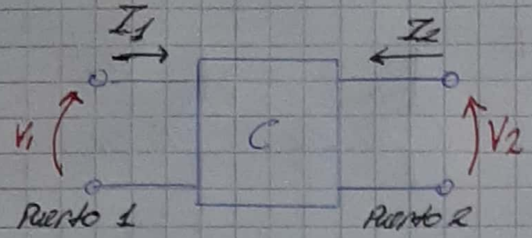
Cuadripolos lineales : Anasto Cap 5



$R_{esp} = C \cdot E$

E ↳ excitación

C ↳ Matriz de parámetros

$$\begin{cases} r_1 = e_1 C_{11} + e_2 C_{12} \\ r_2 = e_1 C_{21} + e_2 C_{22} \end{cases}$$

$E, R \ (N \times 1) \quad (2 \times 1)$

$C \ (N \times N) \quad (2 \times 2)$

Parámetros (E)	Variables dependientes (R)		variables independientes
$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$	$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$	$\longrightarrow$	Y (admitancia)
$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$	$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$	$\longrightarrow$	Z (impedancia)
$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$	$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$	$\longrightarrow$	H (híbridos) (V_o/V_i)
$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$	$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$	$\longrightarrow$	G / γ $\quad I_o/I_i$
$\begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$	$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$	$\longrightarrow$	T ó $ABCD = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$
$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$	$\begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$	$\longrightarrow$	T^{-1}

$$\begin{cases} I = Y \cdot V \\ V = Z \cdot I \end{cases} \Rightarrow I = \underbrace{Y \cdot Z}\, \cdot I$$

$$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow Y \cdot Z = \mathbb{I}$$

$$\boxed{Y^{-1} = Z}$$

de la misma manera $\quad H^{-1} = \gamma$

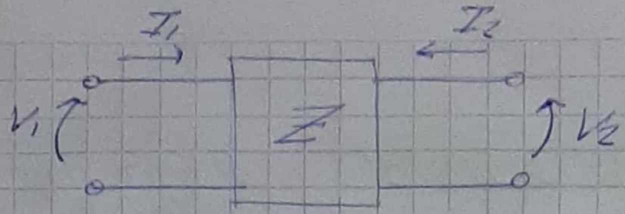
$(T^{-1})^{-1} = T$ (esto es obvio)

DCBA $\quad$ ABCD

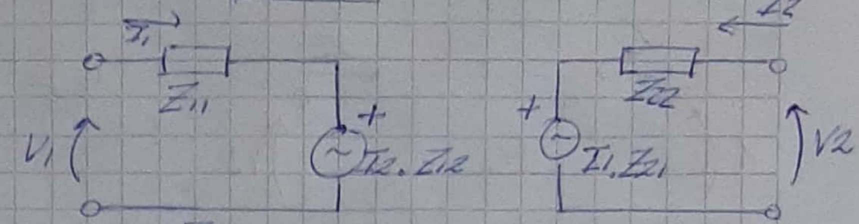
NOTA $\boxed{17:00} \rightarrow \boxed{18:00}$

Parámetros Z

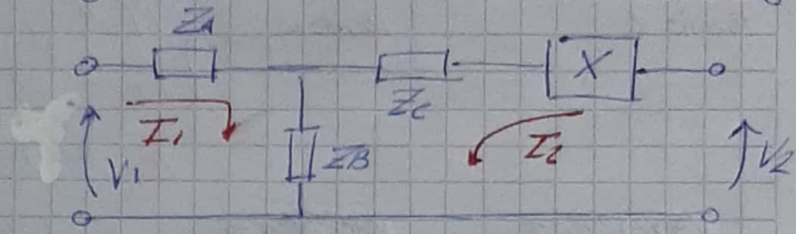
$V = Z \cdot I$



$$\begin{cases} v_1 = i_1 \, Z_{11} + i_2 \, Z_{12} \\ v_2 = i_1 \, Z_{21} + i_2 \, Z_{22} \end{cases}$$



$$\begin{cases} V_1 = I_1 (Z_A + Z_B) + I_2 Z_B \\ V_2 = I_2 (Z_C + Z_B) + I_1 Z_B + X \end{cases}$$



Viendo los 2 modelos se propone:

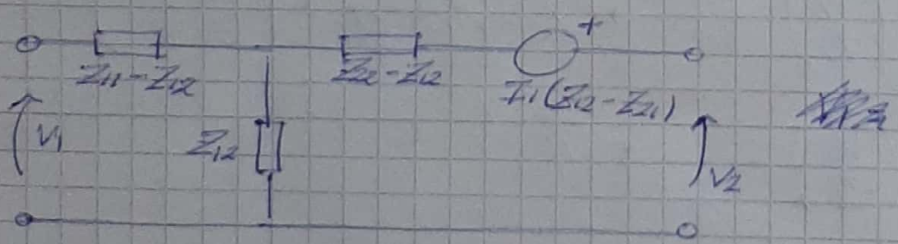
$\boxed{Z_B = Z_{12}}$

$Z_A + Z_B = Z_{11} \Rightarrow \boxed{Z_A = Z_{11} - Z_{12}}$

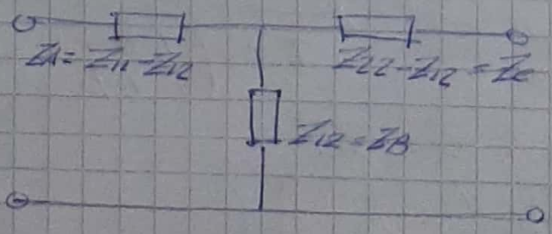
$\boxed{Z_C = Z_{22} - Z_{12}}$ $\Rightarrow$ hasta ahora tiempo

$V_2 = I_2 (Z_{22} - Z_{12} + Z_{12}) + I_1 Z_{12} + I_1 Z_{21} - I_1 Z_{21}$ ~~XX~~

$V_2 = I_1 Z_{21} + I_2 (\underbrace{Z_{22} - Z_{12}}_{Z_C} + \underbrace{Z_{12}}_{Z_B}) + \underbrace{I_1 (Z_{12} - Z_{21})}_{X}$



Cuando $Z_{12} = Z_{21}$



Cuadripolo T
pasivo
Recíproco

Cuadripolo recíproco $\equiv$ pasivo

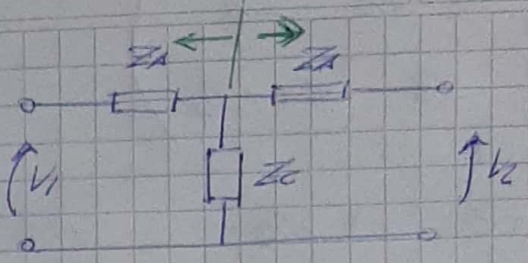
NOTA

HOJA N° 31

FECHA

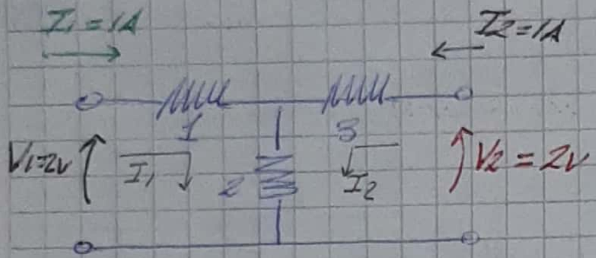
Si $Z_{11} = Z_{22}$

$$\left.\begin{aligned} Z_A &= Z_{11} - Z_{12} \\ Z_B &= Z_{12} \\ Z_C &= Z_{11} - Z_{12} \end{aligned}\right\} Z_A = Z_C$$



cuadripolo simétrico y recíproco.

Red T sencilla



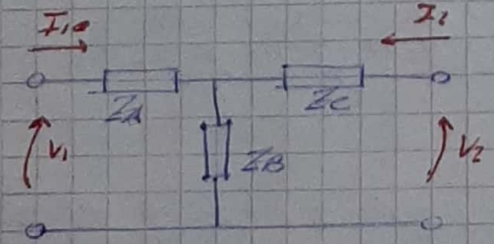
Reciprocidad es que la respuesta en el puerto 1 es la misma que en el puerto 2 cuando se excita desde el puerto 2 o 1 respectivamente con la misma excitación

$$V_1 = I_1 Z_{11} + I_2 Z_{21}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22}$$

$$\left.\begin{aligned} Z_{in} = Z_{11} &= \frac{V_1}{I_1}\Big|_{I_2=0} \\ Z_r = Z_{12} &= \frac{V_1}{I_2}\Big|_{I_1=0} \\ Z_f = Z_{21} &= \frac{V_2}{I_1}\Big|_{I_2=0} \\ Z_{out} = Z_{22} &= \frac{V_2}{I_2}\Big|_{I_1=0} \end{aligned}\right\}$$

