Problems

1. (80 pt. in total)

Assume that we have n data points, $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$. Let the degree of polynomial be d. Then, we want to find $w_0, w_1, w_2, \dots, w_d$ of the polynomial such that

$$\hat{f}(x_1) = w_0 + w_1 x_1 + w_2 x_1^2 + \dots + w_d x_1^d = y_1,
\hat{f}(x_2) = w_0 + w_1 x_2 + w_2 x_2^2 + \dots + w_d x_2^d = y_2,
\hat{f}(x_3) = w_0 + w_1 x_3 + w_2 x_3^2 + \dots + w_d x_3^d = y_3,
\hat{f}(x_4) = w_0 + w_1 x_4 + w_2 x_4^2 + \dots + w_d x_4^d = y_4,
\hat{f}(x_5) = w_0 + w_1 x_5 + w_2 x_5^2 + \dots + w_d x_5^d = y_5,
\vdots$$

$$\hat{f}(x_n) = w_0 + w_1 x_n + w_2 x_n^2 + \dots + w_d x_n^d = y_n.$$

Now, we reformulate the equations into the vector and matrix form. First, let $\mathbf{w} = [w_0, w_1, \cdots, w_d]^T$ and $\mathbf{y} = [y_1, y_2, \cdots, y_n]^T$. Then, the above equations can be rewritten as

$$\hat{f}(x_1) = [1, x_1, x_1^2, x_1^3, \cdots, x_1^d] \cdot egin{bmatrix} w_0 \ w_1 \ w_2 \ w_3 \ dots \ w_d \end{bmatrix} = [1, x_1, x_1^2, x_1^3, \cdots, x_1^d] \mathbf{w} = y_1$$

Similarly, we have,

$$[1, x_2, x_2^2, x_2^3, \cdots, x_2^d] \mathbf{w} = y_2,$$

$$[1, x_3, x_3^2, x_3^3, \cdots, x_3^d] \mathbf{w} = y_3,$$

$$[1, x_4, x_4^2, x_4^3, \cdots, x_4^d] \mathbf{w} = y_4,$$

$$[1, x_5, x_5^2, x_5^3, \cdots, x_5^d] \mathbf{w} = y_5,$$

$$\vdots$$

$$[1, x_n, x_n^2, x_n^3, \cdots, x_n^d] \mathbf{w} = y_n.$$

Then, all equations can be written as the form of linear equation,

$$A\mathbf{w} = \mathbf{y}$$

where A is the stack of $[1, x_i, x_i^2, x_i^3, \dots, x_i^d]$ for $i = 1, \dots, n$. Under this setting, answer the following questions.

1-(a) What is the size of vector w and v? (10nt)

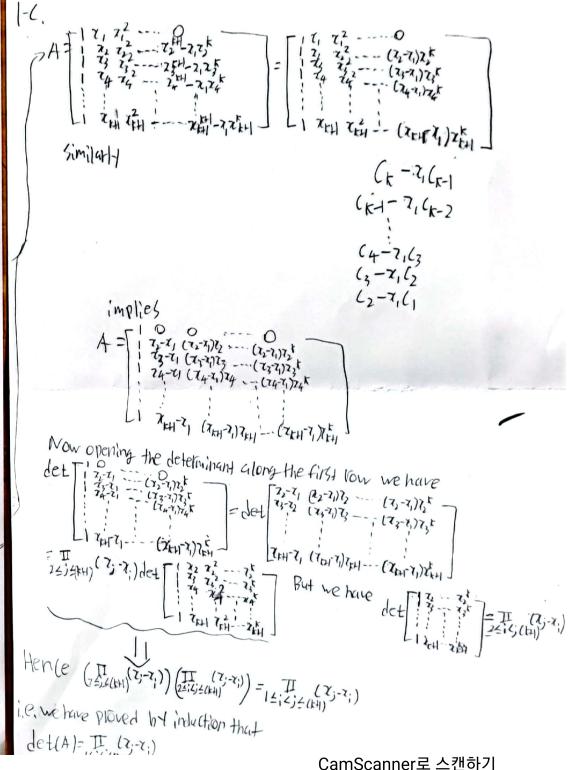
1-(b) What is the size of matrix A? Write A. (10pt)

1-(c) Let d=n, then, A becomes a square matrix. Compute the determinant of A. (40pt in total, Derivation: 30pt, Answer: 10pt, Hint: Vandermonde Matrix.)

1-(d) What is the condition that makes the determinant of A non-zero? (10pt)
As det(A)=1 (7,-7;) we have det A+O() x;+x; for all 15;45 in

1-(e) Assume that the determinant of A is non-zero, then, what is the solution of linear equation, $A\mathbf{w} = \mathbf{y}$, with respect to \mathbf{w} ? (10pt)

If det A to then we have A exists and hence A w= y=> W=A y ... the solution is given by w=A y



2. (20pt)

Suppose that n > d. Then, we cannot compute the inverse of A since A is not a square matrix. In this case, how can we solve the linear equation $A\mathbf{w} = \mathbf{y}$? (Hint: Pseudo Inverse)

AW=Y

Then

where A=) A matrix of order nx(d+1)

W=) Matrix of order (dH) x1 or A vector of size (dH)

Y=) Vector of size n

First we have to write the matrix A included with Yas

then write the matrix A into row echelon form and same operations are applied on y which are applied on A

Let reduced form of A is B and reduced form of Y is X

By educating both sides we find the elements of w