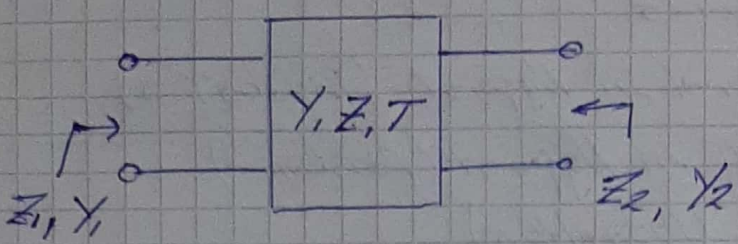
Síntesis de funciones de excitación: (Cap 4 Araujo, Cap 10 Kuo)



$F(s)$: Inmitancia $= \dfrac{P(s)}{Q(s)} \longrightarrow$ Realizabilidad

↳ sea implementable con componentes de valores reales.

↳ sistema causal y estable.

Si $F(s)$ es estable $\Rightarrow$ Funciones Reales y positivas (FPR)

F. transferencia $\longrightarrow F(s) \longrightarrow$ FPR? $\rightarrow$ Z? / $\rightarrow$ Y? } $\boxed{\text{Red circuital}}$

Condiciones necesarias y suficientes de $F(s)$ para ser FPR

1- $F(s)$ no tendrá polos en el semiplano derecho

2- $F(s)$ podría tener polos simples en el $j\omega$, con residuos $\mathbb{R}^+$

3- $\mathrm{Re}\{F(j\omega)\} \geq 0 \ \forall \omega$

Propiedades de las FPR

- Polos y ceros serán reales o estarán dispuestos como complejos conjugados
- $F(s) = \dfrac{P(s)}{Q(s)} \Rightarrow Gr\{P\} - Gr\{Q\} \rightarrow +1, \ 0, \ -1$

  diferencia entre ordenes del den y num.
- Teniendo $F_1(s)$ y $F_2(s)$ FPR

  $\Rightarrow F_3 = F_1 + F_2$ es FPR

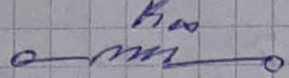
  $\Rightarrow F_3 = F_1 \parallel F_2$ es FPR
- $F(s) = \dfrac{P(s)}{Q(s)}$; si $P(s)$ tiene FPR pares y $Q(s)$ impares o viceversa $\Longrightarrow$ No pueden tener términos faltantes

NOTA $\boxed{21:00} \longrightarrow \boxed{22:00}$

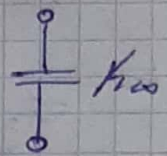
Repaso de residuos

$$F(s) = \frac{P(s)}{Q(s)} = \frac{K_0}{s} + \sum_i^N \frac{K_i}{s+\sigma_i} + \sum_i^M \frac{2K_i s}{s^2+\omega_i} + K_\infty s$$

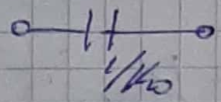
• $F(s) = K_\infty s \rightarrow Z$



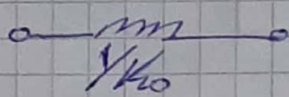
$\rightarrow Y$



• $F(s) = \frac{K_0}{s} \rightarrow Z$

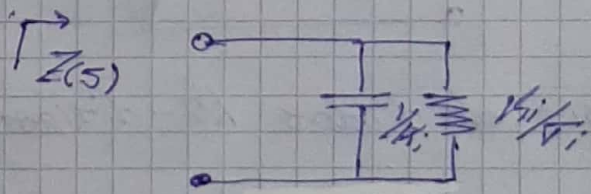


$\rightarrow Y$

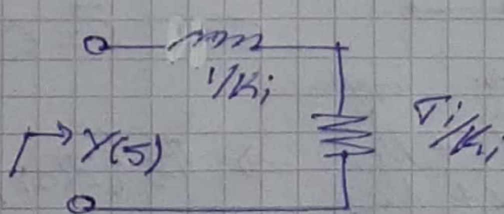


• $F(s) = \frac{K_i}{s+\sigma_i} \rightarrow Z(s) = \frac{1}{s\,1/K_i + \frac{\sigma_i}{K_i}}$ → suma de admitancias

Un polo sobre el eje σ da lugar a un filtro disipativo.



