

3)  $h(k) = [1, 1] \Rightarrow y(k) = x(k) + x(k-1)$

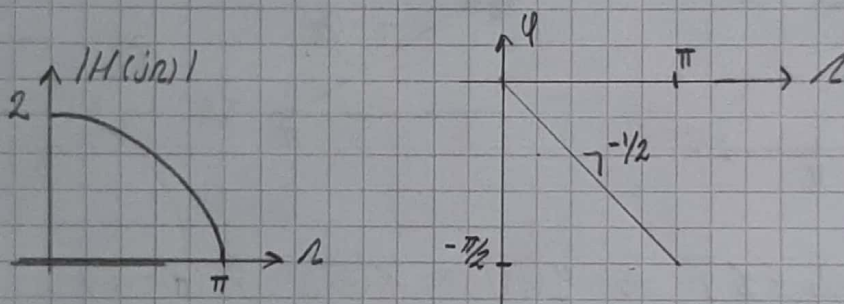
$$H(z) = 1 + z^{-1} = \frac{z+1}{z}$$

$$H(j\Omega) = H(z) \Big|_{z=e^{j\Omega}}$$

$$H(j\Omega) = \frac{e^{j\Omega} + 1}{e^{j\Omega}} = 1 + e^{-j\Omega} = e^{0j} + e^{-j\Omega}$$

$$= e^{-j\Omega/2} (e^{j\Omega/2} + e^{-j\Omega/2})$$

$$\Rightarrow H(j\Omega) = \underbrace{2 \cos(\Omega/2)}_{|H|} \cdot \underbrace{e^{-j\Omega/2}}_{\varphi}$$



$h(k) = [1, 1, 1] \Rightarrow y(k) = x(k) + x(k-1) + x(k-2)$

$$Y(z) = X(z) (1 + z^{-1} + z^{-2})$$

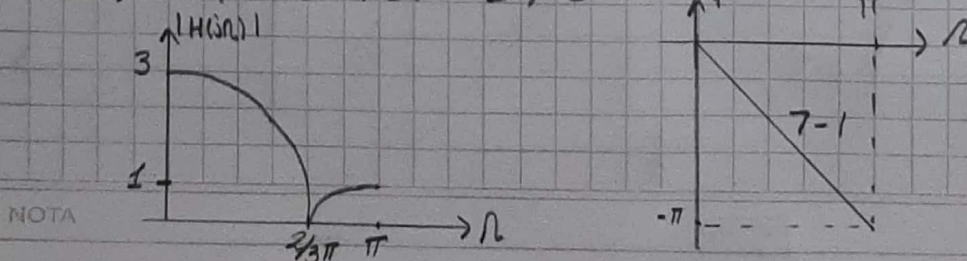
$$H(z) = \frac{z^2 + z + 1}{z^2} \Rightarrow \cos = (-0.5 \pm j0.866)$$

$$H(j\Omega) = H(z) \Big|_{z=e^{j\Omega}}$$

$$H(j\Omega) = 1 + e^{-j\Omega} + e^{-2j\Omega}$$

$$= e^{-j\Omega} (1 + e^{j\Omega} + e^{-j\Omega})$$

$$H(j\Omega) = (2 \cos(\Omega) + 1) e^{-j\Omega}$$



NOTA



## b) Filtro diferenciador

- $h_1(k) = (1, -1)$  de 1º orden

$$y(k) = x(k) - x(k-1] \rightarrow Y(Z) = X(Z)(1 - Z^{-1})$$

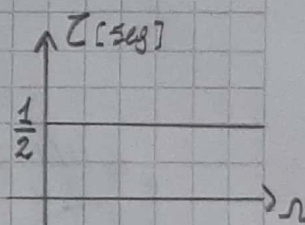
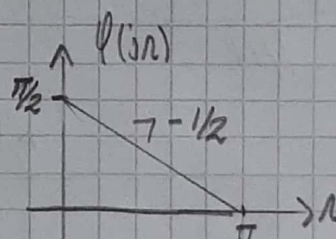
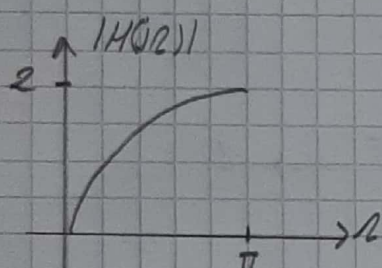
$$H(Z) = 1 - Z^{-1} = \frac{Z - 1}{Z}$$

$$H(j\omega) = H(Z) \Big|_{Z=e^{j\omega}}$$

$$\Rightarrow H(j\omega) = 1 - e^{-j\omega} = e^{-j\omega/2} (e^{j\omega/2} - e^{-j\omega/2})$$

$$H(j\omega) = 2j \sin(\omega/2) e^{-j\omega/2}$$

$$H(j\omega) = 2 \sin(\omega/2) e^{j(\pi/2 - \omega/2)}$$



- $h_2(k) = [1, 0, -1]$   $\rightarrow y(k) = x(k) - x(k-2]$

$$Y(Z) = X(Z)(1 - Z^{-2})$$

$$H(Z) = 1 - Z^{-2} = \frac{Z^2 - 1}{Z^2}$$

$$H(j\omega) = H(Z) \Big|_{Z=e^{j\omega}}$$

$$H(j\omega) = 1 - e^{-j2\omega} = e^{-j\omega} (e^{j\omega} - e^{-j\omega})$$

$$H(j\omega) = 2j \sin(\omega) \cdot e^{-j\omega} = 2 \sin(\omega) e^{j(\pi/2 - \omega)}$$

