state distribution π_i: π_i is a K-element vector (K represents the number of possible levels of the engagement state), with the k^{th} element ($\pi_i(k)$) representing the probability that player i's engagement state in period 1 is k ($\pi_i(k) = P(S_{i1} = k)$).

2. Engagement State Transition (Process 1 in Figure 3): The transitions between the latent engagement states are a sequence of Markovian transitions ($Q_{i,t,t+1}$), which express in a probabilistic manner how the game-play outcome in the previous time period affects the transition of player engagement state. $Q_{i,t,t+1}$ is a K by K matrix, with the element in Row k, Column k' representing the probability of a player transitioning from state k in period t to k' in period $t+1$.

3. State-Dependent Game-Play Activity (Process 2 in Figure 3): The state dependent game-play activity in our research context include two components: (1) the probability that a player plays the game in period t, conditional on her current hidden engagement state ($P(d_{it} = 1|S_{it} = k)$); d_{it} is a binary variable with 1 indicating individual i plays at least one round in period t); and (2) if she decides to play in period t, the conditional distribution of the number of rounds that she would play ($f(r|S_{it} = k)$). Although neither (1) nor (2) is directly observable, the observed user participation decisions in each period are realizations of these probability distributions.

Figure 3 A HMM of Player Participation



Another stochastic process in our model, which is not part of the HMM, is Process 3 in Figure 3. This process captures how the profile of co-players in a game round affects the focal player's game-play outcome. Process 1 captures how players' engagement state evolve as a function of their game-play outcome, whereas Process 3 links the assignment algorithm (which rounds a player is assigned to) to game-play outcome.

In reality, some players may be inherently more interested in or attached to the video game than others. Therefore, the transition of the hidden engagement level may vary across individuals. To capture such unobserved individual heterogeneity, we assume that there are M classes of players. Here, we use C to denote the class an individual belongs to, and $C \in \{1, 2, ...M\}$. Player-level heterogeneity is captured by discrete classes rather than continuous distribution because discrete classes are easier to deal with when it comes to designing the matching algorithm. The initial state distribution and state transition probabilities discussed above are then class-specific, and we modify their corresponding notations to π_i^m and $Q_{i,t,t+1}^m$; however, for identification purposes, we assume that the state-dependent playing activities are not class specific. In other words, if two players are at the same engagement level, then their respective probabilities of playing in the current period are the same, and the numbers of rounds they play are drawn from the same distribution, irregardless of which class each of the two players belongs to.

We next elaborate on the specification of the three components of the HMM and Process 3 described Figure 3.

4.1. Engagement State Transition

As discussed above, we model the transitions between engagement states as a Markov process. The state transition matrix $Q^m_{i,t,t+1}$ for class $C = m$ is defined as

$$Q^m_{i,t,t+1} = \begin{bmatrix} q^m_{it11} & q^m_{it12} & \cdots & q^m_{it1K} \\ q^m_{it21} & q^m_{it22} & \cdots & q^m_{it2K} \\ \vdots & \vdots & \ddots & \vdots \\ q^m_{itK1} & q^m_{itK2} & \cdots & q^m_{itKK} \end{bmatrix}$$

Here, $q^m_{itkk'}$ represents the probability that individual i's engagement state transitions from k to k' at the end of period t, if i belongs to class m. The transition probabilities are affected by individual i's decision to play in period t (d_{it}), and a set of time-varying covariates (denoted as X_{it}), including the total score an individual receives in the current period, the standard deviation of the scores she receives in the same period, the number of weeks in which she has played the game since her last rank increase, and the number of weeks passed since she registered for the game. As noted earlier, we include these variables because they capture the key motivational factors identified in the behavioral literature and the descriptive analyses presented earlier suggest that they all affect players' next period game-play activities. We assume that if individual i does not play in period t, i.e., $d_{it} = 0$, then she will stay at the same engagement state in the next period ($S_{it+1} = S_{it}$), i.e., $Q^m_{i,t,t+1} = I_K$. While this assumption may seem unusual, as we show in one of our robustness checks (see §5.4), relaxing it does not affect our results. However, it does help us considerably in implementing the matching algorithm. If the individual plays in period t, i.e., $d_{it} = 1$, her latent propensity to move from one engagement level to another in period t, Z_{it}, can be expressed as

$$Z_{it} = X'_{it}\eta_k + \epsilon_{it} \tag{1}$$

where η_k is a k-specific vector, with each element representing the marginal effect of the corresponding element in the X_{it} vector on the latent transition propensity. ϵ_{it} captures the unobserved factors that also affect the engagement state transition. In the main model, we impose the restriction that players' engagement states can only transition between adjacent states: when the propensity for transition crosses the threshold for transitioning to a higher state (μ^m_{hk}), the engagement state will move up by 1; when the propensity for transition falls below the threshold for transitioning to a lower state (μ_{lk}^m), the engagement state will move down by 1. Notice that η_k, μ_{hk}^m, and μ_{lk}^m are all k-specific, allowing for the possibility that the same values of X_{it} lead to different transition propensities at different engagement state levels. To capture the possibility that players in a certain class (or classes) are inherently more interested in the game and thus more likely to stay engaged than other players, assuming everything else is equal, we allow the thresholds to be class-specific (μ_{hk}^m and μ_{lk}^m are m-specific). We assume the same η_k for different classes, indicating that the effects of game-play outcome on the latent state transition propensity are common across classes.[6] We further assume the error term, ϵ_{it}, follows the Type I extreme value distribution, and thus the transition probability takes an ordered-logit form. The elements in the $Q_{i,t,t+1}^m$ matrix can then be written as:

If $d_{it} = 0$:

$$q_{itkk'}^m = \begin{cases} 1 & \text{if } k = k' \\ 0 & o.w. \end{cases}$$

If $d_{it} = 1$:

$$q_{itkk'}^m = \begin{cases} 1 - \dfrac{\exp(\mu_{hk}^m - X_{it}'\eta_k)}{1 + \exp(\mu_{hk}^m - X_{it}'\eta_k)} & \text{if } k' = k+1 \\ \dfrac{\exp(\mu_{lk}^m - X_{it}'\eta_k)}{1 + exp(\mu_{lk}^m - X_{it}'\eta_k)} & \text{if } k' = k-1 \\ \dfrac{\exp(\mu_{hk}^m - X_{it}'\eta_k)}{1 + \exp(\mu_{hk}^m - X_{it}'\eta_k)} - \dfrac{\exp(\mu_{lk}^m - X_{it}'\eta_k)}{1 + \exp(\mu_{lk}^m - X_{it}'\eta_k)} & \text{if } k' = k \\ 0 & o.w. \end{cases}$$

where $\mu_{hk}^m = \infty$ for $k = K$ and $\mu_{lk}^m = -\infty$ for $k = 1$.

4.2. The Initial State Distribution

The initial state distribution is typically set to the stationary distribution of the state transition matrix if an HMM has time-invariant transition matrix (MacDonald and Zucchini 1997). However, the state transition matrix in our model contains time-varying variables and players' playing decisions in each period. We calculate the stationary distribution of the transition matrix by solving the equation $\pi_i^m = \pi_i^m \bar{Q}_i^m$ under the constraint $\sum_{k=1}^K \pi_i^m(k) = 1$, where $\bar{Q}_i^m$ is the transition matrix with all time-varying covariates set to zero and under $d_{it} = 1$, as our data set is not left truncated.[7]

[6] We relax two of the assumptions discussed above: (a) we allow transitions to non-adjacent states and (b) we allow η_k to be class-specific. Details are in §5.4.

[7] Alternatively, we take the stationary distribution of the transition matrix with the covariates set to their mean levels. To further ensure the stability of the results, we also estimate the model using several different randomly sampled initial state distributions. The estimation results reported later are robust to the choice of the initial distribution, because at $t = 1$, the probability of moving/staying at medium/high engagement states is very high.

4.3. The State-Dependent Game-Play Behavior

Given a player's state, her participation is assumed to be conditionally independent. We use a hurdle model to capture players' state-dependent game-play behavior. A hurdle model is "a modified count model in which the two processes generating the zeros and the positives are not constrained to be the same" (Cameron and Trivedi 1998). The underlying idea of the hurdle formulation is that a binomial probability model governs the binary outcome of whether a count variate has a zero or a positive realization. If the realization is positive, the "hurdle is crossed", and the conditional distribution of the positives is governed by a truncated-at-zero data model (Mullahy 1986). In our context, we assume that in each period, a player first decides whether to play in this period d_{it}, which depends on the engagement state she is in and the realization of the random error. Then conditional on $d_{it}=1$, she decides on how many rounds to play. For different engagement state k, the probability of playing and the distribution of rounds of playing are different. We denote the probability of playing in engagement state k as $p_k = \exp(\alpha_k)/(1+\exp(\alpha_k))$, and assume the number of rounds played in a period follows a k–specific log-normal distribution $[\ln(r_{it})|S_{it}=k] \sim \mathcal{N}(\gamma_k, \sigma_k^2)$. To ensure identification of the states, we restrict α_k and γ_k to be non-decreasing in k. We further define $a_{it} = (d_{it}, r_{it})$.

