Math 168: HW 5

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2) Newman 7.1

(a) Since G is connected, k-regular and undirected, we know that the sum of each row of its adjacency matrix A is k.
But then $A\vec{1} = \vec{k} = k\vec{1} \Rightarrow \vec{1}$ is an eigenvector of A with eigenvalue k.

(b) We know Katz centraility can be calculated by
$\vec{x} = \alpha A \vec{x} + \vec{1} \Rightarrow x_i = \alpha \sum_j A_{ij} x_j + 1 = \alpha k x_i + 1$
(by symmetry)

$$\Rightarrow \quad x_i = \frac{1}{1-\alpha k} \quad \forall i.$$

(c) Closeness centraility is an example of such a centraity measure. This even makes sense heuristically, since nodes on the boundary of a k-regular graph, while still connected to k other nodes, are further from all the nodes of the graph than one in the center.

2) Newman 7.3

(a) $\vec{x} = \alpha A \vec{x} + \vec{1} \Rightarrow \vec{x} - \alpha A \vec{x} = \vec{1} \Rightarrow \vec{x} = \dfrac{\vec{1}}{\vec{I} - \alpha A}$

Then we can use the Taylor expansion if $\alpha|\lambda|<1$ where λ is the largest eigenvalue of A, which gives us:

$$\vec{x} = \frac{\vec{1}}{\vec{I} - \alpha A} = \vec{1}\sum_{i=0}^{\infty} (\alpha A)^i = \vec{1} + \alpha A \vec{1} + \alpha^2 A^2 \vec{1} + \dots$$

(b) As $\alpha \to 0$, the higher order terms of α disappear, noting this gives us $\vec{x} \approx \vec{1} + \alpha \vec{d}$ where $\vec{d}$ is the degree centrality. $\Rightarrow \dfrac{\vec{x} - \vec{1}}{\alpha} = \vec{d}$. $\therefore$ We care about the relative differences of centrality measures and not their absolute value, noting that $\vec{x}$ is simply an linearly increasing function of $\vec{d}$, Katz centrality is effectively equivalent to degree centrality.

(c) As $\alpha \to 1/k_1$,

$$\vec{n} \to \frac{\vec{I}}{\vec{I} - A/k_1} \Rightarrow \vec{n} = \frac{k_1\vec{I}}{k_1\vec{I} - A}$$

$$\Rightarrow (k_1\vec{I} - A)\vec{n} = k_1\vec{I}$$
$$\Rightarrow k_1\vec{I}\vec{n} - A\vec{n} = k\vec{I}$$
$$\Rightarrow A\vec{n} = k_1\vec{n} - \vec{k_1},$$

which is exactly proportional to the eigenvector centrality found by $A\vec{n} = k_1\vec{n}$.

3) Newman 7.5

Pagerank is defined by $n_i = \alpha \sum_j A_{ij} \frac{n_j}{k_j^{out}} + \beta$, which in our case simplifies to $n_i = \alpha \sum_j A_{ij} n_j + 1$

Any node starts off with a contribution 1 i.e. β, then upon progressing to its parent multiplies its contribution by α and adds another 1, giving $2\alpha + 1$. observing this pattern the contribution is $\alpha^{d_i}\beta$

$\therefore$ PageRank for central node

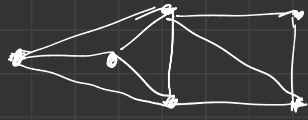
$$\parallel$$

$$\sum_{i=1}^{n} \alpha^{d_i}$$

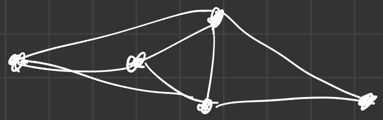
4) Newman 7.8

(a) we start with the whole network, removing any vertices with degree less than 3 iteratively as these will not be members of the 3-core.

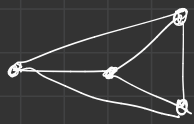
After Iteration 1:



Iteration 2:



So our resulting 3-core is



(b) We can calculate reciprocity as $\frac{1}{m} \sum_i A_{ij} A_{ji}$ where m is the total # of edges. This gives us

$$r = \frac{3}{8}.$$

(c) cosine similarity is the num of common neighbors of A and B divided by the geometric mean of the degrees $\therefore \sigma_{ij} = \frac{2}{\sqrt{4 \cdot 5}} = \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}}$.