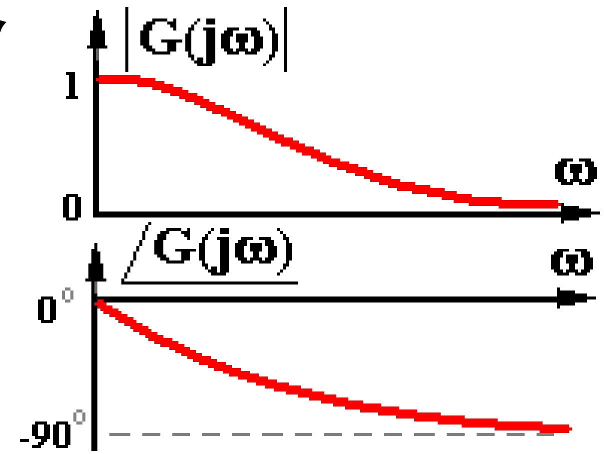
自动控制原理

(第 18 讲)

§5. 线性系统的频域分析与校正

课程回顾 (1)

§5.1.1 频率特性 G(jω) 的定义



$G(j\omega)$ 定义一： $G(j\omega)=|G(j\omega)|\angle G(j\omega)$

$$\begin{cases} |G(j\omega)|=\dfrac{|u_{cs}(t)|}{|u_r(t)|}=\dfrac{1}{\sqrt{1+\omega^2\mathrm{T}^2}} \\ \angle G(j\omega)=\angle u_{cs}(t)-\angle u_r(t)=-\arctan\,\omega\mathrm{T} \end{cases}$$

$G(j\omega)$ 定义二： $G(j\omega)=G(s)\big|_{s=j\omega}$

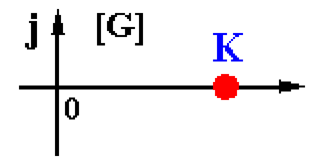
$G(j\omega)$ 定义三： $G(j\omega)=\dfrac{C(j\omega)}{R(j\omega)}$

课程回顾（2）

§5.2 幅相频率特性（Nyquist图）

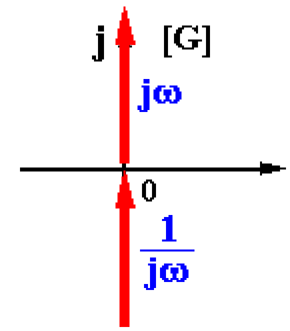
§5.2.1 典型环节的幅相特性曲线

(1) 比例环节 $G(s)=K \quad G(j\omega)=K \quad \begin{cases} |G|=K \\ \angle G=0^\circ \end{cases}$



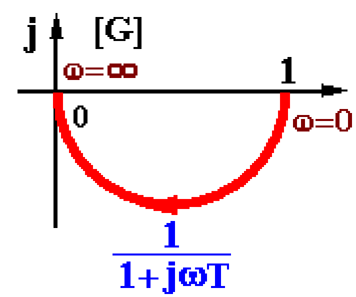
(2) 微分环节 $G(s)=s \quad G(j\omega)=j\omega \quad \begin{cases} |G|=\omega \\ \angle G=90^\circ \end{cases}$

(3) 积分环节 $G(s)=\dfrac{1}{s} \quad G(j\omega)=\dfrac{1}{j\omega} \quad \begin{cases} |G|=1/\omega \\ \angle G=-90^\circ \end{cases}$



(4) 惯性环节 $G(s)=\dfrac{1}{Ts+1}$

$$G(j\omega)=\frac{1}{1+j\omega T} \quad \begin{cases} |G|=\dfrac{1}{\sqrt{1-\omega^2T^2}} \\ \angle G=-\arctan\omega T \end{cases}$$

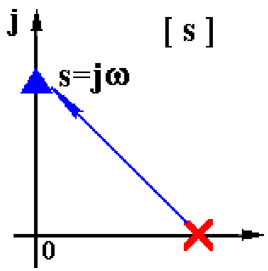


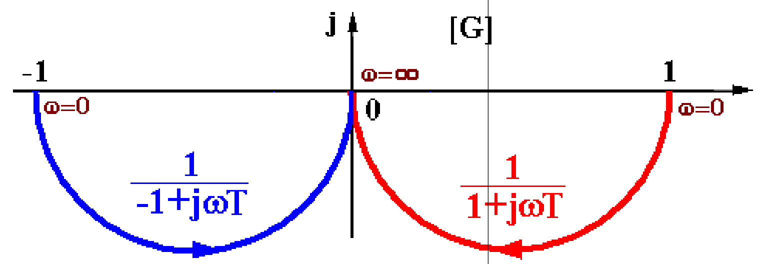
课程回顾（3）

不稳定惯性环节 $G(s)=\dfrac{1}{Ts-1}$

$$G(j\omega)=\frac{1}{-1+j\omega T}$$

$$\begin{cases} |G|=\dfrac{1}{\sqrt{1+\omega^2T^2}} \\ \angle G=-\arctan\dfrac{\omega T}{-1}=-180^\circ+\arctan\omega T \end{cases}$$

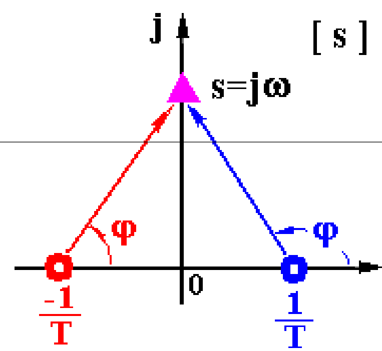


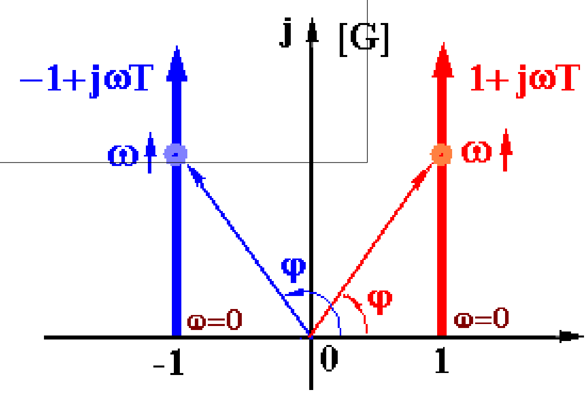


(5) 一阶复合微分 $G(s)=Ts\pm1$

$$G(j\omega)=\pm1+j\omega T$$

$$\begin{cases} |G|=\sqrt{1+\omega^2T^2} \\ \angle G=\begin{cases} \arctan\omega T \\ 180^\circ-\arctan\omega T \end{cases} \end{cases}$$