$\rightarrow Y(s) = \frac{1}{\frac{s}{K_i} + \frac{\sigma_i}{K_i}}$



- $F(s) = \dfrac{2K_i s}{s^2 + \omega_i^2}$; $F(s) = \dfrac{K_i}{s + j\omega_i} + \dfrac{K_i^*}{s - j\omega_i}$

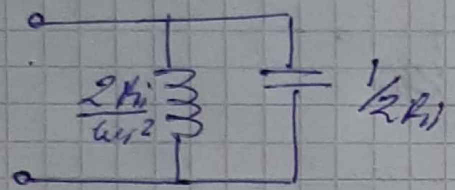
$\rightarrow K_i = K_i^*$; $K_i \in \mathbb{R}$

$$F(s) = \frac{K_i (s - j\omega_i) + K_i (s + j\omega_i)}{s^2 - j^2 \omega_i^2} = \frac{K_i s - j\omega_i K_i + K_i s + j\omega_i K_i}{s^2 + \omega_i^2}$$

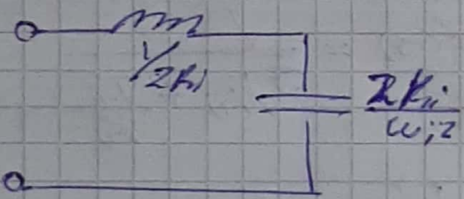
$$F(s) = \frac{2K_i s}{s^2 + \omega_i^2} = \frac{1}{\frac{s^2}{2K_i s} + \frac{\omega_i^2}{2K_i s}}$$

$$F(s) = \frac{1}{s\frac{1}{2K_i} + \frac{1}{s\frac{2K_i}{\omega_i^2}}}$$

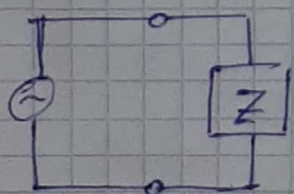
$\rightarrow Z(s)$



$\rightarrow Y(s)$



Síntesis de dipolos no-disipativos



$$\bar{P} = \frac{1}{2} \mathrm{Re}\{Z(j\omega)\} \cdot |I(j\omega)|^2$$

Si $\bar{P} = 0 \Rightarrow \mathrm{Re}\{Z(j\omega)\} = 0$

$$Z(s) = \frac{\overset{\text{par}}{M_1} + \overset{\text{impar}}{N_1}}{M_2 + N_2} \Rightarrow \mathrm{Re}\{Z(s)\} = \mathrm{Par}\{Z(s)\} = \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2}$$

Si $\mathrm{Re}\{Z(s)\} = 0 \Rightarrow M_1 M_2 - N_1 N_2 = 0$

- $M_2 = N_1 = 0$ $\quad Z(s) = \dfrac{M_1}{N_2}$ $\quad \vee$ $\quad$ • $M_1 = N_2 = 0$ $\quad Z(s) = \dfrac{N_1}{M_2}$

NOTA $\boxed{9:30} \rightarrow \boxed{10:30} \rightarrow \boxed{12:00}$ $\quad \boxed{10:50}$

- Si queremos que la inmitancia sea no disipativa

$\Rightarrow F(s) = \frac{Par}{Impar} \vee \frac{Impar}{par}$

Singularidades serán imaginarias. (Polos y ceros)

$$F(s) = \frac{Par}{impar} = \frac{a_4 s^4 + a_2 s^2 + a_0}{b_3 s^3 + b_1 s} = \frac{k_1 (s^2+\omega_2^2)(s^2+\omega_3^2)}{s(s^2+\omega_1^2)}$$

$$F(s) = \frac{impar}{par} = \frac{k\, s(s^2+\omega_1^2)}{(s^2+\omega_2^2)}$$

Si $\begin{array}{l} par \rightarrow +1 \\ impar \rightarrow -1 \end{array} \Rightarrow F(s) = \frac{k_0}{s} + \frac{2k_1 s}{(s^2+\omega_1)^2} + \dots + k_\infty s$

