MATH 168 HW #1

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Newman Chp 6 Exercises

3) 6.1

a) The internet, at the level of autonomous systems: undirected,
cyclic

b) A food web: directed,
approximately acyclic,
approximately planar

c) The stem and branches of a plant:
Undirected, acyclic, planar, tree

d) A spider web:
undirected, cyclic, planar

e) A complete clique of 4 nodes:

undirected, cyclic, planar

f) acyclic directed network:

a schedule of tasks to run an end to end machine learning model in production.

g) A cyclic directed network:
a circuit of resistors with current flowing

h) A tree:
folders in a Mac's Finder or a PC's Windows Explorer

i) Planar network:
Dependicies of classes (like pre or co-requisites) at UCLA

j) Bipartite network:
Diners at a restaurant and their food choices from the menu.

k) Could do BFS by starting from a webpage, cataloguing the "edges" present i.e. hyperlinks, moving on to the webpages (which are our nodes) linked and continue.

l) Nodes can be scientific papers with edges present b/w papers if there is at least one collaborator that worked on both papers. Similarly we can start with a base paper and continue to add papers with edges based on the authors' other works.

m) We note that a bottom-up construction would be ideal for a food-web so as to not miss any involved organisms. Starting with plants and bacterial matter we can create directed edges when an organism is consumed and continue recursively.

n) Again, start with a base friend and add all their friends to the network with edges. Continue by adding those friend's friends and ensure no duplicate nodes are formed. At the end we have to check if all co-workers are present and add those who aren't in the network. (and their friends)

c) A full power grid network would probably be multilayer. We have to start with power plants, adding edges if they supply each other. Then there are distribution networks with transformers as nodes and edges as whether power is transferred between them. Finally, consumers get a layer built with a similar process as above. (there are links between layers)

4) 6.2

A simple network cannot have self or multi edges. Therefore for the max num of edges, each node connects to every other node resulting in $\frac{n(n-1)}{2} = \frac{n^2-n}{2}$ edges.

For min num of edges, we note that a trees have the property of no closed loops i.e. exactly one path between any pair of nodes. This is also what we want since our network is simple and connected (single component)). Since a tree with n nodes has $n-1$ edges, the minimum possible num of edges is $n-1$.

5) 6.4

a) $\vec{k} = A\vec{1}$

b) $m = \frac{1}{2} \sum_{ij} A_{ij}$

c) $N = A^2$

d) num of triangles in network:

$$\text{trace}(A^3)/6$$

2) Dimension Thm:

Let V and W be vector spaces and $T: V \to W$ is a linear transformation.

If V is finite dimensional, then

$$\text{nullity}(T) + \text{rank}(T) = \dim(V).$$

Pf

Using the incomplete base thm, take $(v_1, \dots, v_k)$ a base of $\ker(T) \subseteq V$.

Extending this basis to all of V we have $(v_1, \dots, v_p, v_{p+1}, \dots, v_n)$.

So we have:

$$T(V) = \underbrace{T(v_1) + \dots + T(v_p)}_{0 \because v_1, \dots, v_p \in \ker(T)} + T(v_{p+1}) + \dots + T(v_n)$$

$$T(V) = T(v_{p+1}) + \dots + T(v_n)$$

$\because$ T injective on $\mathrm{span}(v_{p+1}, \dots, v_n)$,

$T(v_{p+1}), \dots, T(v_n)$ is a basis for $T(V)$

$\therefore$ $\dim \ker(T) + \mathrm{rank}\, T = p + (n - (p+1) + 1)$

$= n = \dim V$ $\square$