ECE 461 Mini Project #1

March 28, 2017

Andrew Lubawy, EE
Due: 3/28/2017

1 Two-Transmission-Link Queueing System Simulator and Output

1.1 Code

```
In [1]: %matplotlib inline
        from pylab import *
        import numpy as np
        from collections import deque
```

This section of code implements a queue data type for use in my simulation. It can enqueue which adds an item to the back of the queue and is FIFO for dequeueing. Doing it this way closely represents a real queue and lets me easily track objects in the queue.

```
In [2]: class Queue:
            """A queue for use in simulating M/M/1/k.

            Attributes:
                k (int): Maximum customers allowed in the system.
                departures (list): A sample of departure intervals.
                queue (list): A deque object.
                dropped (int): Number of items dropped because queue was full.
                served (int): Number of items served from queue.
            """

            def __init__(self, k, mu, departures):
                """Forms a queue.

                Args:
                    k (int): Maximum customers allowed in the system.
                    mu (float): Rate out of the queue.
                    departures (int): Number of departure intervals to generate.
                """
```

```python
        self.k = k
        # Generates the departure intervals
        # according to an exponential distribution.
        self.departures = exponential(1/mu, departures)
        self.queue = deque([], k)
        self.dropped = 0
        self.served = 0

    def empty(self):
        """Checks if the queue is empty.

        Returns:
            True if empty, False otherwise.
        """
        return len(self.queue) is 0

    def is_full(self):
        """Checks if the queue is full.

        Returns:
            True if full, False otherwise.
        """
        return len(self.queue) is self.k

    def enqueue(self, item):
        """Adds an item to end of the queue.

        Args:
            item: An item to add to the queue.
        """
        if self.is_full():
            self.dropped += 1
        else:
            self.queue.append(item)

    def dequeue(self):
        """Removes the fist item from the queue."""
        if not self.empty():
            self.served += 1
            return self.queue.popleft()
        return None

    def get_size(self):
        """Get the size of the queue.

        Returns:
            An integer for the size of the queue.
        """
```

```
        return len(self.queue)
```

This next piece of code performs the actual simulation and keeps track of everything. My method generates some arrival interval according to an exponential distribution with relation to λ. I then check if the time is 0 to handle an edge case by immediately adding to the queue. To do that I need to generate a uniformly distributed random variable to compare with ϕ and determine which queue to use. I then add that arrival time to the queue for use later and to simulate a packet entering. With that out of the way I then go through a series of checks to make sure packets are dequeueing according to exponential service times with relation to μ. If anything should be dequeued I can then check its arrival time that I stored when dequeueing with the current time to determine delay while also incrementing a counter to check the number of packets serviced. Next, I'll have to enqueue whatever arrived after all dequeues finished, if any. When doing this I go through an important check. I will increment a dropped counter if the queue I'm adding to is full which helps me determine blocking. If it successfully queues I can then increment a counter which checks how full the queue is at certain points during the runtime. This counter also decreases during dequeues, and it helps me determine the average number of packets in the queue.

