

哈尔滨理工大学

《概率论与数理统计》习题课四

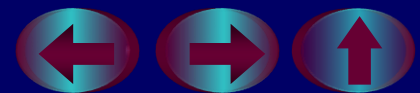


一、填空题

(1)已知 $X \sim N(-2, 0.4^2)$, 则 $E(X + 3)^2 = \underline{1.16}$

解:由均值的性质得

$$\begin{aligned} E(X + 3)^2 &= E(X^2 + 6X + 9) \\ &= E(X^2) + 6E(X) + 9 \\ &= D(X) + E(X)^2 + 6E(X) + 9 \\ &= 0.16 + 4 + 6(-2) + 9 = 1.16 \end{aligned}$$

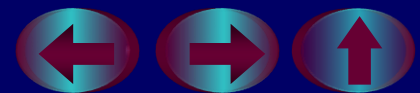


一、填空题

(2) 设 $X \sim N(10, 0.6)$, $Y \sim N(1, 2)$, 且 X 与 Y 相互独立, 则 $D(3X - Y) = \underline{7.4}$

解: 由方差的性质得

$$\begin{aligned} D(3X - Y) &= 9D(X) + D(Y) \\ &= 5.4 + 2 = 7.4 \end{aligned}$$



一、填空题

(3) 设 X 的概率密度为 $f(x) = Ae^{-x^2}$, 则 $D(X) = \underline{\frac{1}{2}}$

$$1 = \int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^{+\infty} Ae^{-x^2} dx$$

$$= A \int_{-\infty}^{+\infty} e^{-x^2} dx = A\sqrt{\pi}$$



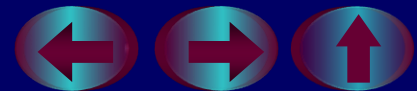
$$A = 1/\sqrt{\pi}$$

$$\int_{-\infty}^{+\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi}$$



$$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$E(X) = \int_{-\infty}^{+\infty} xf(x) dx = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} xe^{-x^2} dx = 0$$



$$D(X) = E(X^2) - E(X)^2 = \frac{1}{2}$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} x^2 e^{-x^2} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{+\infty} x^2 e^{-x^2} dx = -\frac{1}{\sqrt{\pi}} \int_0^{+\infty} x de^{-x^2}$$

$$= -\frac{1}{\sqrt{\pi}} \left[x e^{-x^2} \Big|_0^{+\infty} - \int_0^{+\infty} e^{-x^2} dx \right]$$

$$= \frac{1}{2}$$



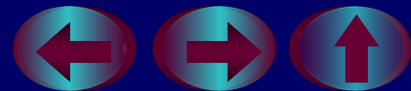
二、选择题

(1) 掷一颗均匀的骰子600次, 那么出现"一点"次数的均值为 B

(A)50 (B)100 (C)120 (D)150

解: 设 X "出现一点的次数", 则 $X \sim b(600, \frac{1}{6})$

$$E(X) = 600 \times \frac{1}{6} = 100$$



二、选择题

(1) 设 X_1, X_2, X_3 相互独立服从参数 $\lambda = 3$ 的泊松分布,

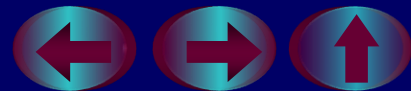
令 $Y = \frac{1}{3}(X_1 + X_2 + X_3)$, 则 $E(Y^2) = \underline{\quad}$

(A) 1 (B) 9 (C) 10 (D) 6

解: $E(Y) = E[\frac{1}{3}(X_1 + X_2 + X_3)] = \frac{1}{3} \times 3 \times \lambda = 3$

$$D(Y) = D[\frac{1}{3}(X_1 + X_2 + X_3)] = \frac{1}{9} \times 3 \times \lambda = 1$$

$$E(Y^2) = D(Y) + E(Y)^2 = 1 + 9 = 10$$



二、选择题

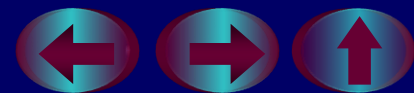
(2) 设 X_1, X_2, X_3 相互独立同服从参数 $\lambda = 3$ 的泊松分布, 令 $Y = \frac{1}{3}(X_1 + X_2 + X_3)$, 则 $E(Y^2) = \underline{\quad}$

(A) 1 (B) 9 (C) 10 (D) 6

$$\text{解: } E(Y) = E\left(\frac{1}{3}(X_1 + X_2 + X_3)\right) = \frac{1}{3} \sum_{k=1}^3 E(X_k) = 3$$

$$D(Y) = D\left(\frac{1}{3}(X_1 + X_2 + X_3)\right) = \frac{1}{9} \sum_{k=1}^3 D(X_k) = 1$$

$$E(Y^2) = D(Y) + E(Y)^2 = 10$$



二、选择题

(3)对于任意两个随机变量 X 和 Y ,若

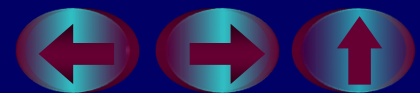
$E(XY) = E(X)E(Y)$,则B

(A) $D(XY) = D(X)D(Y)$ (B) $D(X + Y) = D(X) + D(Y)$

(C) X 和 Y 相互独立 (D) X 和 Y 不相互独立

解: $Cov(X, Y) = E(XY) - E(X)E(Y) = 0$

$$\begin{aligned} D(X + Y) &= D(X) + D(Y) + 2Cov(X, Y) \\ &= D(X) + D(Y) \end{aligned}$$



