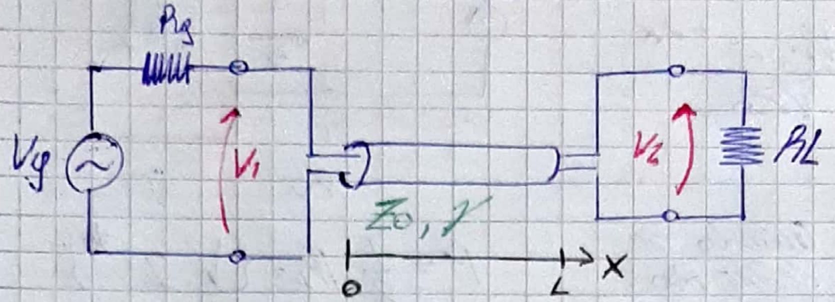
Parámetros S (scattering) ⟶ útil para frec. elevadas.



$$\begin{cases} V(x) = V_i\, e^{-\gamma x} + V_r\, e^{+\gamma x} \\ I(x) = \underbrace{I_i\, e^{-\gamma x}}_{\text{incidente}} - \underbrace{I_r\, e^{+\gamma x}}_{\text{reflejada}} \end{cases}$$

$Z_0 = \sqrt{\frac{Z}{Y}}$ ↳ imp. característica

$\gamma = \sqrt{Z \cdot Y}$ ↳ cte de propagación

$$I(x) = \frac{V_i}{Z_0} e^{-\gamma x} - \frac{V_r}{Z_0} e^{+\gamma x}$$

$$V(x) = V_i\, e^{-\gamma x} + V_r\, e^{+\gamma x}$$

$$\Rightarrow \begin{cases} V(x) + I(x) Z_0 = 2V_i\, e^{-\gamma x} \\ V(x) - I(x) Z_0 = 2V_r\, e^{+\gamma x} \end{cases} \Rightarrow \begin{cases} V_i\, e^{-\gamma x} = \frac{1}{2}\left(V(x) + I(x) Z_0\right) \\ V_r\, e^{+\gamma x} = \frac{1}{2}\left(V(x) - I(x) Z_0\right) \end{cases}$$

Si $x = L$; $Z_L = Z_0$

$$V_r \cdot e^{\gamma L} = \frac{1}{2}\left(V(L) - I(L) Z_0\right)$$

$$V_r\, e^{\gamma L} = \frac{1}{2} I(L) \underbrace{\left(\underbrace{\frac{V(L)}{I(L)}}_{Z_0} - Z_0\right)}_{\emptyset}$$

↳ ⇒ $\emptyset$

cuando la línea está adaptada $Z_L = Z_0$

⇒ $V_r = \emptyset$ ∴ No hay onda reflejada.

onda incidente $a = \frac{1}{2}\left(\frac{V(x)}{\sqrt{R_0}} + \sqrt{R_0}\, I(x)\right)$

onda reflejada $b = \frac{1}{2}\left(\frac{V(x)}{\sqrt{R_0}} - \sqrt{R_0}\, I(x)\right)$

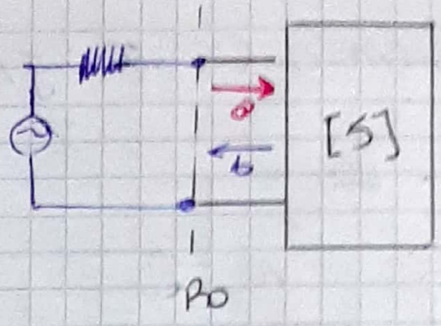
(ambas reflejadas)

↳ R_0 es una constante que se conoce como imp. de referencia.

Si $R_0 = Z_0$

$$a = \frac{V_i \cdot e^{-\gamma x}}{\sqrt{Z_0}}$$

$$b = \frac{V_r \cdot e^{+\gamma x}}{\sqrt{Z_0}}$$



$$\left.\begin{aligned} V(x) &= \sqrt{Z_0}\,(a+b) \\ I(x) &= \frac{1}{\sqrt{Z_0}}\,(a-b) \end{aligned}\right\}$$ cuando se define R_0

$$P = \frac{1}{2}\,\mathrm{Re}\{V \cdot I^*\}$$
↳ pot activa

$$\Rightarrow P = \frac{1}{2}\,\mathrm{Re}\left\{\sqrt{Z_0}\,(a+b) \cdot \frac{1}{\sqrt{Z_0}}\,(a-b)^*\right\}$$

$$P = \frac{1}{2}\,\mathrm{Re}\{\underbrace{a \cdot a^* - b\,b^*}_{\mathrm{Re}} + \underbrace{b \cdot a^* - a \cdot b^*}_{\mathrm{Im}}\}$$

$$P = \frac{1}{2}\left(|a|^2 - |b|^2\right)$$
↳ pot reflejada
↳ pot incidente

otra forma de verlo

$$P = \frac{1}{2}|a|^2\left(1 - \left(\frac{|b|}{|a|}\right)^2\right) = \frac{1}{2}|a|^2\left(1 - |S|^2\right)$$