Parámetros de circuito abierto
o
parámetros Z



Recordar de antes que:

$$\begin{cases} Z_A = Z_{11} - Z_{12} \\ Z_B = Z_{12} \end{cases}$$

$$\underset{FPP}{Z_{11}} = \frac{V_1}{I_1}\Big|_{I_2=0} = Z_A + Z_B$$

$$\underset{Ft}{Z_{21}} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_B$$

$$\underset{Ft}{Z_{12}} = \frac{V_1}{I_2}\Big|_{I_1=0} = Z_B$$

$$\underset{FPP}{Z_{22}} = \frac{V_2}{I_2}\Big|_{I_1=0} = Z_C + Z_B$$

Ft → función transferencia

↳ funciones reales y positivas

NOTA 21:30 → 22:02 22:20 → 23:20

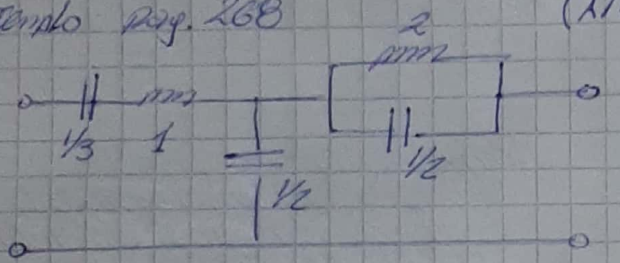
Escaneado con CamScanner

$$Z = \begin{pmatrix} Z_A + Z_B & Z_B \\ Z_B & Z_C + Z_B \end{pmatrix}$$

Polos $Z_B \rightarrow Z_{12}$
$\searrow Z_{21}$

Polos Z_{11}, Z_{22} → Privados $(Z_A)(Z_C)$
→ comunes (Z_B)
↳ los polos de Z_B se comparten con los de Z_A y Z_C

Ejemplo pag. 268 (λPauto)



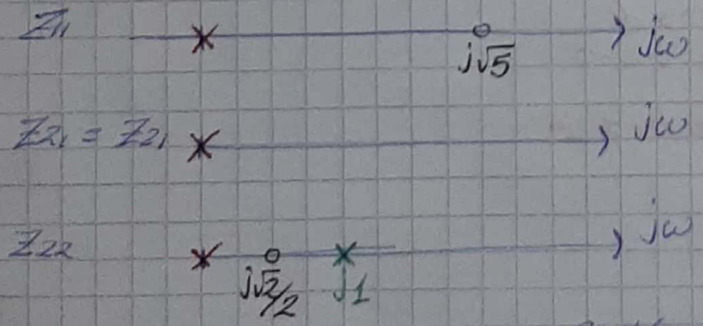
$$Z_{11} = Z_A + Z_B = \frac{3}{s} + s + \frac{2}{s} = \frac{5+s^2}{s}$$

$$Z_{12} = Z_{21} = Z_B = \frac{2}{s}$$

Polo común

$$Z_{22} = Z_C + Z_B = \frac{2}{s} + \frac{1}{\frac{s}{2} + \frac{1}{2s}} = \frac{2}{s} + \frac{2s}{s^2+1} = \frac{4s^2+2}{s(s^2+1)}$$

↳ Polo privado



$$\frac{P(s)}{Q(s)} = \frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \dots$$

residuos que determinamos en cada uno de los polos están asociados con los componentes

$\boxed{k_{11} \cdot k_{22} - k_{12}^2 \geq 0}$ condición de residuos en polos ~~[illegible]~~ comunes

$= 0$ polo compacto.

↳ si no se cumple esta condición ⇒ el circuito no es realizable con estructuras pasivas.

NOTA

Residuos

$k_{11} = \lim_{s\to 0} s. Z_{11}(s) =$

$$Z_{11}(s) = \frac{P(s)}{Q(s)} = \frac{k_0}{s} + \overbrace{\frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \dots + k_\infty s}^{\approx 0}$$