(polo en infinito)

$$F(j\omega) = j\left(-\frac{k_0}{\omega} + \frac{2k_1\omega}{\omega_1^2-\omega^2} + \dots + k_\infty \omega\right)$$

$F(j\omega) = j \longrightarrow X(\omega)$ Reactancia

$\quad \longrightarrow B(\omega)$ Susceptancia

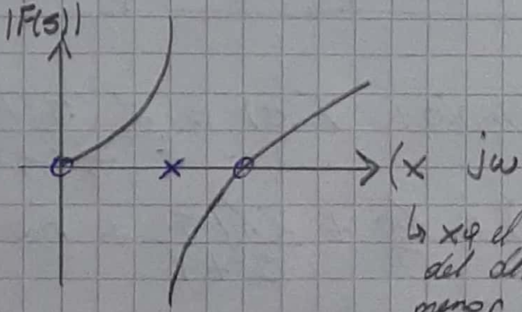
$$\frac{\partial X(\omega)}{\partial \omega} = \frac{k_0}{\omega^2} + \frac{2k_1(\omega_1^2+\omega^2)}{(\omega_1-\omega_2)^2} + k_\infty \Rightarrow \text{siempre} > 0$$

$\oplus \qquad \oplus$

$\Rightarrow F(s)$ siempre creciente con la frecuencia.

ejemplo

$$F(s) = s\,\frac{(s^2+\omega_1^2)}{(s^2+\omega_2^2)}$$



↳ x q el orden del den es menor que el del num.

que $\frac{\partial X(\omega)}{\partial \omega} > 0$ impone una

alternancia de singularidades $\Rightarrow$ o x o x o x

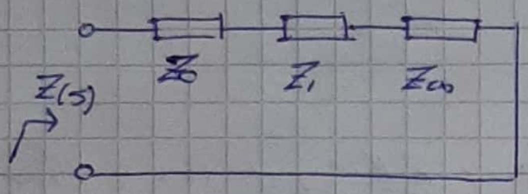
x o x o x o

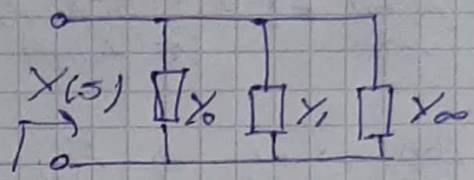
Teorema de Foster

$$F(s) = \frac{P(s)}{Q(s)} = \frac{(s^2+\omega_{n+1}^2)(s^2+\omega_{n+2}^2)+\dots}{s(s^2+\omega_1^2)(s^2+\omega_2^2)+\dots} =$$

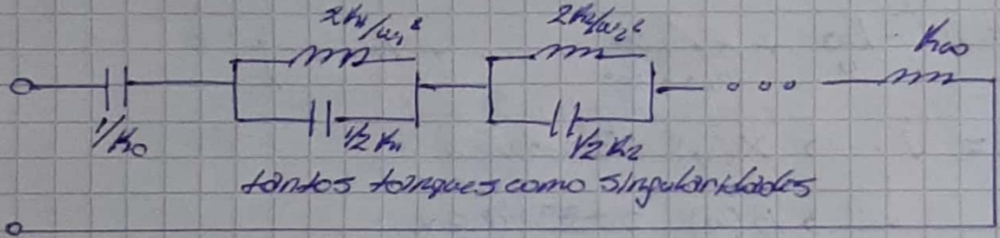
$$= \frac{K_0}{s} + \frac{2K_1 s}{s^2+\omega_1^2} + \dots + K_\infty s$$

$$F(s) \begin{cases} \rightarrow Z(s) \rightarrow \text{serie} \\ \rightarrow Y(s) \rightarrow \text{derivación} \end{cases}$$