The latter portion of the code block contains equations and methods to output the expected metrics and the ones simulated in an easily readable format.

```
In [3]: def simulation(lamb, mu, k, phi, samples):
            """Used to run a simulation of an M/M/1/k network.

            Args:
                lamb (float): The rate into the entire network.
                mu (float): The rate out of the two queues in the network.
                k (int): Maximum number of customers the two queues can handle.
                phi (float): Probability an arrival goes to the first queue.
                samples (int): Number of packets to sample. Defaults to 6000.
            """
            queue1 = Queue(k, mu, samples*2)
            queue2 = Queue(k, mu, samples*2)
            # Counts arrivals to each node.
            queue1_arrivals, queue2_arrivals = 0, 0
            # Count time passed.
            time = 0
            # Indexes for sample space lists.
            i, j, n, m = 0, 0, 0, 0
            # Lists for obtaining average number of packets and time in queue.
            queue1_size, queue2_size = [], []
            queue1_time, queue2_time = [0], [0]
            # Iterate over entire sample of arrivals.
            while queue1.served < samples and queue2.served < samples:
                # Generate an interarrival time.
                arrivals = exponential(1/lamb)
                # Idle state, ignores output rates.
                if time is 0:
                    if random() < phi:
```

```
                queue1_arrivals += 1
                queue1.enqueue(0)
            else:
                queue2_arrivals += 1
                queue2.enqueue(0)
            # Increments time by one arrival interval.
            time += arrivals
        else:
            # Dequeues any packets that should have been processed
            # before the next arrival.
            while queue1.departures[i] <= time:
                t = queue1.dequeue()
                if t is not None:
                    queue1_time.append(queue1.departures[i] - t)
                # Sums the intervals to compare with time since arrival.
                queue1.departures[i+1] += queue1.departures[i]
                i += 1
                if queue1.served > 1000:
                    queue1_size.append(queue1.get_size())
            while queue2.departures[j] <= time:
                t = queue2.dequeue()
                if t is not None:
                    queue2_time.append(queue2.departures[j] - t)
                queue2.departures[j+1] += queue2.departures[j]
                j += 1
                if queue2.served > 1000:
                    queue2_size.append(queue2.get_size())
            # Splits arrivals based on phi probability.
            if random() < phi:
                queue1_arrivals += 1
                queue1.enqueue(time)
            else:
                queue2_arrivals += 1
                queue2.enqueue(time)
            if queue1.served > 1000 or queue2.served > 1000:
                queue1_size.append(queue1.get_size())
                queue2_size.append(queue2.get_size())
            # Increments time by one arrival interval.
            time += arrivals

    # Print the metrics for the queues.
    print_metrics(lamb, mu, k, phi, samples, time,
                  queue1, queue1_arrivals, queue1_size, queue1_time,
                  queue2, queue2_arrivals, queue2_size, queue2_time)

def print_metrics(lamb, mu, k, phi, samples, time,
                  queue1, queue1_arrivals, queue1_size, queue1_time,
                  queue2, queue2_arrivals, queue2_size, queue2_time):
```

```python
"""Prints the metrics for the system, queue1, and queue2.

Args:
    lamb (float): The rate into the entire network.
    mu (float): The rate out of the two queues in the network.
    k (int): Maximum number of customers the two queues can handle.
    phi (float): Probability an arrival goes to the first queue.
    samples (int): Number of packets sampled.
    time: The runtime of the system.
    queue1 (Queue): The first Queue object.
    queue1_arrivals: The number of arrivals into the system.
    queue1_size (list): A list of the number of items in queue at different times.
    queue1_time (list): A list of the delay for each packet that left the system.
    queue2 (Queue): The second Queue object.
    queue2_arrivals: The number of arrivals into the system.
    queue2_size (list): A list of the number of items in queue at different times.
    queue2_time (list): A list of the delay for each packet that left the system.
"""
# Calculate and print results.
# Queue 1.
# Blocking probability.
e_pb1 = eval_blocking(lamb*phi, mu, k)
pb1 = queue1.dropped/queue1_arrivals
# Average delay.
e_et1 = eval_delay(lamb*phi, mu, k, e_pb1)
et1 = average(queue1_time)