↳ Residuo de la sing. nula cuando $s\to 0$

$$\lim_{s\to 0} Z_{11}(s) = \frac{k_0}{s}$$

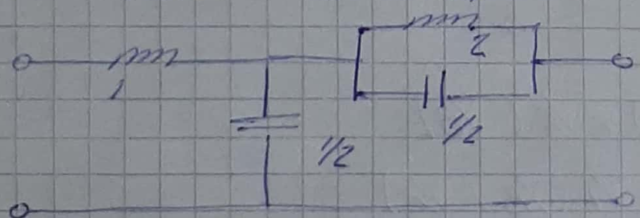
$$\Rightarrow \boxed{k_0 = \lim_{s\to 0} s\, Z_{11}(s)} = \lim_{s\to 0} s. \frac{5+s^2}{s} = 5 = k_{11}$$

$$k_0 = \lim_{s\to 0} s. Z_{12}(s) = \lim_{s\to 0} s. \frac{2}{s} = 2 = k_{21} = k_{12}$$

$$k_0 = \lim_{s\to 0} s\, Z_{22}(s) = \lim_{s\to 0} s. \frac{4s^2+2}{s(s^2+1)} = 2 = k_{22}$$

$$\Rightarrow 5.2 - 2^2 = 6 \geq 0 \quad ✓$$

Se puede forzar el polo compacto



$$Z_{11} = Z_A + Z_B = s + \frac{2}{s} = \frac{s^2+2}{s}$$

$$Z_{12} = Z_{21} = \frac{2}{s}$$

$$Z_{22} = Z_B + Z_C = \frac{2}{s} + \frac{1}{\frac{s}{2}+\frac{1}{2s}} = \frac{4s^2+2}{s(s^2+1)}$$

Residuos

$$k_{11} = \lim_{s\to 0} s. \frac{s^2+2}{s} = 2$$

$$k_{12} = k_{21} = \lim_{s\to 0} s. \frac{2}{s} = 2$$

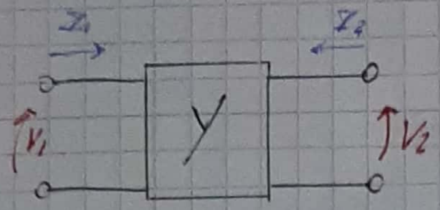
$$k_{22} = \lim_{s\to 0} s. \frac{4s^2+2}{s(s^2+1)} = 2$$

$$\Rightarrow 2.2 - 2^2 = 0$$

↳ $\boxed{\text{polo compacto} \quad s=0}$

NOTA

Parámetros Y (o parámetros de cc)



$I = Y V$

$$\begin{cases} I_1 = V_1 Y_{11} + V_2 Y_{12} \\ I_2 = V_1 Y_{21} + V_2 Y_{22} \end{cases}$$

$Y_i = Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ F.Excitación (FBP)

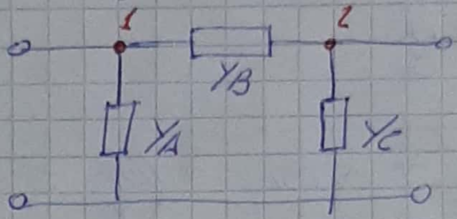
$Y_f = Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ F.T

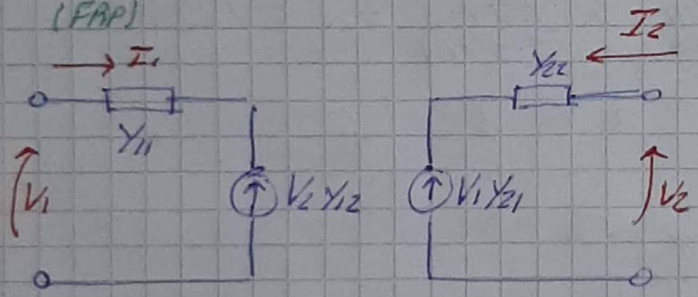
$Y_r = Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ F.T

$Y_o = Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

↳ F. excitación (FBP)

Topología Pi (π)





Nodo 1 : $I_1 = V_1 (Y_A + Y_B) + V_2 (-Y_B)$

Nodo 2 : $I_2 = V_1 (-Y_B) + V_1 (Y_B + Y_C)$

$\boxed{-Y_B = Y_{12} = Y_{21}}$

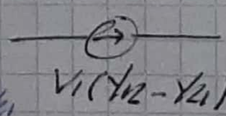
$Y_{11} = Y_A + Y_B \Rightarrow \boxed{Y_A = Y_{11} + Y_{12}}$

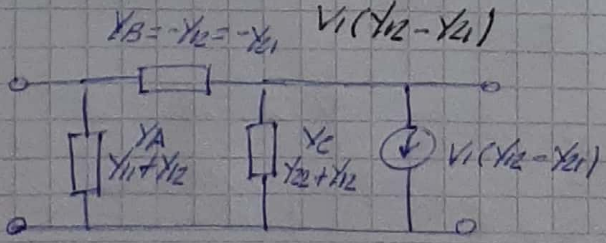
reescribo la ec de nodo 2 $\quad I_2 = V_1 Y_{12} + V_2 (Y_C - Y_{12}) \rightarrow$ Falta Y_{21}

$\Rightarrow I_2 = V_1 Y_{21} \underline{- V_1 Y_{21} + V_1 Y_{12}} + V_2 \underbrace{(Y_C - Y_{12})}_{Y_{22}}$

$\Rightarrow \boxed{Y_C = Y_{22} + Y_{12}}$

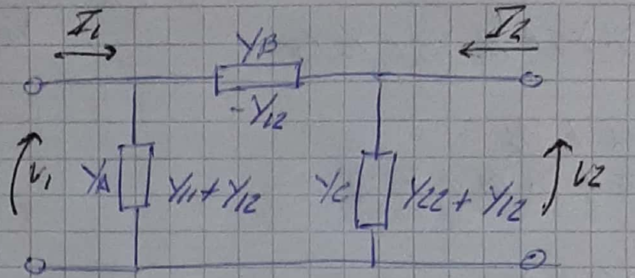
↳





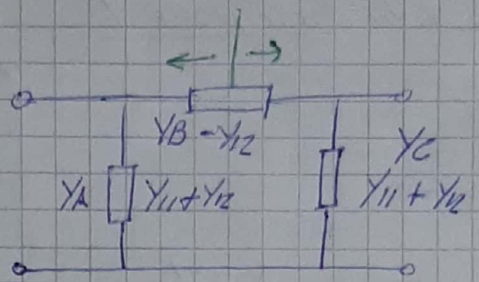
Si $Y_{12} = Y_{21}$

Red recíproca pasiva



Si $Y_{11} = Y_{22}$

Red simétrica



$$Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}$$

$$Y = \begin{pmatrix} Y_A + Y_B & -Y_B \\ -Y_B & Y_C + Y_B \end{pmatrix}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A + Y_B$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_B + Y_C$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_B$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_B$$

$$Y = Z^{-1}$$

$$Y^{-1} = \frac{1}{\Delta Y} \cdot AdJ(Y) = Z$$

$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$AdJ(Y) = \begin{pmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{pmatrix}$$

Si quiero calcular los parámetros Z del cuadripolo Pi →

$$\Rightarrow Z = Y^{-1} = \frac{1}{\Delta Y} AdJ(Y)$$

$$\Delta Y = (Y_A + Y_B)(Y_C + Y_B) - Y_B^2$$

$$\Delta Y = Y_A Y_C + Y_A Y_B + Y_B Y_C + \cancel{Y_B^2} - \cancel{Y_B^2}$$

$$\Rightarrow \Delta Y = Y_A Y_C + Y_A Y_B + Y_B Y_C$$

$$Z = \frac{1}{\Delta Y} \begin{pmatrix} Y_C + Y_B & Y_B \\ Y_B & Y_A + Y_B \end{pmatrix}$$

NOTA

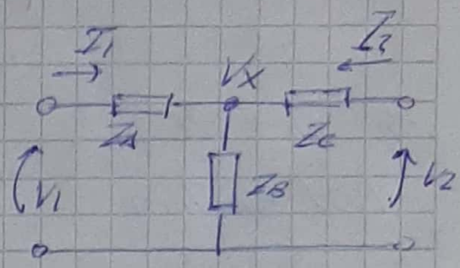
es conv ~~Δ~~ $Y \rightarrow \Delta$ $\quad$ $Y \Rightarrow \Delta$