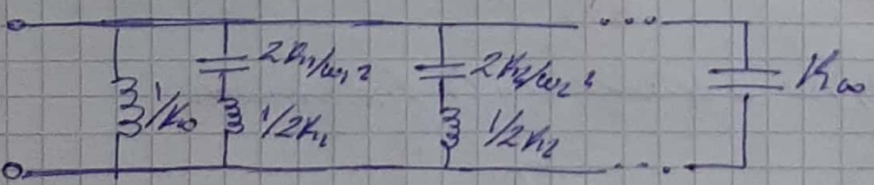




Foster serie

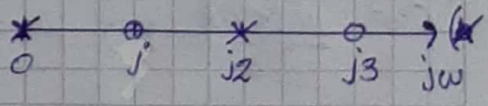


Foster paralelo



Ejemplo: $F(s) = \dfrac{2(s^2+1)(s^2+9)}{s(s^2+4)} = Z(s)$

$$Z(s) = \frac{K_0}{s} + \frac{2K_1 s}{s^2+4} + K_\infty s$$



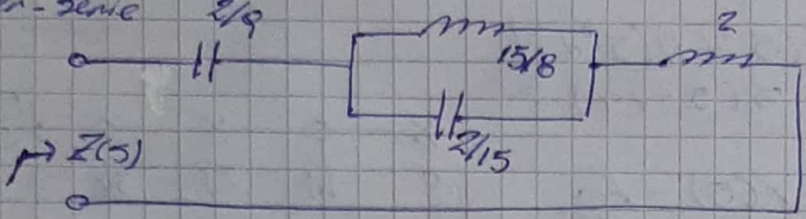
$$K_0 = \lim_{s\to 0} Z(s) \cdot \cancel{s}\, \frac{2(s^2+1)(s^2+9)}{\cancel{s}(s^2+4)} = \frac{2\cdot 1\cdot 9}{4} = \frac{9}{2}$$

$$2K_1 = \lim_{s^2\to -4} \frac{s^2+4}{s} \cdot \frac{2(s^2+1)(s^2+8)}{s\cancel{(s^2+4)}} = \frac{2(-4+1)(-4+8)}{-4}$$

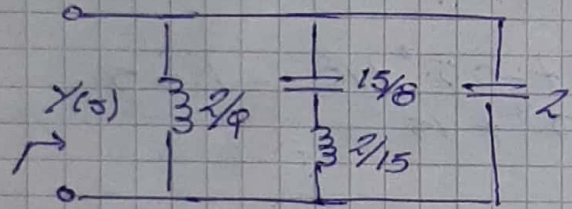
$$2K_1 = \frac{2(-3)(+5)}{-4} = \frac{30}{4} = \frac{15}{2}$$

$$K_\infty = \lim_{s\to\infty} \frac{Z(s)}{s} = \frac{2(s^2+1)(s^2+8)}{s^2(s^2+4)} = 2$$

