课程回顾

奈奎斯特稳定判据 $Z = P - 2N$

Z: 在右半s平面中的闭环极点个数

P: 在右半s平面中的开环极点个数

N: 开环幅相曲线GH(jω)包围[G]平面(-1, j0)点的圈数

$$Z \begin{cases} > 0 & \text{闭环系统不稳定} \\ = 0 & \text{闭环系统稳定} \\ < 0 & \text{有误！} \end{cases}$$

注意问题

1. 当[s]平面虚轴上有开环极点时，奈氏路径要从其右边绕出半径为无穷小的圆弧；[G]平面对应要补充大圆弧
2. N 的最小单位为二分之一

§ 5.4.3 对数稳定判据 (4)

例2 $$G(s)=\frac{1000}{s(s^2+25)(0.2s+1)}=\frac{40}{s[(\frac{s}{5})^2+1](\frac{s}{5}+1)}$$

$$G(j0)=\infty\angle 0^\circ$$

$$G(j0^+)=\infty\angle -90^\circ$$

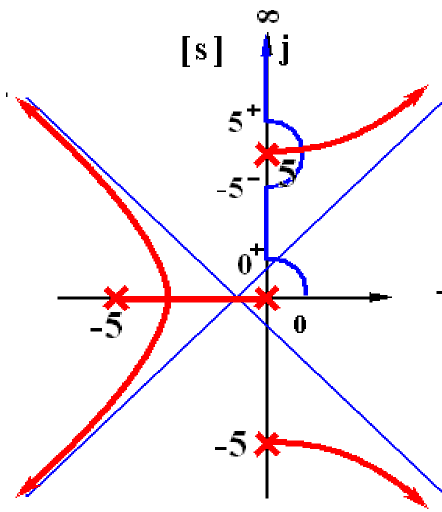
$$G(j5^-)=\infty\angle -135^\circ$$

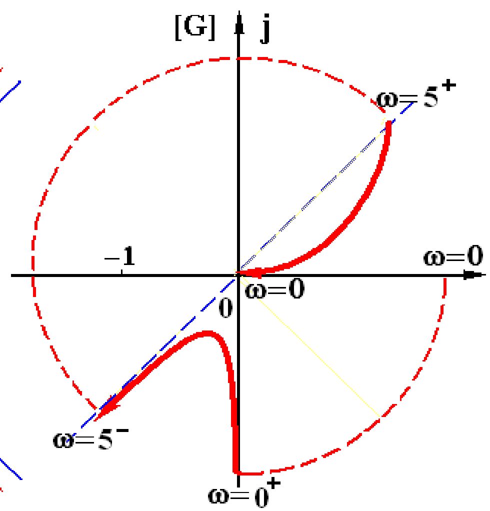
