Measuring the Verdet Constant in SF-59 Glass

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Abstract

Faraday was the first to experimentally observe that light and magnetic fields were related. The Faraday Rotation is a phase shift in the polarization of light due to propagation through a birefringent material in a magnetic field. We observed Faraday Rotation by calculating the change in polarization angle of red (670nm) light from a polarized laser propagating through SF-59 glass in various magnetic fields. After passing through the material the light went through a polarizing filter. The change in the polarization of the laser light resulted in a change in the relative angle between the laser light and the polarizing filter which changed the intensity of light measured by a photodiode placed directly behind the filter. The output of the photodiode was a voltage proportional to the light intensity it measured. We powered the laser with a square wave from a function generator. We then used a lock-in detector to extract only information at the frequency and phase of our signal, thus eliminating dc offsets and noise at other frequencies and phases. Our goal was to determine the Verdet constant, a material-specific property. The Verdet constant gives the angle by with light polarization will change for a given magnetic field strength, wavelength, birefringent material and length of that material. Using two different experiments, we calculated the Verdet constant of a SF-59 glass rod. In our first experiment, we measured the photodiode voltage at various polarization filter angles for four different magnetic fields. Then we plotted voltage versus angle and fitted a curve to the data for each field to find the phase shift between the data sets. Using the phase shift we calculated the Verdet constant to be 19.5 $\pm 0.9 \ \frac{rad}{Tm}$. In the second experiment, we fixed the polarization filter at 45° relative to the light's polarization and measured photodiode voltage for thirteen magnetic fields. As predicted by theory, there was a linear relationship between photodiode voltage and field which we used to calculate the Verdet constant. We obtained a second value of 19.12 $\pm 0.05 \ \frac{rad}{Tm}$. Our values agree with each other within the precision of our measurements and are also in general agreement with the values found by our peers performing similar experiments for the same material.

I. INTRODUCTION

Faraday Rotation refers a phenomenon first observed by Faraday in 1945. It was a landmark discovery, because Faraday theorized and demonstrated the first link between light and magnetism.[2] Faraday discovered that polarized light propagating through certain materials while in a magnetic field experiences a shift in the polarization angle. This effect occurs in materials which have different refractive indices for left circularly polarized (LCP) light and right circularly polarized (RCP) light when they are in a magnetic field. These materials are called birefringent. Polarized light can be written as the vector sum of LCP and RCP components; so when it propagates through a birefringent material, its components experience different phase shifts leading to a total phase shift in the light's polarization.[1] The magnitude of the phase shift depends on four factors: the wavelength of the light, the strength of the magnetic field (B), the distance that the light travels in the material (L) and the properties of the material itself. The phase shift ($\Delta\theta$) which will result from propagation through a specific material, for a given wavelength, magnetic field and length of material, is given by the Verdet constant (v_c) which has units of $\frac{\text{radians}}{\text{tesla meter}}$. Thus the equation for total phase shift is $\Delta\theta = v_c B L$.

In this paper, we measure the Verdet constant of SF-59 glass using the expected linear relationship between phase shift and magnetic field. We also describe a second experiment which we performed to confirm the value we obtained in our first experiment.

II. METHODS

Our experimental setup was based off the TeachSpin teaching manual.[2] We used a TeachSpin FRI-A apparatus consisting of a diode laser, a solenoid, a polarizing filter and photodiode. To observe Faraday Rotation and measure the Verdet constant we used a SF-59 glass rod. SF-59 glass is heavy flint glass with a high lead content. The manufacturers dope silicone glass with lead because the high lead content increases the Verdet constant of the glass, making it easier to observe Faraday Rotation.[3] We placed the glass rod in the center of the solenoid. The length of our rod was 5 cm shorter than the solenoid so that the magnetic field was approximately constant across the length of the rod and we could neglect the severe drop in magnetic field near the edges of the solenoid.

