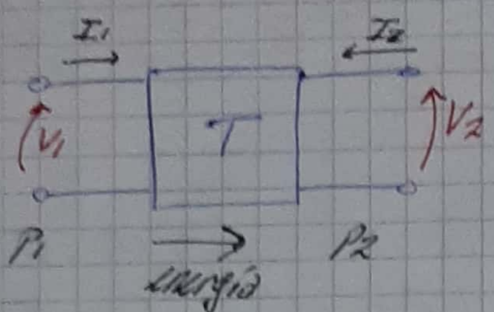
clase 8: cuadripolos

HOJA N° 36

FECHA

Parámetros de transmisión

↳ flujo de energía de un punto hacia el otro



$$\begin{cases} V_1 = V_2 \cdot A + (-I_2) B \\ I_1 = V_2 C + (-I_2) D \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

$$T^{-1} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \qquad T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} \frac{D}{\Delta T} & -\frac{B}{\Delta T} \\ \frac{-C}{\Delta T} & \frac{A}{\Delta T} \end{pmatrix}$$

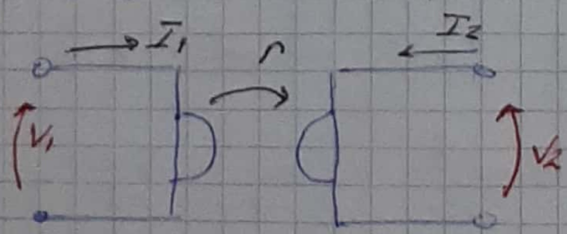
$$T^{-1} \begin{cases} V_2 = A' V_1 + (-I_1) B' \\ I_2 = C' V_1 + (-I_1) D' \end{cases}$$

Simetría $\quad A = D$

Reciprocidad ≈ (Pasividad)

$\Delta T = 1 \Rightarrow AD - CB = 1$

Girador → esto no lo vimos antes (?)



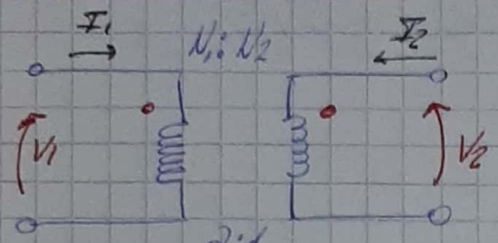
$$\begin{cases} V_2 = I_1 \cdot r \\ V_1 = (-I_2) \cdot r \end{cases}$$

r → cte. de proporcionalidad en ohms.

NOTA $\boxed{17:20}$ →

$$T = \begin{pmatrix} 0 & r \\ 1/r & 0 \end{pmatrix} \qquad \frac{V_1}{I_1} = \underbrace{\left[\frac{(-I_2)}{V_2}\right]}_{\hookrightarrow 1/Z_2} r^2 \Rightarrow Z_1 = \frac{1}{Z_2} r^2$$

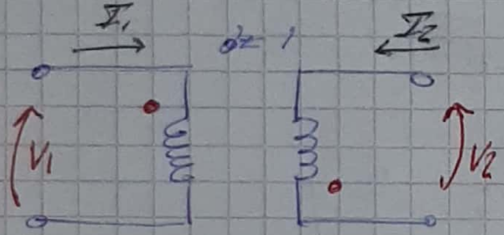
Transformador ideal



↳ bornes homólogos

$$\begin{cases} V_1 = a \cdot V_2 \\ I_1 = \frac{1}{a} \cdot (-I_2) \end{cases} \qquad T = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix}$$

Xq es un dispositivo pasivo $\Rightarrow \boxed{\Delta T = 1}$



Bornes homólogos ↳ no

$$\begin{cases} V_1 = -aV_2 \\ I_1 = -\frac{1}{a}(-I_2) \end{cases} \qquad T = \begin{pmatrix} -a & 0 \\ 0 & -1/a \end{pmatrix}$$

$I_1 = I_2 \, 1/a$ ↳ $\Delta T = 1$

(*)

$$T = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} \rightarrow Z = \begin{pmatrix} \to \infty & \to \infty \\ \to \infty & \to \infty \end{pmatrix}$$

↳ No está definido Z (N/A)

↳ Al igual que Y

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} \longrightarrow I_1 = C\,V_2 + (-I_2)\,D$$

$$\Rightarrow \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = 1/C$$ pero $C = 0 \Rightarrow Z_{21} \to \infty$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} = \frac{Z_{11}}{Z_{21}} = a$$ como $Z_{21} \to \infty \Rightarrow Z_{11} \to \infty$ para que $\frac{Z_{11}}{Z_{21}} = a$