Foster - serie



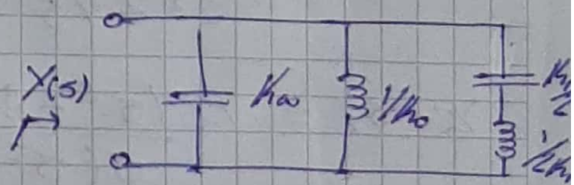
Foster - paralelo



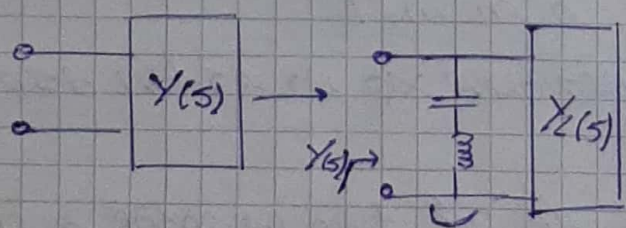
Método de Cauer

- Concepto de remoción

$$F(s) = \frac{2(s^2+8)(s^2+1)}{(s^2+4)\,s}$$

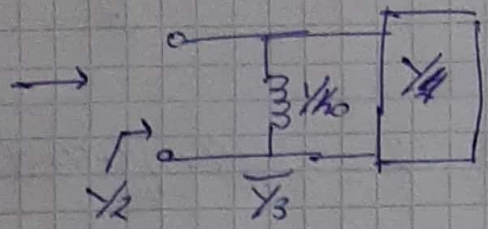


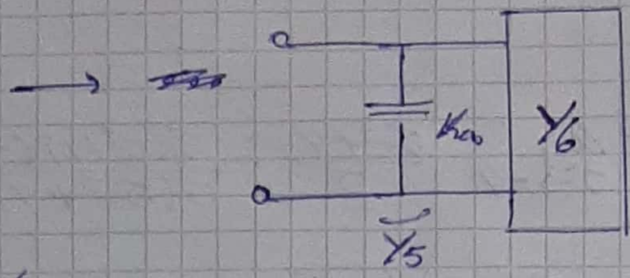
$$Y(s) = \frac{K_0}{s} + K_\infty s + \frac{2K_1 s}{s^2+4}$$



$$Y_2 = Y(s) - Y_1(s)$$

$$Y_1(s) = \frac{2K_1 s}{s^2+4}$$



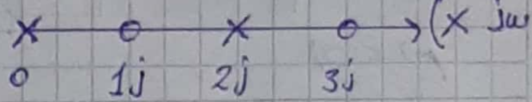


$\Rightarrow Y(s) = Y_1 + Y_3 + Y_5$

El orden de remoción es arbitrario.

¿Qué tal si siempre removemos singularidades en $s \to \infty$?
$\vee$
$s \to 0$

$$F(s) = Y(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$



Cauer I $\quad Y_2(s) = Y(s) - \frac{K_0}{s}$

Cauer II $\quad Y_2(s) = Y(s) - K_{\infty} s$

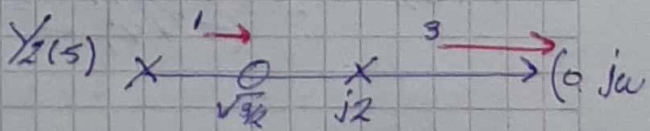
$$\hookrightarrow Y(s) - K_{\infty} s = \frac{2s^4+20s^2+18}{s^3+4s} - K_{\infty} s$$

$$K_{\infty} = \lim_{s\to\infty} \frac{Y(s)}{s} = 2$$

$$\Rightarrow Y(s) - K_{\infty} s = \frac{2s^4+20s^2+18-2s(s^3+4s)}{s^3+4s}$$

$$Y_2(s) = \frac{12s^2+18}{s^3+4s}$$ se cancela el orden mayor y queda en orden 2

$$Y_2(s) = \frac{12(s^2+3/2)}{s(s^2+4)}$$



Los polos se mantienen en su lugar cuando se hace una remoción

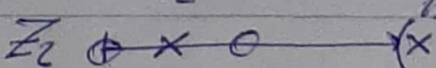
Los ceros se movieron (solo se mueven si no están en cero o en inf)

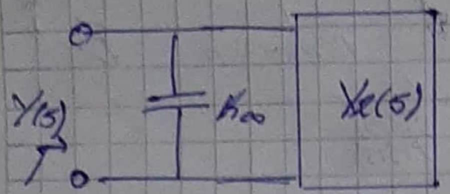
Los ceros no se pueden remover xq no están asociados a los residuos.