三、解答题

(1) 盒中有7个球, 其中4个白球, 3个黑球, 从中任取3个球, 求抽到白球数 X 的期望 $E(X)$ 和方差 $D(X)$.

解: X 的分布率为

X	0	1	2	3
p_k	$\frac{C_3^3}{C_7^3}$	$\frac{C_4^1 C_3^2}{C_7^3}$	$\frac{C_4^2 C_3^1}{C_7^3}$	$\frac{C_4^3}{C_7^3}$

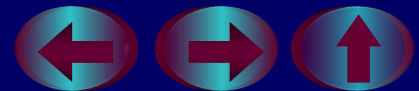
$$E(X) = 12/7$$

$$D(X) = E(X^2) - [E(X)]^2 = \frac{24}{49}$$



三、解答题

(2)有一物品的重量为1克,2克, \dots 10克是等概率的,为用天平称此物品的重量准备了三组砝码,甲组有五个砝码分别为1,2,2,5,10克,乙组为1,1,2,5,10克,丙组为1,2,3,4,10克,只准备用一组砝码放在天平的一个称盘里称重量,问哪一组砝码称重物时所用的砝码数平均最少?



解: X "甲组砝码称重物时所用的砝码数"

Y "乙组砝码称重物时所用的砝码数"

Z "丙组砝码称重物时所用的砝码数"

物品的重量是一个随机变量 U ,

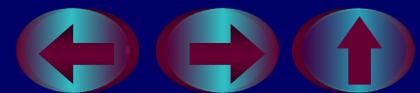
$$U = k \quad (k = 1, 2, \dots, 10),$$

$$P\{U = k\} = 1/10 \quad (k = 1, 2, \dots, 10).$$

$$\{X = 1\} = \{U = 1\} + \{U = 2\} + \{U = 5\} + \{U = 10\}$$

$$\{X = 2\} = \{U = 3\} + \{U = 4\} + \{U = 6\} + \{U = 7\}$$

$$\{X = 3\} = \{U = 8\} + \{U = 9\}$$



X	1	2	3
p_k	$\frac{4}{10}$	$\frac{4}{10}$	$\frac{2}{10}$

Y	1	2	3	4
p_k	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

Z	1	2	3
p_k	$\frac{5}{10}$	$\frac{3}{10}$	$\frac{2}{10}$

$$E(X) = \frac{18}{10}, \quad E(Y) = \frac{20}{10}, \quad E(Z) = \frac{17}{10}$$

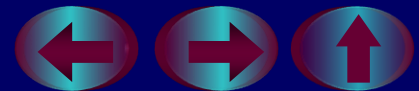


三、解答题

(3)公共汽车起点站于每时的10分,30分,55分发车,该乘客不知发车时间,在每小时内的任意时刻随机到达车站,求乘客候车时间的数学期望.

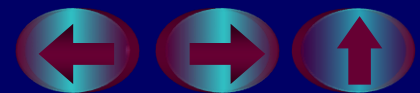
解: X "乘客到站时间" Y "乘客候车时间"

$$X \sim U[0,60] \quad f(x) = \begin{cases} 1/60 & 0 \leq x \leq 60 \\ 0 & \text{其它} \end{cases}$$



$$Y = \begin{cases} 10 - X, & 0 \leq X \leq 10 \\ 30 - X, & 10 < X \leq 30 \\ 55 - X, & 30 < X \leq 55 \\ 70 - X, & 55 < X \leq 60 \end{cases} \quad \square g(X)$$

$$\begin{aligned} E(Y) &= \int_{-\infty}^{+\infty} g(x) f(x) dx \\ &= \frac{1}{60} \left[\int_0^{10} (10 - x) dx + \int_{10}^{30} (30 - x) dx + \right. \\ &\quad \left. \int_{30}^{55} (55 - x) dx + \int_{55}^{60} (70 - x) dx \right] \\ &= 10 \text{分} 25 \text{秒} \end{aligned}$$



三、解答题

(4) 设排球队A和B比赛, 若有一队胜4场, 则比赛宣告结束, 假定A, B在每场比赛中获胜的概率均为 $\frac{1}{2}$, 试求平均需比赛几场才能分出胜负?