NOTA

Trafo Real

Ley de Lenz - Faraday $\longrightarrow V = -N \cdot \frac{d\phi}{dt}$

Ley de Ampere $\longrightarrow \phi \cdot R = N_1 \cdot i_1 \pm N_2 i_2$

↳ reluctancia (relacionado al medio físico)

$$\underbrace{N_1 \frac{\partial \Phi}{\partial t}}_{V_1} = \left[\frac{N_1}{R} \cdot \frac{\partial i_1}{\partial t} + \frac{N_2}{R} \frac{\partial i_2}{\partial t}\right] N_1$$

(tensión en el devanado 1)

$$\Rightarrow V_1 = \frac{N_1^2}{R} \cdot \frac{\partial i_1}{\partial t} + \frac{N_1 N_2}{R} \frac{\partial i_2}{\partial t}$$

$$V_1 = \underbrace{\frac{N_1^2}{R}}_{L_1} s \cdot I_1 + \frac{N_1 N_2}{R} s I_2$$

$$\left\{\begin{array}{l} V_1 = L_1 s I_1 + M s I_2 \\ V_2 = M s I_1 + L_2 s I_2 \end{array}\right.$$

$$\boxed{\begin{array}{c} M = K\sqrt{L_1 L_2} \\ 0 < K \leq 1 \\ K = 1 \text{ acoplamiento perfecto} \end{array}}$$

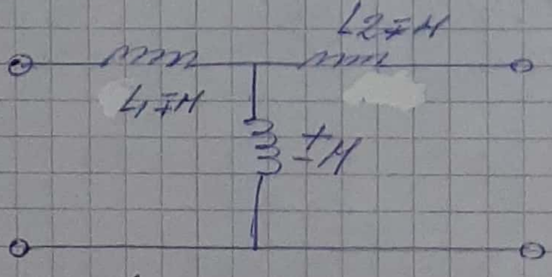
En su forma más general $\left\{\begin{array}{l} V_1 = sL_1 I_1 \pm sM I_2 \\ V_2 = \pm sM I_1 + sL_2 I_2 \end{array}\right.$

$$Z_{TR} = \begin{pmatrix} sL_1 & \pm sM \\ \pm sM & sL_2 \end{pmatrix}$$

↳ depende de los bornes homólogos

+ bornes homólogos coincidentes

− " " NO coincidentes



$$\left\{\begin{array}{l} V_1 = sL_1 I_1 + sM I_2 \\ V_2 = sM I_1 + sL_2 I_2 \end{array}\right.$$

Modelo equivalente transformador real

NOTA

(*) El trafo ideal se utiliza para adaptar impedancias

$$Z_1 = \frac{V_1}{I_1} = \underbrace{\frac{V_2}{-I_2}}_{Z_2} a^2 \Rightarrow \boxed{Z_1 = Z_2\, a^2}$$

~~Ecuaciones del transformador~~ Malla de entrada

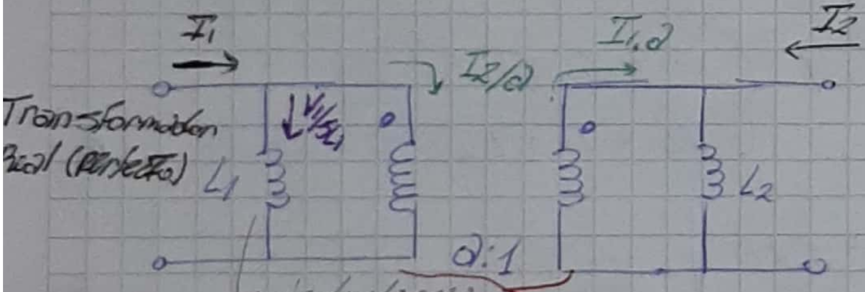
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11}}{Z_{21}} = \frac{L_1}{M} = \frac{L_1}{\sqrt{L_1 L_2}} = \boxed{\sqrt{\frac{L_1}{L_2}} = a}$$

xq $L_1 = \frac{N_1^2}{R}$ y $L_2 = \frac{N_2^2}{R} \Rightarrow \frac{L_1}{L_2} = \cancel{R} \cdot \frac{N_1^2}{N_2^2} = a^2$

$$V_1 - sM I_2 = sL_1 I_1 \Rightarrow I_1 = (-I_2)\frac{\cancel{s}M}{\cancel{s}L_1} + \frac{V_1}{sL_1}$$