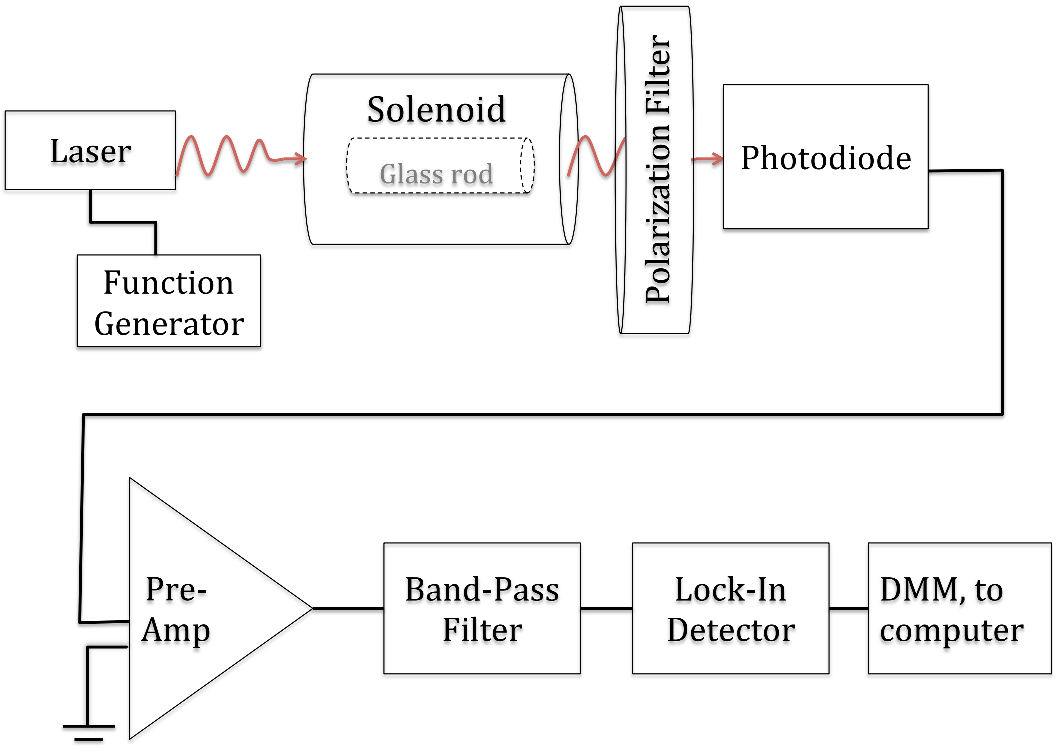

FIG. 1. The set-up of our experiment. The light source was a laser, powered by a function generator. We sent the beam through a solenoid containing the glass rod and then through a polarization filter. A photodiode at the other end measured the light intensity and outputted a voltage. This signal went through a pre-amp, bandpass and lock-in detector to remove offsets due to ambient light and photodiode drift. The much clearer signal was then measured with a multimeter connected to a computer.

We changed the magnetic field through the glass rod by varying the current through the solenoid. Our power source to the solenoid was a Keithley 2230-30-1 Triple Channel DC Power Supply which we used on current control throughout our experiments. We connected the two 30 volt terminals in parallel so that we could obtain a maximum current of 3 amps through the solenoid. We measured the magnetic field using a TEL-Atomic Inc. Smart Magnetic Sensor for a current of 2A and obtained a field of 21.8mT at the center of solenoid decreasing to 20.5mT at the edges of the glass rod. We took the average of this range to be our best value and difference between high and low values divided by two to be our uncertainty. We later used a better approximation by creating a position dependent function of magnetic field ($B(x)$, where x is along the axis of the solenoid) from our measurements of magnetic field. We then integrated over position to obtain a more accurate value for magnetic field. The discrepancy between our two calculated values of magnetic field times length was less than 0.5% and overall uncertainty in magnetic field times length was 4%. Since the discrepancy was far smaller than the uncertainty in either measurement, we determined that the rough approximation was sufficient for our calculations. To obtain field times length values for currents other than 2A, we exploited the linear relationship between magnetic field and current.

After passing through the solenoid, the polarized light from the laser diode (670 nm) passed through a second polarizer before the intensity of the light was measured by a photodiode. The output of the photodiode was a voltage, proportional to intensity of the light, that contained significant noise and offsets due to changes in the intensity of the ambient light of the room and the dc drift of the photodiode (dc drift is due to internal circuitry and is a limitation of the photodiode we cannot control). To remove the noise and offsets from our signal, the 4 volt 670 nm laser diode polarized light source was square-wave modulated at a frequency of 400 Hz and the resulting measured signal from the photodiode was run through a preamp, a bandpass filter and a lock-in detector. The preamp was single ended, the signal with respect to ground. The bandpass filter removed frequencies both higher and lower than our signal and ensured that we had only the first harmonic of the original square wave. Finally, we used a lock-in detector to select the specific frequency and phase of our signal. The final signal had fluctuations in voltage that were two orders of magnitude smaller than voltage measurements, and the signal was stable even when the room lights were turned off and on.