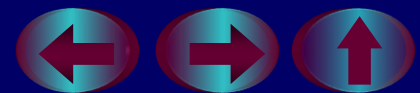
解: 设X"需要比赛的场数"

$$X = 4, 5, 6, 7$$

$$\{X = 5\} = \{A \text{ 胜 } 4 \text{ 场}\} + \{B \text{ 胜 } 4 \text{ 场}\}$$

$$\{A \text{ 胜 } 4 \text{ 场}\}$$

$$= \{A \text{ 在前 } 4 \text{ 场中胜 } 3 \text{ 场, } B \text{ 胜 } 1 \text{ 场}\} \cap \{\text{第 } 5 \text{ 场 } A \text{ 必胜}\}$$



$$P(X = 4) = 2 \times \left(\frac{1}{2}\right)^4 = \frac{1}{8}$$

$$P(X = 5) = 2 \times C_4^3 \left(\frac{1}{2}\right)^3 \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(X = 6) = 2 \times C_5^3 \left(\frac{1}{2}\right)^3 \frac{1}{2} \times \frac{1}{2} = \frac{5}{16}$$

$$P(X = 7) = 2 \times C_6^3 \left(\frac{1}{2}\right)^3 \frac{1}{2} \times \frac{1}{2} = \frac{5}{16}$$

$$E(X) = 4 \times \frac{1}{8} + 5 \times \frac{1}{4} + 6 \times \frac{5}{16} + 7 \times \frac{5}{16} \approx 5.8$$



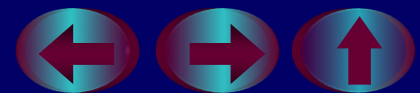
三、解答题

(5) 一袋中有 n 张卡片, 分别记有号码 $1, 2, \dots, n$, 从中有放回地抽出 k 张来, 以 X 表示所得号码之和, 求 $E(X)$ 和 $D(X)$.

解: X_i "抽取第 i 张卡片的号码" $i = 1, 2, \dots, k$

$X_i (i = 1, 2, \dots, k)$ 相互独立, 令 $X = X_1 + X_2 + \dots + X_k$

X_i	1	2	3	...	n
p_k	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$...	$\frac{1}{n}$



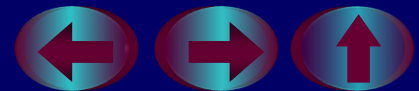
$$E(X_i) = \frac{1}{n}(1 + 2 + \cdots + n) = \frac{n+1}{2}$$

$$E(X) = \sum_{i=1}^k E(X_i) = k \cdot \frac{n+1}{2}$$

$$D(X_i) = E(X_i^2) - E(X_i)^2 = \frac{1}{n} \sum_{i=1}^n i^2 - \frac{(n+1)^2}{4}$$

$$= \frac{1}{n} \frac{n(2n+1)(n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n^2-1}{12}$$

$$D(X) = \sum_{i=1}^k D(X_i) = \frac{k(n^2-1)}{12}$$



四、证明题

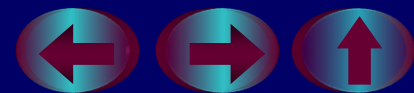
设随机变量 X 的概率密度为 $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < +\infty$,

(1)证明 $E(X) = 0, D(X) = 2$

(2)证明 X 与 $|X|$ 不相互独立

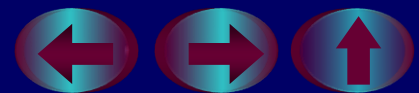
(3)证明 X 与 $|X|$ 不相关.

证 ①
$$E(X) = \int_{-\infty}^{+\infty} xf(x)dx$$
$$= \int_{-\infty}^{+\infty} x \frac{1}{2} e^{-|x|} dx = 0$$



$$\begin{aligned}
 E(X^2) &= \int_{-\infty}^{+\infty} x^2 f(x) dx \\
 &= \frac{1}{2} \int_{-\infty}^{+\infty} x^2 e^{-|x|} dx = \int_0^{+\infty} x^2 e^{-x} dx \\
 &= \left[-x^2 e^{-x} \right]_0^{+\infty} + 2 \int_0^{+\infty} x e^{-x} dx \\
 &= 2 \int_0^{+\infty} x e^{-x} dx \\
 &= 2 \left[-x e^{-x} \right]_0^{+\infty} + 2 \int_0^{+\infty} e^{-x} dx \\
 &= 2
 \end{aligned}$$

故 $D(X) = E(X^2) - [E(X)]^2 = 2$



证明 (2) X 与 $|X|$ 不相互独立, 因为任给 $x > 0$

$$P(X \leq x, |X| \leq x) = P(|X| \leq x) \\ \neq P(X \leq x)P(|X| \leq x)$$

随机变量函数的数学期望

奇函数

$$(3) \quad E(X|X|) = \int_{-\infty}^{+\infty} x|x| \frac{1}{2} e^{-|x|} dx = 0$$

$$Cov(X, |X|) = E(X|X|) - E(X)E(|X|) = 0$$

$$\rho_{XY} = \frac{Cov(X, Y)}{\sqrt{D(X)} \cdot \sqrt{D(Y)}} = 0$$

