Isotropic smoothing of image via Heat equation

import library

In [3]:
```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.colors as colors
from skimage import color
from skimage import io
```

load input image

- filename for the input image is 'barbara_color.jpeg'

In [4]:
```
I0 = io.imread('barbara_color.jpeg')
```

check the size of the input image

In [5]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
num_row     = I0.shape[0]
num_column  = I0.shape[1]
num_channel = I0.shape[2]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I0 = ', num_row)
print('number of columns of I0 = ', num_column)
print('number of channels of I0 = ', num_channel)
```

```
number of rows of I0 =  512
number of columns of I0 =  512
number of channels of I0 =  3
```

convert the color image into a grey image

In [6]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
I = color.rgb2gray(I0)

num_row     = I.shape[0]
num_column  = I.shape[1]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I = ', num_row)
print('number of columns of I = ', num_column)
```

```
number of rows of I =  512
number of columns of I =  512
```

normalize the converted image

- normalize the converted grey scale image so that its maximum value is 1 and its minimum value is 0

In [7]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
I = (I - np.min(I))/(np.max(I) - np.min(I))
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('maximum value of I = ', np.max(I))
print('minimum value of I = ', np.min(I))
```

```
maximum value of I =  1.0
minimum value of I =  0.0
```

define a function to compute the derivative of input matrix in x(row)-direction

- forward difference : $I[x+1, y] - I[x, y]$

In [9]:
```
def compute_derivative_x_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    # x == row y == column
    # I  = [[A],[B],[],..[0]      ]
    # I_ = [[B],[],[],..[0] + [0]]
    I_ = np.vstack([I[1:], I[-1]])

    D = I_ - I
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x, y] - I[x-1, y]$

In [10]:
```
def compute_derivative_x_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    # x == row y == column
    # I  = [[A],[B],[] ,..[0]]
    # I_ = [[A],[A],[B],..[Z]]
    I_ = np.vstack([I[0], I[:-1]])

    D = I - I_
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

define a function to compute the derivative of input matrix in y(column)-direction

- forward difference : $I[x, y+1] - I[x, y]$

In [11]:
```
def compute_derivative_y_forward(I):

    D = np.zeros(I.shape)


    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    I_t = np.transpose(I)
    D_t = np.zeros(I_t.shape)
    I_ = np.vstack([I_t[1:], I_t[-1]])

    D_t = I_ - I_t
    D = np.transpose(D_t)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    return D
```

- backward difference : $I[x, y] - I[x, y-1]$

In [12]:
```
def compute_derivative_y_backward(I):

    D = np.zeros(I.shape)
    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    I_t = np.transpose(I)
    D_t = np.zeros(I_t.shape)

    I_ = np.vstack([I_t[0], I_t[:-1]])

    D_t = I_t - I_
    D = np.transpose(D_t)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++


    return D
```

define a function to compute the laplacian of input matrix

- $\Delta I = \nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$
- $\Delta I = I[x+1, y] + I[x-1, y] + I[x, y+1] + I[x, y-1] - 4 * I[x, y]$
- ΔI = derivative_x_forward - derivative_x_backward + derivative_y_forward - derivative_y_backward

In [13]:
```
def compute_laplace(I):

    laplace = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    laplace = compute_derivative_x_forward(I) - compute_derivative_x_backward(I) + compute_derivative_y_forward(I) - compute_derivative_y_backward(I)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return laplace
```

define a function to compute the heat equation of data I with a time step

- $I = I + \delta t * \Delta I$

In [14]:
```
def heat_equation(I, time_step):

    I_update = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    I_update = I + time_step * compute_laplace(I)

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return I_update
```

run the heat equation over iterations

In [16]:
```
def run_heat_equation(I, time_step, number_iteration):

    I_update = np.zeros(I.shape)

    for t in range(number_iteration):
        # ++++++++++++++++++++++++++++++++++++++++++++++++++
        # complete the blanks
        #
        I = heat_equation(I, time_step)


        #
        # ++++++++++++++++++++++++++++++++++++++++++++++++++
    I_update = I
    return I_update
```

---

functions for presenting the results

In [17]:
```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.imshow(I0)
    plt.show()
```

In [18]:
```
def function_result_02():

    plt.figure(figsize=(8,6))
    plt.imshow(I, cmap='gray', vmin=0, vmax=1, interpolation='none')
    plt.show()
```

In [19]:
```
def function_result_03():