4.4. Factors Affecting Game-Play Outcome (Score)

In the previous analysis, we establish why and how the game-play outcomes X_{it} may affect players' engagement state transition. Since our ultimate goal is to design a matching algorithm that maximizes player retention, we also need to identify which round-level characteristics can affect game-play outcome, quantify their impact, and incorporate them into our matching algorithm. More specifically, we want to establish the connection between a focal player's game-play outcome (round-level score) and the characteristics of other players in the round. Since player rank summarizes a player's experience with the game and also skill level of the player, we build the following model to connect the score an individual receives from a game round and her relative rank level among all players in the game round. Although we consider only player rank in this model, our modeling approach can be readily extended to incorporate more player characteristics (but at the expense of significantly complicating the matching algorithm).

$$\begin{aligned}\ln score_{ir} = \delta_0 + \delta_1 AvgRank_r + \delta_2 Percentile_{ir} + \delta_3 AvgRank_r * Percentile_{ir} \\ + \delta_4 \ln(AvgScore_{ir}^{PREV}) + \delta_5 \ln(TtlScore_{ir}^{PREV}) + \xi_{ir}\end{aligned} \tag{2}$$

Here, $score_{ir}$ represents the score player i receives in round r; $AvgRank_r$ is the average of the ranks of all players in the game round, which captures the mean competence level of players in the round. $Percentile_{ir}$ is the percentile of the focal player's rank at the beginning of the round among all players in round r, which reflects the relative standing of player i in the round in terms of her skill/experience level. $AvgScore_{ir}^{PREV}$ and $TtlScore_{ir}^{PREV}$ represent player i's average score per round in the most recent period in which she played the game and the total score she received in that period, respectively. These two covariates are introduced into the regression to capture players' short-term performance level, which differ from the rank-based variables that capture players' long-term cumulative performance level.

Note that this regression model is independent of the HMM, and thus can be separately estimated. Also note that this analysis is at the round level, not the weekly level.

4.5. The Likelihood of an Observed Sequence of Actions

Let $S_i = (S_{i1}, S_{i2}, ..., S_{iT_i})$ denote player i's hidden state sequences and $a_i = (a_{i1}, a_{i2}, ..., a_{iT_i})$ denote an observed game-play activity sequence. T_i refers to the number of periods between the date when player i signed up for the game and the time when she upgraded to the next version of the game, or the last date observed in the data, whichever occurs first. Given any two consecutive observed actions are linked only through the hidden states, the probability of observing the sequence a_i conditioned on the states in each period can be written as

$$P(a_i|s_i) = \prod_{t=1}^{T_i} P(a_{it}|S_{it}).$$

The joint likelihood of a sequence of activities a_i, conditional on this player belonging to class m (i.e., $C = m$), is given by the sum over all possible paths individual i could take over time among the underlying states:

$$L(a_i|C=m) = P(a_{i1}, a_{i2}, ..., a_{iT_i}|C=m) = \sum_{s_{i1}=1}^{K} \sum_{s_{i2}=1}^{K} \cdots \sum_{s_{iT_i}=1}^{K} [P(S_{i1}=s_{i1}|C=m)P(a_{i1}|S_{i1}=s_{i1})$$
$$\cdot \prod_{t=2}^{T_i} P(S_{it}=s_{it}|S_{it-1}=s_{it-1}, C=m) \cdot \prod_{t=2}^{T_i} P(a_{it}|S_{it}=s_{it})].$$

Following MacDonald and Zucchini (1997), we can rewrite the equation above in a matrix product form that simplifies computation:

$$L(a_i|C=m) = \pi_i^m \tilde{A}_{i1} Q_{i,1,2}^m \tilde{A}_{i2} Q_{i,2,3}^m ... \tilde{A}_{i,T_i-1} Q_{i,T_i-1,T_i}^m \tilde{A}_{i,T_i} \mathbf{1}$$

where $\mathbf{1}$ is a $K \times 1$ vector of ones, and $\tilde{A}_{i,t}$ is a diagonal $K \times K$ matrix with the kth diagonal element, $\tilde{a}_{it|k}$, being the likelihood of observing a_{it} given player i's engagement state in period t is k (i.e., $S_{it} = k$):

$$\tilde{a}_{it|k} = \left\{ \frac{\exp(\alpha_k)}{1+\exp(\alpha_k)} \frac{\phi[(\ln(r_{it}) - \gamma_k)/\sigma_k]}{r_{it}} \right\}^{d_{it}} \cdot \left\{ \frac{1}{1+\exp(\alpha_k)} \right\}^{1-d_{it}}$$

Let τ^m denote the fraction of player population that belongs to class m, such that $\sum_{m=1}^{M} \tau^m = 1$. Then the unconditional likelihood of individual i's sequence of activities a_i is $L(a_i) = \sum_{m=1}^{M} \tau^m L(a_i|C=m)$. The likelihood of observing all individuals' sequence of activities is

$$L(a) = \prod_{i=1}^{N} \sum_{m=1}^{M} \tau^m L(a_i|C=m) = \prod_{i=1}^{N} \sum_{m=1}^{M} \tau^m \pi_i^m \tilde{A}_{i1} Q_{i,1,2}^m \tilde{A}_{i2} Q_{i,2,3}^m ... \tilde{A}_{i,T_i-1} Q_{i,T_i-1,T_i}^m \tilde{A}_{i,T_i} \mathbf{1} \quad (3)$$

5. Estimation and Results

Maximum likelihood estimation (MLE) is used to estimate the HMM. We randomly sample 1,100 of the 1,309 players of whom we have the complete game-play history to calibrate the model, and use data about the remaining 209 players to validate the model. In order to arrive at the "best" model, we estimate HMMs with different combinations of latent classes and engagement state levels. We first estimate a series of homogeneous HMM with up to four possible engagement state levels. The estimation results suggest that the model fit and predictive performance do not improve significantly beyond three engagement levels. Then we expand the model by incorporating up to four latent classes into the three-engagement-state model. In the heterogeneous HMM models, we allow only the threshold parameters μ's to be different across classes. However, as a robustness check, we also estimate a model which allows the parameters associated with the covariates in the X_{it} vector to be different for different classes of players. Finally, we estimate a non-dynamic model (without any time-varying covariates) with three latent states. This model does not allow individuals to move among the engagement states over time. Table 6 summarizes the fit statistics of this set of models.

Table 6 Model Performance

Model	Log-Likelihood	BIC	Validation Log-Likelihood
1 Classes – 2 States	-160,289	320,689	-27,643
1 Classes – 3 States	-155,609	311,394	-26,986
2 Classes – 3 States	-155,300	310,809	-26,906
3 Classes – 3 States	-155,220	310,686	-27,179
4 Classes – 3 States	-155,178	310,637	-26,898
Non-Dynamic – 3 States	-180,734	361,546	-31,051

5.1. Model Comparison

Results reported in Table 6 show that all dynamic models perform better than the non-dynamic model. This confirms that players' engagement state evolves over time. Not surprisingly, as we allow for more engagement state levels, the in-sample model fit improves. Among the models with three engagement states, the in-sample log-likelihood and BIC improves as the number of classes increases. However, after the number of classes goes beyond 2, the in-sample fit improvement is rather small when we add additional classes to the model. Moreover, the "2 Classes – 3 States" model performs almost as well as the "4 Classes – 3 States" model out of sample, and the "3 Classes – 3 States" model has a worse out-of-sample fit than the "2 Classes – 3 States" model and the "4 Classes – 3 States" model. Hence, we select the "2 Classes – 3 States" model as our main model and base our discussion of HMM parameter estimates on this model.

