Recreating Young's Double Slit Experiment

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Abstract

I. INTRODUCTION

The double slit experiment preformed by Thomas Young was seminal because it demonstrated the wave nature of light.

II. METHODS

We used the TeachSpin Two-Slit Interference, One Photon at a Time (TWS1-B) which is a apparatus designed to perform Young's double slit experiment. The apparatus has two light sources, a 670 nm diode laser and a light bulb with a removable green light filter. There are four slit holders spaced throughout the length of the apparatus, see Figure 1. The first slit, the source slit, is eliminates excess light from the light source and approximates a plane wave. Next is the double slit, our double slit had 0.1mm wide slits and center to center separation of 0.353mm. Following the double slit was a wide single slit attached to a micrometer, called the slit blocker. The slit blocker could be moved to block either or both slits of the double slit. Finally there was a detector slit which allowed us to measure the intensity of light (using the laser) or the number of photons (using the light bulb) over a small region of the x-axis. The micrometer attached to the detector slit allowed us to scan along the x-axis. When we worked with the laser as our light source we used the built in photodetector which gave an output voltage that was proportionate to the light intensity. When we used light bulb as our light source we used the built in photo-multiplier tube (PMT) attached to the TeachSpin Pulse Counter / Interval Timer (PCIT1) to count the number of photons.

We began by aligning the slits within our apparatus. We started by using the laser roughly align the source slit and double slit and ensure that the double slit was not tilted. Then we made fine adjustments to the alignment using the light bulb. Throughout our experiments we used a midrange light bulb power setting of 5. We used a green filter on our light bulb to lower the intensity of the light and narrow the range of wavelengths to 541 - 551nm.

Next we optimized the signal-to-noise ratio for our PMT. To do so we held the PMT voltage constant and varied the discriminator threshold. We found that dial value of the PMT voltage was not consistent with the measured value of the PMT voltage using a digital
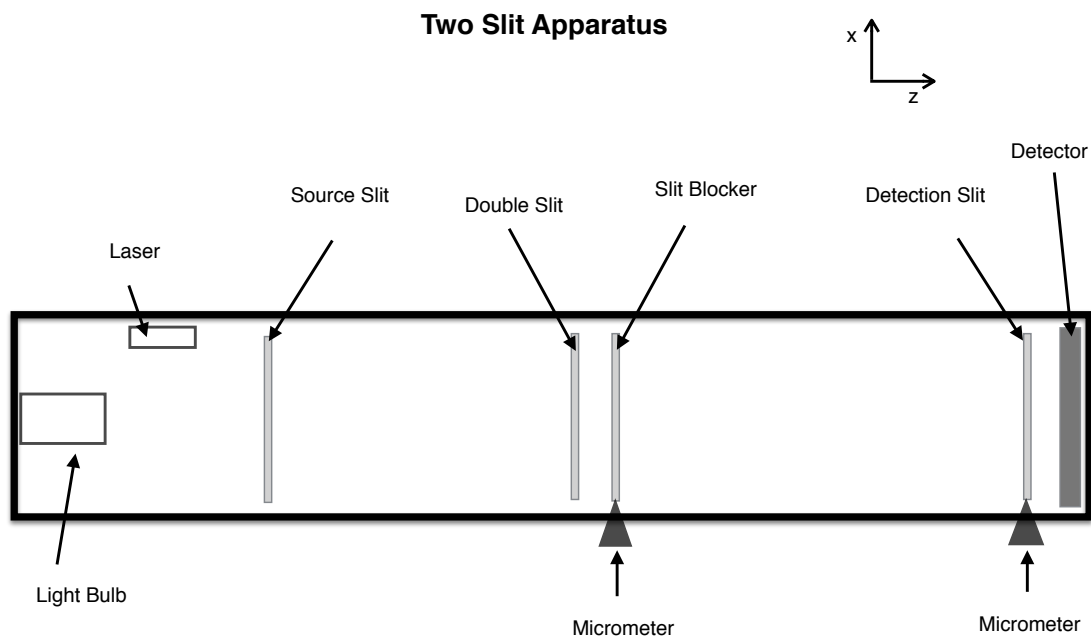
FIG. 1. The experimental set-up of the apparatus we used. The laser can slide into place in front of the light bulb. The source slit is a wide slit that blocks extraneous light from the light source. The slit blocker is another wide slit that can be moved, using the micrometer, to block one of the double slits, effectively turning it into a single slit. The detector slit allowed us to count the number of photon reaching the detector over a small region. The micrometer allowed us to scan across the detector and get a photon count at each x value.

multimeter, so we used the multimeter reading to determine our voltage. For each value of the discriminator threshold we measured four values for our signal plus our noise and four values of just noise with no signal. We then averaged each set of four values and took the ratio to calculate the signal-to-noise. In our case the ratio we actually calculated was signal and noise to noise, this difference should scale our results but not change the overall outcome. The calculated signal-to-noise rations can be seen for various PMT voltages in Figure **??**. The highest values for the PMT voltage, 900 V and 800 V, had a significantly lower signal to noise ratio. The lower values for the PMT voltage, 600 V and 650 V, required small

values for the discriminator threshold and did not have as high a signal-to-noise ratio. The highest signal-to-noise ratio we obtained was for PMT voltage of 750 V and a discriminator threshold of 15.
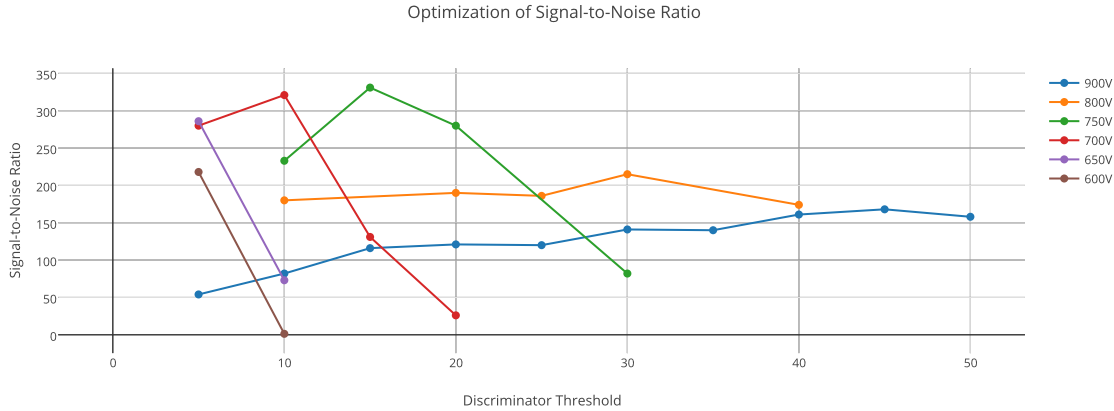

FIG. 2. The signal-to-noise ratios we obtained for a range of discriminator thresholds for various PMT voltages. For the high PMT voltages, 900 V (blue) and 800 V (orange) the signal-to-noise ratio was much lower. For lower PMT voltages, 650 V (purple) and 600 V (brown), the signal-to-noise ratio was again lower. The optimal signal-to-noise ratio was at a PMT voltage of 750 V (green) and a discriminator threshold value of 15.

To collect data we used a Labview program called "TeachSpin Counter V3" that connected to the pulse counter. We then moved the detector slit along the x-axis in increments of 0.05 mm and recorded the number of counts given by the pulse counter. We were careful when collecting multiple sets of data to only scan along the x-axis in one direction and turn the micrometer father than necessary to minimize any backlash effects.