    L = compute_laplace(I)

    plt.figure(figsize=(8,6))
    plt.imshow(L, cmap='gray')
    plt.show()
```

In [20]:
```
def function_result_04():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [21]:
```
def function_result_05():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [22]:
```
def function_result_06():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [23]:
```
def function_result_07():

    L = compute_laplace(I)

    value1 = L[0, 0]
    value2 = L[-1, -1]
    value3 = L[100, 100]
    value4 = L[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [24]:
```
def function_result_08():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [25]:
```
def function_result_09():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

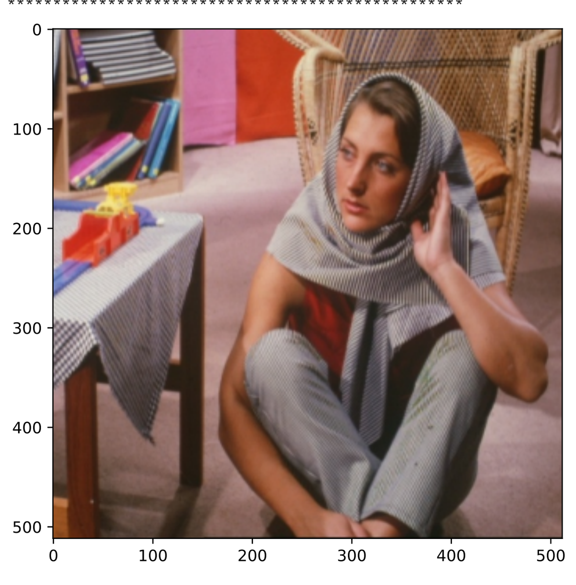
In [26]:
```
def function_result_10():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

---

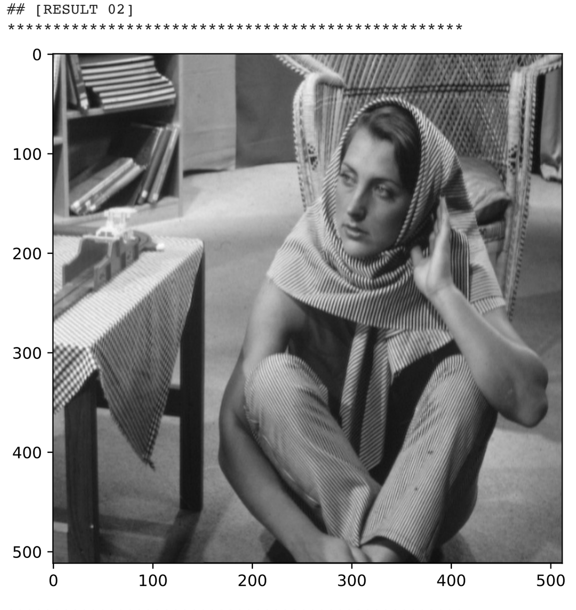
results

In [27]:
```
number_result = 10

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

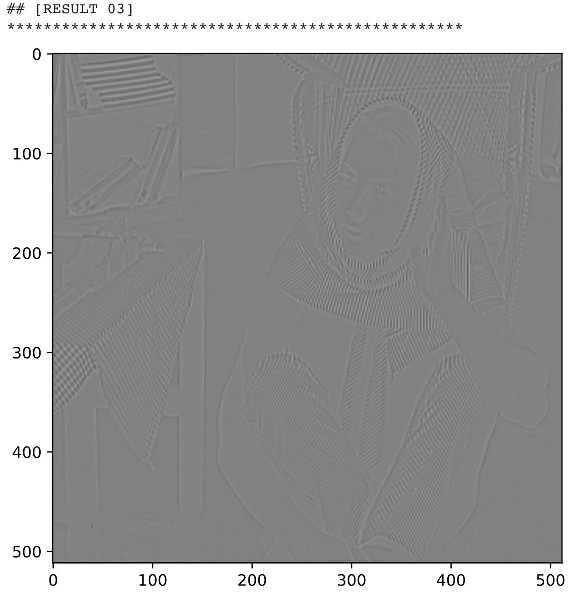
```
**************************************************
## [RESULT 01]
**************************************************
```



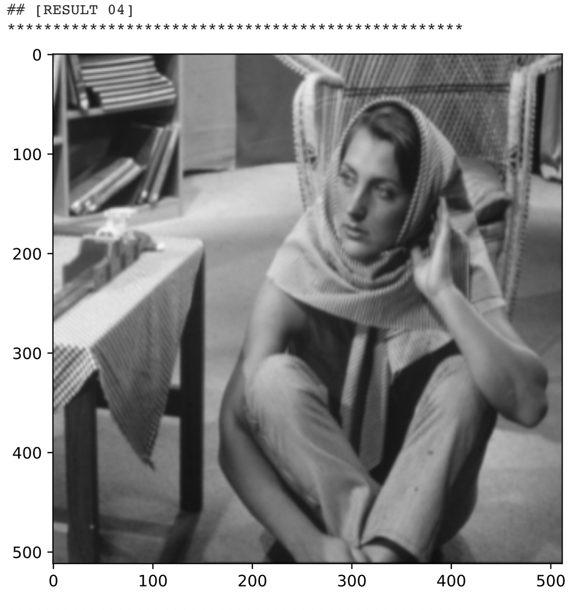