$$G(j5^+)=\infty\angle -315^\circ$$

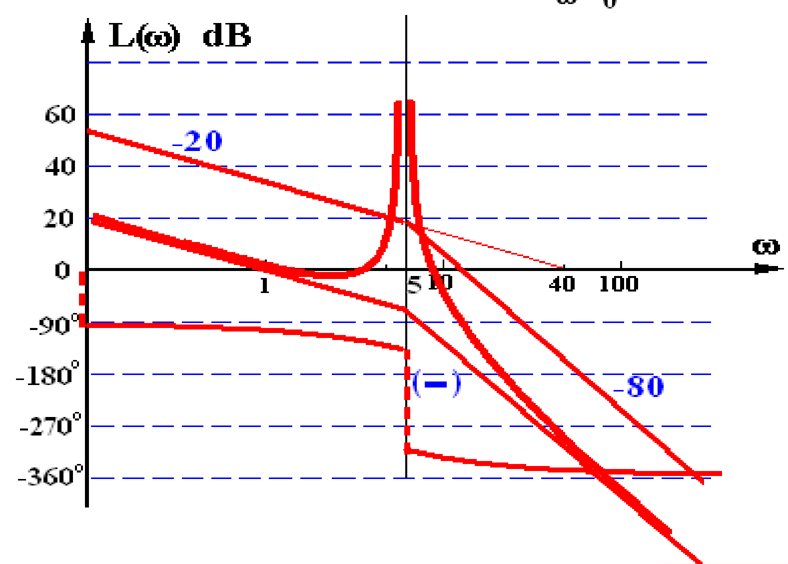
$$G(j\infty)=0\angle -360^\circ$$

$$N=N_+-N_-=0-1=-1$$

$$Z=P-2N=0-2\times(-1)=2$$

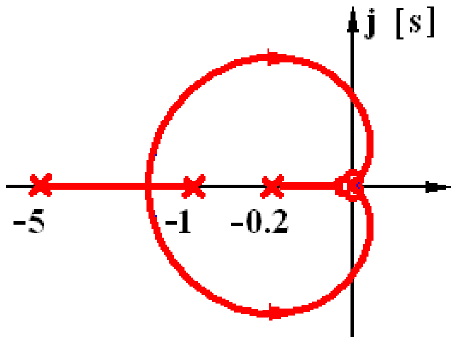


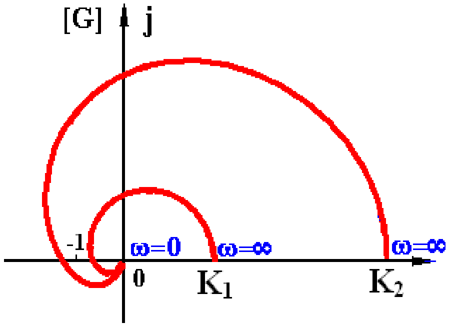




§5.4.3 对数稳定判据 (5)

例3 $G(s)=\dfrac{Ks^3}{(0.2s+1)(s+1)(5s+1)}$





$$G(j0)=0\angle 0^\circ$$

$$G(j0^+)=0\angle 270^\circ$$

$$G(j\infty)=K\angle 0^\circ$$

K_1

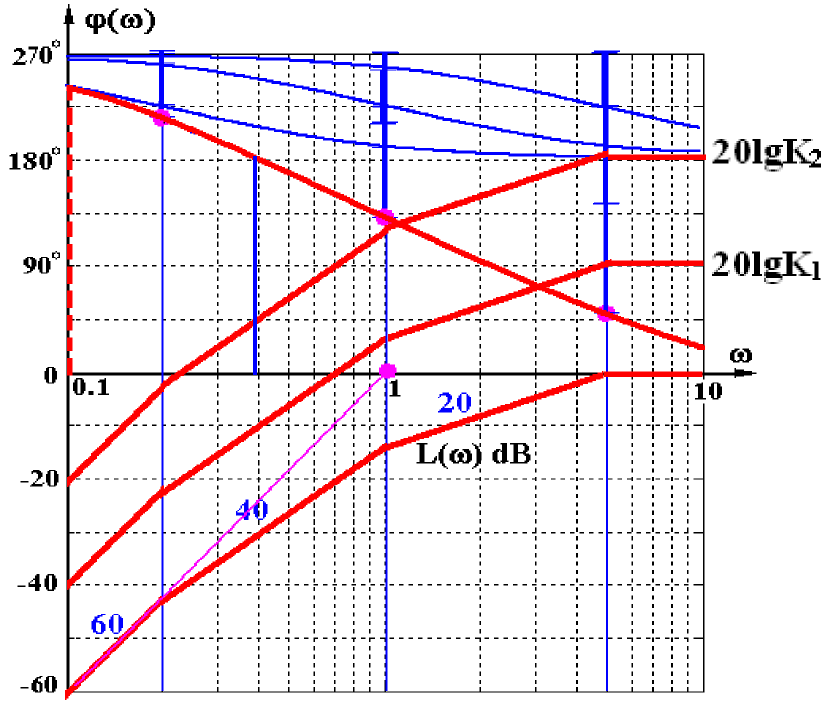
$$N=N_+-N_-=0-0=0$$

$$Z=P-2N=0-2\times 0=0$$

K_2

$$N=N_+-N_-=0-1=-1$$

$$Z=P-2N=0-2\times(-1)=2$$



(第 22讲)