↳ coeficiente de reflexión $= S$

∴ La max transferencia de potencia

$$P = \frac{1}{2}|a|^2$$

Si $S = 0 \Rightarrow$ Adaptación
cuando $R_0 = R_g$

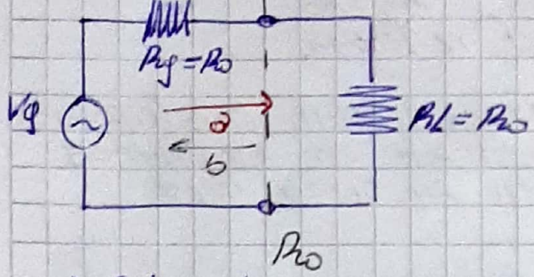
- $$S = \frac{|b|}{|a|} = \frac{V/\sqrt{R_0} - I\sqrt{R_0}}{V/\sqrt{R_0} + I\sqrt{R_0}} = \frac{V - I R_0}{V + I R_0} = \frac{V/I - R_0}{V/I + R_0}$$

→ $Z(x=0)$

$$S = \frac{Z - R_0}{Z + R_0}$$ ∴ cuando $Z(x=0) = R_0$ $S = 0 \Rightarrow$ Adaptación

$$S \begin{cases} +1 & (\text{circ. abierto}) \\ 0 & \text{Adaptación } (R_0 = R_g) \\ -1 & (\text{corto circuito}) \end{cases}$$

NOTA



$$\Rightarrow P = \frac{1}{2}|a|^2$$

$$\Rightarrow P_L = \frac{V^2}{R_L} = \frac{V_g^2}{4R_o}$$

↳ máxima potencia que puede entregar el gen a la carga en adaptación.

↱ Pot. media

$$\bar{P} = \frac{1}{2} P_L = \frac{V_g^2}{8R_o}$$

- $P = \frac{1}{2}|a|^2(1 - |S|^2)$ siendo $S = \frac{Z - R_o}{Z + R_o}$

Para una red pasiva

$$|S|^2 \leq 1$$

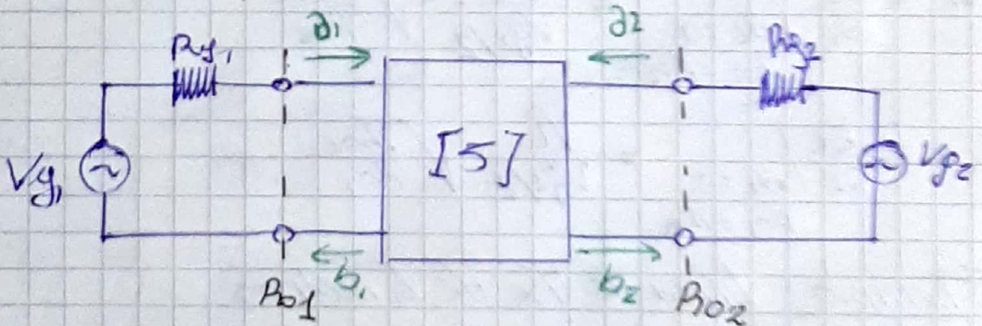
Si $Z \in \mathbb{I}m \Rightarrow S = \frac{jX - R_o}{jX + R_o}$

$\Rightarrow |S| = 1$

$\therefore \bar{P} = 0$

Parámetros S

$$b = S \cdot a$$



$$\begin{cases} b_1 = S_{11} a_1 + S_{12} a_2 \\ b_2 = S_{21} a_1 + S_{22} a_2 \end{cases}$$

↱ coeficiente de reflexión

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2 = 0}$$

↳ $R_{g2} = R_{o2}$

a_2 se considera como onda reflejada en el sistema visto desde el puerto 1 (pasivo V_{g2})

↱ transmisión inversa

$$S_{12} = \frac{b_1}{a_2}\Big|_{\substack{a_1 = 0 \\ R_{g1} = R_{o1}}}$$

~~pasivo V_{g2}~~

NOTA

→ transferencia directa

$$S_{21} = \frac{b_2}{a_1}\Big|_{\substack{a_2=0 \\ R_{g2}=R_{o2}}}$$

$$S_{22} = \frac{b_2}{a_2}\Big|_{\substack{a_1=0 \\ R_{g1}=R_{o1}}}$$

↳ coef. reflexión.

Los parámetros S permiten representar cuadripolos que no ~~p~~ pueden no tener representación en parámetros Z e Y. Esto es xq no se miden en situaciones extremas de carga.

→ tensión en el plano 1

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} = \frac{\left(V_1/\sqrt{R_{o1}} - I_1\sqrt{R_{o1}}\right)}{V_1/\sqrt{R_{o1}} + I_1\sqrt{R_{o1}}} = \frac{V_1 - I_1 R_{o1}}{V_1 + I_1 R_{o1}}$$

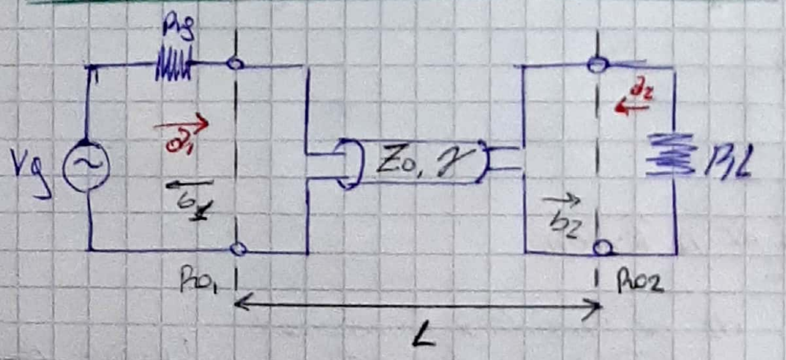
$$S_{11} = \frac{V_1/I_1 - R_{o1}}{V_1/I_1 + R_{o1}} = \frac{Z_1 - R_{o1}}{Z_1 + R_{o1}}$$

$$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0} = \frac{Z_2 - R_{o2}}{Z_2 + R_{o2}}$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{\substack{a_2=0 \\ R_{o2}=R_{g2}}} = \frac{V_2}{V_{g1}/2}\sqrt{\frac{R_{o1}}{R_{o2}}}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{\substack{a_1=0 \\ R_{o1}=R_{g1}}} = \frac{V_1}{V_{g2}/2}\sqrt{\frac{R_{o2}}{R_{o1}}}$$

Ejemplo: Línea de Tx de L metros



$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} = \frac{Z_1 - R_o}{Z_1 + R_o} = 0 \quad \text{está adaptada}$$

$R_L = R_{o2} = R_g = R_{o1} = R_o = Z_o$

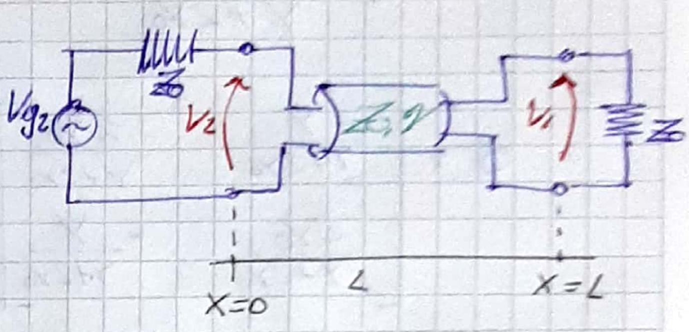
↳ xq es práctico

NOTA

$$S_{22} = \frac{b_2}{a_2}\Big|_{\substack{a_1=0 \\ R_g=Z_0}} = \frac{Z_2 - R_0}{Z_2 + R_0} = 0$$

Si $S_{11} = S_{22} \Rightarrow$ Red simétrica

$$S_{12} = \frac{b_1}{a_2}\Big|_{\substack{a_1=0 \\ R_g=Z_0}} = \frac{V_1}{V_{g2}/2} \cdot 1$$



$$\frac{V_{g2}}{2} = V_2 = V(0)$$

$$V(x) = V_i \cdot e^{-\gamma x} + V_r e^{+\gamma x}$$

$V_1 = V(L)$ como esta línea está adaptada $\Rightarrow V_r = 0$

$$\therefore V(0) = V_i \underbrace{e^{-\gamma \cdot 0}}_{1} \Rightarrow V(0) = V_i = V_g/2$$