§5.2 幅相频率特性（Nyquist）(6)

§5.2.1 典型环节的幅相特性曲线

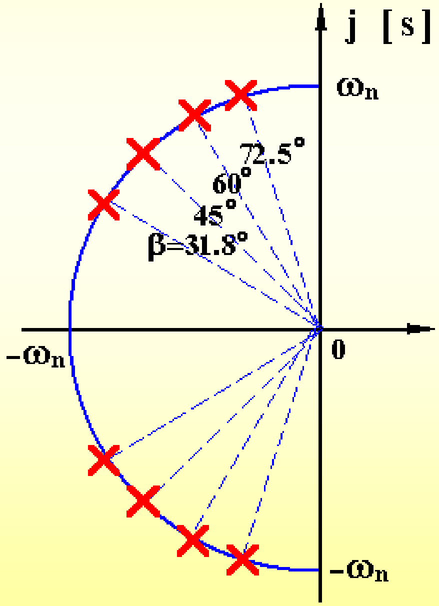
(6) 振荡环节 $G(s)=\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}=\dfrac{1}{(\dfrac{s}{\omega_n})^2+2\xi\dfrac{s}{\omega_n}+1}=\dfrac{\omega_n^2}{(s-\lambda_1)(s-\lambda_2)}$

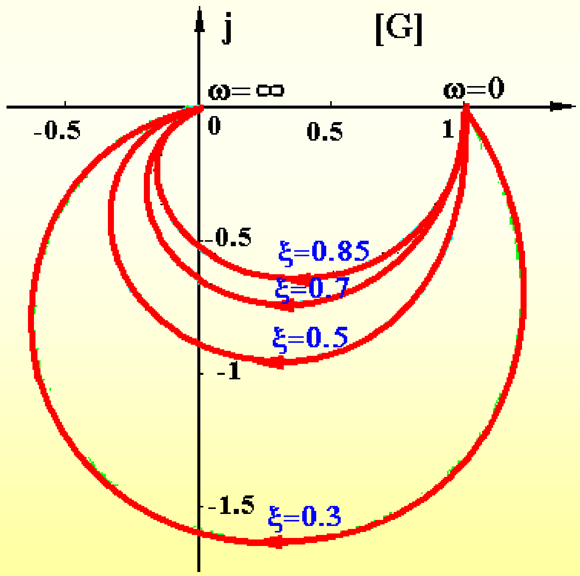
$$G(j\omega)=\frac{1}{1-\dfrac{\omega^2}{\omega_n^2}+j2\xi\dfrac{\omega}{\omega_n}}$$

$$\begin{cases} |G|=\dfrac{1}{\sqrt{[1-\dfrac{\omega^2}{\omega_n^2}]^2+[2\xi\dfrac{\omega}{\omega_n}]^2}} \\ \angle G=-\arctan\dfrac{2\xi\dfrac{\omega}{\omega_n}}{1-\dfrac{\omega^2}{\omega_n^2}} \end{cases}$$

$G(j0)=1\angle 0°$

$G(j\infty)=0\angle -180°$





§ 5.2 幅相频率特性（Nyquist）(7)

谐振频率ω_r 和谐振峰值M_r

$$|G| = 1\Big/\sqrt{[1-\frac{\omega^2}{\omega_n^2}]^2+[2\xi\frac{\omega}{\omega_n}]^2}$$

$$\frac{d}{d\omega}|G| = 0$$

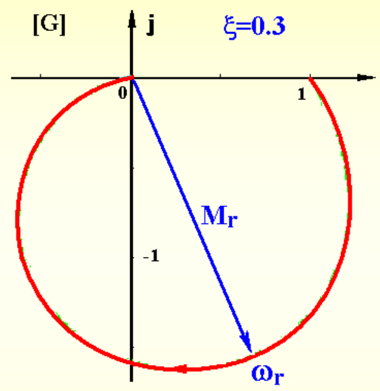
$$\frac{d}{d\omega}\left\{[1-\frac{\omega^2}{\omega_n^2}]^2+[2\xi\frac{\omega}{\omega_n}]^2\right\} = 0$$

$$2[1-\frac{\omega^2}{\omega_n^2}][-2(\frac{\omega}{\omega_n^2})] + 2\,[2\xi\frac{\omega}{\omega_n}](\frac{2\xi}{\omega_n}) = 0$$

$$\frac{4\omega}{\omega_n^2}[-1+\frac{\omega^2}{\omega_n^2}+2\xi^2] = 0$$

$$\frac{\omega^2}{\omega_n^2} = 1-2\xi^2$$

$$\begin{cases} \omega_r = \omega_n\sqrt{1-2\xi^2} \\ M_r = |G(j\omega_r)| = \dfrac{1}{2\xi\sqrt{1-\xi^2}} \end{cases}$$



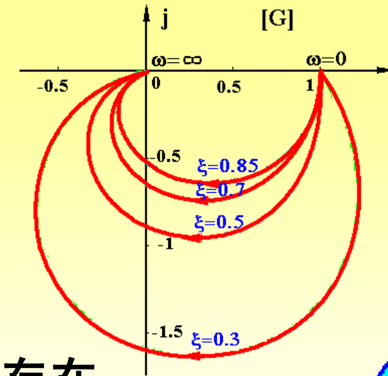
例4:当 $\xi = 0.3,\ \omega_n = 1$，时

$$\begin{cases} \omega_r = 1\times\sqrt{1-2\times 0.3^2} = 0.9055 \\ M_r = \dfrac{1}{2\times 0.3\sqrt{1-0.3^2}} = 1.832 \end{cases}$$

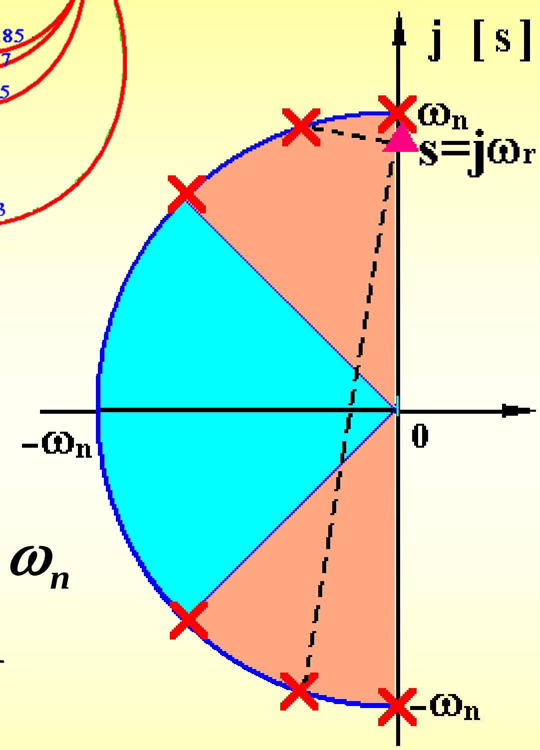
§ 5.2 幅相频率特性（Nyquist）(8)

谐振频率 $\omega_r = \omega_n \sqrt{1-2\xi^2}$

谐振峰值 $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$



$$\begin{cases}
\xi > 0.707 \ (\beta < 45^\circ) & 1-2\xi^2 < 0 & \omega_r,\ M_r \text{不存在} \\
\xi = 0.707 \ (\beta = 45^\circ) & 1-2\xi^2 = 0 & \begin{cases} \omega_r = 0 \\ M_r = 1 \end{cases} \\
0 < \xi < 0.707 \ (45^\circ < \beta < 90^\circ) & 1-2\xi^2 > 0 & \begin{cases} \omega_r = \omega_n\sqrt{1-2\xi^2} < \omega_n \\ M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}} > 1 \end{cases} \\
\xi = 0 \ (\beta = 90^\circ) & 1-2\xi^2 = 1 & \begin{cases} \omega_r = \omega_n \\ M_r = \infty \end{cases}
\end{cases}$$



§ 5.2 幅相频率特性 (Nyquist) (9)

$G(j\omega)$ ⇔ 幅相特性

例5 系统的幅相曲线如图所试，求传递函数。

$$\begin{cases} |G| = \dfrac{K}{\sqrt{[1-\dfrac{\omega^2}{\omega_n^2}]^2+[2\xi\dfrac{\omega}{\omega_n}]^2}} \\ \angle G = -\arctan\dfrac{2\xi\dfrac{\omega}{\omega_n}}{1-\dfrac{\omega^2}{\omega_n^2}} \end{cases}$$

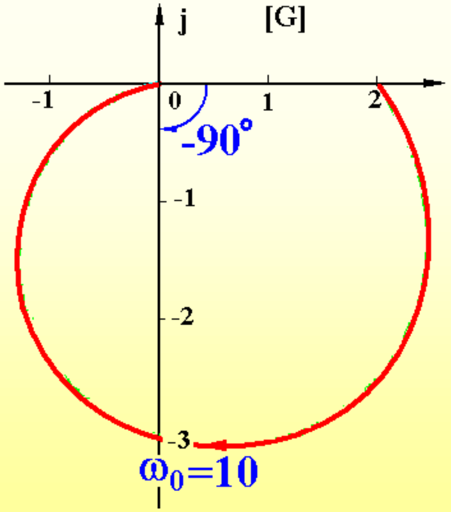
由曲线形状有 $G(s)=\dfrac{K}{\dfrac{s^2}{\omega_n^2}+2\xi\dfrac{s}{\omega_n}+1}$

由起点： $G(j0)=K\angle 0°$ $\quad K=2$

由φ(ω_0)： $\angle G(j\omega_0)=-90°$ $\quad \omega_0=\omega_n=10$

由|G(ω_0)|： $|G(\omega_0)|=3\overset{\omega_0=\omega_n}{=}\dfrac{K}{2\xi}=\dfrac{2}{2\xi}$ $\quad \xi=\dfrac{1}{3}$

$$G(s)=\frac{2\times10^2}{s^2+2\times\dfrac{1}{3}\times10s+10^2}=\frac{200}{s^2+6.67s+100}$$



§ 5.2 幅相频率特性 (Nyquist) (10)

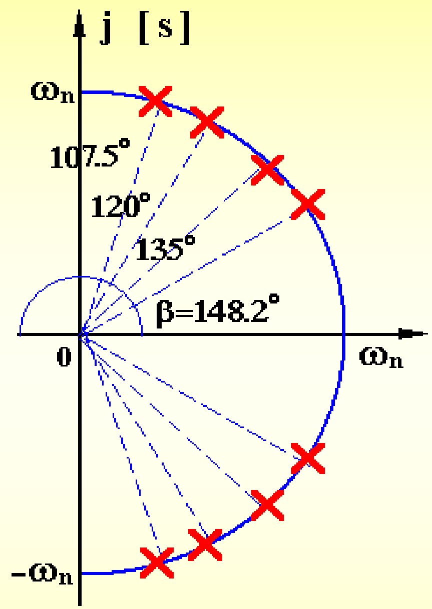
不稳定振荡环节 $G(s)=\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

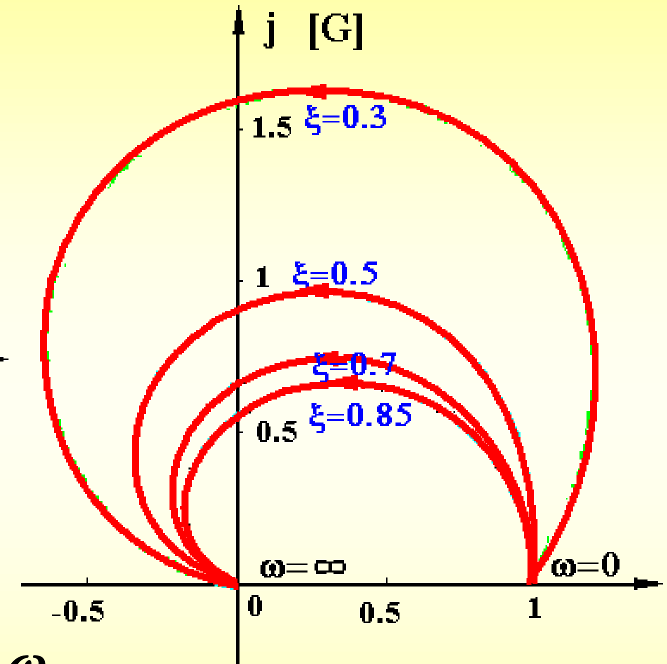
$$G(j0)=1\angle-360^\circ$$
$$G(j\infty)=0\angle-180^\circ$$

$$G(s)=\frac{1}{(\frac{s}{\omega_n})^2+2\xi\frac{s}{\omega_n}+1}$$

$$G(j\omega)=\frac{1}{1-\frac{\omega^2}{\omega_n^2}+j2\xi\frac{\omega}{\omega_n}}$$

$$\begin{cases} |G|=\dfrac{1}{\sqrt{[1-\frac{\omega^2}{\omega_n^2}]^2+[2\xi\frac{\omega}{\omega_n}]^2}} \\ \angle G=-\arctan\dfrac{-2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}=-360^\circ+\arctan\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}} \end{cases}$$



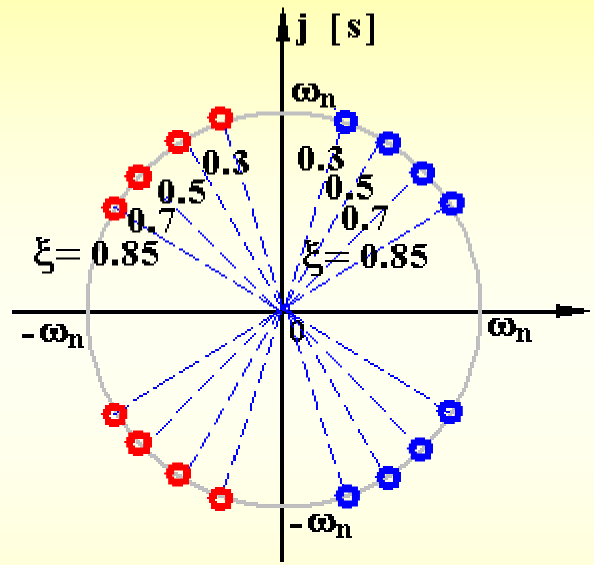


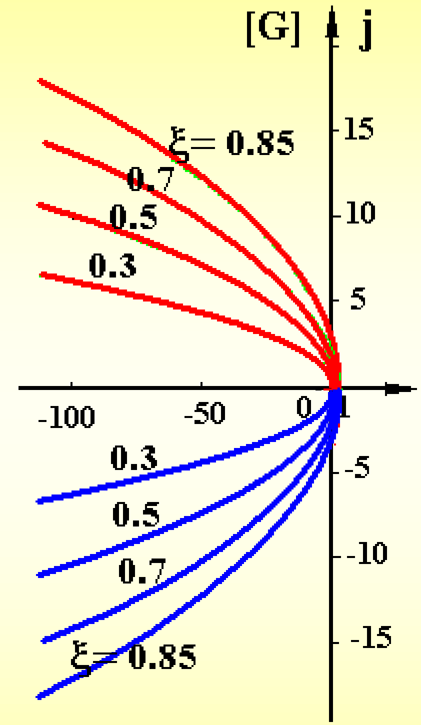
§ 5.2 幅相频率特性 (Nyquist) (11)

(7) 二阶复合微分 $G(s)=T^2s^2 \pm 2\xi Ts+1 \overset{T=1/\omega_n}{=} (\frac{s}{\omega_n})^2 \pm 2\xi\frac{s}{\omega_n}+1$

$$G(j\omega)=1-\frac{\omega^2}{\omega_n^2} \pm j2\xi\frac{\omega}{\omega_n}$$

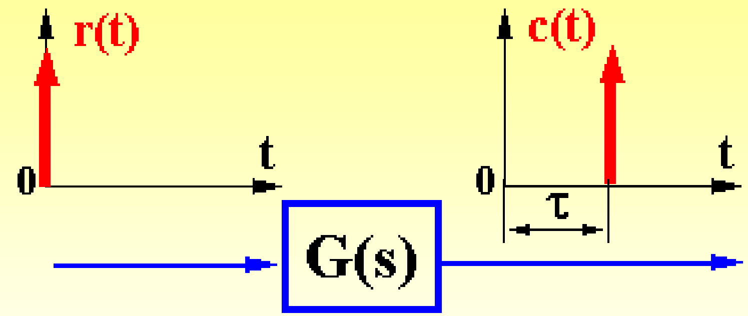
$$\begin{cases} |G| = \sqrt{[1-\frac{\omega^2}{\omega_n^2}]^2+[2\xi\frac{\omega}{\omega_n}]^2} \\ \angle G^+ = \arctan\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}} \\ \angle G^- = \arctan\dfrac{-2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}} = 360-\arctan\dfrac{2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}} \end{cases}$$





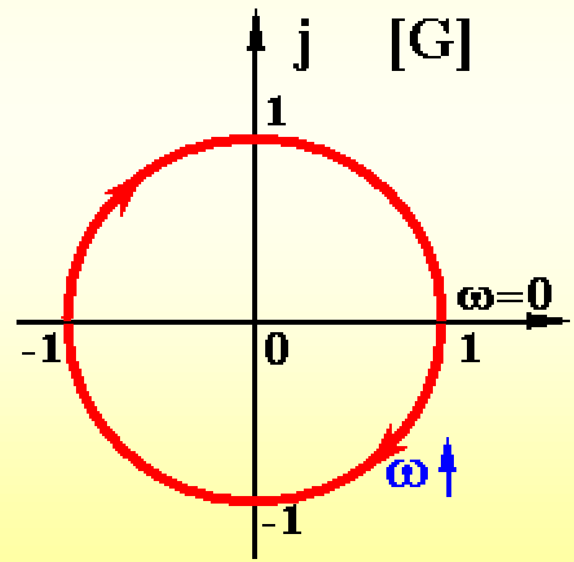
§ 5.2 幅相频率特性 (Nyquist) (12)