# Average number of packets in system.
rho = phi*lamb/mu
e_n1 = (rho/(1-rho))-((k+1)*rho**(k+1)/(1-rho**(k+1)))
n1 = average(queue1_size)
# Throughput.
e_thru1 = e_n1/e_et1
thru1 = n1/et1
# Queue 2.
# Blocking probability.
e_pb2 = eval_blocking(lamb*(1-phi), mu, k)
pb2 = queue2.dropped/queue2_arrivals
# Average delay.
e_et2 = eval_delay(lamb*(1-phi), mu, k, e_pb2)
et2 = average(queue2_time)
# Average number of packets in system.
rho = (1-phi)*lamb/mu
e_n2 = (rho/(1-rho))-((k+1)*rho**(k+1)/(1-rho**(k+1)))
n2 = average(queue2_size)
# Throughput.
e_thru2 = e_n2/e_et2
thru2 = n2/et2
# Whole system.
```

```
# Blocking probability.
e_pb = phi*e_pb1 + (1-phi)*e_pb2
pb = (queue1.dropped+queue2.dropped)/(queue1_arrivals + queue2_arrivals)
# Average delay.
e_et = phi*e_et1 + (1-phi)*e_et2
et = average(queue1_time+queue2_time)
# Average number of packets in system.
e_n = phi*e_n1 + (1-phi)*e_n2
n = average(queue1_size+queue2_size)
# Throughput.
e_thru = average([e_thru1, e_thru2])
thru = n/et

print("\nSimulation of two M/M/1/{0} queues with phi={1}:\n".format(k,phi))
# Whole system.
system_metrics = {'expected_blocking':e_pb, 'blocking':pb, 'expected_delay':e_et,
          'expected_number':e_n, 'number':n, 'expected_throughput':e_thru, 'throug
print("\tSystem:")
print("\t\tBlocking probability:\n\t\t\tExpected: ", e_pb)
print("\t\t\tSimulated: ", pb)
print("\t\tAverage delay in seconds:\n\t\t\tExpected: ", e_et)
print("\t\t\tSimulated: ", et)
print("\t\tAverage number of packets:\n\t\t\tExpected: ", e_n)
print("\t\t\tSimulated: ", n)
print("\t\tThroughput in packets/second:\n\t\t\tExpected: ", e_thru)
print("\t\t\tSimulated: ", thru)
# Queue 1.
queue1_metrics = {'expected_blocking':e_pb, 'blocking':pb, 'expected_delay':e_et,
          'expected_number':e_n, 'number':n, 'expected_throughput':e_thru, 'throug
print("\n\tQueue 1:")
print("\t\tBlocking probability:\n\t\t\tExpected: ", e_pb1)
print("\t\t\tSimulated: ", pb1)
print("\t\tAverage delay in seconds:\n\t\t\tExpected: ", e_et1)
print("\t\t\tSimulated: ", et1)
print("\t\tAverage number of packets:\n\t\t\tExpected: ", e_n1)
print("\t\t\tSimulated: ", n1)
print("\t\tThroughput in packets/second:\n\t\t\tExpected: ", e_thru1)
print("\t\t\tSimulated: ", thru1)
# Queue 2.
queue2_metrics = {'expected_blocking':e_pb, 'blocking':pb, 'expected_delay':e_et,
          'expected_number':e_n, 'number':n, 'expected_throughput':e_thru, 'throug
print("\n\tQueue 2:")
print("\t\tBlocking probability:\n\t\t\tExpected: ", e_pb2)
print("\t\t\tSimulated: ", pb2)
print("\t\tAverage delay in seconds:\n\t\t\tExpected: ", e_et2)
print("\t\t\tSimulated: ", et2)
print("\t\tAverage number of packets:\n\t\t\tExpected: ", e_n2)
print("\t\t\tSimulated: ", n2)
```

```
    print("\t\tThroughput in packets/second:\n\t\t\tExpected: ", e_thru2)
    print("\t\t\tSimulated: ", thru2)

    f, (ax1, ax2) = subplots(1, 2, sharey=True)
    f.suptitle("Distribution of Packets in Queue as a Factor of Runtime")
    ax1.hist(queue1_size)
    ax1.set_title("Queue 1")
    #ax1.fill_between(range(0,len(queue1_size)), queue1_size)
    ax1.tick_params(
        axis='y',           # changes apply to the x-axis
        which='both',       # both major and minor ticks are affected
        bottom='off',       # ticks along the bottom edge are off
        top='off',          # ticks along the top edge are off
        labelleft='off') # labels along the bottom edge are off'''
    ax1.set_ylabel("Runtime")
    ax1.set_xlabel("Packets in Queue")
    ax2.hist(queue2_size)
    ax2.set_title("Queue 2")
    #ax2.fill_between(range(0,len(queue2_size)), queue2_size)
    ax2.tick_params(
        axis='y',           # changes apply to the x-axis
        which='both',       # both major and minor ticks are affected
        bottom='off',       # ticks along the bottom edge are off
        top='off',          # ticks along the top edge are off
        labelleft='off') # labels along the bottom edge are off
    ax2.set_ylabel("Runtime")
    ax2.set_xlabel("Packets in Queue")
    show()

def eval_blocking(lamb, mu, k):
    """Finds the blocking probability of a queue.

    Args:
        lamb (float): The rate into the queue.
        mu (float): The rate out of the queue.
        k (int): Maximum number of customers able to be in the queue.
    """
    rho = lamb/mu
    return rho**k*((1-rho)/(1-rho**(k+1)))

def eval_delay(lamb, mu, k, pb):
    """Finds the average delay of a queue.

    Args:
        lamb (float): The rate into the queue.
        mu (float): The rate out of the queue.
        k (int): Maximum number of customers able to be in the queue.
        pb (float): The blocking probability for the queue.
```

```
    """
    rho = lamb/mu
    en = (rho/(1-rho))-((k+1)*rho**(k+1)/(1-rho**(k+1)))
    return en/(lamb*(1-pb))
```