5.2. HMM Results

The estimation results for the "2 Classes – 3 States" HMM are reported in Table 7. In the table, we label the three engagement states as "low", "medium" and "high". The first set of parameters reported in the table govern the state-dependent game-play behavior. The state-specific intercepts in the logit probability of playing (α_k) are -3.212, 0.936 and 4.542 for the low, medium and high engagement states, respectively. These numbers can be translated into probabilities of 0.039, 0.718, and 0.989 of playing in a given week for the low, medium and high engagement states, respectively. The gap in the probability of playing is a lot larger between the low and the medium engagement states than between the medium and the high engagement states. Interestingly, when it comes to the number of rounds a player will play if she decides to play in a given period, the difference between the medium and the high engagement states is larger than that between the low and the medium engagement states – the conditional mean of the logarithm of the number of rounds played for the low, medium and high engagement states are 1.521, 1.791 and 3.334 respectively. The estimated variance of the logarithm of the number of rounds a player would play for the low, medium and high engagement state is 1.093, 1.013 and 0.713, respectively (calculated by taking the exponential of the $\ln(\sigma_k^2)$). Combining the mean and variance estimates for the low-, medium- and high-state log-normal distributions, we can approximate the expected number of rounds a player would play in a given week: 7.9 for the low engagement state, 9.9 for the medium engagement state, and 40 for the high engagement state.

The second set of parameters of interest are the parameters governing the state transition process, $\eta's$, and the threshold parameters, $\mu's$. $\eta_{.,k}$ capture the effects of the time-varying motivation measures/proxies we consider in this model – the total score a player receives in the current period, the standard deviation of the scores among different rounds played in the current period, the number of weeks the player has played since she last leveled up, and the number of weeks since the player started playing the game – on the transition of the engagement state. Note that in the model, we take the logarithm of all four covariates because all are highly skewed.

First, somewhat surprisingly, we find that obtaining higher scores does not always help keep players engaged. The positive effect of the total score on the propensity of moving to/staying in a higher engagement state exists only when players are in either the low or the high engagement state. When players are in the medium engagement state, a higher total score can slightly reduce their propensity to move to a higher engagement state. Also, the magnitude of the positive effect of the total score is larger for players who are currently highly engaged than those who are in the low engagement state. The standard deviation of the scores a player receives from different rounds she plays in a period also has differential effects on individuals' propensity of moving towards a higher engagement state. For players in the low or medium engagement state, higher variations in the game-play outcome actually increases the propensity of moving to a higher engagement state. However, the effect of such variation is negative for individuals who are in the high engagement state. These results suggest that the sense of being challenged motivates only players who are not highly engaged, but not those who are already highly engaged. If we look at the joint effects of the total and the standard deviation of scores a player receives in a week on her engagement state evolution, we see that for players who are currently highly engaged, continuous reinforcement of their sense of achievement and self-efficacy is important to keep them engaged, and the sense of being challenged discourages their continuous play. For players who are in the low engagement state, both sense of achievement and sense of being challenged can motivate their future participation. However, players who are in the medium engagement state enjoy being challenged, even at the expense of receiving a lower total score.

The number of weeks a player has played the game without improving her rank [ln(No. weeks same rank)] negatively affects her propensity to move towards higher engagement states. This result suggests that, in general, the lack of achievement or progress discourages players from staying engaged. This finding is consistent with those reported in the literature that the sense of achievement motivates individuals to continuously play the video game (Sherry et al. 2006). It is worth noticing, though, that this negative effect is not statistically significant in the case of the low engagement state, but is significant for individuals in either the medium or the high engagement state, indicating that engaged players seem to care more about leveling up, probably because they play the game more and thus expect more frequent rank improvements. If they fail to level up, they get more frustrated. In contrast, those who are in the low engagement state probably do not have as strong a desire to level up, and thus staying at the same rank may be less frustrating to them. Although both the total score and the number of weeks staying at the same rank reflect the "(lack of) achievement," the former is a short-term measure, whereas the latter is a longer-term measure.

The number of weeks since a player started playing the game is estimated to have a negative impact on players' propensity to move towards a higher engagement state, reflecting a natural decrease in players' engagement level. This result is consistent with the decay pattern we observe in the data. Also, the size of the negative effect is larger for individuals in a lower engagement state than those in a higher engagement state. This is not surprising because individuals who are in the high engagement state are still very interested and highly engaged in the game, and so their decay rate is likely to be slower.

The remaining parameters are the class-specific parameters. First, the estimated value of τ is 0.648, which corresponds to a proportion of $e^{0.648}/(1+e^{0.648}) = 0.657$ of players who belong to Class 1. As we see in the Table 7, both the $\mu'_{lk}s$, the threshold of moving down to a lower engagement state, and the μ_{hk}'s, the threshold of moving up to a higher engagement state are smaller for Class 2 players than those for Class 1 players. This indicates that given the same past game-play outcome, Class 2 players are less likely to drop down to a lower engagement state, and more likely to move up to a higher engagement state. In other words, Class 2 players are more inherently interested in the game, and tend to stay engaged for a longer period of time, all else being equal.

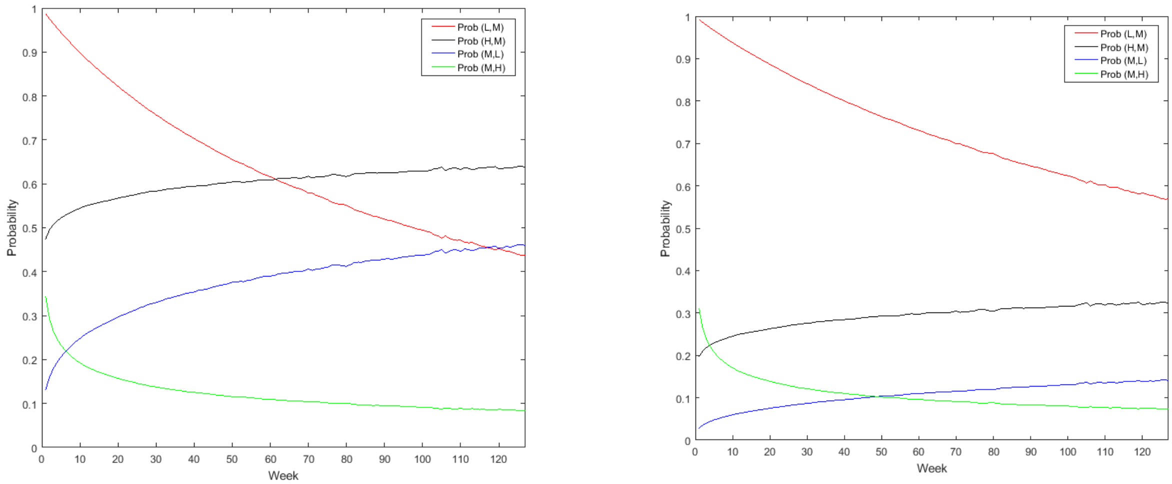
To better illustrate how the engagement state transition probabilities for the two classes of players evolve overtime, we calculate numerically the transition probabilities from Week 1 to Week 127, for an "average" Class 1 player and an "average" Class 2 player, respectively, conditional on $d_{it} = 1$. In the calculation, we fix the ln(total score), ln(std score), and ln(no. weeks same rank) at their corresponding sample means. The results are displayed in Figure 4. In the figure, we see that the probability of moving from the low to the medium engagement state (the Prob(L,M) curve) and the probability of moving from the medium to the high engagement state (the Prob(M,H) curve) are decreasing over time. For Class 1 players, the two probabilities drop from 0.98 and 0.34 to 0.44 and 0.09, respectively. The probability for Class 2 players to move from the medium to the high engagement state evolves in the same direction as that for Class 1 players. However, the probability of moving from the low to the medium engagement state decreases at a smaller rate for Class 2 players than for Class 1 players - in the 127^{th} week, Prob(L,M) for Class 2 players remains around 0.6.

The probabilities of moving from a higher to a lower engagement state vary significantly between the two classes of players. The probability of moving from the high to the medium engagement state increases from 0.47 to 0.63 for Class 1 players, but increases from 0.20 to 0.32 only for Class 2 players. The probability of dropping from the medium to the low engagement state increases from 0.13 to 0.46 for Class 1 players, but the same probability only goes from 0.03 to 0.14 for Class 2 players. This set of results show that although the difference between Class 1 and Class 2 in the probability of moving up to a higher engagement state is not very large, Class 2 players tend to stay longer in either the medium or the high engagement states once they reach these states. Also, it seems that the medium engagement state is the most "sticky" engagement state, as the probability of moving from the low to the medium engagement state is higher than the probability of moving from the medium to the low engagement state; movement from the high to the medium engagement state also dominates the movement in the opposite direction.