(8) 延迟环节 $G(s)=e^{-\tau s}$



$$\begin{cases} r(t)=\delta(t) \\ c(t)=k(t)=\delta(t-\tau) \end{cases}$$

$$\begin{cases} R(s)=1 \\ C(s)=e^{-\tau s} \end{cases} \qquad G(s)=\frac{C(s)}{R(s)}=e^{-\tau s}$$



$$G(j\omega)=e^{-j\omega\tau} \qquad \begin{cases} |G|=1 \\ \angle G=-\tau\omega \end{cases}$$

§ 5.2 幅相频率特性 (Nyquist) (13)

典型环节的幅相频率特性

(1) $G(j\omega)=K$

(2) $G(j\omega)=j\omega$

(3) $G(j\omega)=1/j\omega$

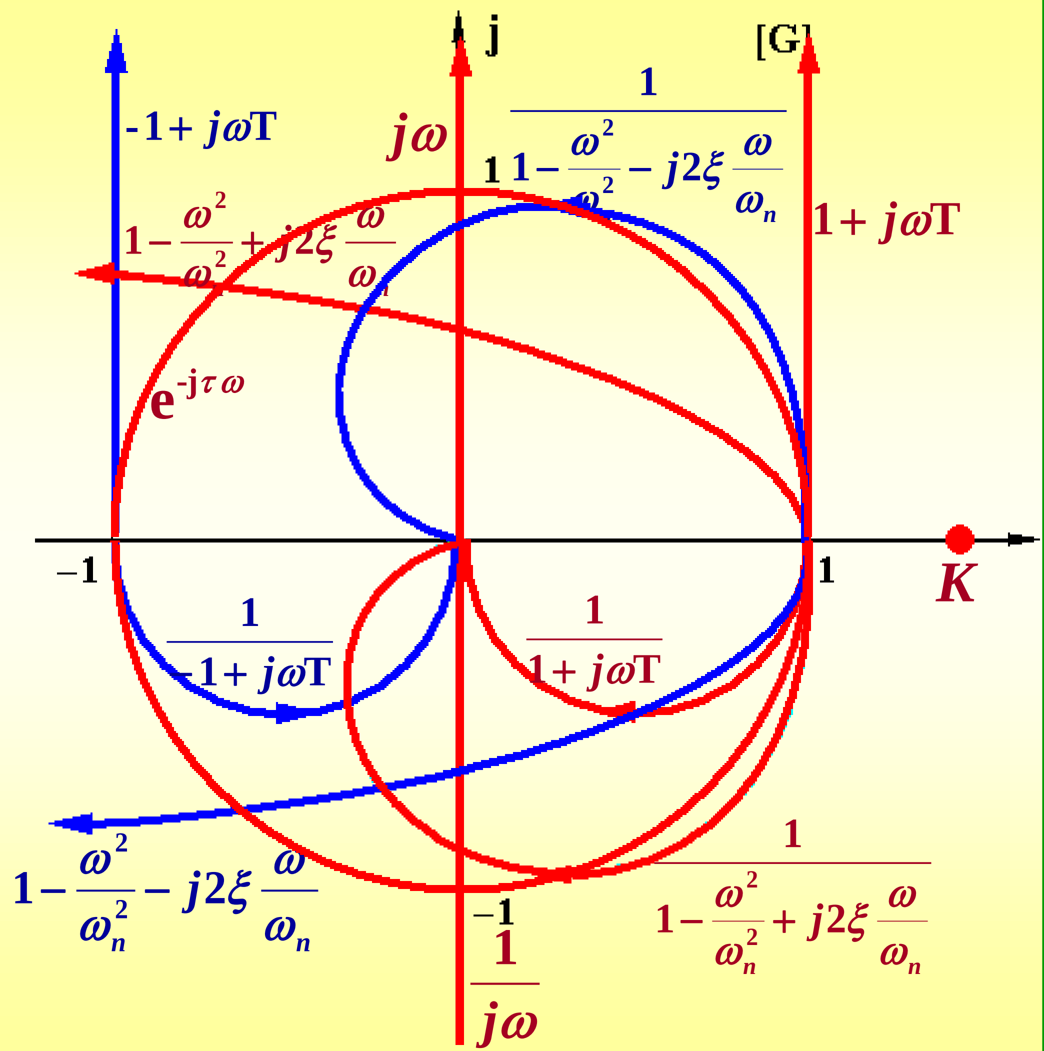
(4) $G(j\omega)=1/(\pm 1+j\omega \mathrm{T})$

(5) $G(j\omega)=\pm 1+j\omega \mathrm{T}$

(6) $G(j\omega)=1\left/\left(1-\frac{\omega^2}{\omega_n^2}\pm j2\xi\frac{\omega}{\omega_n}\right)\right.$

(7) $G(j\omega)=1-\frac{\omega^2}{\omega_n^2}\pm j2\xi\frac{\omega}{\omega_n}$

(8) $G(j\omega)=e^{-j\tau\omega}$



§5.3.2 开环系统的幅相频率特性 (1)

§5.2.2 开环幅相特性曲线的绘制

$$G(s)=\frac{K(\tau_1 s+1)\cdots(\tau_m s+1)}{s^v(T_1 s+1)\cdots(T_{n-v}s+1)}=\frac{K\prod_{i=1}^{m}(\tau_i s+1)}{s^v\prod_{j=1}^{n-v}(T_j s+1)}$$

$$\begin{cases} |G(\omega)|=\dfrac{K\prod_{i=1}^{m}|1+j\tau_i\omega|}{|\omega|^v\prod_{j=1}^{n-v}|1+jT_j\omega|}=\dfrac{K\prod_{i=1}^{m}\sqrt{1+\tau_i^2\omega^2}}{\omega^v\prod_{j=1}^{n-v}\sqrt{1+T_j^2\omega^2}} \\ \varphi(\omega)=\angle G(j\omega)=\sum_{i=1}^{m}\angle(1+j\tau_i\omega)-v\times 90^\circ-\sum_{j=1}^{n-v}\angle(1+jT_j\omega) \\ \qquad =\arctan\tau_1\omega+\cdots+\arctan\tau_m\omega \\ \qquad\quad -v\times 90^\circ-\arctan T_1\omega-\cdots-\arctan T_{n-v}\omega \end{cases}$$

§5.2.2 开环系统的幅相频率特性 (2)

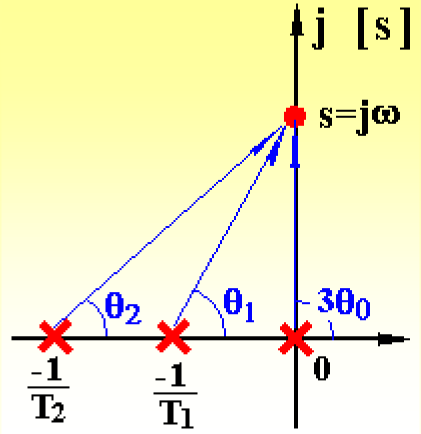
§5.2.2 开环系统幅相特性曲线的绘制

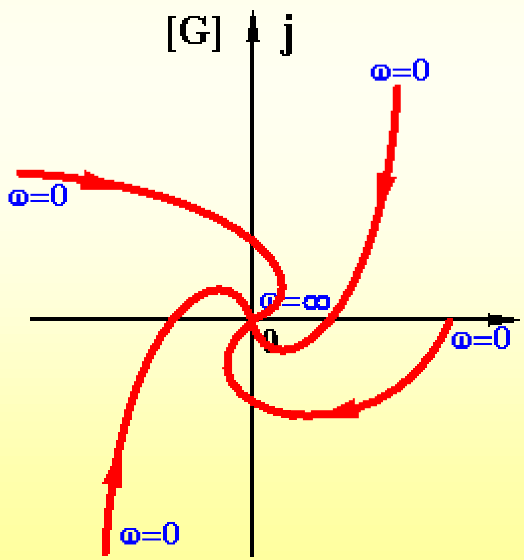
例6 $G(s)=\dfrac{K}{s^{v}(\mathrm{T}_1 s+1)(\mathrm{T}_2 s+1)}=\dfrac{K/(\mathrm{T}_1\mathrm{T}_2)}{s^{v}(s+1/T_1)(s+1/\mathrm{T}_2)}$

v	$G(j\omega)$	$G(j0)$	$G(j\infty)$
0	$\dfrac{K}{(1+j\omega\mathrm{T}_1)(1+j\omega\mathrm{T}_2)}$	$K\angle 0^\circ$	$0\angle -180^\circ$
I	$=\dfrac{\frac{K}{\mathrm{T}_1\mathrm{T}_2}}{\quad}$	$\infty\angle -90^\circ$	$0\angle -270^\circ$
II	$=\dfrac{\frac{K}{\mathrm{T}_1\mathrm{T}_2}}{\quad}$	$\infty\angle -180^\circ$	$0\angle -360^\circ$
III	$\dfrac{K}{\mathrm{T}_1\mathrm{T}_2}$	$\infty\angle -270^\circ$	$0\angle -450^\circ$

$$=\frac{\frac{K}{\mathrm{T}_1\mathrm{T}_2}}{(j\omega)^3(j\omega+\frac{1}{\mathrm{T}_1})(j\omega+\frac{1}{\mathrm{T}_2})}$$

起点 终点 $0\angle -90^\circ(n-m)$





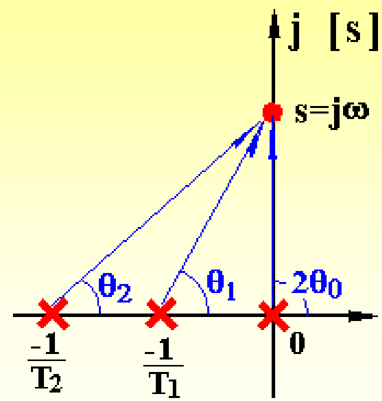
§5.2.2 开环系统的幅相频率特性（3）

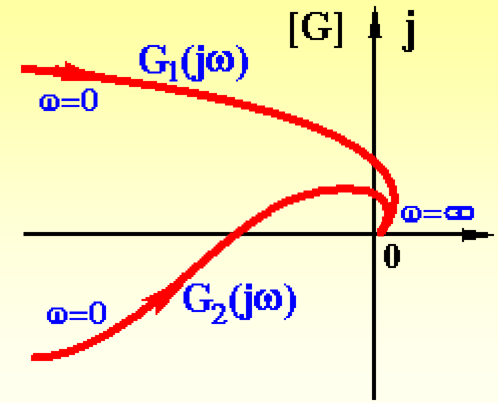
例7 $G_1(s)=\dfrac{K}{s^2(T_1s+1)(T_2s+1)}$

$$G_1(j0)=\infty\angle-180^\circ$$

$$\Big\downarrow\quad |G_1|\downarrow\quad \angle G_1\downarrow$$

$$G_1(j\infty)=0\angle-360^\circ$$





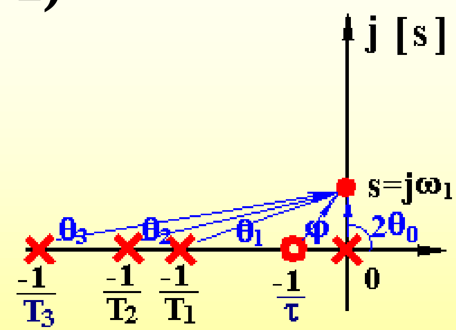
$$G_2(s)=\frac{K(\tau s+1)}{s^2(T_1s+1)(T_2s+1)(T_3s+1)}$$

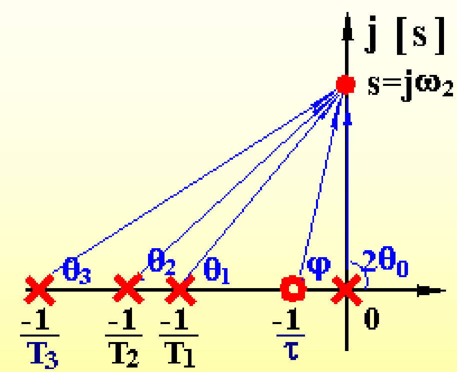
$$G_2(j0)=\infty\angle-180^\circ$$

$$|G_2|\downarrow\quad \angle G_2\uparrow\ (>-180^\circ)$$

$$|G_2|\downarrow\quad \angle G_2\downarrow\ (<-180^\circ)$$

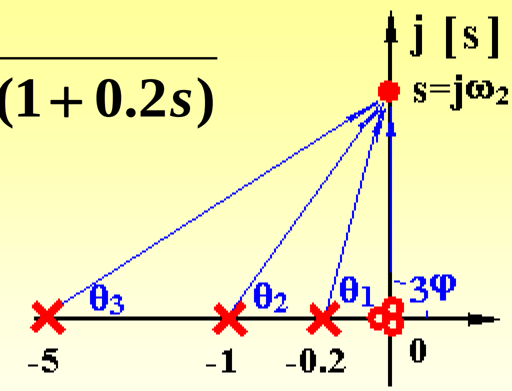
$$G_2(j\infty)=0\angle-360^\circ$$





§5.2.2 开环系统的幅相频率特性 (4)

例8 $$G(s)=\frac{s^3}{(s+0.2)(s+1)(s+5)}=\frac{s^3}{(1+5s)(1+s)(1+0.2s)}$$



$$G(j0)=0\angle+270^\circ$$

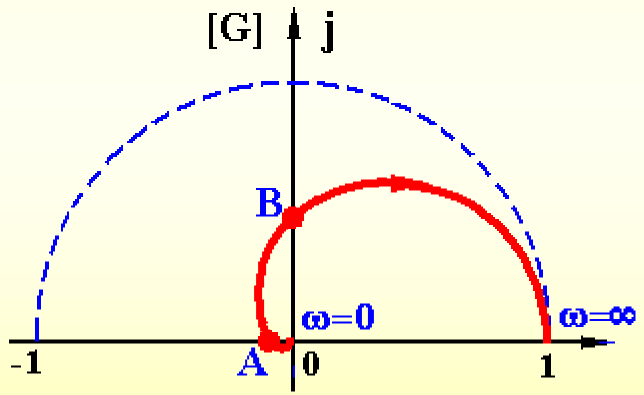
$$\downarrow \quad |G|\uparrow \quad \angle G\downarrow$$

$$G(j\infty)=1\angle 0^\circ$$

$$G(j\omega)=\frac{-j\omega^3(1-j5\omega)(1-j\omega)(1-j0.2\omega)}{(1+25\omega^2)(1+\omega^2)(1+0.04\omega^2)}$$

$$=\frac{-\omega^4(6.2-\omega^2)-j\omega^3(1-6.2\omega^2)}{(1+25\omega^2)(1+\omega^2)(1+0.04\omega^2)}$$

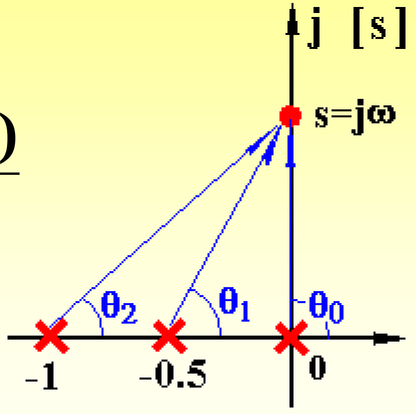
$$=X+jY$$



$$\text{A:}\begin{cases}\omega_A=1/\sqrt{6.2}=0.402\\ G(j\omega_A)=-0.0267+j0\end{cases}\qquad \text{B:}\begin{cases}\omega_B=\sqrt{6.2}=2.49\\ G(j\omega_B)=0+j0.412\end{cases}$$

§5.2.2 开环系统的幅相频率特性 (5)

例9 $G(s)=\dfrac{5}{s(s+1)(2s+1)}$ 画G(jω)曲线。

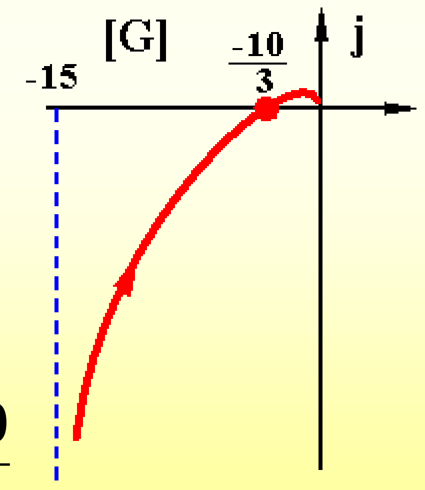


解 $G(j\omega)=\dfrac{5}{j\omega(1+j\omega)(1+j2\omega)}=\dfrac{-j5(1-j\omega)(1-j2\omega)}{\omega(1+\omega^2)(1+4\omega^2)}$

$$=\frac{-15}{(1+\omega^2)(1+4\omega^2)}-j\frac{5(1-2\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$G(j0)=\infty\angle-90^\circ$$

$$G(j\infty)=0\angle-270^\circ$$



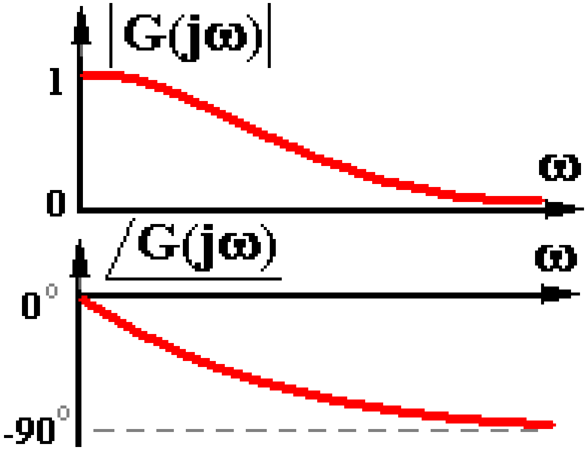
渐近线： $\mathrm{Re}[G(j0)]\Rightarrow-15$

与实轴交点： $\mathrm{Im}[G(j\omega)]=0 \quad\Rightarrow\quad \omega=1/\sqrt{2}=0.707$

$$\mathrm{Re}[G(j0.707)]=\frac{-15}{(1+0.5)(1+4\times0.5)}=-\frac{10}{3}$$

课程小结 (1)