The final signal from the lock-in detector was then measured with a Keithley 2100 DMM. To facilitate data collection we used a computer program (helpfully provided by our instructors) called “Keithley DC Incremental Write.” The program would record a specified number of values for voltage then average them to obtain a single data point. Therefore, our results for each point were the mean of the measured points at that intensity with an uncertainty given as the standard deviation of the mean.

For our first experiment, which we called the changing theta experiment, we measured the photodiode voltage (which is proportional to light intensity) for a full rotation of the polarizing filter in increments of 10° while holding magnetic field constant. Since the photodiode voltage was proportional to intensity we expected it to depend on the relative angle just as intensity would. This dependance is given by the function $V = A\cos^2(\theta + C)$ where θ is the relative angle and A and C are both fit parameters. We started with no current through the solenoid and an angle of 90° between the polarized laser and the polarizing filter. The difference in angle between the polarization of the laser and the polarization of the filter is referred to as the relative angle throughout this paper. We found where the relative angle was 90° by observing where our intensity and measured voltage were at a minimum. We then measured the voltage every 10° for a single rotation (360°). For this experiment we set the computer program to average over 16 values for each data point. We repeated this measurement for currents of I = 1A, I = 2A and I = 3A.

For our second (changing field) experiment, we set our relative angle to 45°. We chose this angle because at 45° the slope of the voltage (intensity) versus angle plot is steepest: see Figure 2. This means that a change in polarization angle leads to the largest change in intensity when the relative angle is 45°. We then measured the photodiode voltage for currents 0A, ± 0.5A, ±1A, ±1.5A, ±2A, ±2.5A and ±3A. For each change in current the initial voltage reading would drift as the solenoid heated and resistance changed. We allowed the voltage to stabilize before starting data collection at each point. For this experiment we set the computer program to average over 100 values for each data point.

III. RESULTS

From the changing theta experiment, we got 36 data points for each current, i.e. 0A, 1A, 2A, and 3A, shown in Figure 2. The red points are the measurements taken without a magnetic field, the blue points were taken with 1A of current passing through the solenoid, the green points with the current at 2A, and the orange points at 3A. The curve fits are the function $V = A\cos^2(\theta + C)$, with A held constant at 6.2845 V (obtained from the first curve fit), and C calculated by the curve fit. We are interested in the change in C between curve fits because this gives us the change in the relative angle caused by applying a range of magnetic fields. The uncertainty in relative angle is due to difficulty reading the increments on the polarizing filter. The uncertainty in photodiode voltage is also dominated by the uncertainty in relative angle which we propagated through the equation $V = A\cos^2(\theta + C)$

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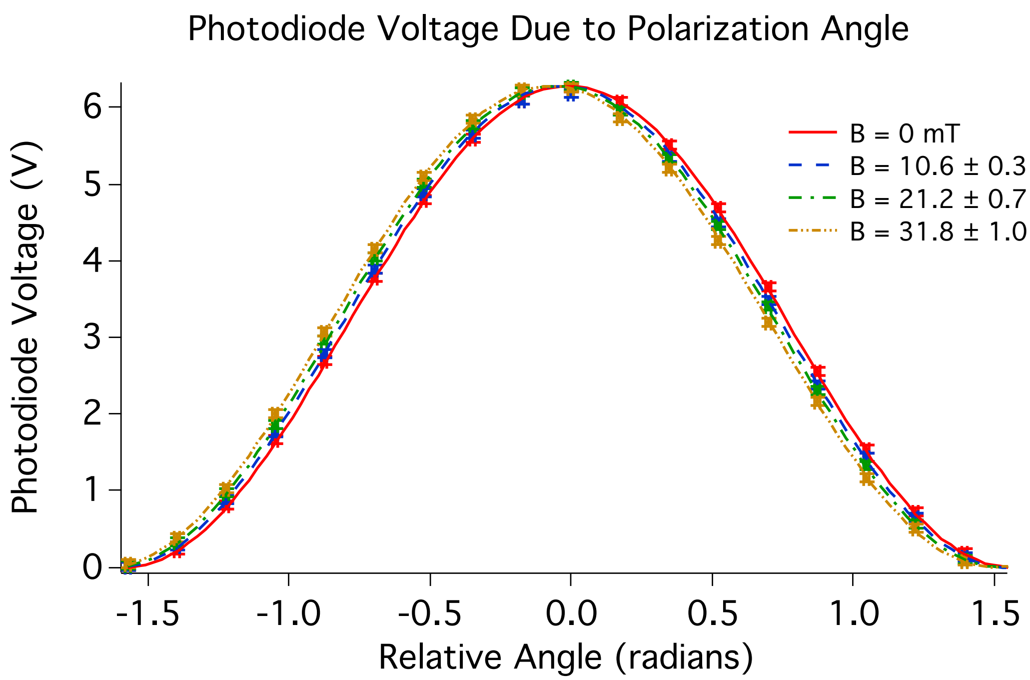