III.   RESULTS

We collected three sets of data using the double slit arrangement. Figure 3 shows a representative selection of the data we collected. The alignment of our slits was not exact because the minima at smaller x values do not reach zero. However, the vertical offset was not linear so it could not easily be subtracted off.

Using the slit blocker we were able to obtain single slit data from both the near and the far slits. We found further evidence of the misalignment of our slits. The intensity of the
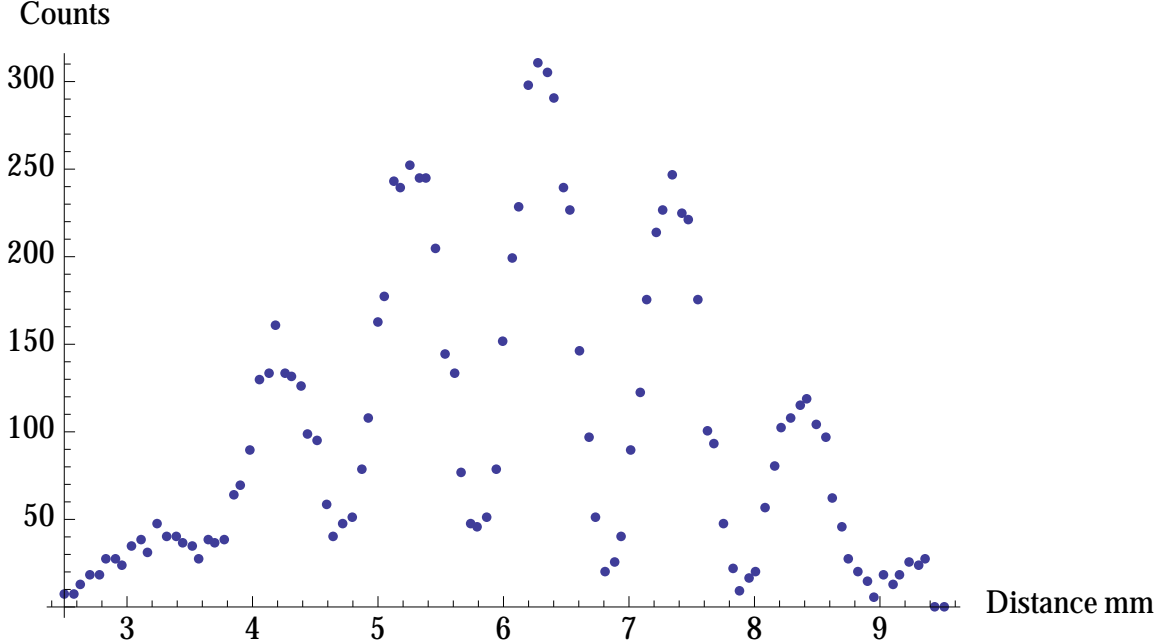
4

FIG. 3. Data collected for the double slit arrangement. Our alignment was imperfect because the minima on the right side do not reach zero.

near slit, Figure 4, was one fourth of the intensity of the double slit arrangement which in is agreement with the theory. However the intensity of the far slit, Figure 5, was twice the intensity of the near slit and nearly twice the intensity predicted by theory. The discrepancy was mostly likely caused by our inability to exactly align the slits. It is also possible that the increase in intensity was caused by a reflection off the wall the apparatus that was was going around the silt instead of through them.

IV. ANALYSIS

The purpose of our experiment was to observe the quantum mechanical wave-like properties of a single photon. We wanted to see a diffraction pattern when only a single photon was passing through the slits which would suggest that the single photon was interfering with itself.

To make our results significant we need to show that there is only one photon in the apparatus at a time. The distance from the source slit to the detector slit is 88 ± 1 cm. Light travels at 2.998×10^{10} cm/s. So the time that a photon spends in the apparatus, $t = L/c$, is equal to 2.94 ± 0.03 ns. We collected data in one second periods. The most
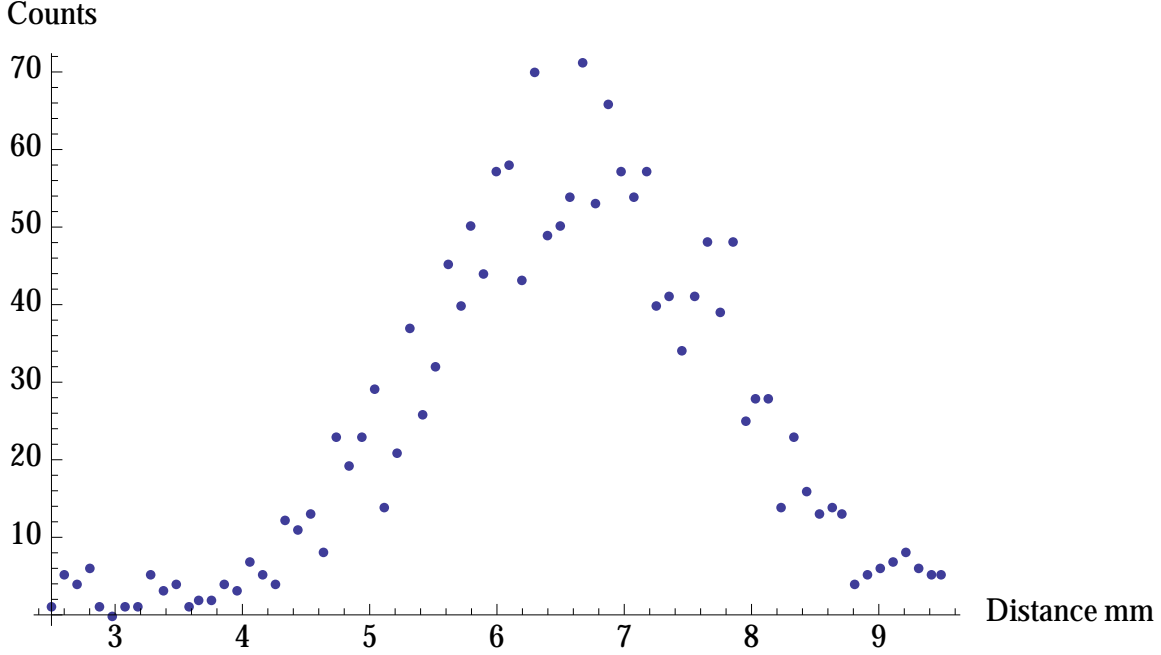
5

FIG. 4. Data collected for a single slit arrangement using the near slit. There appears to be a large uncertainty, note the wide band of data points. The intensity of the near single slit is approximately one fourth the intensity of the double slit.

counts we got in a one second period was 350. That means that on a average one photon arrives at the detector every 2.86 ms. There is only photons in the apparatus for 1029 ± 10 ns out of the second. That means that 99.9999% of the time the apparatus is empty. The chances of there being more than one photon in the apparatus at a time are negligible.

We used two different models to fit to our double slit data, the Fraunhofer model and the Fresnel model. We did all of our data fitting in Mathematica and the code we used can be found in the appendix. Figure 6-8 are our double slit data with fits using the Fraunhofer model. The χ^2 and reduced χ^2 values for both the Fraunhofer and Fresnel fits can be found in Table I.