$$I_1 = (-I_2)\underbrace{\frac{M}{L_1}}_{1/a} + \underbrace{\frac{V_1}{sL_1}}$$

↳ compuesto x 2 corrientes



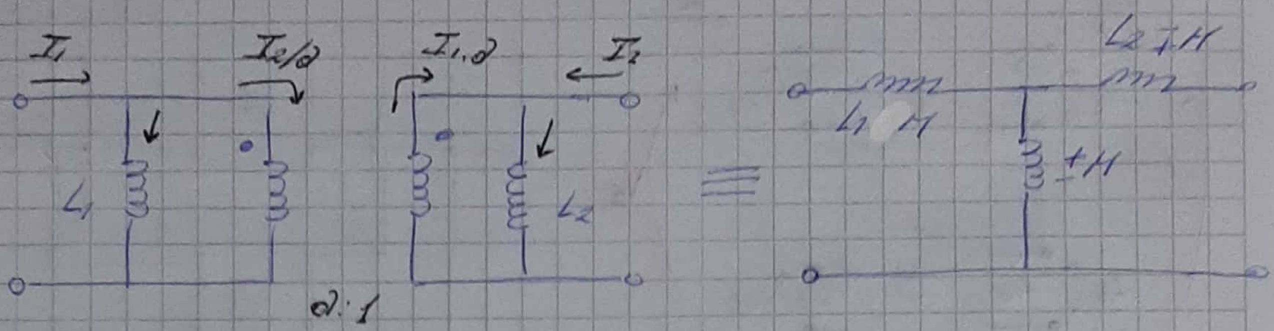
↳ inductancia de magnetización (Principal, establece el flujo magnético)

ideal

Malla de salida

$$V_2 - sM I_1 = sL_1 I_2 \Rightarrow I_2 = \frac{\cancel{s}M(-I_1)}{\cancel{s}L_2} + \frac{V_2}{sL_2}$$

$$I_2 = \underbrace{\frac{\sqrt{L_1 L_2}}{L_2}}_{a}(-I_1) + \frac{V_2}{sL_2}$$



Parámetros T del Trafo real

$$T_{TR} = \begin{pmatrix} a & 0 \\ \frac{1}{\pm sM} & 1/a \end{pmatrix} \quad \text{siendo} \quad Z_{TR} = \begin{pmatrix} sL_1 & \pm sM \\ \pm sM & sL_2 \end{pmatrix}$$

Partiendo de $\begin{cases} V_1 = I_1 Z_{11} + I_2 Z_{12} \\ V_2 = I_1 Z_{21} + I_2 Z_{22} \end{cases}$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} \Rightarrow Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{1}{C} \Rightarrow C = \frac{1}{Z_{21}}$$

$$\boxed{C = \frac{1}{\pm sM}}$$

$$D = \frac{I_1}{-I_2}\Big|_{V_2=0} = \frac{Z_{22}}{Z_{21}} = \frac{L_2}{M} = \frac{1}{a}$$

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0} \quad ; \quad I_1 = (-I_2)\frac{Z_{22}}{Z_{21}}$$

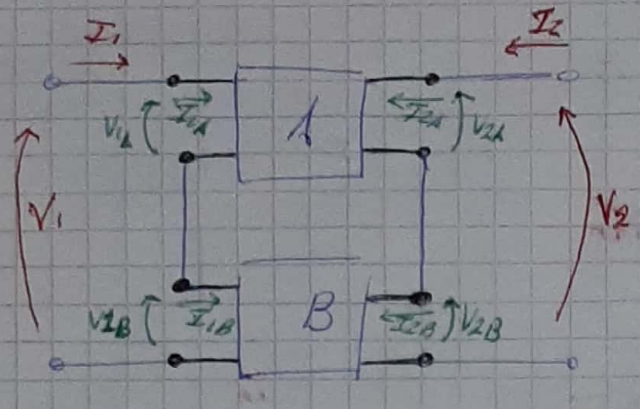
$$V_1 = (-I_2) \cdot \frac{Z_{11} Z_{22}}{Z_{21}} + I_2 Z_{12}$$

$$\Rightarrow \frac{V_1}{-I_2} = \frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12}$$

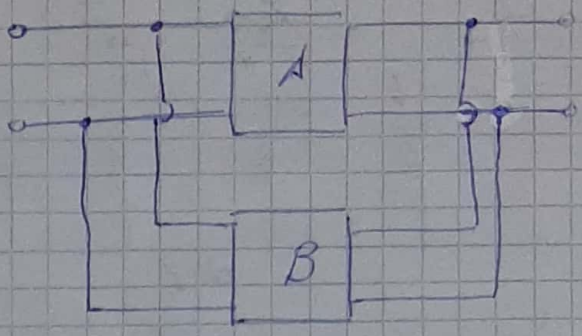
$$\Rightarrow B = \frac{s^2 L_1 L_2}{sM} - (\pm sM) = \frac{sL_1 L_2}{\sqrt{L_1 L_2}} - (\pm sM)$$

$$B = s\underbrace{\sqrt{L_1 L_2}}_{M} - sM = 0$$

Interconexión de cuadripolos

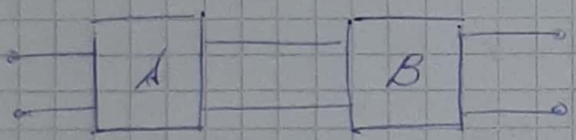


serie - serie



Paralelo - Paralelo

- serie - Paralelo
- paralelo - serie



cascada

serie - serie

- $I_{1A} = I_{1B}$
- $I_{2A} = I_{2B}$

} corrientes iguales

$V_1 = V_{1A} + V_{1B}$

$V_2 = V_{2A} + V_{2B}$

$V = Z \cdot I$ → $V = V_A + V_B$

$V_A = Z_A \, I_A$

$V_B = Z_B \, I_B$

} $I_A = I_B$

siendo $I_A = \begin{pmatrix} I_{1A} \\ I_{2A} \end{pmatrix}$; $I_B = \begin{pmatrix} I_{1B} \\ I_{2B} \end{pmatrix}$

$V_A + V_B = Z_A I_A + Z_B \cdot I_B$

$V = \underbrace{(Z_A + Z_B)}_{Z} \cdot I \implies \boxed{Z = Z_A + Z_B}$

Paralelo - Paralelo

- $V = V_A = V_B$
- $I = I_A + I_B$

$\begin{cases} I_A = Y_A \, V_A \\ I_B = Y_B \, V_B \end{cases} \implies I = Y_A V_A + Y_B \cdot V_B$

$I = \underbrace{(Y_A + Y_B)}_{Y} \, V$

NOTA

$\boxed{Y = Y_A + Y_B}$

Cascada

$P_{1A} = \begin{pmatrix} V_{1A} \\ I_{1A} \end{pmatrix}$ $\quad P_{2A} = \begin{pmatrix} V_{2A} \\ -I_{2A} \end{pmatrix}$ $\quad P_{1B} = \begin{pmatrix} V_{1B} \\ I_{1B} \end{pmatrix}$ $\quad P_{2B} = \begin{pmatrix} V_{2B} \\ -I_{2B} \end{pmatrix}$

$P_{1A} = P_1 \qquad P_{2A} = P_{1B} \qquad P_{2B} = P_2$

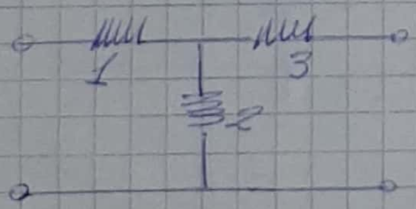
$$\begin{cases} P_{1A} = T_A \, P_{2A} \\ P_{1B} = T_B \, P_{2B} \end{cases} \Rightarrow P_1 = \underbrace{T_A \, . \, T_B}_{T} \, P_2$$

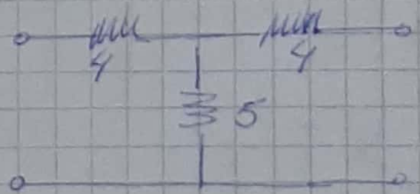
$$\boxed{T = T_A \, . \, T_B}$$

- Serie-paralelo $\quad H = H_A + H_B$
- Paralelo-serie $\quad Y' = Y'_A + Y'_B$

Ejemplos

Int. serie-serie





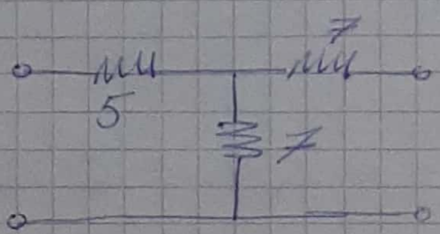
$Z_A = \begin{pmatrix} 3 & 2 \\ 2 & 5 \end{pmatrix}$ $\qquad Z_B = \begin{pmatrix} 9 & 5 \\ 5 & 9 \end{pmatrix}$

↳ no simétrico
recíproca

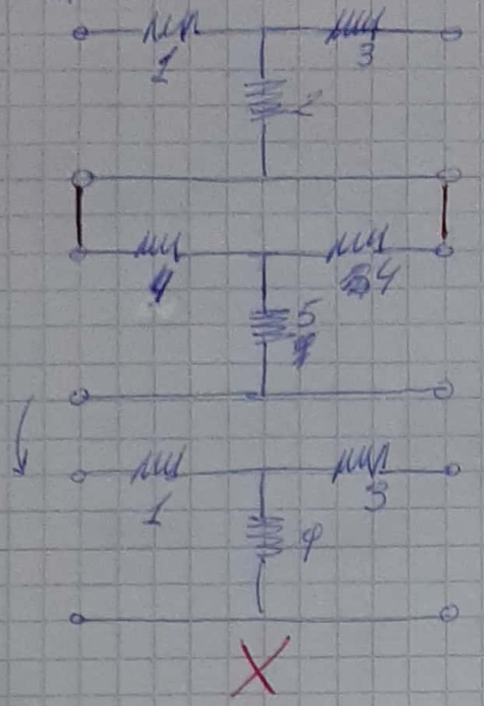
↳ simétrico
recíproca

$$Z = Z_A + Z_B = \begin{pmatrix} 12 & 7 \\ 7 & 14 \end{pmatrix}$$

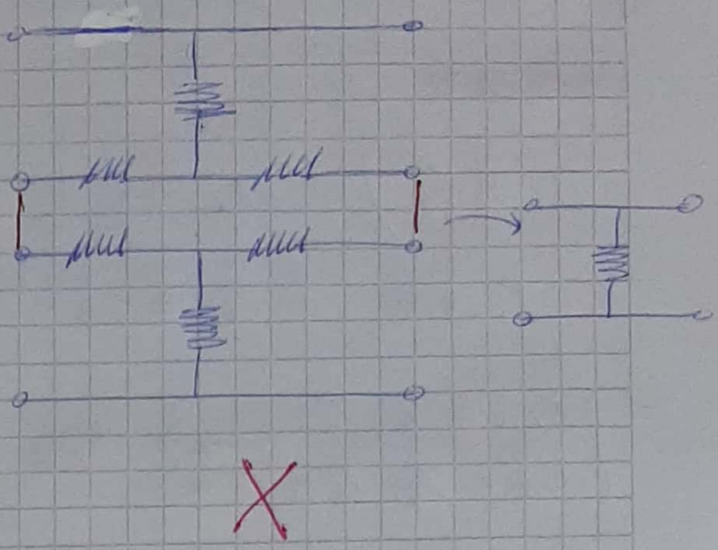
↳



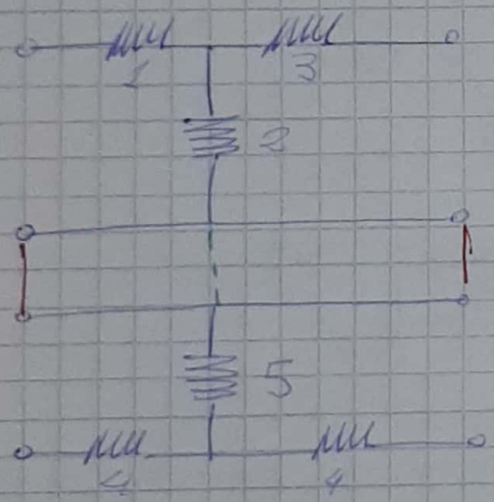
Intento 1:



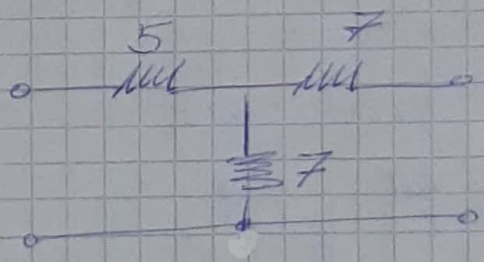
Intento 2:



Intento 3:



$\equiv$



$$Z = \begin{pmatrix} 12 & 7 \\ 7 & 14 \end{pmatrix}$$

¿Porque falló I_1 e I_2?

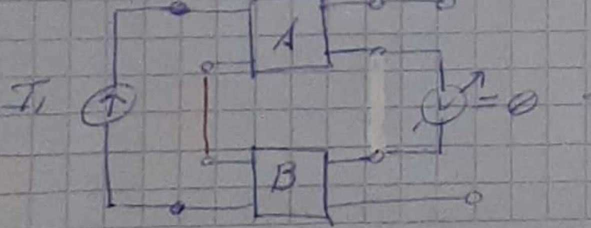
$$Z_{11} = Z_{11A} + Z_{11B}$$

$$Z_{11A} = \frac{V_{1A}}{I_{1A}}\Big|_{I_{2A}=0} \qquad Z_{11B} = \frac{V_{1B}}{I_{1B}}\Big|_{I_{2B}=0}$$

Pruebas de Brune para la interconexión serie

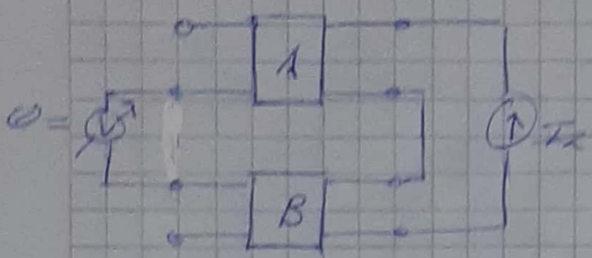
↳ permite verificar rápidamente el cumplimiento de las condiciones de corriente

$\leftarrow I_2 = 0$



$\rightarrow$ si se cumple $\Rightarrow I_1 = I_{1A} = I_{1B}$

NOTA

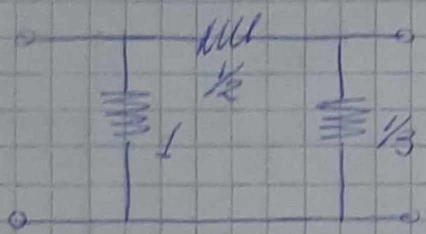


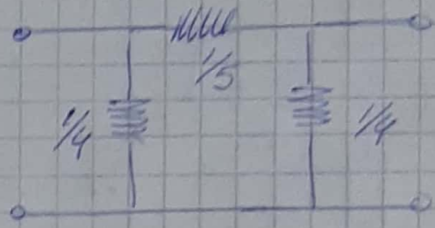
→ Si se cumple que $V = 0$

⇒ $I_2 = I_{2A} = I_{2B}$

Si al medir tensión $V \neq 0$ ⇒ forzar un c.c ~~[illegible]~~ no soluciona el problema ⇒ habría que interconectar el circuito de otra manera.

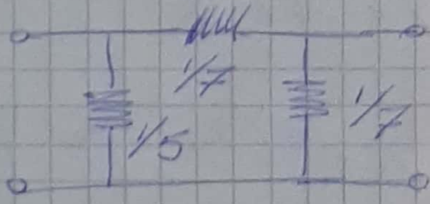
Paralelo - Paralelo

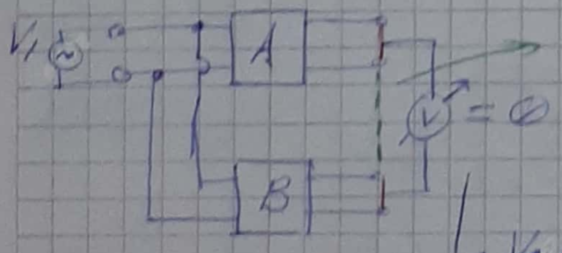




$$Y = \begin{pmatrix} 3 & -2 \\ -2 & 5 \end{pmatrix} \qquad Y = \begin{pmatrix} 9 & -5 \\ -5 & 9 \end{pmatrix}$$

$$Y = Y_A + Y_B = \begin{pmatrix} 12 & -7 \\ -7 & 14 \end{pmatrix}$$



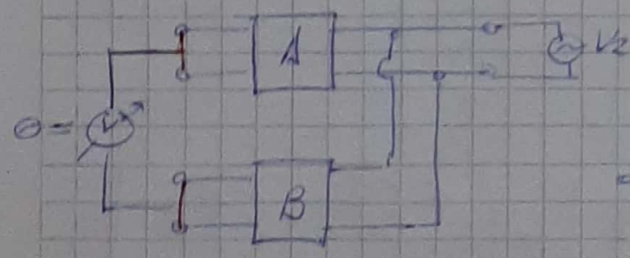


solo cc si $V = 0$

En este caso, el test de Brune permite verificar el cumplimiento de la condición de tensión ~~[illegible]~~

$V_{2A} = V_{2B} = V_2$ ⇒ $I_1 = I_{1A} + I_{1B} = V_1 (Y_{11A} + Y_{11B})$

$I_2 = I_{2A} + I_{2B} = V_2 (Y_{22A} + Y_{22B})$

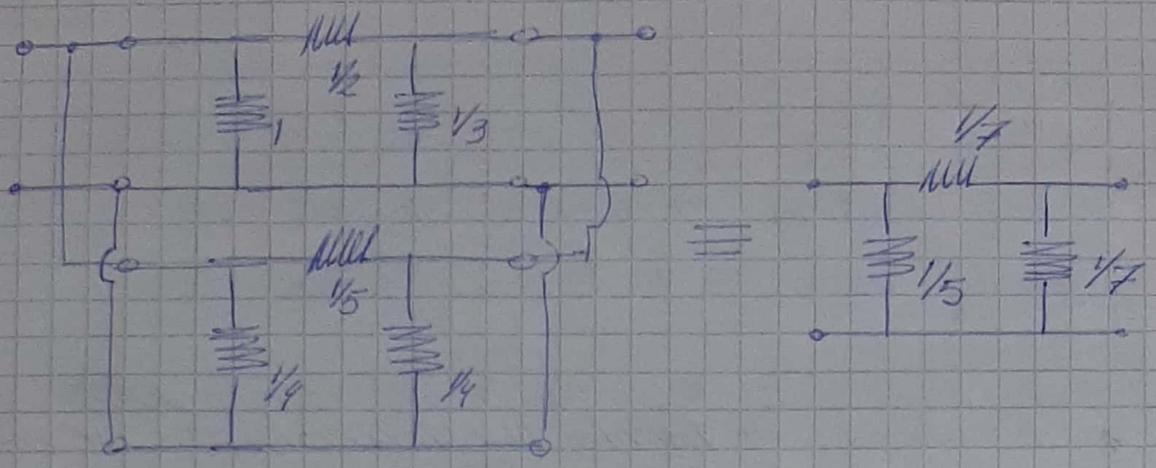


Si $V_1 = V_{1A} = V_{1B} = 0$

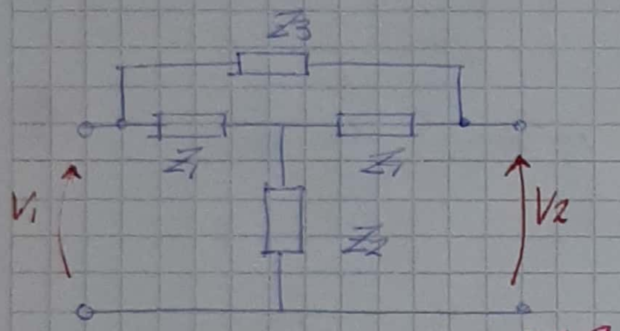
$$\Rightarrow \begin{cases} I_1 = I_{1A} + I_{1B} = V (Y_{12A} + Y_{12B}) \\ I_2 = I_{2A} + I_{2B} = V_2 (Y_{22A} + Y_{22B}) \end{cases}$$

Si $V \neq 0$ La interconexión no se puede hacer de esa forma. Habría que interconectarlo distinto.