FIG. 2. Photodiode voltage (proportional to light intensity) plotted against relative angle from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ radians. The red points are the measurements taken without a current in the solenoid, the blue points were taken with 1A of current, the green points with the current at 2A, and the orange points at 3A. The curve fits are the function $V = A\cos^2(\theta + C)$. Faraday Rotation explains the phase shifts between the data sets at different magnetic fields values.

From the measurements at a 45° relative angle while changing magnetic field, we obtained the results shown in Table I. Uncertainty in voltage is again primarily due to uncertainty in relative angle.

IV. ANALYSIS

A. Changing Theta Experiment

We analyzed the data from both experiments separately to calculate two values for the Verdet Constant. For the changing theta experiment, we used IgorPro to fit the function $V = A\cos^2(\theta + C)$ to the data in Figure 2. We then calculated the magnetic field from the applied current and our measurements of magnetic field for a current of 2A. The phase

TABLE I. Voltage readings from the photodiode for various currents through the solenoid for the changing field experiment, with the magnetic fields produced by those currents for a fixed relative polarizer angle of 45°.

Current (A)	Field (mT)	Error Field (mT)	Voltage (V)	Error Voltage (V)
-3	-31.8	-1	2.822	0.05
-2.5	-26.5	-0.9	2.887	0.05
-2	-21.2	-0.7	2.951	0.05
-1.5	-15.9	-0.5	3.016	0.05
-1	-10.6	-0.3	3.082	0.05
-0.5	-5.3	-0.15	3.145	0.05
0	0	0	3.211	0.05
0.5	5.3	0.15	3.272	0.05
1	10.6	0.3	3.338	0.05
1.5	15.9	0.5	3.404	0.05
2	21.2	0.7	3.468	0.05
2.5	26.5	0.9	3.534	0.05
3	31.8	1	3.599	0.05

shift values are the difference between the C-value of each curve fit and the C-value of the zero-field curve fit (that is, the horizontal shift of the plot resulting from the applied field). The results are in Table II. The uncertainty in the BL was calculated from the estimates of error in our magnetic field measurements and length measurements for the glass rod. Uncertainty came primarily from lack of precision on the part of the magnetic field sensor and inaccuracy in determining the sensor's position within the closed solenoid. There was some uncertainty in the length measurement, but it was a much smaller percentage of our best value than the percentage uncertainty in magnetic field.

We then plotted the phase shifts versus the magnetic field multiplied by the length of our rod, which gave us Figure 3. From the theory we know that $\Delta\theta = v_c BL$ therefore we would expect our data to be linear and the Verdet constant to be the slope. A linear fit matches our data well, passing through every point within uncertainty. From the slope of the linear

TABLE II. Curve fit data and calculated phase shifts for changing angle experiment, with magnetic fields calculated from the applied currents.

I (A)	BL (mT cm)	δBL (mT cm)	C (rad)	δC (rad)	$\Delta\theta$ (rad)	$\delta\Delta\theta$ (rad)
0	0	0	0.0088553	0.001	0	0.002
1	108	4	0.030849	0.0014	0.022	0.0024
2	215	8	0.049599	0.0013	0.0407	0.0023
3	323	12	0.072754	0.0015	0.0639	0.0025

fit we obtained a value for the Verdet constant of $19.5 \pm 0.9\frac{rad}{Tm}$.