```
**************************************************
## [RESULT 02]
**************************************************
```



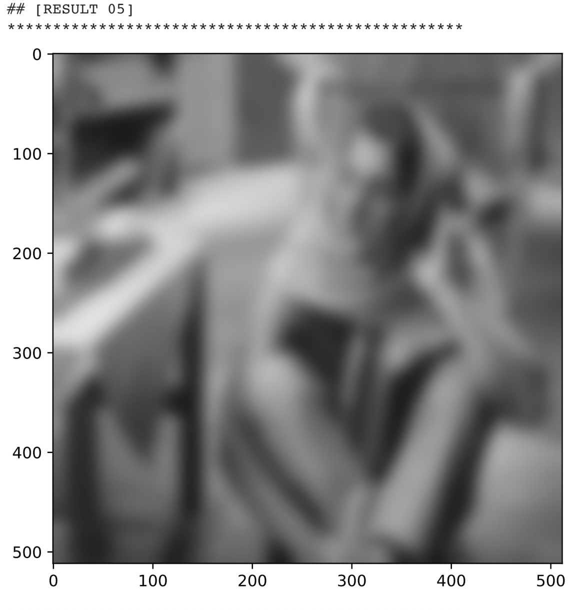

```
**************************************************
## [RESULT 03]
**************************************************
```



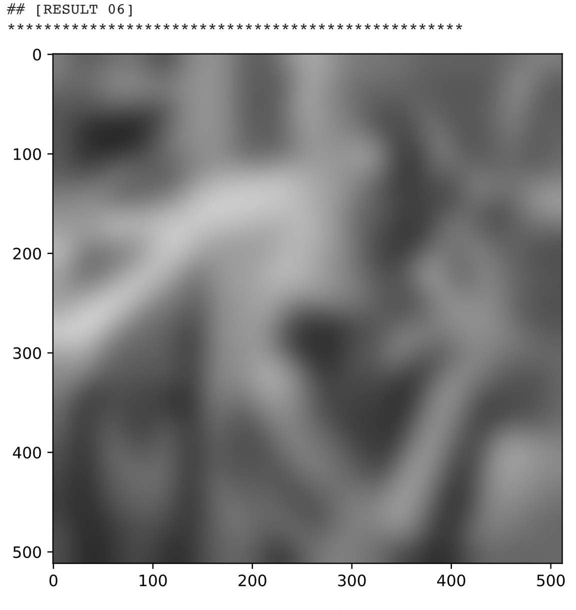

```
**************************************************
## [RESULT 04]
**************************************************
```




```
**************************************************
## [RESULT 05]
**************************************************
```




```
**************************************************
## [RESULT 06]
**************************************************
```




```
**************************************************
## [RESULT 07]
**************************************************
value1 =  0.38873832902073646
value2 =  0.3503440089528004
value3 =  -0.12053874833921746
value4 =  -0.05899756869858053
**************************************************
## [RESULT 08]
**************************************************
value1 =  0.11620040458063048
value2 =  0.157486029006043
value3 =  0.5094184630641501
value4 =  0.5927687759993218
**************************************************
## [RESULT 09]
**************************************************
value1 =  0.5888968793394928
value2 =  0.4001111466554692
value3 =  0.3666490919303763
value4 =  0.6016268928857371
**************************************************
## [RESULT 10]
**************************************************
value1 =  0.4900069282490228
value2 =  0.40789065395948143
value3 =  0.3499314993278954
value4 =  0.6313890906075725
```

In [ ]:

In [ ]: