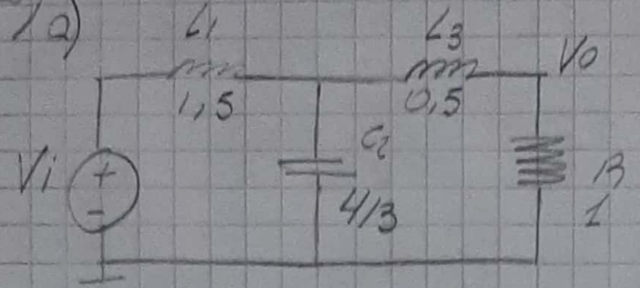
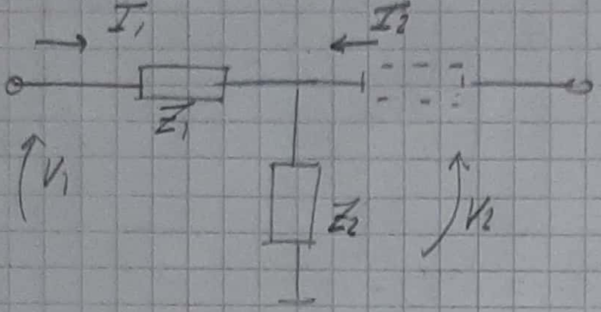
2) a)





$$Z = \begin{pmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 \end{pmatrix}$$

$$Z \begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$T \begin{cases} A = \left.\dfrac{V_1}{V_2}\right|_{-I_2=0} & B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0} \\ C = \left.\dfrac{I_1}{V_2}\right|_{-I_2=0} & D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0} \end{cases}$$

$A|_{-I_2=0} \Rightarrow \dfrac{V_1}{Z_{11}} = \dfrac{V_2}{Z_{21}} \Rightarrow A = \dfrac{Z_{11}}{Z_{21}}$

$B|_{V_2=0} \Rightarrow V_1 = Z_{11} I_1 + Z_{12} I_2$

$0 = Z_{21} I_1 + Z_{22} I_2 \rightarrow I_1 = -\dfrac{Z_{22} I_2}{Z_{21}}$

$\Rightarrow V_1 = Z_{11} \dfrac{(-Z_{22} I_2)}{Z_{21}} + Z_{12} I_2$

$V_1 = -I_2 \dfrac{(Z_{11} Z_{22} - Z_{12} \cdot Z_{21})}{Z_{21}} \Rightarrow B = \dfrac{\Delta Z}{Z_{21}}$

$C|_{-I_2=0} = \dfrac{I_1}{V_2} = \dfrac{1}{Z_{21}}$

$D|_{V_2=0} = \dfrac{I_1}{-I_2} \qquad 0 = Z_{21} I_1 + Z_{22} I_2$

$\Rightarrow -\dfrac{I_1}{I_2} = \dfrac{Z_{22}}{Z_{21}}$

$$T = \begin{pmatrix} Z_{11}/Z_{21} & \Delta Z/Z_{21} \\ 1/Z_{21} & Z_{22}/Z_{21} \end{pmatrix}$$

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$$Z_1 = \begin{pmatrix} sL_1 + \frac{1}{sC_2} & \frac{1}{sC_2} \\ 1/sC_2 & 1/sC_2 \end{pmatrix} \qquad \Delta Z_1 = \left(sL_1 + \frac{1}{sC_2}\right)\frac{1}{sC_2} - \frac{1}{(sC_2)^2}$$

$$\Delta Z_1 = \frac{sL_1}{sC_2} = \frac{L_1}{C_2}$$

$$T_1 = \begin{pmatrix} \frac{sL_1 + \frac{1}{sC_2}}{1/sC_2} & \frac{L_1/C_2}{1/sC_2} \\ sC_2 & +1 \end{pmatrix} = \begin{pmatrix} s^2L_1C_2 + 1 & sL_1 \\ sC_2 & -1 \end{pmatrix}$$

$$Z_2 = \begin{pmatrix} sL_3 + B & B \\ B & B \end{pmatrix} \qquad \Delta Z_2 = (sL_3 + B)B - B^2$$

$$\Delta Z_2 = sL_3 B$$

$$T_2 = \begin{pmatrix} \frac{sL_3 + B}{B} & sL_3 \\ 1/B & +1 \end{pmatrix}$$

$$T = T_1 . T_2$$

$$T = \begin{pmatrix} s^2L_1C_2 + 1 & sL_1 \\ sC_2 & +1 \end{pmatrix}\begin{pmatrix} \frac{sL_3 + B}{B} & sL_3 \\ 1/B & +1 \end{pmatrix}$$

$$T = \begin{pmatrix} (s^2L_1C_2 + 1)\frac{(sL_3 + B)}{B} + \frac{sL_1}{B} & (s^2L_1C_2 + 1)sL_3 - sL_1 \\ sC_2\frac{(sL_3 + B)}{B} + \frac{1}{B} & sC_2 . sL_3 + 1 \end{pmatrix}$$

$$A = \frac{V_1}{V_2} = \frac{s^3L_1L_3C_2 + s^2L_1C_2B + sL_3 + B + sL_1}{B}$$

$$A = \frac{V_1}{V_2} = \frac{s^3 + s^2\,B/L_3 + s\left(\frac{L_3 + L_1}{L_1L_3C_2}\right) + \frac{B}{L_1L_3C_2}}{\frac{B}{L_1L_3C_2}}$$