1.2 Results

1.2.1 Configuration 1:

- $\mu 1 = 5\,packets/sec$
- $\mu 2 = 5\,packets/sec$
- $\lambda = 8\,packets/sec$
- $buffer = 20$
- $\phi = 0.4, 0.5, 0.6$

```
In [4]: simulation(8, 5, 21, 0.4, 6000)
        simulation(8, 5, 21, 0.5, 6000)
        simulation(8, 5, 21, 0.6, 6000)
```

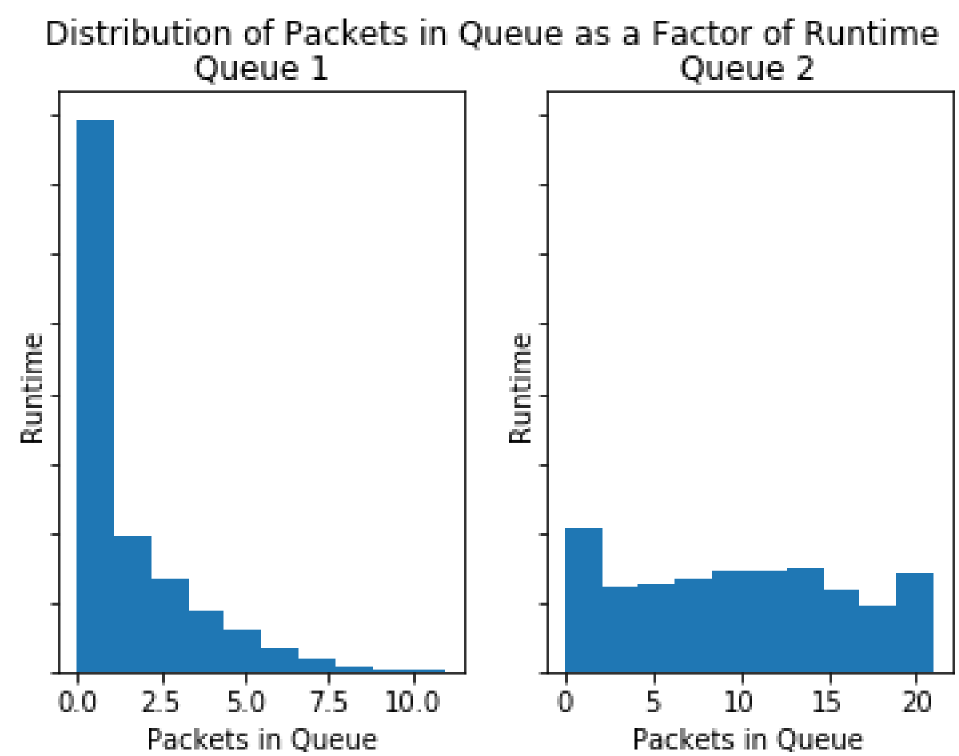
```
Simulation of two M/M/1/21 queues with phi=0.4:

        System:
                Blocking probability:
                        Expected:  0.017195624672635433
                        Simulated:  0.020029168692270297
                Average delay in seconds:
                        Expected:  1.364629618009292
                        Simulated:  1.45998980657
                Average number of packets:
                        Expected:  6.037810579705681
                        Simulated:  5.85768933682
                Throughput in packets/second:
                        Expected:  3.93121750131
                        Simulated:  4.01214399612

        Queue 1:
                Blocking probability:
                        Expected:  3.062708051975666e-05
                        Simulated:  0.0
                Average delay in seconds:
                        Expected:  0.5551982286722792
                        Simulated:  0.501615432239
                Average number of packets:
                        Expected:  1.7765799186285607
                        Simulated:  1.65331440102
                Throughput in packets/second:
```

```
                Expected:  3.199901993342337
                Simulated:  3.29597993753

        Queue 2:
                Blocking probability:
                        Expected:  0.028638956400712554
                        Simulated:  0.03313495254946115
                Average delay in seconds:
                        Expected:  1.9042505442339672
                        Simulated:  2.10918024677
                Average number of packets:
                        Expected:  8.878631020423763
                        Simulated:  9.93263798349
                Throughput in packets/second:
                        Expected:  4.662533009276579
                        Simulated:  4.70924094737
```