§5. 线性系统的频域分析与校正

§5.1　频率特性的基本概念
§5.2　幅相频率特性（Nyquist图）
§5.3　对数频率特性（Bode图）
§5.4　频域稳定判据
§5.5　稳定裕度
§5.6　利用开环频率特性分析系统的性能
§5.7　闭环频率特性曲线的绘制
§5.8　利用闭环频率特性分析系统的性能
§5.9　频率法串联校正

(第 22 讲)

§5.5 稳定裕度

§5.5.1 稳定裕度的定义

§5.5.2 稳定裕度的计算

§ 5.5 稳定裕度 (1)

系统动态性能

⇩

稳定程度	稳定边界	稳定程度
时域（t）	虚轴	阻尼比 ξ
频域（ω）	(−1, j0)	到(−1, j0)的距离
		稳定裕度
		(开环频率指标)

§ 5.5 稳定裕度 (2)

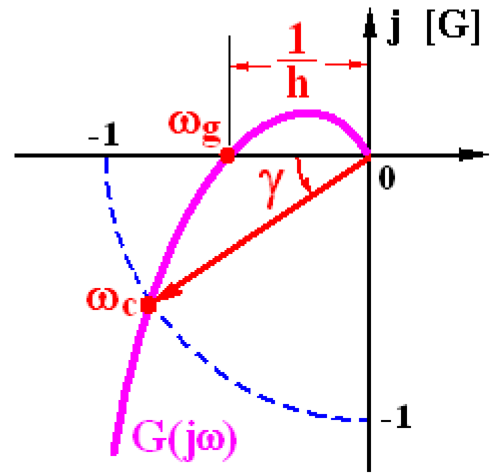
§ 5.5.1 稳定裕度的定义

截止频率 ω_c $\quad |G(j\omega_c)| = 1$

相角裕度 γ $\quad \gamma = 180° + \angle G(j\omega_c)$

相角交界频率 ω_g $\quad \angle G(j\omega_g) = -180°$

幅值裕度 h $\quad h = \dfrac{1}{|G(j\omega_g)|}$

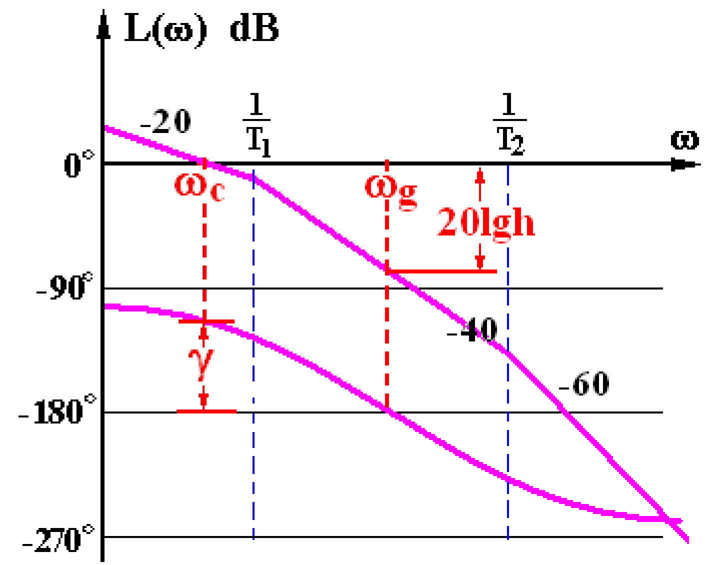


γ, h 的几何意义

γ, h 的物理意义

$\left\{\begin{matrix} \gamma \\ h \end{matrix}\right.$ 系统在 $\left\{\begin{matrix} \text{相角} \\ \text{幅值} \end{matrix}\right.$ 方面的稳定储备量

一般要求 $\left\{\begin{matrix} \gamma > 40° \\ h > 2 \end{matrix}\right.$



§5.5 稳定裕度 (3)