NOTA



EJemplo de interconexión - Método de bisección



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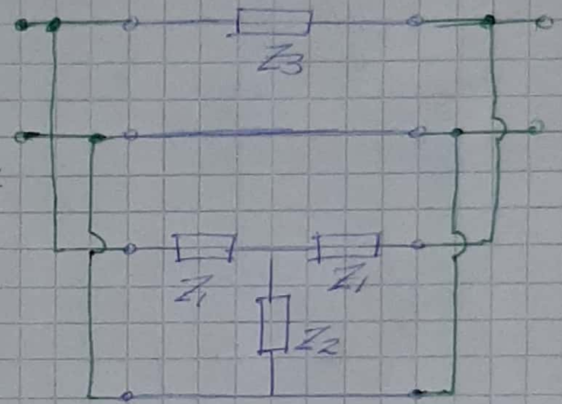


Z?

se piensa como 2 cuadripolos en paralelo

① $Y \Rightarrow Z = Y^{-1}$

$Y = Y_A + Y_B$

$$Y_A = \begin{pmatrix} Y_3 & -Y_3 \\ -Y_3 & Y_3 \end{pmatrix} \qquad Y_B = Z_B^{-1} \; ; \; Z_B = \begin{pmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{pmatrix}$$

$$Y_B = Z_B^{-1} = \frac{1}{\Delta Z_B} \begin{pmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_1 + Z_2 \end{pmatrix}$$

$$\Delta Z_B = (Z_1 + Z_2)^2 - Z_2^2 = Z_1^2 + 2 Z_1 Z_2$$

$$Y_B = \frac{\begin{pmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_1 + Z_2 \end{pmatrix}}{Z_1^2 + 2 Z_1 Z_2}$$

$$Y = Y_A + Y_B = \begin{pmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{pmatrix} + \begin{pmatrix} \frac{Z_1 + Z_2}{\Delta Z} & -\frac{Z_2}{\Delta Z} \\ \frac{-Z_2}{\Delta Z} & \frac{Z_1 + Z_2}{\Delta Z} \end{pmatrix}$$

NOTA

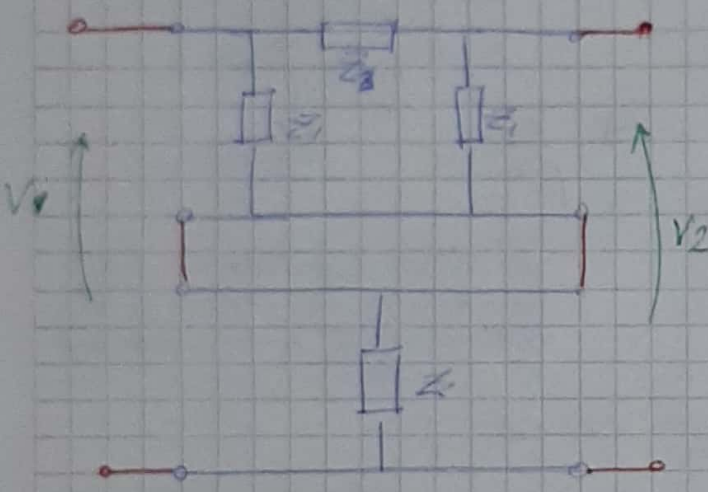
$$Y_{22} = Y_{11} = \frac{1}{Z_3} + \frac{Z_1 + Z_2}{\Delta Z} = Y_3 + \frac{\frac{Y_1 + Y_2}{\cancel{Y_1 Y_2}}}{\frac{1}{Y_1}\left(\frac{2Y_1 + Y_2}{\cancel{Y_1 Y_2}}\right)} = \boxed{Y_3 + \frac{Y_1(Y_1 + Y_2)}{2Y_1 + Y_2}}$$

$$Y_{12} = Y_{21} = -\frac{1}{Z_3} - \frac{Z_2}{\Delta Z} \Rightarrow -Y_{12} = Y_3 + \frac{\cancel{1/Y_2}}{\frac{1}{Y_1}\left(\frac{2Y_1 + Y_2}{Y_1 \cancel{Y_2}}\right)}$$

$$-Y_{12} = -Y_{21} = \boxed{Y_3 + \frac{2Y_1^2}{Y_1 + Y_2}}$$

∴ es muy largo plantearlo de esta manera xq ahora tiempo por hacer la inversa de la matriz Y.