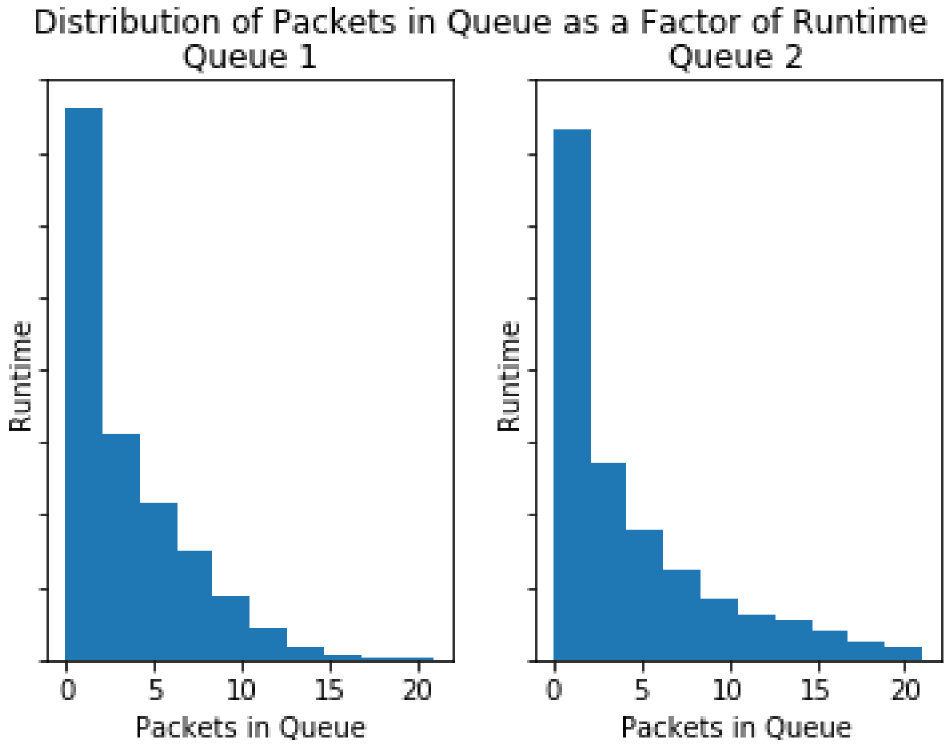

```
Simulation of two M/M/1/21 queues with phi=0.5:

        System:
                Blocking probability:
```

```
                Expected:  0.0018583868822537061
                Simulated:  0.0012650754828371427
        Average delay in seconds:
                Expected:  0.9609012148031506
                Simulated:  0.994559118709
        Average number of packets:
                Expected:  3.836461954361675
                Simulated:  4.05892834813
        Throughput in packets/second:
                Expected:  3.99256645247
                Simulated:  4.08113331

Queue 1:
        Blocking probability:
                Expected:  0.0018583868822537061
                Simulated:  0.00016655562958027982
        Average delay in seconds:
                Expected:  0.9609012148031506
                Simulated:  0.849751181109
        Average number of packets:
                Expected:  3.836461954361675
                Simulated:  3.62284859067
        Throughput in packets/second:
                Expected:  3.992566452470985
                Simulated:  4.26342283625

Queue 2:
        Blocking probability:
                Expected:  0.0018583868822537061
                Simulated:  0.0023919357594396036
        Average delay in seconds:
                Expected:  0.9609012148031506
                Simulated:  1.14341016933
        Average number of packets:
                Expected:  3.836461954361675
                Simulated:  4.49730441324
        Throughput in packets/second:
                Expected:  3.992566452470985
                Simulated:  3.9332380749
```



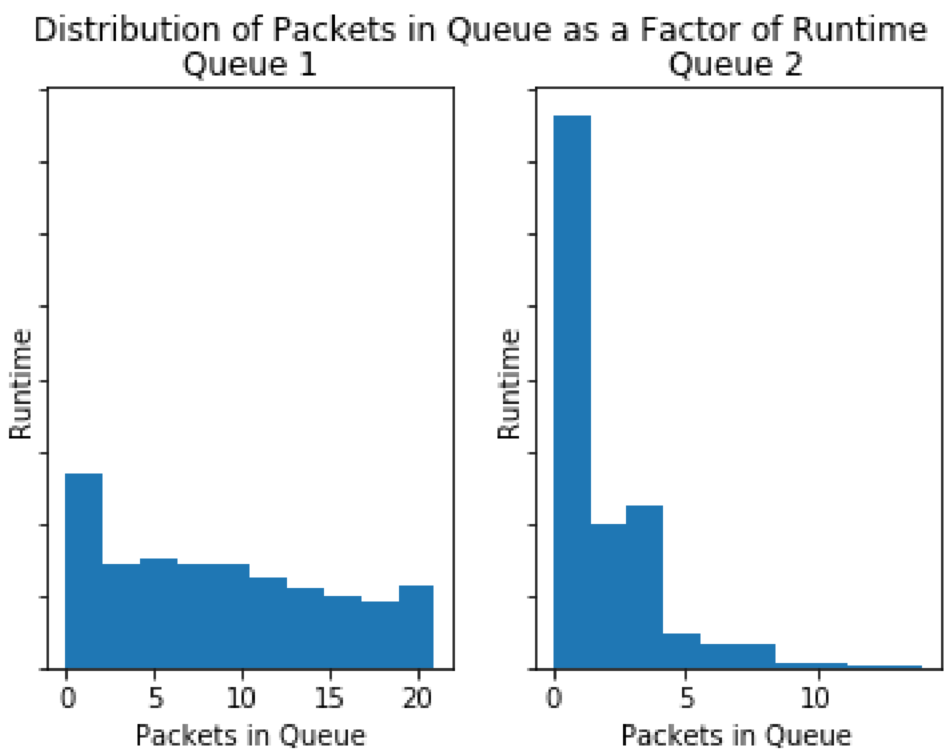

```
Simulation of two M/M/1/21 queues with phi=0.6:

        System:
                Blocking probability:
                        Expected:  0.017195624672635433
                        Simulated:  0.015041289815178922
                Average delay in seconds:
                        Expected:  1.364629618009292
                        Simulated:  1.30802423098
                Average number of packets:
                        Expected:  6.037810579705681
                        Simulated:  5.39846403781
                Throughput in packets/second:
                        Expected:  3.93121750131
                        Simulated:  4.12718962686

        Queue 1:
                Blocking probability:
                        Expected:  0.028638956400712554
                        Simulated:  0.02478134110787172
                Average delay in seconds:
                        Expected:  1.9042505442339672
```

```
                Simulated:  1.83092439405
        Average number of packets:
                Expected:  8.878631020423763
                Simulated:  8.87163426926
        Throughput in packets/second:
                Expected:  4.662533009276579
                Simulated:  4.84543998546

Queue 2:
        Blocking probability:
                Expected:  3.062708051975666e-05
                Simulated:  0.0
        Average delay in seconds:
                Expected:  0.5551982286722792
                Simulated:  0.522758453342
        Average number of packets:
                Expected:  1.7765799186285607
                Simulated:  1.74207973321
        Throughput in packets/second:
                Expected:  3.199901993342337
                Simulated:  3.3324754905
```