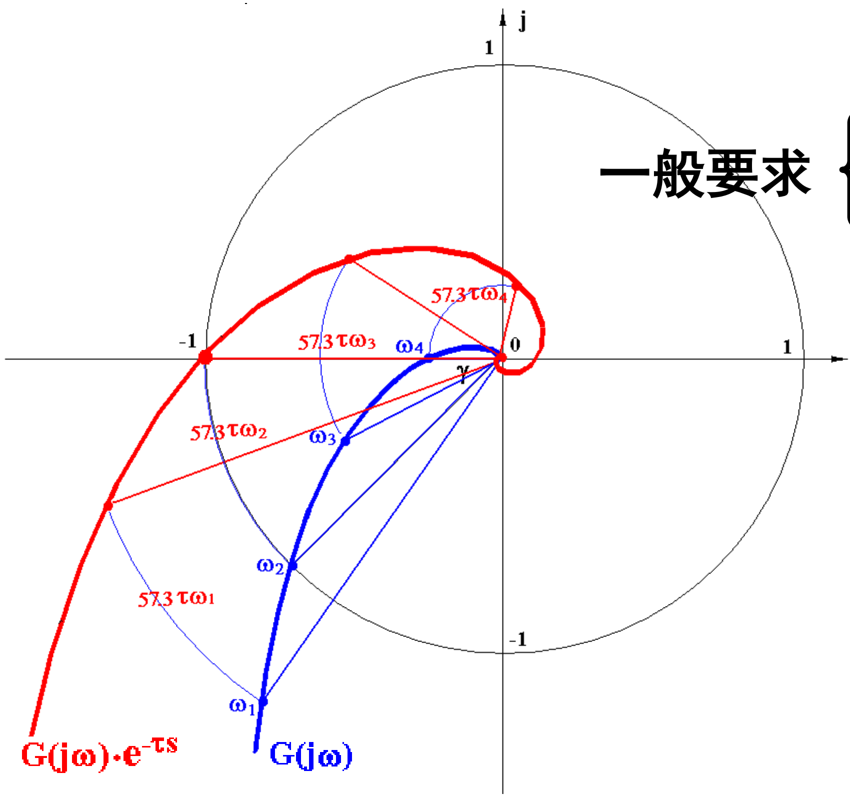
相角裕度的物理意义：

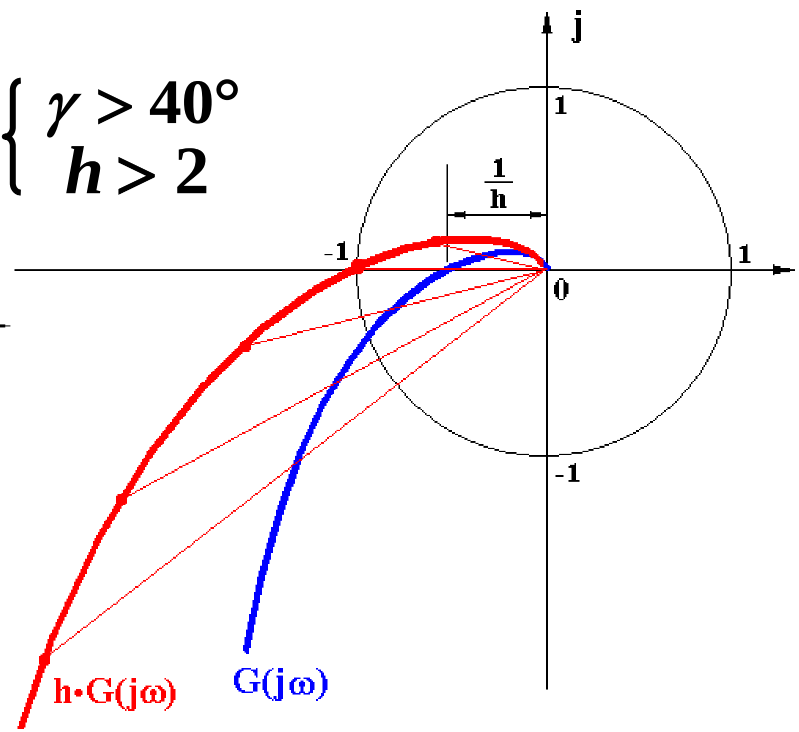
系统在相角上距离临界稳定还具有的储备量

幅值裕度的物理意义：

系统在增益上距离临界稳定还具有的储备量

一般要求 $\begin{cases} \gamma > 40^\circ \\ h > 2 \end{cases}$



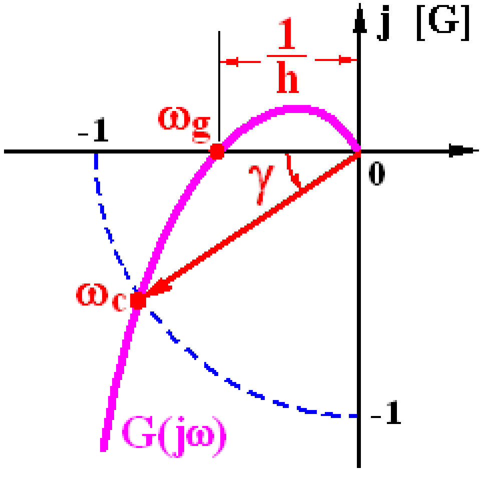


§ 5.5 稳定裕度 (3)

§ 5.5.2 稳定裕度的计算

例4 $G(s)=\dfrac{5}{s(\frac{s}{2}+1)(\frac{s}{10}+1)}=\dfrac{100}{s(s+2)(s+10)}$，求 γ，h。

解法I：由幅相曲线求 γ，h。



(1) 令 $\left|G(j\omega_c)\right|=1=\dfrac{100}{\omega_c\sqrt{\omega_c^2+2^2}\sqrt{\omega_c^2+10^2}}$

$$\omega_c^2[\omega_c^4+104\omega_c^2+400]=10000$$

试根得 $\omega_c=2.9$

$$\gamma=180°+\angle G(j\omega_c)=180°+\varphi(2.9)$$
$$=180°-90°-\arctan\frac{2.9}{2}-\arctan\frac{2.9}{10}$$
$$=90°-55.4°-16.1°=18.5°$$