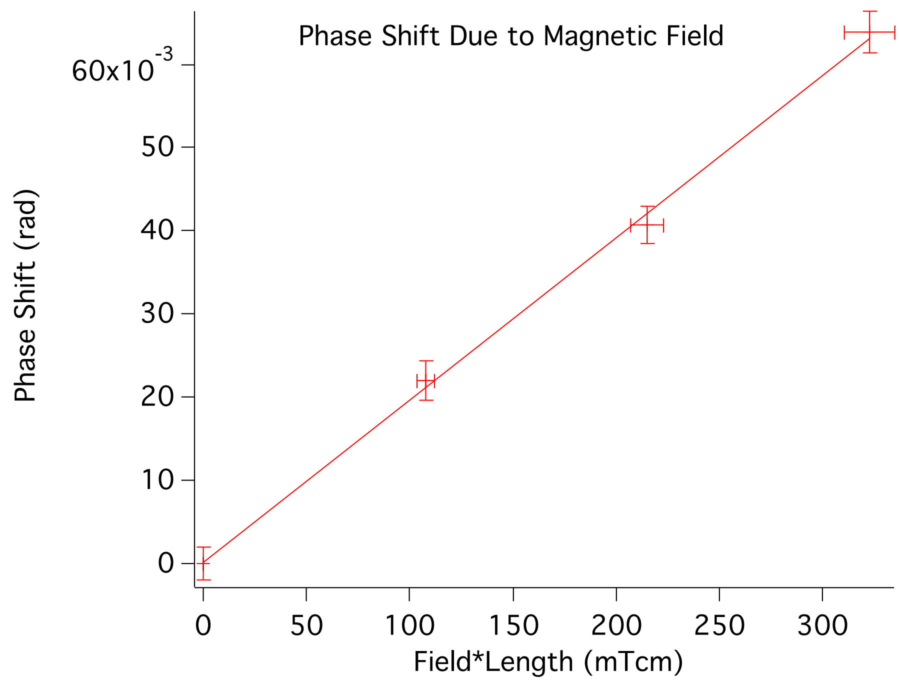

FIG. 3. A plot of Table II, applied magnetic field (times length of the birefringent material) versus the resulting phase shift in the polarization of the light. The slope of a linear fit of this plot gives a value for the Verdet constant of the glass rod.

B. Changing Field Experiment

To get the Verdet constant from our changing field experiment, we first calculated the values of magnetic field times length for currents from -3A to 3A in steps of 0.5As, see

Table I. As before we calculated the field from our measured field for 2A and the linear relationship between field and current. We multiplied the magnetic field by the length of our glass rod (L = 10.15 cm) to get values for BL.

We then plotted the results of Table I in Figure 4. From the slope of the linear fit we got $\frac{\delta V}{\delta BL} = 0.0012018 \pm 1.8 \times 10^{-6} \frac{V}{mT\ cm}$

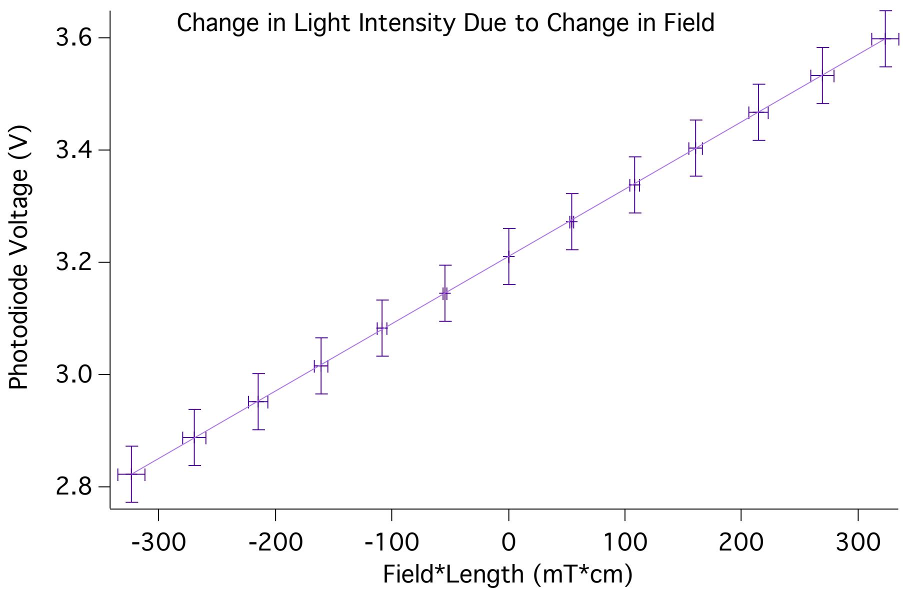

FIG. 4. A plot of Table I, the applied magnetic field (times the length of the refracting material) versus the voltage measured by the photodiode for that field. The slope of a linear curve fit to this plot can be used to calculate the Verdet constant of the material.

