Question 1 : 0.1 Computer Problems

1. Use the function nest to evaluate P(x) = 1 + x + · · · + x50 at x = 1.00001. (Use the Matlab ones command to save typing.) Find the error of the computation by comparing with

the equivalent expression Q(x) = (x51 − 1)/(x − 1).

Soln:

By nesting function, P (1.00001) = 51.012752082749990

By plugging x in polynomial, Q(x) = 51.0127520827452

The absolute error = |P(x)- Q(x)| = 4.70E-12

The Relative error = |[P(x)- Q(x)]/ Q(x)| = 9.21E-14

```
% Function: denote function as nest
function y=nest(d,c,x,b)
if nargin<4. b=zeros(d,1);end
y = c(d+1);
for i = d:-1:1
    y = y.*(x-b(i))+c(i);
end

% Input: degree d of polynomial,
% array of d+1 coefficients c (constant term first),
% x-coordinate x at which to evaluate, and
% array of d base points b, if needed
% Output: value y of polynomial at x

d = 50
c = ones(51,1)
x = 1.00001
b = zeros(d,1)

y = nest(d,c,x,b)
y

% Function: denote function as q
function z=q(x)
z = (x^51-1)/(x-1);
% Input: x
% Output: value y of polynomial at x
z = q(x)
z
```

Question 6: 1.1 Computer Problems

3. Use the Bisection Method to locate all solutions of the following equations. Sketch the function by using Matlab's plot command and identify three intervals of length one that contain a root. Then find the roots to six correct decimal places. (a) 2x3 − 6x − 1 = 0

soln:

Use plot command, we find three intervals containing roots:

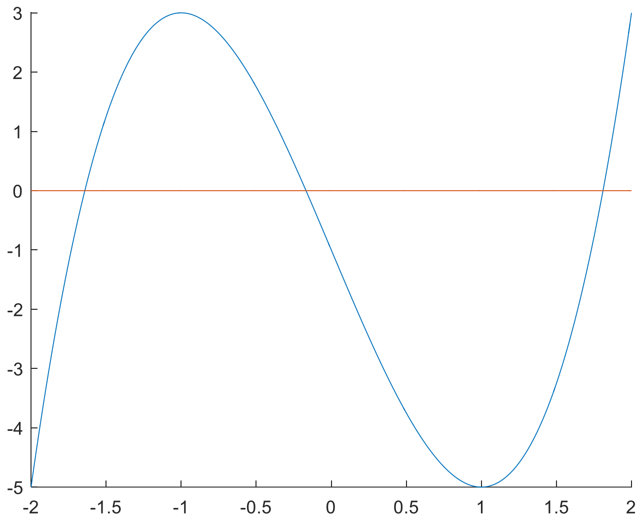
[-2,-1], [-0.5,0.5], [1,2]

Use bisection method, we can solve roots to be:

X1 = -1.641784

X2 = -0.168254

X3 = 1.810038




```
% Function: denote function as bisect
function bisect(f,a,b,tol)
if sign(f(a))*sign(f(b))>=0
    disp('error')
end
fa=f(a);
fb=f(b);
while (b-a)>tol
    c=(a+b)/2;
    fc=f(c);
    if f(c)==0
        break
    end
    if sign(f(a))*sign(f(c))<0
        b=c;fb=fc;
    else
```

```
        a=c;fa=fc;
    end
end
xc=(a+b)/2;
end

% Plot the graph between x: [-2,2]

x=-2:0.01:2;
F=zeros(1,length(x));
for i=1:length(x)
F(i)=2*x(i)^3-6*x(i)-1;
end
y=zeros(1,length(x));
figure
hold on
plot(x,F)
plot(x,y)

f=@(x)2*x^3-6*x-1
a=-2;
b=-1;
tol=10^-7;
if sign(f(a))*sign(f(b))>=0
    disp('error')
end
while (b-a)>tol
    c=(a+b)/2;
    if f(c)==0
        break
    end
    if sign(f(a))*sign(f(c))<0
        b=c;
    else
        a=c;
    end
end
xc=(a+b)/2
```

Question 8: 1.2 Computer Problems

Example 1.3 showed that g(x) = cosx is a convergent FPI. Is the same true for g(x) = cos2 x?

Find the fixed point to six correct decimal places, and report the number of FPI steps needed.

Discuss local convergence, using Theorem 1.6.

Soln:

It is true for g(x) = cos2 x

Find the fixed point to be 0.291927

And the number of FPI steps to be 401

Assume that g is continuously differentiable, that g(r) = r, and that S = |g(r)| < 1. Then Fixed-Point Iteration converges linearly with rate S to the fixed point r for initial guesses sufficiently close to r.

```
% fpi function
function xc=fpi(g,x0,tol)
x(1)=x0;
x(2)=g(x(1));
i=1;
while abs(x(i+1)-x(i))>tol
    x(i+2)=g(x(i+1));
    i=i+1;
end
xc=g(x(i+1));
i
end

g=@(x)cos(x)^2
tol=10^(7)
r=fpi(g,0,tol)
```