1.2.2 Configuration 2:

- $\mu 1 = 5\, packets/sec$
- $\mu 2 = 5\, packets/sec$
- $\lambda = 8\, packets/sec$
- $buffer = 5$
- $\phi = 0.4, 0.5, 0.6$

```
In [5]: simulation(8, 5, 6, 0.4, 6000)
        simulation(8, 5, 6, 0.5, 6000)
        simulation(8, 5, 6, 0.6, 6000)
```

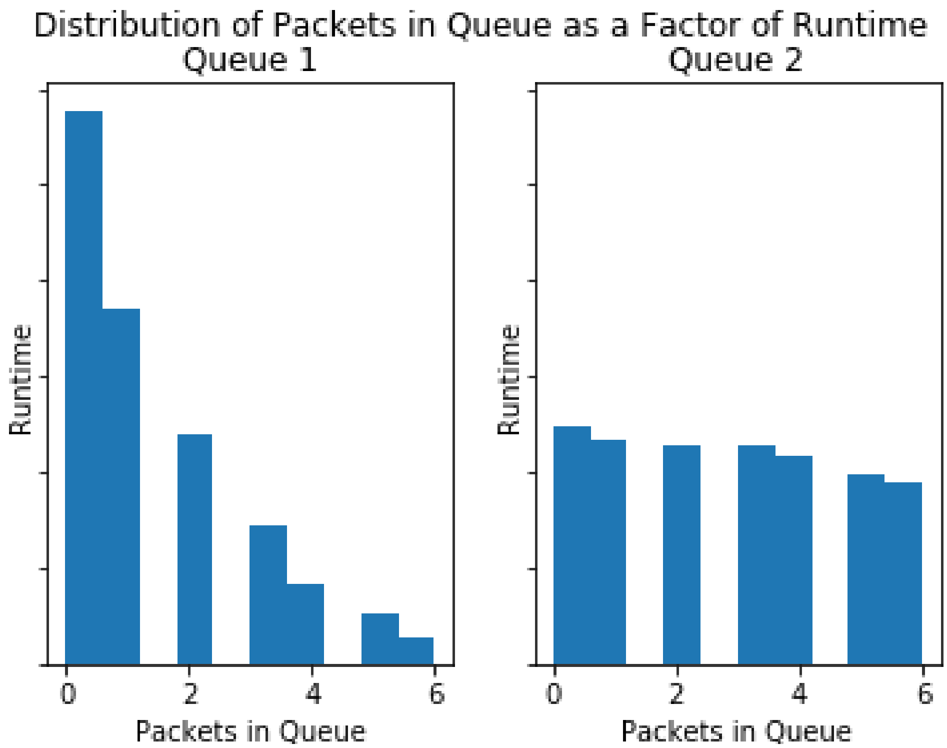
```
Simulation of two M/M/1/6 queues with phi=0.4:

        System:
                Blocking probability:
                        Expected:  0.08593320245468473
                        Simulated:  0.07779564130046446
                Average delay in seconds:
                        Expected:  0.5925298378368264
                        Simulated:  0.570751222008
                Average number of packets:
                        Expected:  2.2844636689798294
                        Simulated:  2.11256837804
                Throughput in packets/second:
                        Expected:  3.65626719018
                        Simulated:  3.7013821374

        Queue 1:
                Blocking probability:
                        Expected:  0.025877098450538878
                        Simulated:  0.019296254256526674
                Average delay in seconds:
                        Expected:  0.46700718583965284
                        Simulated:  0.437186603875
                Average number of packets:
                        Expected:  1.4557516637266275
                        Simulated:  1.3661066863
                Throughput in packets/second:
                        Expected:  3.117193284958276
                        Simulated:  3.1247679462

        Queue 2:
                Blocking probability:
                        Expected:  0.12597060512411531
                        Simulated:  0.11574142247091739
                Average delay in seconds:
                        Expected:  0.6762116058349422
```

```
                Simulated:  0.66690172198
        Average number of packets:
                Expected:  2.836938339148631
                Simulated:  2.83654657579
        Throughput in packets/second:
                Expected:  4.195341095404246
                Simulated:  4.25332021541
```



