Measuring the Verdet Constant in SF-59 Glass

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Abstract

We measured the intensity of a laser propagating through SF-59 glass in a magnetic field while rotating a polarizing lens or varying a magnetic field. We observed Faraday Rotation in both cases and measured the Verdet constant of the SF-59 glass rod to be 19.5 $\pm 0.9 \frac{1}{Tm}$ and 19.12 $\pm 0.05 \frac{1}{Tm}$.

I. INTRODUCTION

II. METHODS

Our experimental setup was based off the TeachSpin teaching manual. We used a Teach-Spin FRI-A apparatus consisting of a diode laser, a solenoid, a polarizing lens and photodetector. To observe Faraday Rotation and measure the Verdet constant we used a SF-59 glass rod. SF-59 glass is heavy flint glass with a high lead content. The manufacturers of this glass claim that the high lead content increases the Verdet constant of the glass making it easier to observe Faraday Rotation.¹

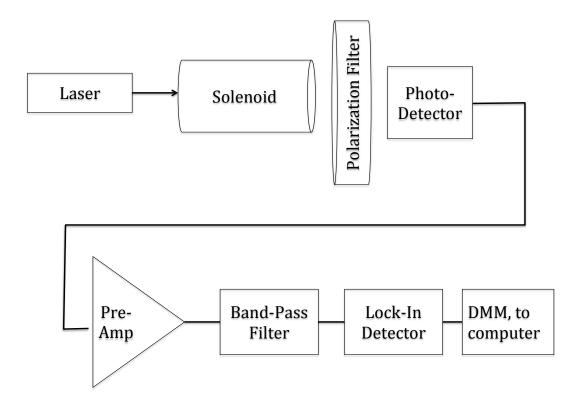


FIG. 1. The set-up of our experiment. The light source was a laser, modulated by a function generator. We sent the beam through a solenoid containing a rod of material, and next a polarization filter. A photodetector at the other end measured the light intensity. This signal went through a lock-in detector, and from there to the computer.

Our power source was a Keithley 2230-30-1 Triple Channel DC Power Supply which we

used on current control throughout our experiments. We connected the two 3 volt terminals in parallel so that we could obtain a total current of 3 amps through the solenoid. By varying the current through the solenoid we could change the magnetic field through the glass rod. We measured the magnetic field using a TEL-Atomic Inc. Smart Magnetic Sensor for a current of 2A and obtained a field of 21.8mT at the center of solenoid decreasing to 20.5mT at the edges of the glass rod. We took the average of this rage to be our best value and difference between high and low values divided by two to be our uncertainty. To obtain field values for other currents we exploited the linear relationship between magnetic field and current. We used a Rigol DG1022A function generator to modulate the laser at at frequency of 400Hz. The signal from the photodiode was run through a preamp, a bandpass filter and a lock-in detector. We used lock-in detector to stabilize our measurements and filter out ambient light from the room along with other forms of systematic error. The output from the lock-in detector was then measured with a Keithley 2100 DMM. To facilitate data collection we used a computer program (helpfully provided by our instructors) called "Keithley DC Incremental Write." The program would record 16 values for voltage, average them and give the result with uncertainty as one data point. For our changing theta experiment, we measured the voltage while rotating the polarizing lens. We started with no current and an angle of 90° between the polarized laser and the polarizing lens, so that our intensity (and measured voltage) was at a minimum. This was our relative angle of 0°. We then measured the voltage every 10° for a single rotation (360°). We repeated this this measurement for currents of I = 1A, I = 2A and I = 3A. For our changing field experiment, we set our relative angle to 45° we then measured the voltage for currents $0A, \pm 0.5A, \pm 1A, \pm 1.5A,$ $\pm 2A$, $\pm 2.5A$ and $\pm 3A$. For this experiment we set the computer program to average over 100 values for each data point.

III. RESULTS

We performed the experiment to measure the Verdet constant of our material in two different ways, and so obtained two different sets of data. From the measurement of light intensity due to changing polarization filter angle in a constant magnetic field, for currents of 0A, 1A, 2A, and 3A, we obtained the results shown in Figure 2.