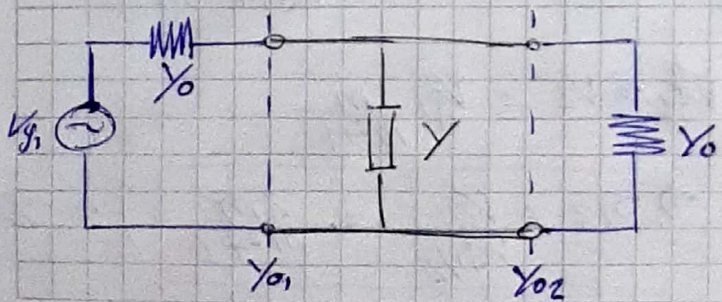
$$\Rightarrow V(L) = \frac{V_{g2}}{2} \cdot e^{-\gamma L} = V_1$$

$$S_{12} = \frac{V_1}{V_{g2}/2} = \frac{V_{g2}/2 \cdot e^{-\gamma L}}{V_{g2}/2} = e^{-\gamma L}$$

$S_{12} = S_{21} \Rightarrow$ red recíproca

$$S = \begin{bmatrix} 0 & e^{-\gamma L} \\ e^{-\gamma L} & 0 \end{bmatrix}$$

EJ: Admitancia



$$S_{11} = \frac{Z_1 - R_{01}}{Z_1 + R_{01}} = \frac{Y_{01} - Y_1}{Y_{01} + Y_1}$$

$$Y_{01} = Y_{02} = Y_0$$

$$\therefore S_{11} = \frac{Y_0 - (Y + Y_0)}{Y_0 + Y + Y_0}$$

$$S_{11} = \frac{-Y}{2Y_0 + Y}$$

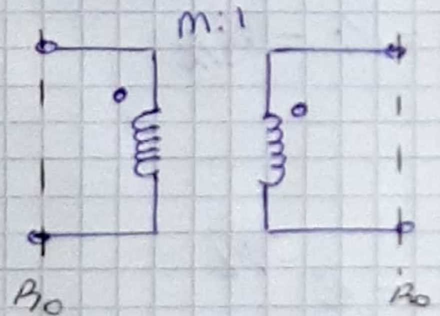
$$S_{22} = \frac{Y_0 - Y_2}{Y_0 + Y_2} = \frac{Y_0 - (Y + Y_0)}{Y_0 + Y + Y_0} = \frac{-Y}{2Y_0 + Y}$$

$$S_{12} = S_{21} \Rightarrow S_{21} = \frac{V_2}{V_{g1}/2}$$

$$V_2 = V_{g1} \frac{Y_0}{Y + 2Y_0} \Rightarrow S_{21} = \frac{2Y_0}{Y + 2Y_0}$$

$$S = \begin{bmatrix} \frac{-Y}{Y+2Y_0} & \frac{2Y_0}{Y+2Y_0} \\ \frac{2Y_0}{Y+2Y_0} & -\frac{Y}{Y+2Y_0} \end{bmatrix}$$

EJ Transformador ideal



$$\begin{cases} V_1 = m V_2 \\ I_1 = \frac{1}{m} I_2 \end{cases}$$

$$\frac{V_1}{I_1} = \frac{V_2}{I_1} m^2$$

$$\Rightarrow Z_1 = Z_2 m^2$$

$$S_{11} = \frac{Z_1 - R_0}{Z_1 + R_0} = \frac{R_0 m^2 - R_0}{R_0 m^2 + R_0} = \cancel{R_0} \frac{m^2 - 1}{m^2 + 1}$$

$$S_{22} = \frac{Z_2 - R_0}{Z_2 + R_0} = \frac{R_0/m^2 - R_0}{R_0/m^2 + R_0} = \frac{1 - m^2}{1 + m^2} = (-1) \frac{m^2 - 1}{m^2 + 1}$$

$S_{11} = -S_{22} \Rightarrow$ Red antisimétrica.

$$S_{12} = \frac{V_2}{V_{g1}/2} \; ; \; V_2 = \frac{V_1}{m} \quad y \quad V_1 = V_{g1} \frac{R_0 m^2}{R_0 m^2 + R_0}$$

$$V_1 = V_{g1} \cdot \frac{m^2}{m^2 + 1} \Rightarrow \cancel{V_{g1}} \; V_2 = V_{g1} \frac{m}{m^2 + 1}$$

$$\Rightarrow S_{12} = \frac{V_2}{V_{g1}/2} = \frac{2m}{m^2 + 1} = S_{21}$$

xq es una red pasiva y recíproca

NOTA 21:20 → 3:40