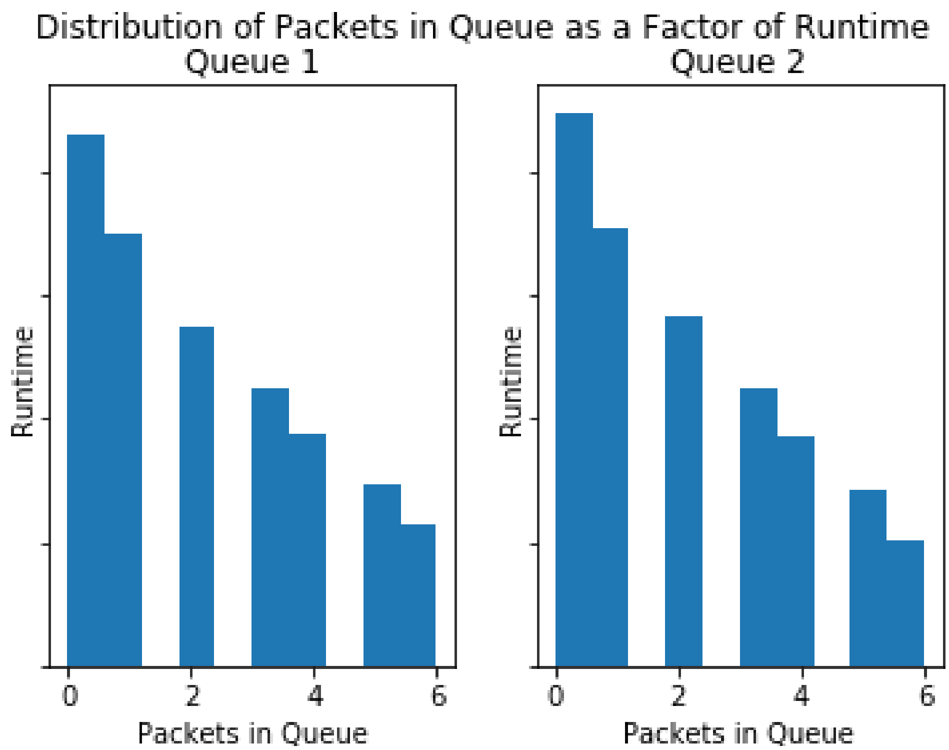

```
Simulation of two M/M/1/6 queues with phi=0.5:

    System:
        Blocking probability:
                Expected:  0.06634165303445037
                Simulated:  0.06473528033734187
        Average delay in seconds:
                Expected:  0.5736664064532917
                Simulated:  0.569222109882
        Average number of packets:
                Expected:  2.1424337150353896
                Simulated:  2.13064558021
        Throughput in packets/second:
```

```
            Expected:  3.73463338786
            Simulated:  3.74308296046

Queue 1:
        Blocking probability:
                Expected:  0.06634165303445037
                Simulated:  0.06521739130434782
        Average delay in seconds:
                Expected:  0.5736664064532917
                Simulated:  0.576761290252
        Average number of packets:
                Expected:  2.1424337150353896
                Simulated:  2.1618898732
        Throughput in packets/second:
                Expected:  3.734633387862198
                Simulated:  3.74832692439

Queue 2:
        Blocking probability:
                Expected:  0.06634165303445037
                Simulated:  0.06425452276980662
        Average delay in seconds:
                Expected:  0.5736664064532917
                Simulated:  0.561719362811
        Average number of packets:
                Expected:  2.1424337150353896
                Simulated:  2.0995972382
        Throughput in packets/second:
                Expected:  3.734633387862198
                Simulated:  3.73780463557
```