Table 7 Estimation Results for the HMM

Parameter	Description	Low	Medium	High
α_k	Intercept in the logit probability of playing	-3.212(0.023)	0.936 (0.030)	4.542(0.142)
γ_k	Mean of ln(no. rounds played)	1.521(0.021)	1.791(0.119)	3.334(0.113)
$ln(\sigma_k^2)$	Ln(variance of ln(no. rounds played))	0.089(0.013)	0.013 (0.006)	-0.338(0.010)
η_{1k}	Ln(total score)	0.237(0.037)	-0.059(0.014)	1.506(0.073)
η_{2k}	Ln(std score)	0.068(0.021)	0.109(0.016)	-0.960(0.089)
η_{3k}	Ln(no. weeks same rank)	-0.057(0.078)	-0.141(0.037)	-0.130(0.060)
η_{4k}	Ln(no. weeks since sign-up)	-0.960(0.077)	-0.352(0.022)	-0.120(0.049)
	Class 1			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-1.683(0.096)	8.518(0.580)
μ_{hk}	Threshold of moving to a higher state	-1.406(0.210)	0.989(0.093)	∞
	Class 2			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-3.283(0.280)	7.237(0.105)
μ_{hk}	Threshold of moving to a higher state	-2.161(0.093)	0.920 (0.087)	∞
τ	Intercept in the logit proportion of Class 1 players	0.609(0.122)		
N			1,109	
log-likelihood			-155,300	
BIC			310,809	

Figure 4   Transition Probabilities Overtime if $d_{it} = 1$



(a) Class 1

(b) Class 2

5.3. Relationship Between Score and Rank Distribution within a Round

We then separately estimate Equation (2), which connects the round-level score with the rank distribution of players playing in the same round and the focal player's short-term performance level. We experiment with four alternative specifications. Specification (1) considers only the percentile of the rank of the focal player among all players in the round ($Percentile_{ir}$) and the average rank of all players in the round ($AvgRank_r$) as independent variables. Specification (2) incorporates the interaction effect between $Percentile_{ir}$ and $AvgRank_r$. Specifications (3) and (4) further include the two short-term performance measures, $\ln(AvgScore_{ir}^{PREV})$ and $\ln(TtlScore_{ir}^{PREV})$, which are defined in §4.4. The results (Table 8) show that the higher a player's rank percentile, the higher score she can expect to receive from a round, all else being equal. This result is not surprising, because a player with a high rank relative to other players in the same round typically has an advantage in the round, and thus can receive a higher score. We also find that when the average rank of all players in the game round is higher, the score a player in the round can receive also tends to be higher. One potential explanation is that when the overall skill level of players in a game round is high, the round typically lasts longer and provides more opportunities for a player to increase her score. We compare the model fit statistics among the four models, and find that the model fit improves significantly after we add $\ln(AvgScore_{ir}^{PREV})$ and $\ln(TtlScore_{ir}^{PREV})$ into the regression model. However, the fit does not further improve after the interaction term between $Percentile_{ir}$ and $AvgRank_r$ is introduced into the models. For parsimony, we use results of specification (3) for the design of the matching algorithm, which is elaborated in the next section.[8]

[8] As a robustness check, we also estimate this regression using (1) data from only PC rounds, and (2) data from only console rounds. The estimation results are not sensitive to the choice of the sample.

Table 8 Regression of ln(score)

	Dependent variable: $\ln(Score_{ir})$			
	(1)	(2)	(3)	(4)
$Intercept$	6.828***	6.624***	3.998***	3.984***
	(0.005)	(0.010)	(0.018)	(0.020)
$Percentile_{ir}$	0.950***	1.252***	0.484***	0.511***
	(0.005)	(0.014)	(0.006)	(0.015)
$AvgRank_r$	0.011***	0.015***	0.004***	0.004***
	(0.0001)	(0.0002)	(0.0001)	(0.0002)
$Percentile_{ir} * AvgRank_r$		−0.007***		−0.001**
		(0.0003)		(0.0003)
$\ln(AvgScore_{ir}^{PREV})$			0.446***	0.445***
			(0.003)	(0.003)
$\ln(TtlScore_{ir}^{PREV})$			−0.023***	−0.022***
			(0.001)	(0.001)
Observations	588,983	588,983	588,983	588,983
R^2	0.082	0.083	0.123	0.123
Adjusted R^2	0.082	0.083	0.123	0.123
Residual Std. Error	1.116	1.116	1.091	1.091
	(df = 588980)	(df = 588979)	(df = 588978)	(df = 588977)
F Statistic	26,279.880***	17,707.810***	20,699.220***	16,560.330***
	(df = 2; 588980)	(df = 3; 588979)	(df = 4; 588978)	(df = 5; 588977)

Note: *p<0.1; **p<0.05; ***p<0.01

5.4. Robustness Checks

In this section, we describe a series of robustness checks that we carried out to ensure that our empirical results are reliable. For the sake of brevity, we describe these briefly below - the detailed result for each robustness check is included in the Appendix. Overall, we find that our results are generally reliable to various sets of player data and model structures.

1. *Cohort Differences*: The video game company classified the players in our sample into two cohorts, based on the time they started to play the video game. Players who registered between October 25, 2011 and November 24, 2011 are defined as Cohort 1, and those who registered between January 1, 2012 and January 31, 2012 are defined as Cohort 2. Cohort 1 players purchased the game within one month of its release, and can be viewed as "early adopters;" Cohort 2 players can be viewed as "followers." To ensure that the game-play behavior of Cohort 1 players is not different from that of Cohort 2 players, we examine whether individuals' latent class membership is correlated with the cohorts. As shown in Figure 5, Cohort 1 players seem to have a lower probability of belonging to Class 1, the class whose engagement level tends to be lower, as compared to Cohort 2. However, the difference does not seem to be material.[9] In addition, we also estimate the "2

[9] We also explore whether players' class membership is correlated with their characteristics. Specifically, we regress the posterior probability of players belonging to Class 1 on a set of observed player characteristics - user age, whether a player belongs to Cohort 1, is from the U.S., and is a PC player. The results confirm that Cohort 2 players have a slightly higher probability of belonging to Class 1.

Classes – 3 States" HMM with the game-play data for only individuals in the Cohort 1 (here we allow two classes within Cohort 1). The estimation results based on the Cohort 1 sub-sample are similar to the estimation results based on the full sample, which suggests little heterogeneity in the evolution of the engagement state between Cohort 1 players and Cohort 2 players.[10]

2. *Console Players Only*: Our main model does not explicitly control for potential behavioral differences between PC players and console players. Since console players are more likely to use Quick Match, we estimate the "2 Classes – 3 States" HMM with data associated with console players only. The estimation results remain largely unchanged.

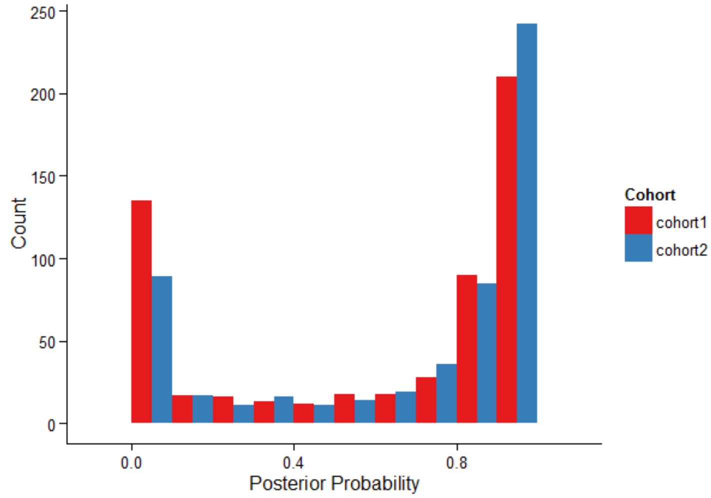
3. *Movement Across States*: We only allow players to move between two adjacent states. In an alternative model, we allow players to drop from the high engagement state directly to the low state to capture the possibility that they suddenly lose interest in the game completely. The estimated probability of falling to the low state from the high state is very small (≤ 0.001 when all X_{it} variables are set to 0). The estimates of other parameters in this model are similar to those in the main model.

4. *No Play Periods and Engagement States*: We estimate a "2 Classes – 3 States" HMM, in which players are allowed to move between engagement states in weeks when they do not play the game. Specifically, we assume that when $d_{it} = 0$, player i's latent propensity to move from engagement level k to another level in period t is $Z_{it} = \eta_{0k} + \eta_{4k} \cdot \ln(\textit{no. weeks since sign-up}) + \epsilon_{it}$, where η_{0k} determines the baseline transition propensity for state level k, and $\ln(\textit{no. weeks since sign-up})$ affects the transition propensity in the same way as it does in weeks when player i plays the game. The estimated η_{1k}, η_{2k}, η_{3k} and η_{4k} in this model all have the same signs and similar magnitudes as those in the main model. This model generally predicts larger probabilities of moving towards higher engagement states when $d_{it} = 1$ than the main model does, because when players are allowed to move between engagement states in weeks when $d_{it} = 0$, they tend move towards lower engagement states. The estimates of other parameters in this model are similar to those in the main model.