Si quiero sacar el cero de inf tengo que pasar a una conf. de impedancia

NOTA

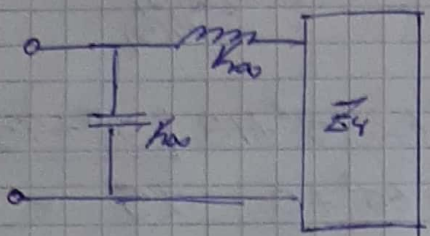
$$\frac{1}{Y_2} = Z_2 = \frac{s^3+4s}{12s^2+18}$$





$$Y_2 = \frac{12s^2+18}{s^3+4s}$$

$$\Rightarrow \frac{1}{Y_2} = Z_2 = \frac{s^3+4s}{12s^2+18}$$



$$\rightarrow Z_4 = Z_2 - K_{2,\infty}\, s$$

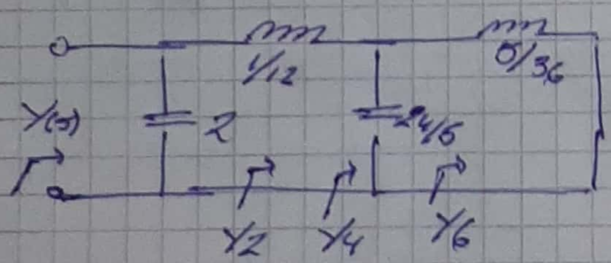
$$K_{2,\infty} = \lim_{s\to\infty} \frac{Z_2}{s} = \frac{1}{12}$$

$$\Rightarrow Z_4 = \frac{5/2\, s}{12s^2+18}$$

$$Y_4 = \frac{12s^2+18}{5/2\, s} \rightarrow Y_6 = Y_4 - K_{4,\infty}\, s$$

$$K_{4,\infty} = \lim_{s\to\infty} \frac{Y_4}{s} = \frac{24}{5}$$

$$Y_6 = \frac{12s^2+18}{5/2\, s} - \frac{24}{5} s \Rightarrow Y_6 = \frac{36}{5}\frac{1}{s}$$



Se sintetiza removiendo $K_\infty s$

$$C_c = C_f + 1$$

→ cant. de singularidades críticas (son todas las singularidades menos las que están en 0 y en inf)

↳ cant. de componentes

$$\frac{P(s)}{Q(s)} = \frac{C(s)\,Q(s)}{Q(s)} + \frac{R(s)}{Q(s)} \Rightarrow F(s) = \underset{\uparrow\ \text{remoción}}{C(s)} + \underset{\uparrow\ F_2}{\frac{R(s)}{Q(s)}} \to \text{Red}$$

$$Y_2 \longrightarrow \frac{1}{Y_2} = Z_2 \Rightarrow F(s) = C(s) + \frac{1}{\frac{Q(s)}{R(s)}}$$

$$= C_1(s) + \frac{1}{C_2(s) + \frac{R_2(s)}{R(s)}}$$

$$F(s) = \frac{2s^4 + 20s^2 + 18}{s^3 + 4s}$$

$Y(s) \nearrow$

$$\begin{array}{r|l} 2s^4 + 20s^2 + 18 & s^3 + 4s \to \text{nuevo dividendo} \\ -\ 2s^4 + 8s^3 + 0 & 2s \to \text{capacitor } 2 \\ \hline 0 + \underbrace{12s^2 + 18}_{R(s)} & \end{array}$$

↳ nuevo cociente

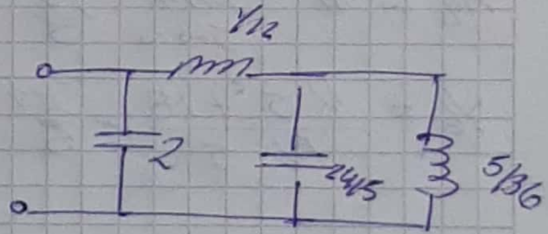
Ahora

$$\begin{array}{r|l} s^3 + 4s & 12s^2 + 18 \\ -\ s^3 + 3/2\, s & 1/12\, s \to \text{inductor } 1/12 \\ \hline 5/2\, s & \end{array}$$