1 频率特性 G(jω) 的定义



$G(j\omega)$ 定义一： $G(j\omega)=|G(j\omega)|\angle G(j\omega)$

$$\begin{cases} |G(j\omega)|=\dfrac{|u_{cs}(t)|}{|u_r(t)|}=\dfrac{1}{\sqrt{1+\omega^2T^2}} \\ \angle G(j\omega)=\angle u_{cs}(t)-\angle u_r(t)=-\arctan\omega T \end{cases}$$

$G(j\omega)$ 定义二： $G(j\omega)=G(s)\big|_{s=j\omega}$

$G(j\omega)$ 定义三： $G(j\omega)=\dfrac{C(j\omega)}{R(j\omega)}$

课程小结（2）

典型环节的幅相频率特性

(1) $G(j\omega)=K$

(2) $G(j\omega)=j\omega$

(3) $G(j\omega)=1/j\omega$

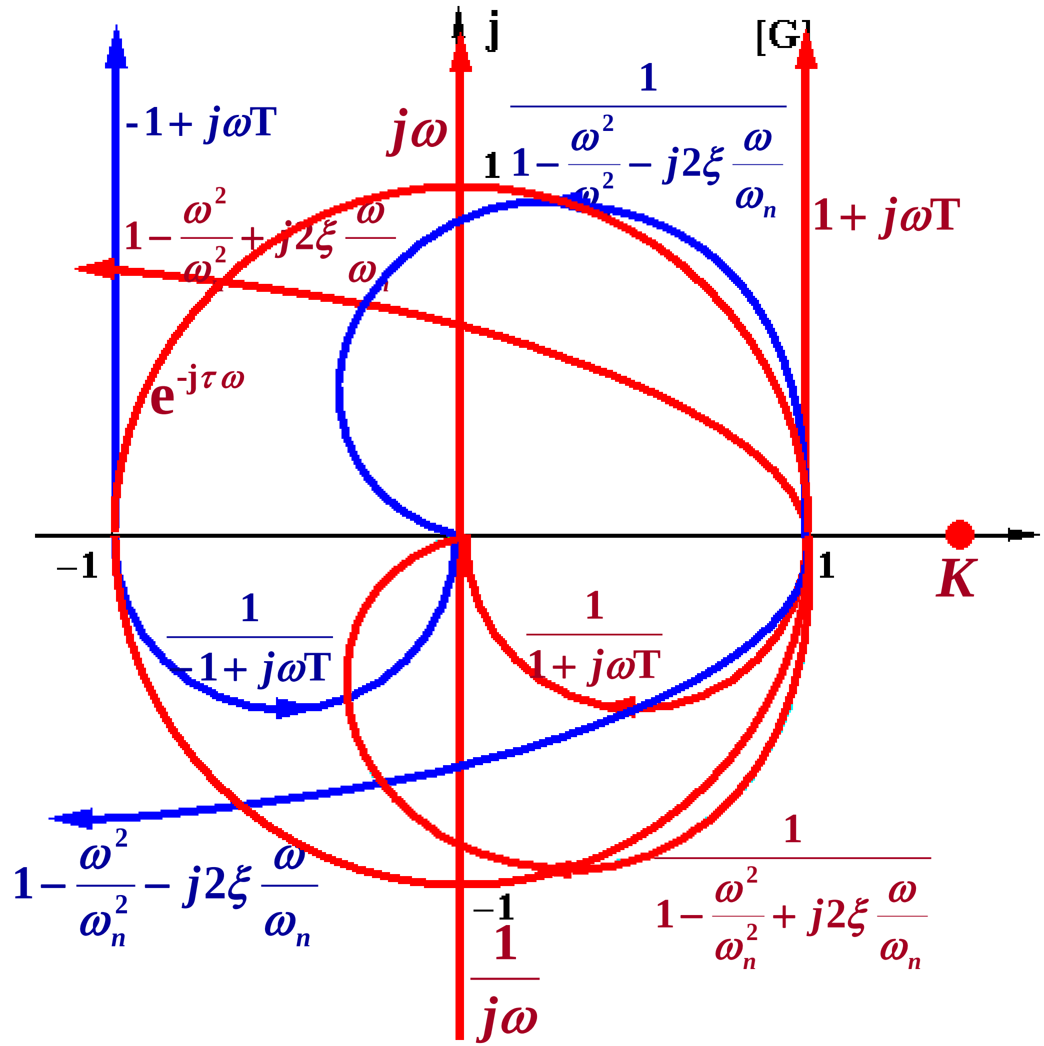
(4) $G(j\omega)=1/(\pm 1+j\omega T)$

(5) $G(j\omega)=\pm 1+j\omega T$

(6) $G(j\omega)=1\bigg/\left(1-\dfrac{\omega^2}{\omega_n^2}\pm j2\xi\dfrac{\omega}{\omega_n}\right)$

(7) $G(j\omega)=1-\dfrac{\omega^2}{\omega_n^2}\pm j2\xi\dfrac{\omega}{\omega_n}$

(8) $G(j\omega)=e^{-j\tau\omega}$



课程小结（3）

§5.2　幅相频率特性（Nyquist图）

§5.2.1　典型环节的幅相特性曲线

§5.2.2　系统的开环幅相特性曲线

(1) 确定幅相曲线的起点$G(j0)$ 和终点 $G(j\infty)$；

(2) 幅相曲线的中间段由s平面零、极点矢量随 $s=j\omega$的变化规律概略绘制；

(3) 必要时可以求出$G(j\omega)$ 与实/虚 轴的交点。