5. *Class-Specific $\eta's$*: In the main model, we only allow the threshold parameters ($\mu's$) to be class-specific. However, it is possible that the different classes of players react differently to the past game-play outcomes. We attempt to estimate a "2 Classes – 3 States" HMM that allows $\eta's$ also to be class-specific, but we ran into convergence and stability issues, most likely due to the large number of parameters and the highly nonlinear model. We are able to estimate a model with two latent classes and two engagement state levels and this model (in-sample log-likelihood: -159,930; BIC: 320,049; validation log-likelihood: -27,555) does not perform as well as the "2 Classes – 3 State" main model both in- and out-of-sample.

[10] Note that the estimated fraction of Class 1 players equals $\exp(\tau)/(1 + \exp(\tau))$, not τ. For example, when $\tau = 0.4$, the fraction is 0.598; when $\tau = 0.6$, the fraction is 0.645.

Figure 5 **Histogram of the Posterior Probability of Belonging to Class 1 by Cohort**



6. Matching Algorithm

In the second stage of our two-stage data analytic approach, we develop a matching algorithm based on the estimated HMM. Specifically, we construct a learning-based matching algorithm to maximize overall player retention, and then numerically test the performance of the proposed algorithm against two algorithms commonly used in practice.

6.1. The "Learn-and-Exploit" Approach to Matching

The objective of the matching algorithm is to maximize the expected number of rounds played by a player during a fixed period of time beginning from when she starts to play the game. Let G_{rt} denote the set of candidate rounds for the r^{th} round in week t and let $\overline{Score}$ denote the deterministic part of the scoring function (Equation 2 excluding ξ_{ir}). Throughout the remainder of this section, we will use $\overline{Score}(g, i)$ to denote the deterministic score that player i will get if she is assigned to candidate round g.

Our proposed algorithm is called "Learn-and-Exploit Matching." This algorithm is motivated by the observation that players in a higher engagement state are more likely to play the game, and to play more rounds on average. Therefore, maximizing a player's game-play activity is equivalent to maximizing the probability that she stays in/moves towards the high engagement state in every period. The results presented in §5 suggest that player heterogeneity and engagement state leads to differential responses to game-play outcomes. Hence, an effective algorithm should take into account both the player's current engagement state and her class. In Table 7, the coefficient of the term Ln(std score) is positive for the low/medium-state players, but negative for high-state players, indicating that variation in game-play outcomes keeps low/medium-state players engaged but discourages high-state players from staying engaged. Therefore, when maximizing the probability of low/medium-state players transitioning to a higher engagement state, we need to find a balance between maximizing the total score in each period and maintaining a reasonable level of score variation among scores a player received in different rounds played in the same period. In other words, we need to find a balance between achievement and challenge – two of the fundamental motivations driving gamer engagement. This motivates us to develop a matching algorithm that alternately matches low/medium-state players to a round that guarantees the highest expected score in the round and a round that leads to a lower excepted score to create sufficient variability among the scores collected during a period. Since both a player's engagement state and class are unobservable, we need to infer or "learn" her engagement state and class from her observed game-play activity in previous periods and then "exploit" that learning to get the best possible match. We label our algorithm Learn-and-Exploit Matching. We first describe the algorithm, and then provide the intuition behind why it works.

The purpose of the first step is to learn player i's engagement state at the beginning of *last* week (the time period) to help us infer her current engagement state at the beginning of week t. To do so, we propose an approximate learning scheme that infers the true engagement state of player i at the beginning of week $t-1$ using only the number of rounds she played in that week. The intuition is as follows: If player i plays a lot of rounds during week $t-1$, she is likely to start week $t-1$ with a High engagement state; if, on the other hand, she only plays a few rounds, she is likely to start with a Low engagement state. Thus, the number of rounds played in the previous week has good predictive power to help us infer player i's engagement state at the beginning of the current week. In the second step, we update our belief regarding player i's true class. In theory, we can calculate the posterior probability that player i belongs to a certain class conditioned on *all* the observed outcomes starting from week 1 (i.e., $O_{i1}, O_{i2}, \ldots, O_{it}$) where O_{it} denotes the observed outcomes of player i in period t. While this is conceptually straight-forward, the actual computation involves a series of multi-dimension integrations in continuous space. This is highly burdensome computationally. Since both speed and simplicity of a matching algorithm are of utmost importance to many game companies, instead of performing an exact inference calculation, we resort to an approximate inference scheme based on the idea of maximum likelihood. In particular, we will act as if player i's true engagement state at the beginning of week $v \leq t-1$ is given by $\hat{s}_{iv}$ as defined in this step. In the third step, we infer the most likely engagement state at the beginning of week t given $\hat{s}_{i,t-1}$, $O_{i,t-1}$, and c_{it}. We then use both $\tilde{s}_{it}$ (our inferred engagement state at the beginning of week t) and c_{it} (our most up-to-date guess on the true class of player i) to make a matching decision in the fourth step.

Learn-and-Exploit Matching

For week $t \geq 1$, do:

1. If $t \geq 2$, calculate $\hat{s}_{i,t-1} = \arg\max_k f(r_{i,t-1}|S_{i,t-1} = k)$
2. If $t \geq 3$, calculate $c_{it} = \arg\max_c \hat{L}_{i,t-1}(c)$, where

$$\hat{L}_{i,t-1}(c) \;=\; \prod_{v=1}^{t-2} P(S_{i,v+1} = \hat{s}_{i,v+1}|S_{iv} = \hat{s}_{iv}, O_{iv}, c)^{d_{iv}};$$

   otherwise, set c_{it} equals the most likely class (by prior probability).

3. Calculate $\tilde{s}_{it} = \arg\max_k P(S_{it} = k|S_{i,t-1} = \hat{s}_{i,t-1}, O_{i,t-1}, c_{it})$
4. For odd round $r = 1, 3, \ldots$, match i to game g_{irt}, where

$$g_{irt} = \arg\max_{g \in G_{rt}} \overline{Score}(g, i).$$

   For even round $r = 2, 4, \ldots$, match i to game g_{irt}, where

$$g_{irt} = \arg\min_{g \in G_{rt}} \left| \overline{Score}(g, i) - (1 - \Delta_t(\tilde{s}_{it}, c_{it})) \cdot \max_{g' \in G_{rt}} \overline{Score}(g', i) \right|.$$

Based on our estimated model, two key variables directly affect the probability that player i will continue playing in week $t+1$: total score accumulated during week t and the standard deviation of scores received from individual rounds she plays in that week. Since these two elements do not always go in the same direction (e.g., increasing total scores may not yield the largest standard deviation), the challenge is to properly balance their impacts on player i's transition. We address this challenge by introducing a new set of parameters $\Delta_t(k, c)$ that can be properly tuned using offline simulation-based optimization. Given $\Delta_t(k, c)$, our matching algorithm works as follows: For *odd* rounds $1, 3, 5, \ldots$, we match player i to a candidate round that yields the highest possible score in the round; for *even* rounds $2, 4, 6, \ldots$, we match player i to a candidate round that yields a score closest to $(1 - \Delta_t(\tilde{s}_{it}, c_{it}))$ times the highest possible score in the round. If $\Delta_t(\tilde{s}_{it}, c_{it}) = 0$, we always match player i to a round that guarantees the highest possible score in the round. Note that the parameter $\Delta_t(\tilde{s}_{it}, c_{it})$ plays the role of ensuring player i has sufficient variation across all rounds she plays in week t. Although the prescribed algorithm alternates between odd and even rounds, we want to emphasize that this is not the only way to create variation in outcomes, because many types of randomization can achieve that. We use this alternating odd-even mechanism as an illustration of one such mechanism. Finally, in its most general form, the value of $\Delta_t(., .)$ can be different for different t. However, increasing the number of different Δ increases exponentially the complexity of the corresponding offline optimization. In the numerical experiments discussed below, we only test the algorithm where we use the same Δs at all t.

It is also important to understand some of the context that characterizes the description of our Learn-and-Exploit Matching algorithm. First, as noted above, we use approximate learning and inference schemes. Second, we match one player at a time. Third, we assume that the matching outcome impacts the performance of only the focal player (the one who is being matched). Fourth, we do not allow for the possibility of deferring matches, i.e., we do not wait to find "better" rounds for the focal player match. Finally, we assume multiple rounds are available for matching whenever the focal player wants to play. The data suggests that the range of available rounds is from 100 to 3000. The more rounds available, the better the match. We set the number of available rounds to the lowest number (100). Relaxing any of these assumptions is likely to provide better matches and therefore a higher level of improvement. Thus, the results we present are conservative and should be interpreted as the lower bound in terms of improvement in retention.