$$G(s)=\frac{5}{s(\frac{s}{2}+1)(\frac{s}{10}+1)}=\frac{100}{s(s+2)(s+10)}$$

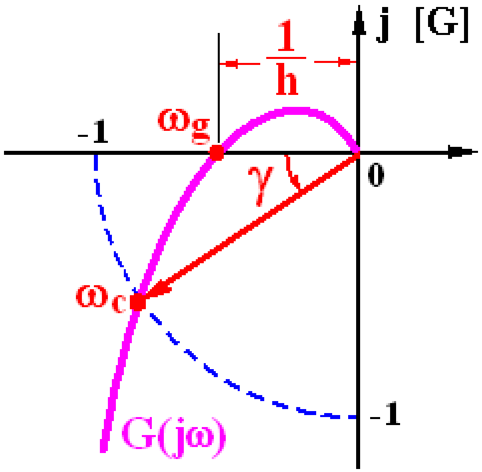
§ 5.5 稳定裕度 (4)

(2. 1) 令 $\varphi(\omega_g)=-180^\circ$

$$=-90^\circ-\arctan\frac{\omega_g}{2}-\arctan\frac{\omega_g}{10}$$

可得 $\arctan\frac{\omega_g}{2}+\arctan\frac{\omega_g}{10}=90^\circ$

$$\frac{\frac{\omega_g}{2}+\frac{\omega_g}{10}}{1-\frac{\omega_g^2}{20}}=\tan 90^\circ \Rightarrow \begin{aligned}\omega_g^2&=20\\ \omega_g&=4.47\end{aligned}$$



$$h=\frac{1}{|G(j\omega_g)|}=\frac{\omega_g\sqrt{\omega_g^2+2^2}\sqrt{\omega_g^2+10^2}}{100}\overset{\omega_g=4.47}{=}2.4 \quad (7.6\ \text{dB})$$

$$G(s)=\frac{5}{s(\frac{s}{2}+1)(\frac{s}{10}+1)}=\frac{100}{s(s+2)(s+10)}$$

§ 5.5 稳定裕度 (5)