$T \longrightarrow \pi$

$$\Rightarrow Z_{11} = \frac{Y_C + Y_B}{Y_A Y_C + Y_A Y_B + Y_C Y_B} \quad ; \quad Z_{12} = \frac{Y_B}{Y_A Y_C + Y_A Y_B + Y_C Y_B} = Z_{21}$$

$$Z_{22} = \frac{Y_A + Y_B}{Y_A Y_C + Y_A Y_B + Y_C Y_B}$$

Parámetros $Z \rightarrow Y$ de un cuadripolo T



$$\begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{cases}$$

$$Y_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = [Z_A + (Z_B // Z_C)]^{-1} = \frac{Z_B + Z_C}{Z_A Z_B + Z_B Z_C + Z_A Z_C}$$

$$Y_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = [Z_C + (Z_B // Z_A)]^{-1} = \frac{Z_A + Z_B}{Z_A Z_B + Z_B Z_C + Z_A Z_C}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad I_1 = \frac{V_x}{Z_A}$$

$$-V_x = V_2 \frac{Z_A // Z_B}{(Z_A // Z_B) + Z_C}$$

$$\Rightarrow Y_{12} = \left[\frac{-Z_A // Z_B}{(Z_A // Z_B) + Z_C} \cdot \frac{1}{Z_A}\right]$$

$$Y_{21} = \frac{-Z_C // Z_B}{(Z_C // Z_B) + Z_A} \cdot \frac{1}{Z_C}$$

otra manera es usando la matriz

$$Z = \begin{pmatrix} Z_A + Z_B & Z_B \\ Z_B & Z_B + Z_C \end{pmatrix} \Rightarrow \Delta Z = (Z_A + Z_B)(Z_B + Z_C) - Z_B^2$$

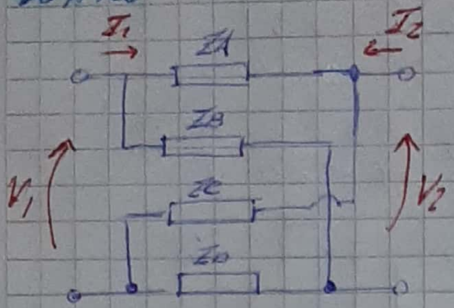
$$\Delta Z = Z_A Z_B + Z_B Z_C + Z_A Z_C$$

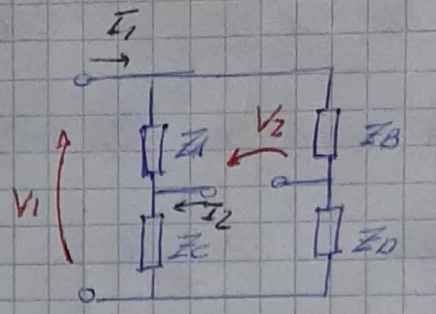
$$Y = Z^{-1} = \frac{1}{\Delta Z} \begin{pmatrix} Z_B + Z_C & -Z_B \\ -Z_B & Z_A + Z_B \end{pmatrix}$$

↳ mucho más fácil y rápido.

NOTA

Lattice





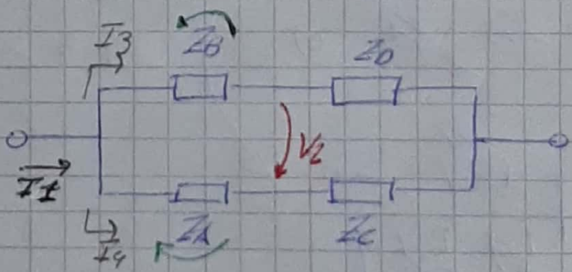
$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = (Z_A+Z_C)//(Z_B+Z_D) = \frac{(Z_A+Z_C)(Z_B+Z_D)}{Z_A+Z_B+Z_C+Z_D}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(Z_A+\cancel{B}Z_B)(Z_C+Z_D)}{\Sigma Z}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_{21}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Rightarrow$$



$$\hookrightarrow I_1 = \frac{V_1}{Z_{11}}$$

$\frac{1}{I_1}$

$$\Rightarrow Z_{21} = \left.\frac{V_2}{V_1} \cdot Z_{11}\right|_{I_2=0} = \left(\frac{Z_C}{Z_C+Z_A} - \frac{Z_D}{Z_D+Z_B}\right) Z_{11}$$

$$\Rightarrow Z_{21} = \frac{Z_C(Z_D+Z_B) - Z_D(Z_C+Z_A)}{\cancel{(Z_A+Z_C)(Z_D+Z_B)}} \cdot \frac{\cancel{(Z_A+Z_C)(Z_B+Z_D)}}{Z_A+Z_B+Z_C+Z_D}$$

$$Z_{21} = \frac{\cancel{Z_C Z_D} + Z_C Z_D - \cancel{Z_D Z_C} - Z_D Z_A}{\Sigma Z} = \boxed{\frac{Z_C Z_D - Z_D Z_A}{\Sigma Z}}$$

NOTA $+3h$ $\boxed{TOTAL = 5h\,30min}$