6.2. Numerical Experiments

To evaluate the performance of the proposed Learn-and-Exploit Matching algorithm, we benchmark the algorithm against two alternative algorithms that are most commonly used in multi-player online video games – Random Matching and Closest Rank Matching.

1. Random Matching. As the name suggests, during each round, this algorithm matches a player to a candidate round *randomly*. It is the simplest algorithm among all, as it does not take into account either the observable characteristics of the player (such as her rank) or the unobserved characteristics of the player (such as her class and engagement state).

2. Closest Rank Matching. Unlike Random Matching, which is completely oblivious to characteristics of the player, the Closest Rank Matching algorithm matches a player to a candidate round in which the average rank of all existing players is closest to her own rank, in absolute value. This algorithm is motivated by the belief that a player's engagement will be high if she plays in a round with other players of similar skill levels.

In our numerical experiments, we measure the average number of rounds played by a randomly generated set of 500 players during a period of 104 weeks or two years. Each player is characterized by her class (generated randomly according to the estimated prior probability in §5) and joins the system in week 1 (thus, their initial ranks are all zero). We treat the randomly generated 500 players as if they are 500 independent Monte Carlo simulations. The sequence underlying our numerical experiments is as follows:

Step 1. Generate 500 random players and index them with i.

Step 2. Generate 100 random candidate rounds for each round to be played, for 300 rounds per week.[11]

Step 3. For week $t = 1$ to 104 and $i = 1$ to 500, do:

1. At the beginning of week t, generate the decision to play d_{it};
2. If $d_{it} = 1$, do:
    i. Generate the number of rounds r_{it} to play in week t;
    ii. For each round, match player i to one of the 100 pre-generated candidate rounds using a matching algorithm.

Step 4. Output the average number of rounds played in 104 weeks across 500 simulated players:

$$\text{AVG} = \frac{1}{500} \sum_{t=1}^{104} \sum_{i=1}^{500} r_{it}.$$

We report the results of our numerical experiments in Tables 9 and 10. The plot of average number of rounds played per week for the three algorithms can be seen in Figure 6. Four aspects are worth highlighting: First, in terms of the number of rounds played during a two-year period, Random Matching performs the worst among all the algorithms tested. This is perhaps not too surprising given that Random Matching completely ignores all player characteristics, both observable and unobservable, when doing the matching. Second, the idea that a player enjoys playing with peers who have a similar skill level may not be completely unfounded. Both the numbers in Table 9 and the plot in Figure 6 suggest that Closest Rank Matching outperforms Random Matching, especially during the first thirty weeks. During the later weeks, Random Matching and Closest Rank Matching seem to have a relatively comparable performance. Third, in a two year span, Learn-and-Exploit Matching increases the expected number of weeks of playing and the expected number of rounds played by 4.04% and 7.79% of the best numbers that Random Matching and Closest Rank Matching produce, respectively, which highlights the potential benefit of using our HMM results, together with learning the player's unobservable characteristics, in matching assignment. Fourth, Table 10 and Figure 6 also provide an interesting result that most of the improvement of Learn-and-Exploit Matching over the other two matching algorithms happen in later weeks as there does not seem to be much improvement during the first 30 weeks. The intuition behind this result is as follows. During the early periods,

[11] In our experiments, we only generate 300 rounds per week, because, per our results in Section 5, the probability that a player plays more than 300 rounds per week is very small.

Table 9 **The average number of playing weeks and rounds played in two years for different algorithms.**

	Algorithm	Number of playing weeks	Number of rounds played
Starting week 1	Random	28.84	629.42
	Closest Rank	29.23	642.23
	Learn-and-Exploit	30.40	692.25
Starting week 27	Random	17.25	357.67
	Closest Rank	17.34	357.42
	Learn-and-Exploit	18.45	402.60
Starting week 53	Random	10.03	198.26
	Closest Rank	10.04	195.20
	Learn-and-Exploit	10.75	223.91
Starting week 79	Random	4.61	83.88
	Closest Rank	4.66	83.10
	Learn-and-Exploit	5.01	96.67

Table 10 **Percentage improvement of Learn-and-Exploit over the best of Random and Closest Rank.**

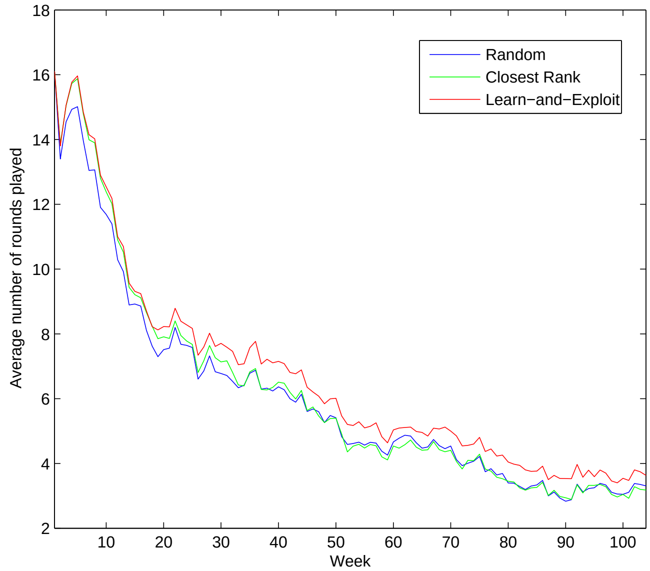
	Number of playing weeks	Number of rounds played
Starting week 1	4.04	7.79
Starting week 27	6.42	12.56
Starting week 53	7.07	12.94
Starting week 79	7.38	15.25

most players are still in a high engagement state and matching assignments have little impact on their state transition. In later periods, players' engagement level is significantly affected by the loss of curiosity, resulting from the longer period since registration. At the same time, the effects of the two main factors considered in the Learn-and-Exploit Matching algorithm, i.e., the total score a player collects in a period and the standard deviation of scores she receives from individual rounds she plays in that period, become stronger. This provides the "bump" in play probability and the number of rounds played in later weeks (conditional on play).

In order to measure the economic impact of the improvements in the total number of rounds played and the number of active weeks (defined as weeks in which a player played at least one round) reported above, we use the link between these two numbers and a player's propensity to upgrade to the next version of the game. In the following logistic regression, we use the total number of rounds played and/or the number of active weeks to predict whether a player will eventually upgrade to the next version of the game. The results (Table 11) suggest that both the total number of rounds played and the number of active weeks positively affect the probability of upgrade. Due to the high correlation (0.90) between ln(No. of active weeks) and ln(No. of rounds played), we can assess the impact of changing only one of them at a time. We plug in the improvement numbers from above (4.04% in active weeks or 7.79% in rounds played)[12] and find that there is a 1.3% increase in the upgrade probability. Given that the game had 9.5 million players and the average

[12] In our setting, the upgrade was released 87 weeks after the launch of the current version of the game.

Figure 6 Plot of average number of rounds played.



cost of the upgrade was \$60, this additional play results in increased revenue of about \$7.4 million. To put this in context, this number represents 1% of the total revenues from this game to date. This represents a very conservative estimate – both from the substantive (retention has many other benefits, e.g., increased in-game purchases, higher ad revenue) and the methodological point of view (as noted above).

Table 11 Logistic Regression of the Probability of Upgrading to the New Version

	Dependent variable: $I(Upgrade)$		
Intercept	-3.537^{***}	-3.912^{***}	-3.499^{***}
	(0.259)	(0.322)	(0.321)
ln(No. of Weeks Played)	0.977^{***}		1.005^{***}
	(0.078)		(0.160)
ln(No. of Rounds Played)		0.598^{***}	-0.022
		(0.054)	(0.109)
Observations	1,309	1,309	1,309
Log Likelihood	−771.051	−792.273	−771.032
Akaike Inf. Crit.	1,546.103	1,588.545	1,548.064
Note:			*p<0.1; **p<0.05; ***p<0.01

7. Discussion and Conclusion

In the online video game industry, while it is commonly believed that players' engagement level is directly affected by their motivations, game-play experience and outcomes, there has been little empirical research to support this belief. Based on the gaming literature, we use data-based proxies to capture the motivations of achievement/competence, self-efficacy, challenge and curiosity; we link them to engagement and explore the impact of dynamically evolving engagement on a critical business outcome – customer retention.