↳ nuevo cociente

$$\begin{array}{r|l} 12s^2 + 18 & 5/2\, s \\ -\ 12s^2 \ \cancel{18} & 24/5\, s \to \text{capacitor } 24/5 \\ \hline 18 & \end{array}$$

$$\begin{array}{r|l} 5/2\, s & 18 \\ -\ 5/2\, s & 5/36\, s \to \text{inductor } 5/36 \\ \hline 0 & \end{array}$$



Si en vez de remover singularidades en ∞ las remuevo en cero

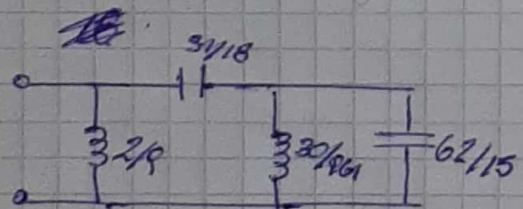
Ordeno el polinomio de menor a mayor

$$\begin{array}{r|l} 18 + 20s^2 + 2s^4 & 4s + s^3 \\ 18\ \ 9/2\, s^2 + 0 & 9/2\, \frac{1}{s} \to \text{inductor } 2/9 \end{array}$$

$$\begin{array}{r|l} 4s + s^3 & 31/2\, s^2 + 2s^4 \\ -\ 4s + 16/31\, s^3 & 8/31\, 1/s \to \text{capacitor } 31/8 \\ \hline 15/31\, s^3 & \end{array}$$

$$\begin{array}{r|l} 31/2\, s^2 + 2s^4 & 15/31\, s^3 \\ -\ 31/2\, s^2 + 0 & 961/30\, 1/s \to \text{inductor } 30/961 \\ \hline 2s^4 & \end{array}$$

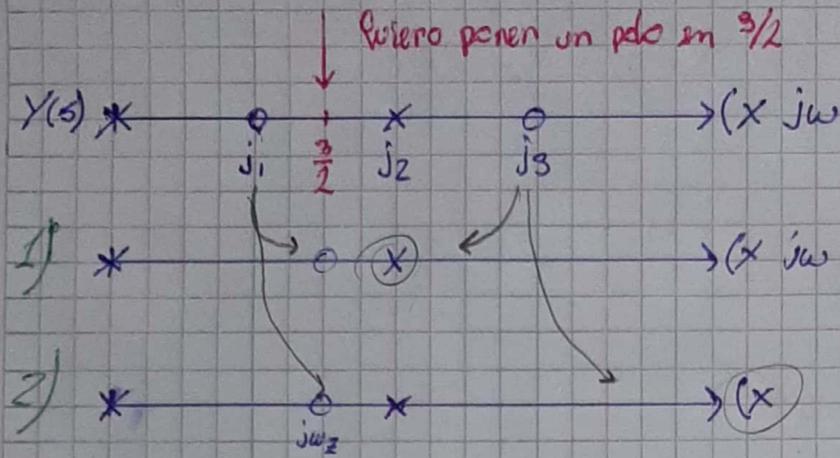
$$\begin{array}{r|l} 15/31\, s^3 & 2s^4 \\ \hline 0 & 15/62\, 1/s \to \text{capacitor } 62/15 \end{array}$$



NOTA

Remociones parciales

↳ Fijar un cero en una determinada frecuencia.



Puedo hacer una remoción parcial del polo en j2 y los ceros se atraen hacia j2.

remoción parcial de k_∞

caso 2) $Y(s) - k'_\infty s = Y_2$ siendo $k'_\infty < k_\infty$

$$Y(s) - k'_\infty s \Big|_{s = j\omega_z} = Y_2(s) = 0$$

$$k'_\infty = \frac{Y(s)}{s} \Big|_{s = j\omega_z}$$

$$\Rightarrow Y_2 \Big|_{s = j\omega_z} = Y(s) - \frac{2k_1' s}{s^2 + \omega_1^2} \qquad k_1' < k_1$$

$$2k_1' = Y(s) \cdot \frac{(s^2 + \omega_1^2)}{s} \Big|_{s = j\omega_z}$$

NOTA