From the measurements of changing magnetic field, with a constant polarization filter

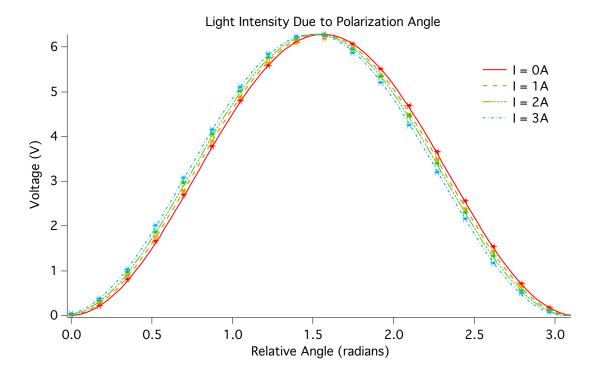


FIG. 2. Photo-detector voltage (proportional to light intensity) plotted against relative angle (the angle between the filter and the lasers polarization) from 0 to 2pi radians. The red points are the measurements taken without a magnetic field, the orange points were taken with 1A of current passing through the solenoid, the green points with the current at 2A, and the blue points at 3A. The curve fits are the function $V = A\cos^2(\theta + C) - A$, with A held constant at 6.2845 V (obtained from the first curve fit), and C calculated by the curve fit.

angle, for currents from -3A to 3A in steps of 0.5A, we obtained the results shown in Table I.

IV. ANALYSIS

We analyzed the data from both experiments separately to obtain two estimates for the Verdet Constant. For the changing theta experiment, we used IgorPro to fit the function $V = A\cos^2(\theta + C) - A$ to the data in Figure 2. We then calculated the magnetic field from the applied current and our measurements for field at 2A, and the phase shift from the difference between the c-values of the curve fits. The results are in Table II.

We then plotted the phase shift versus the magnetic field multiplied by the length of our

TABLE I. Voltage readings from the photodetector for various currents applied to the solenoid.

| Current (A) | Voltage (V) | Error Voltage (V) |
|-------------|-------------|-------------------|
| -3 | 2.822 | 1.30E-10 |
| -2.5 | 2.887 | 5.47E-10 |
| -2 | 2.951 | 3.72 E-10 |
| -1.5 | 3.016 | 3.78E-11 |
| -1 | 3.082 | 8.98E-10 |
| -0.5 | 3.145 | 9.03E-10 |
| 0 | 3.211 | 1.18E-09 |
| 0.5 | 3.272 | 1.16E-10 |
| 1 | 3.338 | 2.05E-09 |
| 1.5 | 3.404 | 2.13E-09 |
| 2 | 3.468 | 6.96E-10 |
| 2.5 | 3.534 | 1.12E-09 |
| 3 | 3.599 | 1.05E-09 |

TABLE II. Curve fit data and calculated phase shifts for changing angle experiment, with magnetic fields calculated from the applied currents.

| I (A) | BL (mT cm) | $\delta BL \ (mT \ cm)$ | C (rad) | $\delta C \text{ (rad)}$ | $\Delta\theta$ (rad) | $\delta\Delta\theta$ (rad) |
|-------|------------|-------------------------|-----------|--------------------------|----------------------|----------------------------|
| 0 | 0 | 0 | 0.0088553 | 0.001 | 0 | 0.002 |
| 1 | 108 | 4 | 0.030849 | 0.0014 | 0.022 | 0.0024 |
| 2 | 215 | 8 | 0.049599 | 0.0013 | 0.0407 | 0.0023 |
| 3 | 323 | 12 | 0.072754 | 0.0015 | 0.0639 | 0.0025 |

rod, which gave us Figure 3. From the theory we know that $\Delta\theta = v_c BL$ therefore we would expect our data to be linear and the Verdet constant to be the slope.

From the linear fit of our plot we obtained a slope $19.5 \pm 0.9 \frac{1}{Tm}$.

To get the Verdet constant from our changing field experiment, we first calculated the values of magnetic field times length for currents from -3A to 3A in steps of 0.5As. As before we calculated the field from our measured field for 2A and the linear relationship between

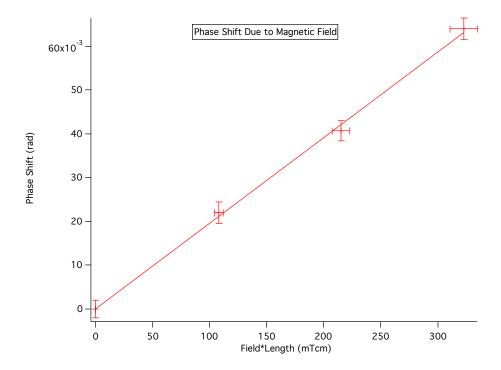


FIG. 3. A plot of Table II, applied magnetic field (times length of the refracting material) versus the resulting phase shift in the polarization of the laser beam. The slope of a linear curve fit of this plot gives a value for the Verdet constant of the material.

field and current. The results are in Table III.