We propose a novel two-stage data analytic modeling approach. In the first stage, we construct a HMM of players' game-play dynamics and estimate it using 1,309 players' game-play history over a 29-month period. Consistent with findings from the existing theoretical and experimental studies on video game design, we find that in-game competence and self-efficacy, the sense of being challenged and the (lack of) sense of achievement can all affect players' engagement levels and its evolution. More interestingly, our results reveal that the effects of these factors vary across players at different engagement levels. While competence and self-efficacy encourage players in a low(high) engagement state to become(remain) engaged, it has a slight negative effect on the future engagement level of players who are currently in a medium engagement state. Being challenged positively affects the future engagement level of players who are currently less engaged, but negatively affects the future engagement level of players who are currently highly engaged. These results suggest that highly engaged players prefer to consistently receive high scores and enjoy the feeling of achievement while players at a medium engagement level care more about the sense of being challenged. For players in a low engagement state, both achievement and the sense of challenge have a modest positive effect on their future engagement level. Moreover, the number of weeks a player has played without being able to level up negatively affects players' future engagement level no matter what their current engagement level is, reinforcing the importance of the sense of achievement to player retention. The data also suggests a natural decay in players' engagement level, but the speed of the decay is much slower among highly engaged players, and faster among players who have already shown signs of losing interest. Finally, our results suggest that there is significant heterogeneity in interest and engagement level across players.

In the second stage, we use the estimates from the first stage to develop and calibrate a dynamic matching algorithm, Learn-and-Exploit Matching, which incorporates each player's time-invariant inherent interest in the game and her time-varying engagement level, both inferred from her individual-level game-play history. This algorithm uses different policies to assign players in different (predicted) engagement states. We benchmark our proposed algorithm against two algorithms most commonly used in industry – Random Matching and Closest Rank Matching. Our numerical experiments show that by incorporating players' engagement state and their latent interest level into the assignment algorithm, the video game company can improve player retention. This improvement in retention is economically significant in terms of increasing the firm's revenue.

In conclusion, our paper is one of the first studies to use behavioral data to model the evolution of gamer engagement states based on motivational factors. It shows that these motivations have a differential impact on customers at different engagement levels in the online video game industry. Specifically, we find that high-, medium- and low-engagement-state players are motivated differentially by achievement and challenge. The novel two-stage data analytic modeling approach we develop demonstrates how statistical modeling and optimization techniques can be seamlessly integrated to generate business insights and assist decision making in real-time. Finally, we think that our approach – using observables to model the evolution of engagement (an unobservable) and its link to business outcomes that can be optimized – can act as a general template for modeling customer-firm interactions where engagement plays a big role.

Our study has several limitations. First, we look at gamer data for only one game from an important game genre. Second, we examine only one aspect of the play experience – matching – to increase engagement while ignoring factors such as game interface and reward structure. Third, we cannot formally establish the connection between player engagement/retention and player spending on in-game purchases (an important revenue stream for the company). Fourth, we cannot incorporate social interactions among players into our model. Most these limitations are a function of the data available to us. Finally, our proposed matching algorithm does not incorporate the entire context in which matching is done (e.g., it does not consider how the matched player will directly impact the game-play outcome and state transition of all players in the matched round), as our objective is to show how the firm can leverage engagement in matching to obtain better outcomes in real time. We hope future research can address these limitations.

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References

Abhishek, V. and P. Fader, and K. Hosanagar. 2015. Media Exposure through the Funnel: A Model of Multi-Stage Attribution, *working paper*.

Amabile, T. and S. Kramer. 2011. Three Secrets of the Video Game Designer. *Harvard Business Review*, August 2011.

Anderson Jr., E. G., G. G. Parker, B. Tan. 2014. Platform Performance Investment in the Presence of Network Externalities. *Information Systems Research*, **25(1)**: 152–172.

Ayabakan, S., I. Bardhan, and E. Zheng. 2016. What Drives Patient Readmissions? A new Perspective from the Hidden Markov Model Analysis. *Proceedings of International Conference on Information Systems 2016*.

Bartle, R.A. 2004. Designing Virtual Worlds. Indianapolis, IN: New Riders Publishing.

Boyle, E. A., T. M. Connolly, T. Hainey, and J. M. Boyle. 2012. Engagement in Digital Entertainment Games: A Systematic Review. *Computers in Human Behavior*, **28(3)**: 771-780.

Cameron, A. C. and P. K. Trivedi. 1998. *Regression Analysis of Count Data.* New York: Cambridge University Press.

Chen, W., X. Wei, and K. X. Zhu. 2017. Engaging Voluntary Contributions in Online Communities: A Hidden Markov Model. *Management Information Systems Quarterly, forthcoming.*

Chu, L. Y., and H. Zhang. 2011. Optimal Preorder Strategy with Endogenous Information Control. *Management Science*, **57(6)**:1055–1077.

Deterding, S. 2012. Gamification: Designing for Motivation. *Interactions*, **19(4)**: 127–165.

Dou, Y., M. F. Niculescu, and D. J. Wu. 2013. Engineering Optimal Network Effects via Social Media Features and Seeding in Markets for Digital Goods and Services. *Information Systems Research*, **24(1)**:164–185.

Dube, J. H., G. J. Hitsch, and P.K. Chintagunta. 2010. Tipping and Concentration in Markets with Indirect Network Effects. *Marketing Science*, **29(2)**: 216-249

Gaudiosi. J., 2012. New Reports Forecast Global Video Game Industry Will Reach $82 Billion By 2017. Forbes. Retrieved from http://www.forbes.com/sites/johngaudiosi/2012/07/18/new-reports-forecasts-global-video-game-industry-will-reach-82-billion-by-2017/

GameAnalytics. 2015. What Analyzing 400+ Games Has Taught Us. Retrieved from http://blog.gameanalytics.com/blog/what-analysing-400-games-taught-us.html.

Hao, L., X. Li, Y. Tan, and J. Xu. 2011. The Economic Role of Rating Behavior in Third-Party Application Market, *Proceedings of International Conference on Information Systems 2011.*

Hartmann, T., Y.Jung, and P. Vorderer. 2012. What Determines Video Game Use? *Journal of Media Psychology* **24(1)**: 19–30.

Hofacker, C. F., K. Ruyter, N.H. Lurie, P. Manchanda, and J. Donaldson. 2015. Gamification and Mobile Marketing, *Journal of Interactive Marketing*, forthcoming.

Klimmt, C., and T. Hartmann. 2006. Effectance, Self-efficacy, and the Motivation to Play Video Games. *Playing Video Games*, New York, 133-145.

Krishnamurthy, V. 2016. *Partially Observed Markov Decision Processes.* Cambridge University Press.

Kwon, H. E., H. So, S. P. Han, and W. Oh. 2016. Excessive Dependence on Mobile Social Apps: A Rational Addiction Perspective. *Information Systems Research*, **27(4)**: 919–939.

Lahiri, A., and D. Dey. 2013. Effects of Piracy on Quality of Information Goods. *Management Science*, **59(1)**: 245–264.

Liu, H. 2010. Dynamics of Pricing in the Video Game Console Market:Skimming or Penetration? *Journal of Marketing Research*, **47(3)**:428-43.

Malone, T. W. 1981. Toward a Theory of Intrinsically Motivating Instruction, *Cognitive Science*, **4**: 333-369.

MacDonald, I. L., and W. Zucchini. 1997. *Hidden Markov and Other Models for Discrete – Valued Time Series*. London: Chapman and Hall.

Mullahy, J. 1986. Specification and testing of some modified count data models. *Journal of Econometrics*, **3(3)**:, 341–365.

Nair, J. 2007. Intertemporal price discrimination with forward-looking Consumers: Application to the US Market for Console Video-Games *Quantitative Marketing and Economics* **5(3)**: 239–292.

Netzer, O., J. M. Lattin, and V. Srinivasan. 2008. A Hidden Markov Model of Customer Relationship Dynamics. *Marketing Science*, **27(2)**: 185–204.

Newzoo. 2017. The Global Games Market Will Reach $108.9 Billion in 2017. Retrieved from https://newzoo.com/insights/articles/the-global-games-market-will-reach-108-9-billion-in-2017-with-mobile-taking-42/.

Niculescu, M. F., D. J. Wu. 2014. Economics of Free Under Perpetual Licensing: Implications for the Software Industry. *Information Systems Research*, **25(1)**: 173–199.

Ryan, R. M., C. S. Rigby, and A. Przybylski. 2006. The Motivational Pull of Video Games: A Self-Determination Theory Approach. *Motivation and Emotion*, **30(4)**: 344–360.