2 ↳ se puede hacer la interconexion serie



$$Z = Z_A + Z_B$$

$$Z_A = Y_A^{-1}$$

$$Z_B = \begin{pmatrix} Z_2 & Z_2 \\ Z_2 & Z_2 \end{pmatrix}$$

$$Y_A = \begin{pmatrix} Y_1 + Y_3 & -Y_3 \\ -Y_3 & Y_1 + Y_3 \end{pmatrix}$$

$$Z_A = \frac{1}{\Delta Y_A} \begin{pmatrix} Y_1 + Y_3 & Y_3 \\ Y_3 & Y_1 + Y_3 \end{pmatrix}$$

$$\Delta Y_A = Y_1^2 + 2Y_1 Y_3 = Y_1(Y_1 + 2Y_3)$$

$$Z = \begin{pmatrix} \frac{Y_1 + Y_3}{\Delta Y_A} & \frac{Y_3}{\Delta Y_A} \\ \frac{Y_3}{\Delta Y_A} & \frac{Y_1 + Y_3}{\Delta Y_A} \end{pmatrix} + \begin{pmatrix} Z_2 & Z_2 \\ Z_2 & Z_2 \end{pmatrix}$$

$$Z_{11} = Z_2 + \frac{Y_1 + Y_3}{Y_1(Y_1 + 2Y_3)} = Z_2 + \frac{\frac{Z_1 + Z_3}{\cancel{Z_1 Z_3}}}{\frac{1}{Z_1}\left(\frac{2Z_1 + Z_3}{\cancel{Z_1 Z_3}}\right)}$$

NOTA

$$\Rightarrow Z_{11} = Z_2 + \frac{Z_1(Z_1+Z_3)}{2Z_1+Z_3} = Z_{22}$$

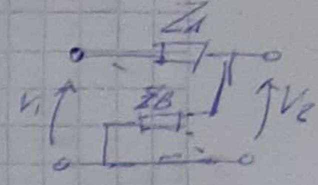
$$Z_{12} = Z_{21} = Z_2 + \frac{Y_3}{Y_1(Y_1+2Y_3)} = Z_2 + \frac{1/Z_3}{\frac{1}{Z_1}\left(\frac{2Z_1+Z_3}{Z_1Z_3}\right)}$$

$$Z_{12} = Z_{21} = Z_2 + \frac{Z_1^2}{2Z_1+Z_3}$$

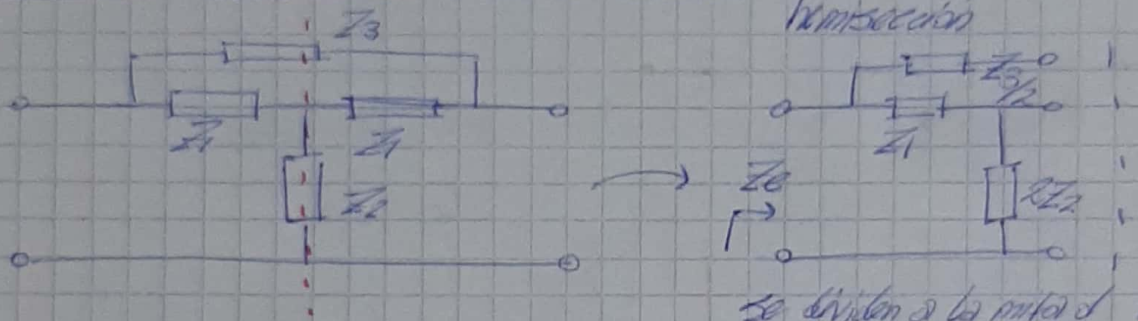
¿Pasar a un circuito balanceado?

Simétrica $\Rightarrow Z_{T_S} = Z_{LT} \longrightarrow \begin{pmatrix} \frac{Z_A+Z_B}{2} & '' \\ \frac{Z_B-Z_A}{2} & '' \end{pmatrix}$

$Y_{TS} = Y_{LT}$



Método de Bisección $\rightarrow$ solo si simétrica



Se dividen a la mitad las impedancias en serie y se duplican las impedancias en paralelo.

Impedancia en extremo de carga en cc

- $Z_e^{cc} = \frac{Z_3/2 \cdot Z_1}{Z_3/2 + Z_1} = \frac{Z_3 Z_1}{2Z_1+Z_3} \longrightarrow \boxed{Z_A = Z_e^{cc}}$

- $Z_e^{ca} = Z_1 + 2Z_2 \longrightarrow \boxed{Z_B = Z_e^{ca}} \longrightarrow$ de un lattice

$$\begin{cases} Z_A = Z_{11} - Z_{21} \\ Z_B = Z_{11} + Z_{21} \end{cases} \rightarrow \text{lattice}$$

NOTA