We plotted the results of Table III in Figure 4. From the slope of the linear fit of this plot we got $\frac{\delta V}{\delta BL}$

Since, as shown above $v_c = \frac{\delta \theta}{\delta BL}$, we can use the chain rule for derivatives and get:

$$\frac{\delta V}{\delta BL} = \frac{\delta V}{\delta \theta} \frac{\delta \theta}{\delta BL} = \frac{\delta V}{\delta \theta} v_c.$$

Therefore $v_c = \frac{\frac{\delta V}{\delta BL}}{\frac{\delta V}{\delta \theta}}$.

From Figure 4 we got $\frac{\delta V}{\delta BL} = 0.0012018 \pm 1.8 \times 10^{-6}$. To get $\frac{\delta V}{\delta \theta}$, we took the derivative of the function $V = A \cos^2(\theta) - A$ at $\theta = \pi/4$ and obtained $\frac{\delta V}{\delta \theta} = -A$, and from the fit to the zero field data in Figure 2, $-A = 6.285 \pm 0.008V$.

Dividing these two results, we get $v_c = \frac{\frac{\delta V}{\delta BL}}{\frac{\delta V}{\delta \theta}} = 19.12 \pm 0.05 \frac{1}{Tm}$.

V. DISCUSSION

All of our data fit the expected relationships. In Figure 2 the intensity of the light, given by voltage from the photodetector, showed a $\cos^2 \theta$ relationship with relative angle between

TABLE III. The fields due to the currents in table I applied to our solenoid, with the resulting voltage measured by the photodetector.

| Magnetic Field * Length (mT*cm) | Error B*L (mT*cm) | Voltage (V) | Error Voltage (V) |
|---------------------------------|-------------------|-------------|-------------------|
| -323 | 12 | 2.822 | 1.30E-10 |
| -269 | 10 | 2.887 | 5.47E-10 |
| -215 | 8 | 2.951 | 3.72E-10 |
| -161 | 6 | 3.016 | 3.78E-11 |
| -108 | 4 | 3.082 | 8.98E-10 |
| -54 | 2 | 3.145 | 9.03E-10 |
| 0 | 0 | 3.211 | 1.18E-09 |
| 54 | 2 | 3.272 | 1.16E-10 |
| 108 | 4 | 3.338 | 2.05E-09 |
| 161 | 6 | 3.404 | 2.13E-09 |
| 215 | 8 | 3.468 | 6.96E-10 |
| 269 | 10 | 3.534 | 1.12E-09 |
| 323 | 12 | 3.599 | 1.05E-09 |

the laser and the polarizing filter as was expected. For the changing theta experiment the relationship between phase shift and magnetic field had a constant, positive slope, Figure 3, which is reassuring since slope should be the Verdet constant, a constant positive value. For the changing field experiment we expect intensity to depend linearly on field because a change in field causes a proportional change in phase shift and the relationship between intensity, i.e. voltage, and phase shift to be linear for small changes in phase. The values that we obtained for the Verdet constant from both experiments agreed with each other within uncertainty.

The biggest sources of error in our experiments came from our measurement of the magnetic field and the imprecision of our angle measurement on the polarizing filter. To decrease error in future experiments we would suggest using a more accurate magnetic field sensor

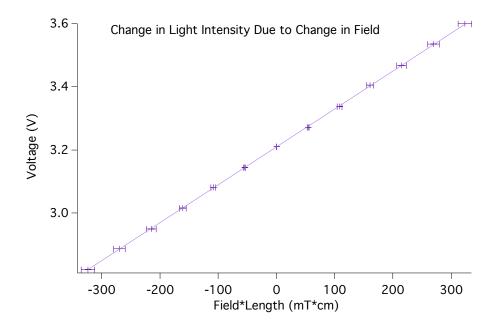


FIG. 4. A plot of Table III, the applied magnetic field (times the length of the refracting material) versus the voltage measured by the photodetector for that field. The slope of a linear curve fit to this plot can be used to calculate the Verdet constant of the material.

and to measure the magnetic field for all currents used in the experiment.

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¹ Jonathan F. Reichert, Faraday Rotation: Instructor's Guide to TeachSpin's FRI-A Apparatus

² Hans Bach and Norbert Neuroth, The Properties of Optical Glass 2nd edition (Springer Science and Business Media, Mainz, Germany,1998)