The Fraunhofer model using diffraction and assumes that the detection location is far aways from the place of diffraction. For the double slit arrangement it gives the function

$$I(x) = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 (\cos \beta)^2 \tag{1}$$

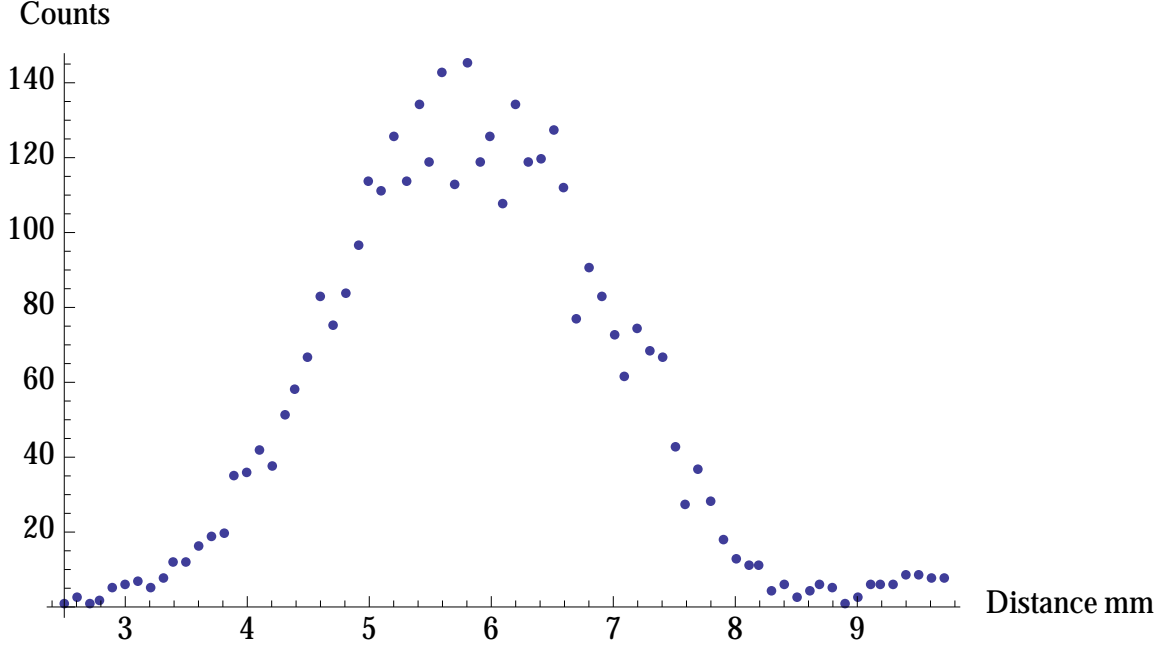were I_0 is the maximum intensity. α and β are defined as

6

FIG. 5. Data collected for a single slit arrangement using the far slit. There is again a large uncertainty evident from the wide line of data points. The intensity of this slit is nearly twice the intensity of the near slit. This is mostly likely due to misalignment or reflected light from outside the slit.

$$\alpha = \frac{a\pi \sin\left(\frac{x-x_0}{R}\right)}{\lambda} \tag{2}$$

$$\beta = \frac{d\pi \sin\left(\frac{x-x_0}{R}\right)}{\lambda} \tag{3}$$

where λ is the wavelength of the light, a is the width of each slit, d is the center-to-center slit separation and R is the distance from the double slit to the detector. We defined x to be the axis that we scanned along with the detector slit and x_0 is a fit parameter to adjust for the offset of the central peak. Because the α contains a sine function, it can be zero. This causes problems since α is in the denominator of a fraction making Equation 1 undefined. To avoid the undefined places in Equation 1 we used a sinc function which is continuous and takes the limit as α approaches zero.

We also used the Fresnel model to fit all of our data. The Fresnel model sums over all of the possible paths that the photon could take. The functions for the Fresnel model are
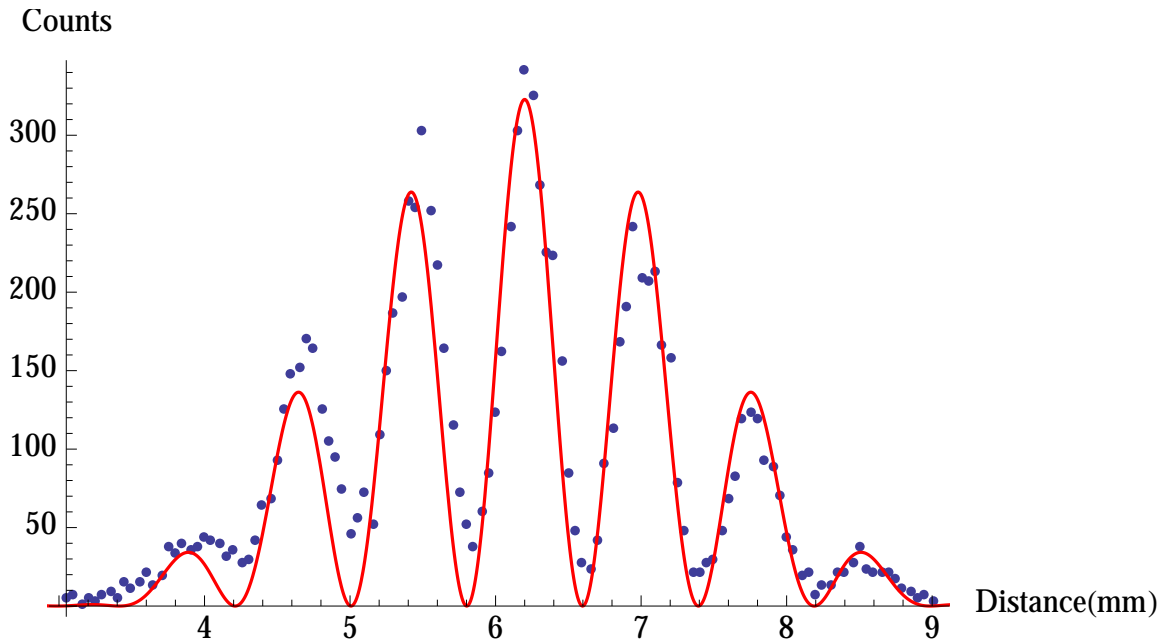
FIG. 6. Double slit data from our first run with a fit using the Fraunhofer model. The fit is much better on the right side than the left due to the alignment issues previously discussed.
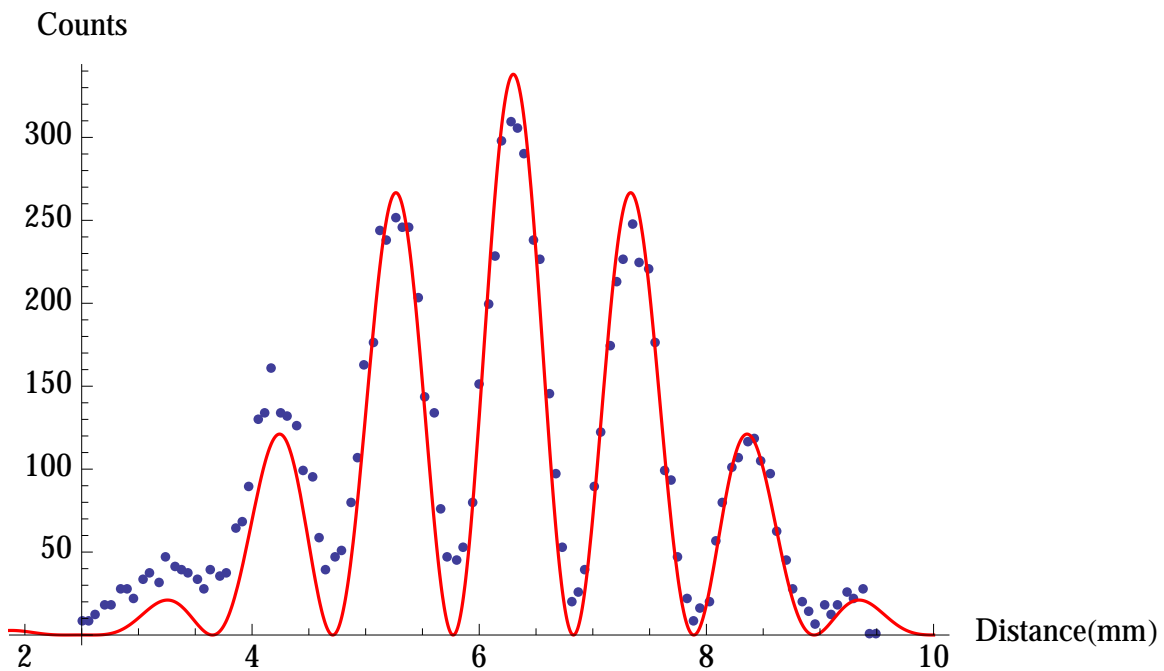

FIG. 7. Double slit data from our second run with a fit using the Fraunhofer model. The fit is again much better on the right side than the left due primarily to the misalignment.
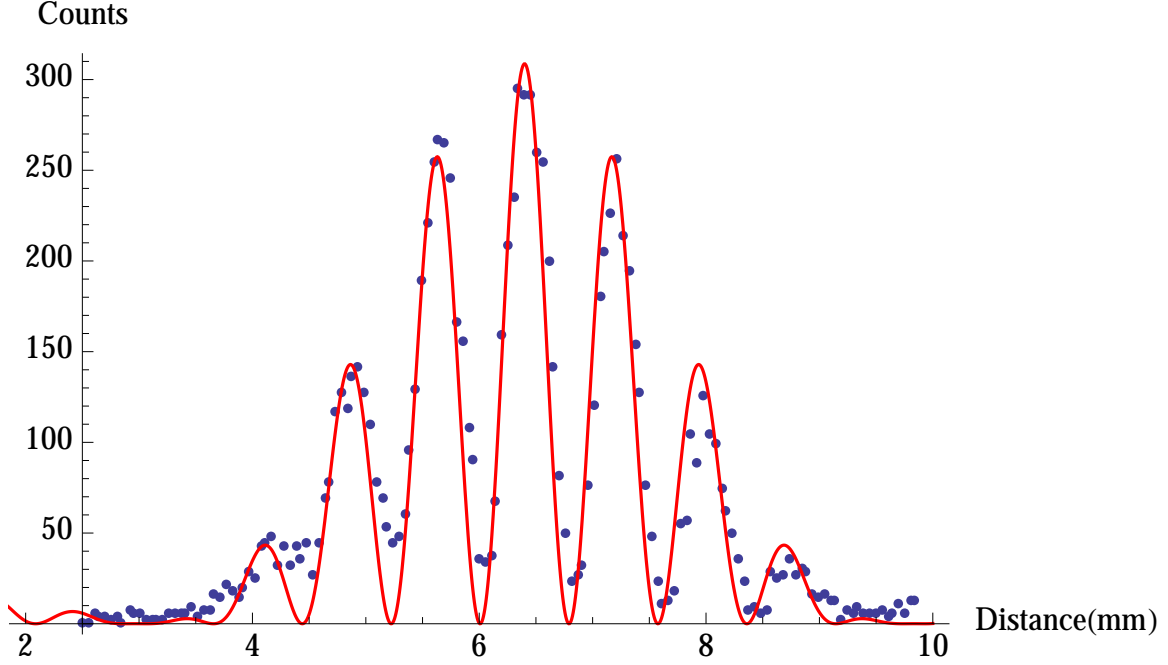
FIG. 8. Double slit data from our third run with a fit using the Fraunhofer model. This fit is slightly better on the left side than the right.

$$I_{\text{near}} = e^{\frac{2\pi I D_1}{\lambda}} e^{\frac{2\pi I D_2}{\lambda}} \int_{\frac{d-a}{2}}^{\frac{d+a}{2}} e^{\frac{2\pi I (w-y_1)^2}{2D_1\lambda}} e^{\frac{2\pi I (y_1-z)^2}{2D_2\lambda}} dy_1 \tag{4}$$

$$I_{\text{far}} = e^{\frac{2\pi I D_1}{\lambda}} e^{\frac{2\pi I D_2}{\lambda}} \int_{\frac{-d-a}{2}}^{\frac{-d+a}{2}} e^{\frac{2\pi I (w-y_2)^2}{2D_1\lambda}} e^{\frac{2\pi I (y_2-z)^2}{2D_2\lambda}} dy_2 \tag{5}$$

where the variables are defined as seen in Figure 9. D_1 and D_2 are the distance from the source slit to the double slit and from the double slit to the detector slit respectively. w is the displacement of the source slit from the light source. Since we aligned our source slit with our light source w should be zero. y_1 and y_2 are the displacements of the double-slit slits from the center of source slit. These values are integrated to account for the width of each slit. z was dependent on the location the detector slit and was related by $z = x - x_0$ to our x value.

We integrated then added Equations 4 and 5. To obtain an intensity we multiplied by the complex conjugate, then we took the real part of the resulting function and fit it to our data. The Fresnel model does not account for the intensity of the light so we needed to multiply by 8,500 to 10,000 to achieve the amplitudes necessary to match our data sets.
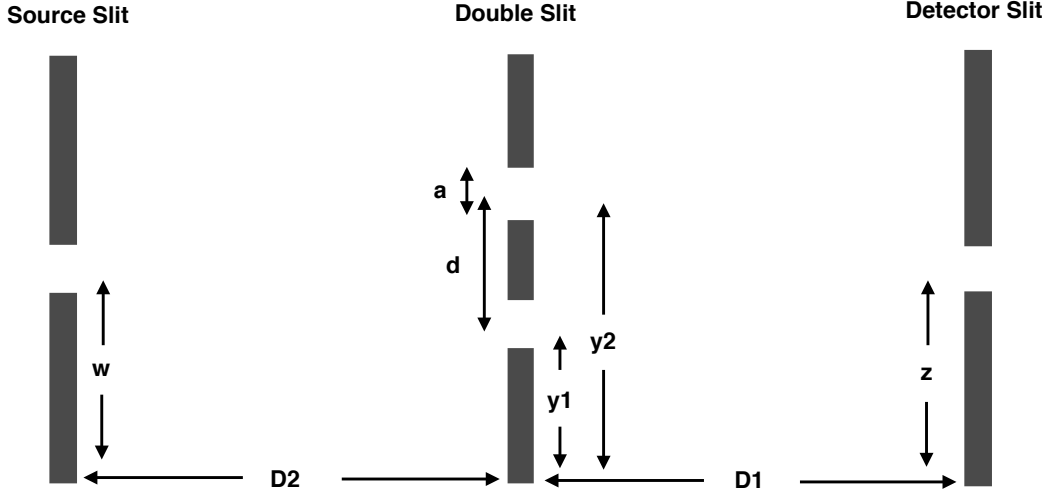
9

The parameters used for the Fresnel model fit

FIG. 9. The set up of our double slit experiment showing the variables used for the Fresnel model fit. D_1 and D_2 are the distance from the source slit to the double slit and from the double slit to the detector slit respectively. w is the displacement of the source slit from the light source. Since we aligned our source slit with our light source w should be zero. y_1 and y_2 are the displacements of the double-slit slits from the center of source slit. These values are integrated to account for the width of each slit. z was dependent on the location the detector slit and was related by $z = x - x_0$ to our x value.

For the single slit data that we collected we were able to do a Fraunhofer model fit. The Fraunhofer model for a single slit gives the same function as Equation 1 without the double slit diffraction. That leaves

$$I(x) = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \tag{6}$$

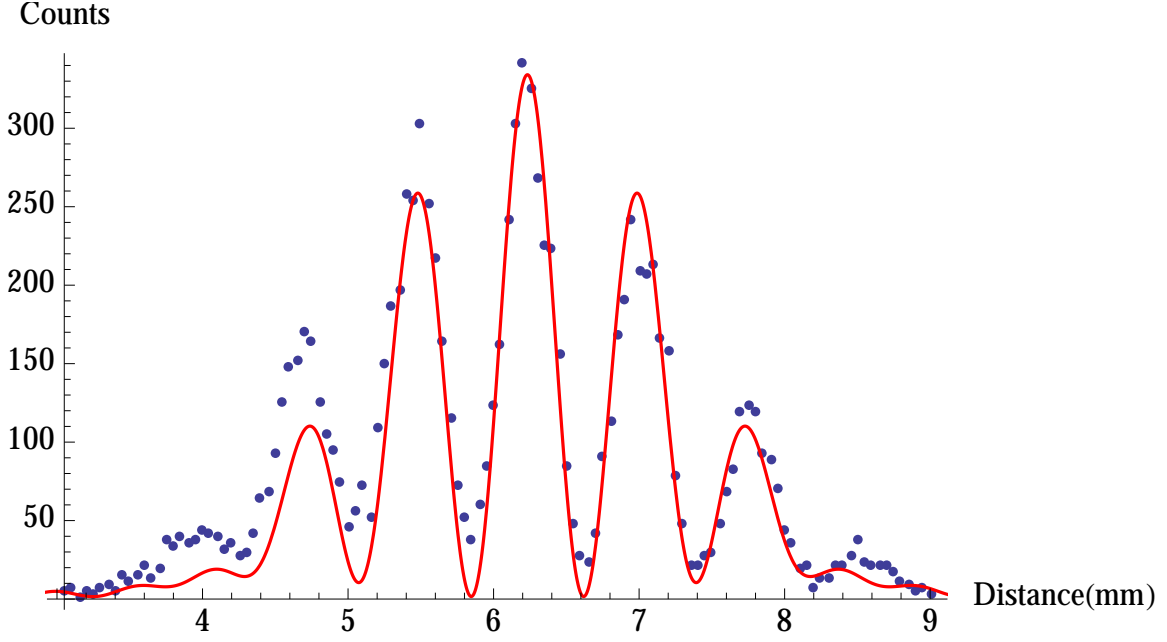where α is once again

10

FIG. 10. The fit to our first data set using the Fresnel model.
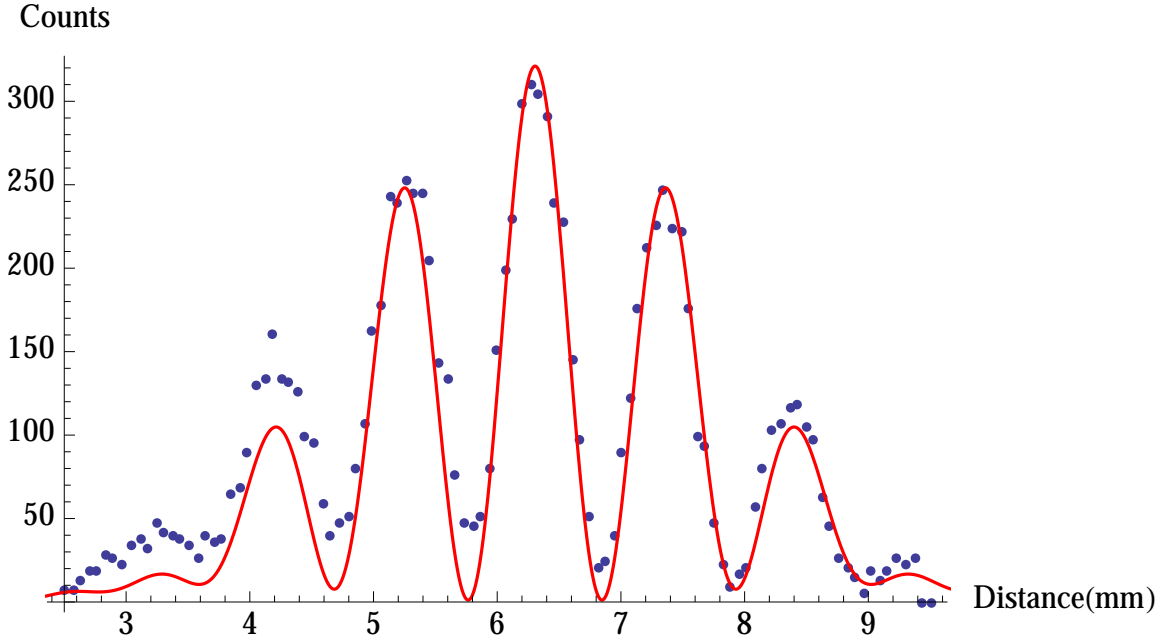

FIG. 11. The fit to our second data set using the Fresnel model.

$$\alpha = \frac{a\pi \sin\left(\frac{x-x_0}{R}\right)}{\lambda} \tag{7}$$

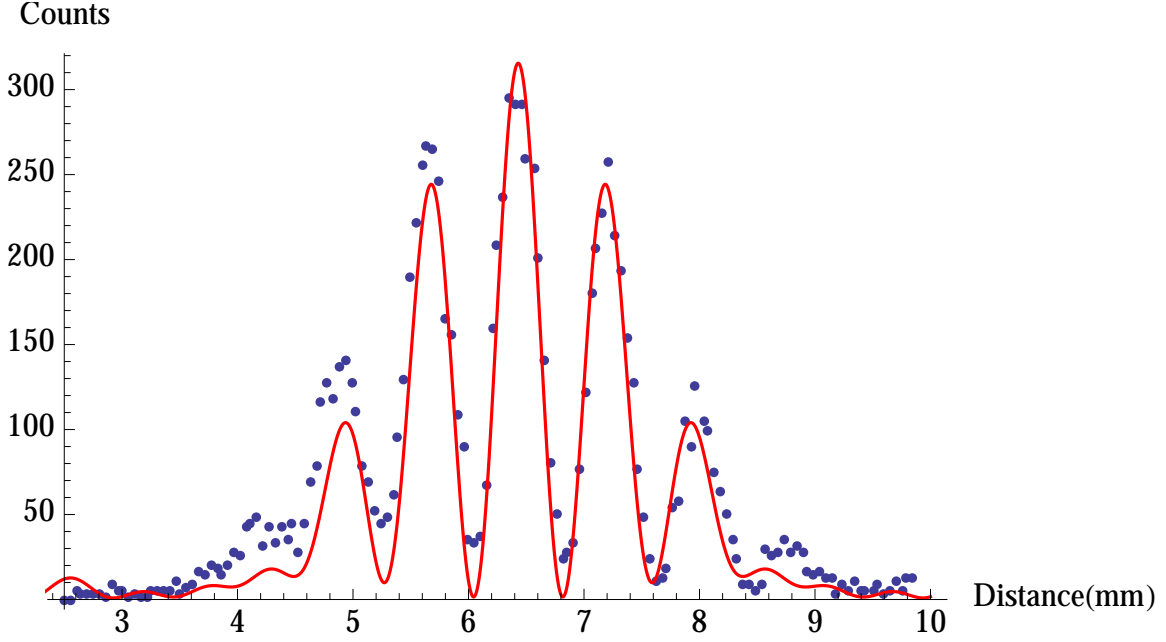Fitting Equation 6 to our single slit data we got Figure 13 and Figure 14. As discussed

FIG. 12. The fit to our third data set using the Fresnel model.

TABLE I. The χ^2 and reduced χ^2 values for both the Fraunhofer and Fresnel fits for all three sets of data.

Data Set	Fraunhofer χ^2	Fraunhofer Reduced χ^2	Fresnel χ^2	Fresnel Reduced χ^2
1	510	73	374	53
2	488	70	376	54
3	856	122	857	122

above the amplitude for our far slit data, Figure **??**, is larger than expected most likely due to slit misalignment. Our near slit data, Figure 13, agreed with our expectations for amplitude being one fourth the amplitude of the double slit fit.

We attempted to fit our single slit data using the Fresnel model by only summing over a single slit, however the Mathematica would not compile and we did not have time to problem solve the code error.
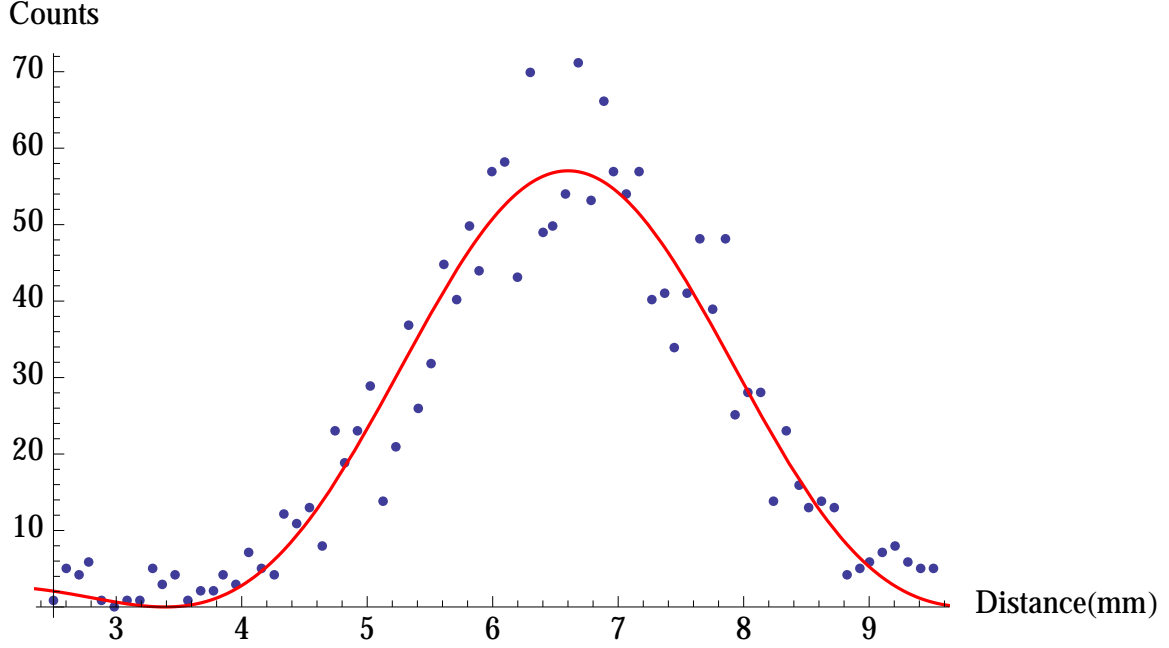
FIG. 13. The fit to our near slit data using the Fraunhofer model. The amplitude meets expectations by being one fourth the amplitude of the double slit fit.
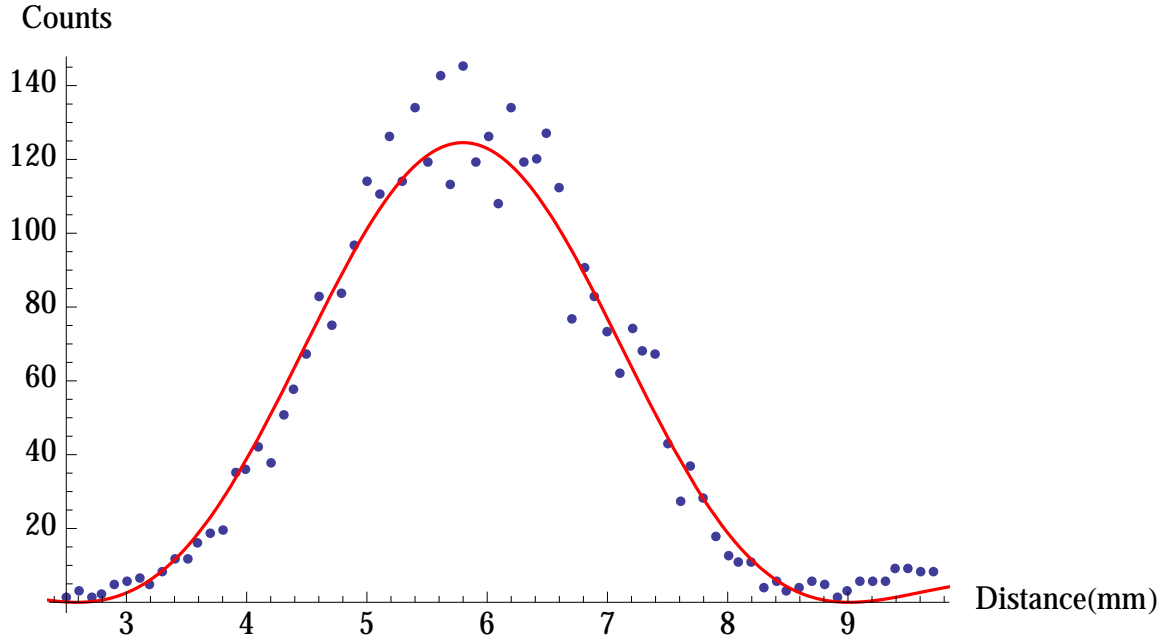

FIG. 14. The fit to our far slit data using the Fraunhofer model. The amplitude is much larger than expected due to slit misalignment.

V. DISCUSSION

The largest limiting factor in our experiment was the alignment of our slits. Aligning the slits was difficult because the light from the light bulb was too dim to see at the double slit. To align the slits we had to use the laser because it was bright enough to see all the way to the detection slit. However, this method is far from perfect because the laser was moveable and did not have the same positioning as the light bulb. So even if the slits were aligned using the laser, they would not necessarily be aligned when we switched to the light bulb.

To better align the slits we would have needed to take data then realign the slits and take data again until the our data showed us the slits were aligned. Alignment could be seen when all the minima of the diffraction pattern reached zero. This method of fine alignment would have required more time than we had. Fine alignment would also have been difficult because we were adjusting the slits with our fingers so such small, precise movements would have been difficult.

For future experiments we would suggest taking the time to make the fine adjustment necessary to ensure better alignment. And if possible use a apparatus that allows fine adjustments with something other than ones fingers.