Roquilly, C. 2011. Control over Virtual Worlds by Game Companies: Issues and Recommendations. *MIS Quarterly*, **35(3)**: 653-671.

Sahoo, N., P. V., Singh, and T. Mukhopadhyay. 2012. A Hidden Markov Model for Collaborative Filtering, *Management Information Systems Quarterly*, **36(4)**: 1329–1356

Salen, K., and E. Zimmerman. 2004. *Rules of Play: Game Design Fundamentals*. Cambridge, MA: MIT Press.

Shankar, V., B. Bayus. 2003. Network effects and competition: An empirical analysis of the home video game industry. *Strategic Management Journal*, **24(4)**: 375-384.

Sherry, J. L., K. Lucas, B. S. Greenberg, and K. Lachlan. 2006. Video Game Uses and Gratifications as Predictors of Use and Game Preference. *Playing Video Games*, New York, 213-224.

Singh, P.V. , Y. Tan, N. Youn. 2011 A Hidden Markov Model of Developer Learning Dynamics in Open Source Software Projects. *Information Systems Research*, **22(4)** 790–807.

Voss, K. E., E. R. Spangenberg, B. Grohmann. 2003. Measuring the Hedonic and Utilitarian Dimensions of Consumer Attitude. *Journal of Marketing Research*, **40(3)**: 310-320.

Wu, C., Y. Chen, Y. Cho. 2013. Nested Network Effects in Online Free Games with Accessory Selling. *Journal of Interactive Marketing*, **27(3)**: 158-171.

Yan, L., and Y. Tan. 2014. Feeling blue? Go online: An Empirical Study of Social Support among Patients. *Information Systems Research*, **25(4)** 690–709.

Zhang, H. 2010. Partially observable Markov decision processes: A geometric technique and analysis. *Operations Research*, **58(1)**: 214-228.

Zhou, Y. 2016. Bayesian Estimation of a Dynamic Model of Two-Sided Markets: Application to the US Video Game Industry. *Management Science, Articles in Advance*.

Zhu, F. and M. Iansiti. 2012. Entry into platformbased markets. *Strategic Management Journal*, **33:1**: 88–106.

Zhu, F. and X., Zhang. 2010. Impact of online consumer reviews on sales: The moderating role of product and consumer characteristics. *Journal of Marketing*, **74(2)**: 133–148.

Appendix: Robustness Check Results

Table A.1 Estimation Results for the HMM (2 Classes, 3 States, Cohort 1 Only)

Parameter	Description	Low	Medium	High
α_k	Intercept in the logit probability of playing	-3.169(0.028)	0.925 (0.036)	4.127(0.150)
γ_k	Mean of log(no. rounds played)	1.486(0.026)	1.774(0.126)	3.284(0.123)
$log(\sigma_k^2)$	Log(variance of log(no. rounds played))	0.080(0.016)	0.004 (0.007)	-0.344(0.011)
η_{1k}	Ln(total score)	0.178(0.046)	-0.044(0.019)	1.288(0.083)
η_{2k}	Ln(std score)	0.110(0.025)	0.077(0.014)	-0.833(0.110)
η_{3k}	Ln(No. weeks same rank)	-0.100(0.095)	-0.047(0.045)	-0.091(0.075)
η_{4k}	Ln(No. weeks since sign-up)	-0.968(0.079)	-0.344(0.027)	-0.184(0.056)
	Class 1			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-1.044(0.130)	6.736(0.639)
μ_{hk}	Threshold of moving to a higher state	-1.847(0.270)	0.940(0.097)	∞
	Class 2			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-3.221(0.287)	6.970(0.132)
μ_{hk}	Threshold of moving to a higher state	-1.802(0.104)	0.922(0.101)	∞
τ	Intercept in the logit proportion of Class 1 players	0.618(0.160)		
N			674	
log-likelihood			-102,768	
BIC			205,340	

Table A.2 Estimation Results for the HMM (2 Classes, 3 States, Console Only)

Parameter	Description	Low	Medium	High
α_k	Intercept in the logit probability of playing	-3.290(0.025)	0.910 (0.032)	4.636(0.147)
γ_k	Mean of log(no. rounds played)	1.571(0.023)	1.875(0.122)	3.447(0.120)
$log(\sigma_k^2)$	Log(variance of log(no. rounds played))	0.095(0.014)	0.023 (0.007)	-0.334(0.011)
η_{1k}	Ln(total score)	0.236(0.039)	-0.071(0.016)	1.503(0.078)
η_{2k}	Ln(std score)	0.047(0.023)	0.123(0.016)	-1.040(0.091)
η_{3k}	Ln(No. weeks same rank)	-0.164(0.082)	-0.055(0.039)	-0.083(0.063)
η_{4k}	Ln(No. weeks since sign-up)	-0.861(0.079)	-0.340(0.023)	-0.072(0.051)
	Class 1			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-1.569(0.107)	8.242(0.639)
μ_{hk}	Threshold of moving to a higher state	-1.227(0.216)	1.110(0.099)	∞
	Class 2			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-3.221(0.287)	6.970(0.132)
μ_{hk}	Threshold of moving to a higher state	-1.802(0.104)	0.940(0.097)	∞
τ	Intercept in the logit proportion of Class 1 players	0.434(0.122)		
N			890	
log-likelihood			-132,629	
BIC			265,051	

Table A.3 Estimation Results for the HMM (Allowing for Transitioning from H to L)

Parameter	Description	Low	Medium	High
α_k	Intercept in the logit probability of playing	-3.210(0.023)	0.939 (0.032)	4.739(0.196)
γ_k	Mean of log(no. rounds played)	1.516(0.021)	1.801(0.090)	3.340(0.129)
$log(\sigma_k^2)$	Log(variance of log(no. rounds played))	0.089(0.013)	0.013 (0.006)	-0.340(0.009)
η_{1k}	Ln(total score)	0.185(0.037)	-0.165(0.023)	1.178(0.059)
η_{2k}	Ln(std score)	0.083(0.021)	0.053(0.012)	-1.182(0.086)
η_{3k}	Ln(No. weeks same rank)	-0.052(0.079)	-0.151(0.038)	-0.081(0.057)
η_{4k}	Ln(No. weeks since sign-up)	-0.992(0.078)	-0.319(0.023)	-0.030(0.040)
	Class 1			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	0.199(0.196)	3.614(1.251)(HM)
				-7.957(2.211)(HL)
μ_{hk}	Threshold of moving to a higher state	-1.924(0.217)	2.666(0.096)	∞
	Class 2			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-1.423(0.180)	2.249(0.689) (HM)
				-8.206(1.919)(HL)
μ_{hk}	Threshold of moving to a higher state	-2.658(0.0269)	2.468(0.198)	∞
τ	Intercept in the logit proportion of Class 1 players	0.400(0.139)		
N			1,109	
log-likelihood			-155,242	
BIC			310,708	

Table A.4 Estimation Results for the HMM (2 Classes, 3 States, Allowing for Transitions in No-Play Periods)

Parameter	Description	Low	Medium	High
α_k	Intercept in the logit probability of playing	-3.224(0.027)	0.911 (0.039)	4.673(0.151)
γ_k	Mean of log(no. rounds played)	0.742(0.032)	1.775(0.231)	3.335(0.127)
$log(\sigma_k^2)$	Log(variance of log(no. rounds played))	0.024(0.016)	-0.045 (0.007)	-0.351(0.009)
η_{0k}	No-play period intercept	-2.823(0.339)	-1.576(0.132)	7.452 (0.896)
η_{1k}	Ln(total score)	0.185(0.049)	-0.153(0.016)	1.428(0.072)
η_{2k}	Ln(std score)	0.022(0.031)	0.157(0.013)	-0.905(0.088)
η_{3k}	Ln(No. weeks same rank)	-0.105(0.099)	-0.203(0.042)	-0.112(0.055)
η_{4k}	Ln(No. weeks since sign-up)	-0.802(0.052)	-0.221(0.021)	-0.182(0.045)
	Class 1			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-2.481(0.101)	7.790(0.545)
μ_{hk}	Threshold of moving to a higher state	-1.700(0.327)	0.830(0.102)	∞
	Class 2			
μ_{lk}	Threshold of moving to a lower state	$-\infty$	-3.188(0.331)	6.770(0.122)
μ_{hk}	Threshold of moving to a higher state	-2.611(0.130)	0.786 (0.107)	∞
τ	Intercept in the logit proportion of Class 1 players	0.448(0.096)		
N			1,109	
log-likelihood			-154,566	
BIC			309,344	