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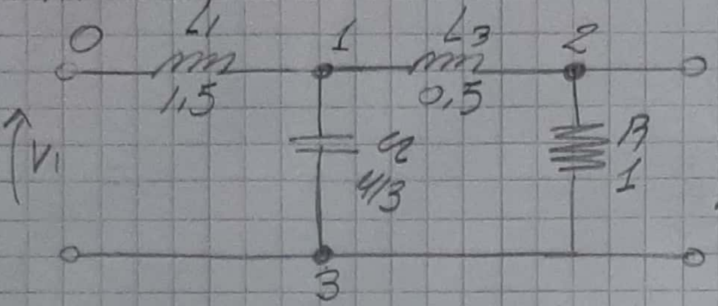
$$\Rightarrow T(s) = \frac{V_2}{V_1} = \frac{R/L_1L_3C_2}{s^3 + s^2\,R/L_3 + s\,\frac{L_3+L_1}{L_1L_3C_2} + \frac{R}{L_1L_3C_2}}$$

b) MAI



$$Y_{MAI} = \begin{pmatrix} \frac{1}{sL_1} & -\frac{1}{sL_1} & 0 & 0 \\ -\frac{1}{sL_1} & \frac{1}{sL_1} + sC_2 + \frac{1}{sL_3} & -\frac{1}{sL_3} & -sC_2 \\ 0 & -\frac{1}{sL_3} & \frac{1}{sL_3} + \frac{1}{R} & -\frac{1}{R} \\ 0 & -sC_2 & -1/R & sC_2 + \frac{1}{R} \end{pmatrix}$$

c)

$$A_{mn}^{ij} = \mathrm{sgn}(i-j)\,\mathrm{sgn}(m-n)\,\frac{\underline{Y}_{ij}^{mn}}{\underline{Y}_{mn}^{mn}} = \frac{V_{ij}}{V_{mn}}$$

$ij = 23 \; ; \; mn = 03$

$$\underline{Y}_{23}^{03} = (-1)^{(8)}\left[\frac{1}{sL_1}\cdot\frac{1}{sL_3} - 0\right] = \frac{1}{s^2L_1L_3}$$

$$\underline{Y}_{03}^{03} = (-1)^6\left[\left(\frac{1}{sL_1} + sC_2 + \frac{1}{sL_3}\right)\left(\frac{1}{sL_3} + \frac{1}{R}\right) - \left(\frac{1}{sL_3}\right)^2\right]$$

$$\underline{Y}_{03}^{03} = \frac{s^3L_1L_3C_2 + s^2L_1C_2R + s(L_1+L_3) + R}{s^2L_1L_3R}$$

NOTA

$$\Rightarrow T_{03}^{23} = \frac{R}{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + s(L_1+L_3) + R}$$

$$T_{03}^{23} = \frac{R/L_1L_3C_2}{s^3 + s^2 \, R/L_3 + s\left(\frac{L_1+L_3}{L_1L_3C_2}\right) + \frac{R}{L_1L_3C_2}}$$

Bonus 1

$$Z_{03} = \frac{\underline{Y}_{mn}^{mn}}{\underline{Y}_{n}^{n}} = \frac{\underline{Y}_{03}^{03}}{\underline{Y}_{3}^{3}}$$

↳ como todos los cofactores de 1° orden son iguales uso el más conveniente

→ dejo las filas y col. que tengan más 0s

uso $\underline{Y}_2^2 = (-1)^4 \cdot \frac{1}{sL_1}\left[\left(\frac{1}{sL_3}+\frac{1}{R}\right)\left(sC_2+\frac{1}{R}\right) - \left(\frac{1}{R}\right)^2\right]$

$$\underline{Y}_2^2 = \frac{1}{sL_1}\left[\frac{sC_2}{sL_3} + \frac{1}{sL_3R} + \frac{sC_2}{R} + \cancel{\left(\frac{1}{R}\right)^2} - \cancel{\left(\frac{1}{R}\right)^2}\right]$$

$$\underline{Y}_2^2 = \frac{1}{sL_1}\left[\frac{sC_2R + 1 + s^2C_2L_3}{sL_3R}\right] = \frac{s^2C_2L_3 + sC_2R + 1}{s^2L_1L_3R}$$

$$\Rightarrow Z_{03} = \frac{\dfrac{s^3L_1L_3C_2 + s^2L_1C_2R + s(L_1+L_3) + R}{\cancel{s^2L_1L_3R}}}{\dfrac{s^2C_2L_3 + sC_2R + 1}{\cancel{s^2L_1L_3R}}}$$

$$Z_{03} = \frac{s^3L_1L_3C_2 + s^2L_1C_2R + s(L_1+L_3) + R}{s^2L_3C_2 + sC_2R + 1}$$

NOTA

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