Another potential difficulty was that our third data set had much higher χ^2 and reduced χ^2 values than than our first and second data sets. Further the χ^2 and reduced χ^2 values for third data set was same for both the Fraunhofer and Fresnel fits. Where as for our first and second data sets the χ^2 and reduced χ^2 values for the Fresnel fit was smaller than the χ^2 and reduced χ^2 values for the Fraunhofer fit. It appears that we had more noise around the edges of the diffraction pattern for our third data set than for the other two data sets. It is possible that this greater noise is reducing the accuracy of both our fits.

VI. CONCLUSION

VII. APPENDIX

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```
In[1]:=
    SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
    counts1 = Import["2014_double_slit_bulb_counts.csv"];
    counts1;

In[4]:=
    θ = (x - x0) / R;
    α = π * a * Sin[θ] / λ;
    β = π * d * Sin[θ] / λ;

    i₂ = i0 * (Sinc[α])² * Cos[β]²;

In[8]:= x0 = 6.2;
    a = 0.085;
    d = 0.343;
    R = 500;
    λ = .000546;

In[13]:= fit1 = NonlinearModelFit[counts1, i₂, {i0}, x];
    plot1 = Plot[fit1[x], {x, -10, 10}, PlotRange → All, PlotStyle → Red];
    Show[ListPlot[counts1], plot1, AxesLabel → {Distance [mm], Counts}]
```
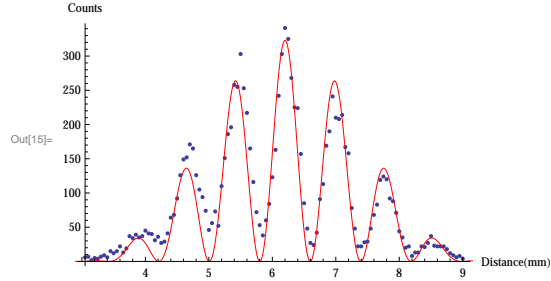
Out[15]=

```
In[16]:= ChiSq = Σ(j=1 to 120) ( fit1["FitResiduals"][[j]] / (2 (√counts1[[j, 2]] - √1.68)) )²

    RedChiSq = ChiSq / 7
```

Out[16]= 510.333

Out[17]= 72.9047

FIG. 15. Mathematica code for the Fraunhofer model fit to our first double slit data set.

[†] dluntzma@smith.edu

[1] Jonathan F. Reichert, *TeachSpin Instruction Manuals: Two-Slit Interference, One Photon at a*

```mathematica
In[1]:= SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
        counts2 = Import["20141122_double_slit_bulb_counts2.csv"];
        counts2;

In[4]:= θ = (x - x0) / R;
        α = π * a * Sin[θ] / λ;
        β = π * d * Sin[θ] / λ;

        i₂ = i0 * (Sinc[α])² * Cos[β]²;

In[8]:= x0 = 6.3;
        a = 0.09;
        d = 0.335;
        R = 650;
        λ = .000546;

In[13]:= fit2 = NonlinearModelFit[counts2, i₂, i0, x];
         Normal[fit2];
         plot2 = Plot[fit2[x], {x, -10, 10}, PlotRange → All, PlotStyle → Red];
         Show[ListPlot[counts2], plot2,
          PlotRange → {{2, 10}, All}, AxesLabel → {Distance [mm], Counts}]
```
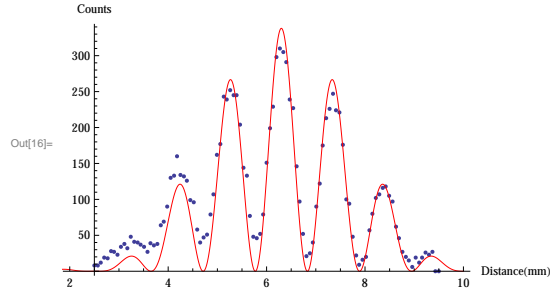


```mathematica
In[17]:= ChiSq2 = Σ_{j=1}^{105} ( fit2["FitResiduals"][[j]] / (2 (√(counts2[[j, 2]]) - √1.68)) )²

         RedChiSq2 = ChiSq2 / 7
Out[17]= 487.614

Out[18]= 69.6591
```

FIG. 16. Mathematica code for the Fraunhofer model fit to our second double slit data set.

Time (TWS1 - B), Pulse Counter / Interval Timer (PCIT1) Rev. 1.0, (2013)

```
In[1]:=  SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
         counts3 = Import["20141122_double_slit_bulb_counts3.csv"];
         counts3;

In[4]:=  θ = (x - x0) / R;
         α = π * a * Sin[θ] / λ;
         β = π * d * Sin[θ] / λ;

         i₂ = i0 * (Sinc[α])² * Cos[β]²;

In[8]:=  x0 = 6.4;
         a = 0.09;
         d = 0.383;
         R = 550;
         λ = .000546;

In[13]:= fit3 = NonlinearModelFit[counts3, i₂, i0, x];
         Normal[fit3];
         plot3 = Plot[fit3[x], {x, -10, 10}, PlotRange → All, PlotStyle → Red];
         Show[ListPlot[counts3], plot3,
          PlotRange → {{2, 10}, All}, AxesLabel → {Distance [mm], Counts}]
```
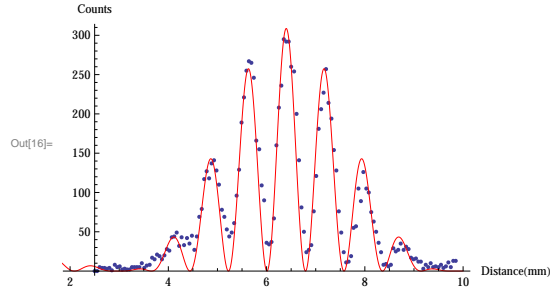

$$\text{In[17]:=} \quad \text{ChiSq3} = \sum_{j=1}^{146} \left(\frac{\text{fit3["FitResiduals"][[j]]}}{2 \left(\sqrt{\text{counts3[[j, 2]]}} - \sqrt{1.68} \right)} \right)^2$$

```
         RedChiSq3 = ChiSq3 / 7

Out[17]= 855.847

Out[18]= 122.264
```

FIG. 17. Mathematica code for the Fraunhofer model fit to our third double slit data set.

17

```
it = E^(2 π * I * D1 / λ) * E^(2 π * I * D2 / λ) ∫[d/2-a/2]^[d/2+a/2] E^(2 π I (w-y1)² / (2 * D1 * λ)) * E^(2 π I (y1-z)² / (2 * D2 * λ)) dy1;