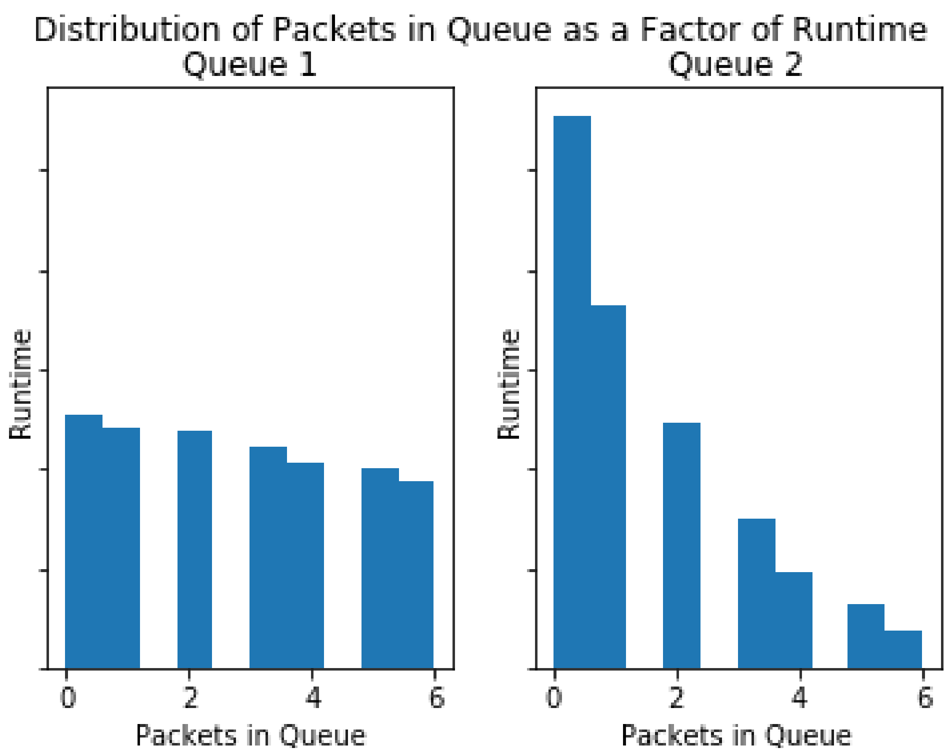

```
Simulation of two M/M/1/6 queues with phi=0.6:

        System:
                Blocking probability:
                        Expected:  0.08593320245468473
                        Simulated:  0.08432366305487592
                Average delay in seconds:
                        Expected:  0.5925298378368264
                        Simulated:  0.580957535758
                Average number of packets:
                        Expected:  2.2844636689798294
                        Simulated:  2.14123498806
                Throughput in packets/second:
                        Expected:  3.65626719018
                        Simulated:  3.68569965318

        Queue 1:
                Blocking probability:
                        Expected:  0.12597060512411531
                        Simulated:  0.12492711370262391
                Average delay in seconds:
                        Expected:  0.6762116058349422
```

```
        Simulated:  0.669751437781
Average number of packets:
        Expected:  2.836938339148631
        Simulated:  2.79766059196
Throughput in packets/second:
        Expected:  4.195341095404246
        Simulated:  4.1771625026

Queue 2:
        Blocking probability:
                Expected:  0.025877098450538878
                Simulated:  0.02356020942408377
        Average delay in seconds:
                Expected:  0.46700718583965284
                Simulated:  0.461911019664
        Average number of packets:
                Expected:  1.4557516637266275
                Simulated:  1.46788200199
        Throughput in packets/second:
                Expected:  3.117193284958276
                Simulated:  3.1778458177
```



1.3 Conclusions

The value of ϕ has an interesting impact on the system's behavior. As can be seen from the graphs, if ϕ is not equal then there is a queue being underutilized with too much runtime spent with too few packets in queue. This affects throughput negatively as can be seen from the results. Buffer size also plays an important role as it changes the probability that a packet will be blocked from entering the queue. An decrease in the buffer size will also negatively impact the throughput as the queue will not be able to utilize the full capability of the service rate.

From, the results of the simulations, the system with a buffer size of 20 and a ϕ of 0.5 is the best configuration. This is obvious from the observed results especially from the viewpoint of throughput. A larger buffer will allow better utilization of the service time with fewer blockings happening, and a ϕ of 0.5 will make sure a queue is not being underutilized. This at first seemed contrary because while this queue has a higher throughput, it's delay was actually worse than the queue with a buffer size of 5. However, that is because the queue can only hold 6 packets. The queue will have less delay because at most there will only be 5 packets ahead of another reducing the delay, but the queue itself is actually unreliable. It will block packets more often and it won't be able to fully utilize its service time the way the size 20 buffer can.