Since, as discussed above, $v_c = \frac{d\theta}{dBL}$, we can use the chain rule for derivatives and get:

$$\frac{dV}{dBL} = \frac{dV}{d\theta}\frac{d\theta}{dBL} = \frac{dV}{d\theta}v_c \tag{1}$$

Therefore

$$v_c = \frac{dV}{dBL} \div \frac{dV}{d\theta} \tag{2}$$

From Figure 4 we got $\frac{dV}{dBL} = 0.0012018 \pm 1.8 \times 10^{-6}$. To get $\frac{dV}{d\theta}$, we took the derivative of the function $V = A\cos^2(\theta)$ at $\theta = \pi/4$ and obtained $\frac{dV}{d\theta} = -A$, and from the fit to the zero field data in Figure 2, $A = 6.285 \pm 0.008 \frac{V}{mT\ cm}$.

Dividing these two results, we get $v_c = \frac{dV}{dBL} \div \frac{dV}{d\theta} = 19.12 \pm 0.05 \frac{rad}{Tm}$. This result agrees with the value we obtained from the changing theta experiment within uncertainty.

V. DISCUSSION

All of our experimental results are consistent with theoretical expectations. In Figure 2 the intensity of the light, given by voltage from the photodiode, showed a $\cos^2\theta$ relationship with the relative angle as was expected. For the changing theta experiment the relationship between phase shift and magnetic field had a constant, positive slope (as shown in Figure 3), which is reassuring since this slope should be the Verdet constant, a constant positive value. For the changing field experiment we expected intensity to depend linearly on field because a change in field causes a proportional change in phase shift, which results in a change in the relative angle between the laser's polarization and the polarizing filter, which changes the measured intensity, i.e. voltage. This change in voltage due to phase shift is approximately linear for small changes in phase, and therefore the overall relationship between field and photodiode voltage is linear. In addition to our data showing the expected relationships between variables, the values that we obtained for the Verdet constant from both experiments agreed with each other within uncertainty. Furthermore they are within the range of values obtained by our peers performing similar experiments.

The biggest sources of error in our experiments came from our measurement of the magnetic field and the imprecision of our angle measurement on the polarizing filter. To decrease error in future experiments we would suggest using a more accurate magnetic field sensor, and to measure the magnetic field for all currents used in the experiment. A more precise profile of magnetic field through the solenoid would also mean that integration of the field along the length of the material would give a more precise value, whereas in our experiment the integration and averaging were about equivalent in terms of precision due to the large error in B. Finally, a polarizing filter that showed smaller increments for angle would help lower the uncertainty in relative angle, which would also give us smaller propagated uncertainty in voltage.

VI. CONCLUSION

Both of our experiments allowed us to calculate values for the Verdet constant. But that does not mean that both methods are equally good. The changing theta experiment relies on there being enough of a phase shift in the data points for the various magnetic fields to have significant horizontal shifts between the curve fits of photodiode voltage versus relative angle. If the shifts are too small, they may not be distinguishable within uncertainty. In our experiment we did have significant phase shifts between data sets because we were working with a material specifically manufactured to increase the magnitude of the Faraday Rotation. However, this method would not be a good choice to use for a material which produced a smaller Faraday Rotation. Additionally, as the larger uncertainty suggests, this method is not as precise as the changing field experiment.

The changing field experiment has its own drawbacks. To calculate the Verdet constant we assumed that the relative polarization angle between the laser beam and the filter was 45° for each value of magnetic field. This is not precisely true, as the relative polarization angle changes when the magnetic field causes the light's polarization to shift. If the actual relative angle divated too much from 45°, then the changing field method would not be accurate. However, the ability to measure the Verdet consent for materials with smaller Faraday Rotations might be an acceptable trade-off for the loss in accuracy due to this approximation. It could also be corrected for by changing the filter angle for each current, or by using a more complicated calculation which takes into account the changing value of $\frac{dV}{d\theta}$.

Both methods could be modified to measure the Verdet constant for different wavelengths of light. Repeating the measurement taken at various magnetic field for an entirely different wavelength would allow the difference in the Verdet constant to be calculated for various wavelengths.

Calculating the Verdet constant of materials and being able to quantify Faraday Rotation has a number of applications, including in astronomy, optics and chemistry. Faraday Rotation is used to map the magnitude of magnetic fields in space and to build optical 'switches' that allow light to pass through in one direction but severely attenuate it when it moves in the other direction. Even without the modern applications, Faraday Rotation will always be significant as the first experimental evidence of a connection between magnetism and light which paved the way for our understanding of electromagnetism.

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