ib = E^(2 π * I * D1 / λ) * E^(2 π * I * D2 / λ) ∫[-d/2-a/2]^[-d/2+a/2] E^(2 π I (w-y2)² / (2 * D1 * λ)) * E^(2 π I (y2-z)² / (2 * D2 * λ)) dy2;
```

In[3]:=
```
i = (it + ib);
coni = Conjugate[i];
func = i * coni;
```

In[6]:=
```
D1 = 380;
D2 = 500;
λ = .000546;
z = x - 6.2;
d = .353;
a = .1;
```
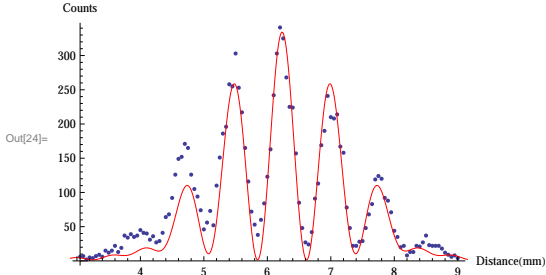
In[12]:=
```
func;
realfunc = 9000 * Re[func];
```

In[14]:=
```
SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
counts1 = Import["2014_double_slit_bulb_counts.csv"];
counts1;
```

In[17]:=
```
fit1 = NonlinearModelFit[counts1, realfunc, {{w, 0}}, x];
```

In[23]:=
```
plot1 = Plot[fit1[x], {x, 0, 10}, PlotRange → All, PlotStyle → Red];
Show[ListPlot[counts1], plot1, AxesLabel → {Distance [mm], Counts}]
```

Out[24]=

In[20]:=
```
fit1["FitResiduals"];
```

In[21]:=
$$\text{ChiSq} = \sum_{j=1}^{120} \left(\frac{\text{fit1["FitResiduals"][[j]]}}{2 \left(\sqrt{\text{counts1}[[j, 2]]} - \sqrt{1.68} \right)} \right)^2$$

Out[21]= 374.246

FIG. 18. Mathematica code for the Fresnel model fit to our first double slit data set.

```
In[1]:=
    it = E^(2π*I*D1/λ) * E^(2π*I*D2/λ) ∫[d/2-a/2]^[d/2+a/2] E^(2πI(w-y1)²/(2*D1*λ)) * E^(2πI(y1-z)²/(2*D2*λ)) dy1;

    ib = E^(2π*I*D1/λ) * E^(2π*I*D2/λ) ∫[-d/2-a/2]^[-d/2+a/2] E^(2πI(w-y2)²/(2*D1*λ)) * E^(2πI(y2-z)²/(2*D2*λ)) dy2;

In[3]:=
    i = (it + ib);
    coni = Conjugate[i];
    func = i * coni;

In[6]:= D1 = 380;
    D2 = 700;
    λ = .000546;
    z = x - 6.3;
    d = .353;
    a = .1;

In[12]:= func;
    realfunc = 8500 * Re[func];

In[17]:= SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
    counts2 = Import["20141122_double_slit_bulb_counts2.csv"];
    counts2;

In[20]:= fit2 = NonlinearModelFit[counts2, realfunc, {{w, 0}}, x];

In[29]:= plot2 = Plot[fit2[x], {x, 0, 10}, PlotRange → All, PlotStyle → Red];
    Show[ListPlot[counts2], plot2,
     AxesLabel → {Distance [mm], Counts}, PlotRange → {{2.5, 9.5}, All}]
```


FIG. 19. Mathematica code for the Fresnel model fit to our second double slit data set.

```mathematica
it = E^(2π*I*D1/λ) * E^(2π*I*D2/λ) ∫_(d/2-a/2)^(d/2+a/2) E^(2π I (w-y1)²/(2*D1*λ)) * E^(2π I (y1-z)²/(2*D2*λ)) dy1;

ib = E^(2π*I*D1/λ) * E^(2π*I*D2/λ) ∫_(-d/2-a/2)^(-d/2+a/2) E^(2π I (w-y2)²/(2*D1*λ)) * E^(2π I (y2-z)²/(2*D2*λ)) dy2;
```

In[3]:=

```mathematica
i = (it + ib);
coni = Conjugate[i];
func = i * coni;
```

In[6]:=

```mathematica
D1 = 380;
D2 = 500;
λ = .000546;
z = x - 6.4;
d = .353;
a = .1;
```

In[12]:=

```mathematica
func;
realfunc = 8500 * Re[func];
```

In[14]:=

```mathematica
SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
counts3 = Import["20141122_double_slit_bulb_counts3.csv"];
counts3;
```

In[19]:=

```mathematica
fit3 = NonlinearModelFit[counts3, realfunc, {{w, 0}}, x];
```

In[22]:=

```mathematica
plot3 = Plot[fit3[x], {x, 0, 10}, PlotRange → All, PlotStyle → Red];
Show[ListPlot[counts3], plot3,
  AxesLabel → {Distance [mm], Counts}, PlotRange → {{2.5, 10}, All}]
```

Out[23]=

FIG. 20. Mathematica code for the Fresnel model fit to our third double slit data set.

20

```
In[1]:= SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
        countsnear2 = Import["20141122_single_slit_near2.csv"];
        countsnear2;
        ListPlot[countsnear2, AxesLabel → {Distance (mm), Counts}];

In[5]:= θ = (x - x0) / R;
        α = π * a * Sin[θ] / λ;
        β = π * d * Sin[θ] / λ;


        i₂ = i0/4 * (Sinc[α])² ;


        x0 = 6.6;
        a = 0.085;
        d = 0.343;
        R = 500;
        λ = .000546;


In[14]:= fitnear2 = NonlinearModelFit[countsnear2, i₂, {i0}, x];
         Normal[fitnear2];

In[16]:= plotnear2 = Plot[57.037926753383644` Sinc[489.0757794050044` Sin[1/500 (-6.6` + x)]]²,

           {x, 0, 10}, PlotStyle → Red];

         Show[ListPlot[countsnear2], plotnear2, AxesLabel → {Distance [mm], Counts}]
```
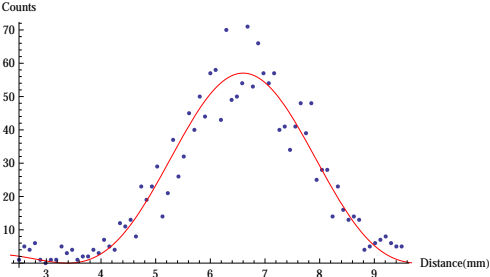

FIG. 21. Mathematica code for the Fraunhofer model fit to our near single slit data set.

```
In[1]:= SetDirectory["/Users/danikaluntz-martin/Desktop/Advanced Lab/DoubleSlit-ED"];
        countsfar1 = Import["20141122_single_slit_far.csv"];
        countsfar1;
        ListPlot[countsfar1, AxesLabel → {Distance (mm), Counts}];

In[5]:= θ = (x - x0) / R;
        α = π * a * Sin[θ] / λ;
        β = π * d * Sin[θ] / λ;


        i₂ = i0/4 * (Sinc[α])² ;


        x0 = 5.8;
        a = 0.085;
        d = 0.343;
        R = 500;
        λ = .000546;


In[18]:= fitfar1 = NonlinearModelFit[countsfar1, i₂, {i0}, x];
         plotfar1 = Plot[fitfar1[x], {x, -10, 10}];
         plotfarb = Plot[0.21642509498661616 (575.4539524532704 + 0.0023980364459083407 )

             Sinc[489.0757794050044 Sin[1/500 (-5.8 + x)]]]² , {x, 0, 10}, PlotStyle → Red];

         Show[ListPlot[countsfar1], plotfarb, AxesLabel → {Distance [mm], Counts}]
```
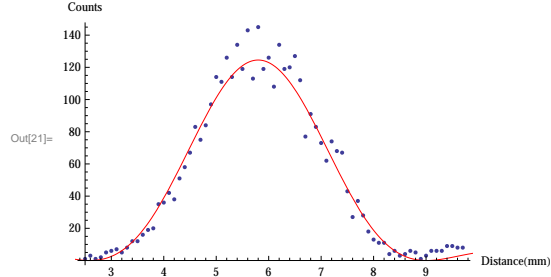

FIG. 22. Mathematica code for the Fraunhofer model fit to our far single slit data set.