(2. 2) 将G(jω)分解为实部、虚部形式

$$G(j\omega)=\frac{100}{j\omega(2+j\omega)(10+j\omega)}$$

$$=\frac{-1200\omega-j100(20-\omega^2)}{\omega(4+\omega^2)(100+\omega^2)}=G_X+jG_Y$$

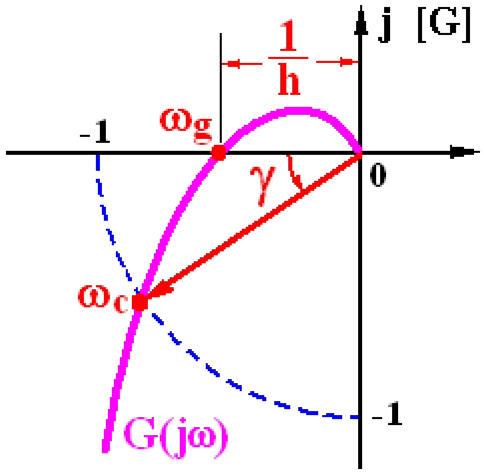
令 $\mathrm{Im}[G(j\omega)]=G_Y=0$

得 $\omega_g=\sqrt{20}=4.47$

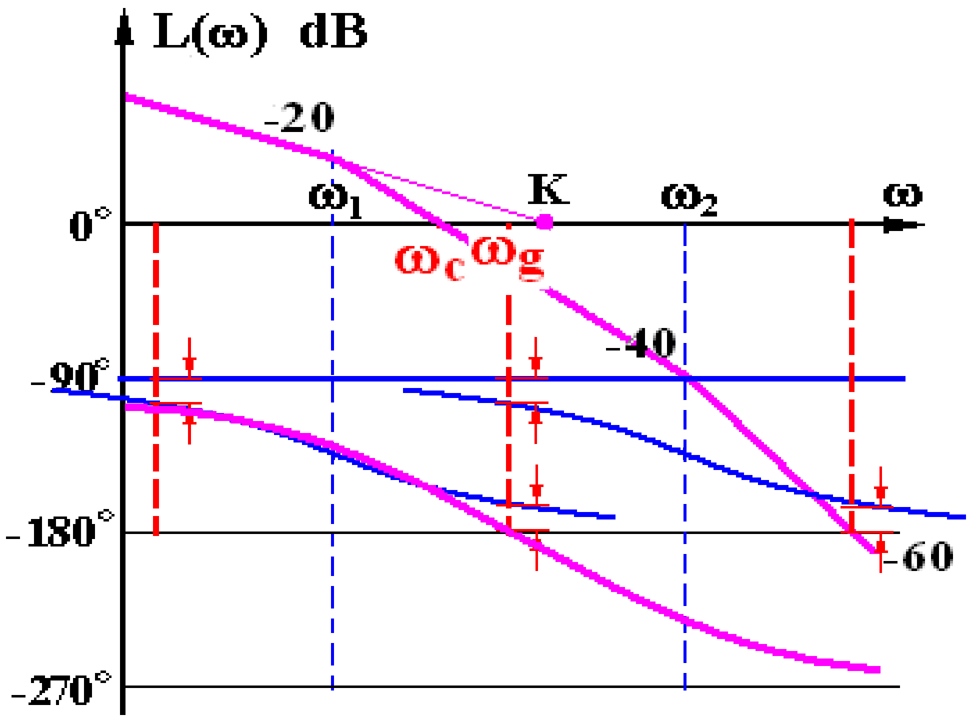
代入实部 $G_X(\omega_g)=-0.4167$

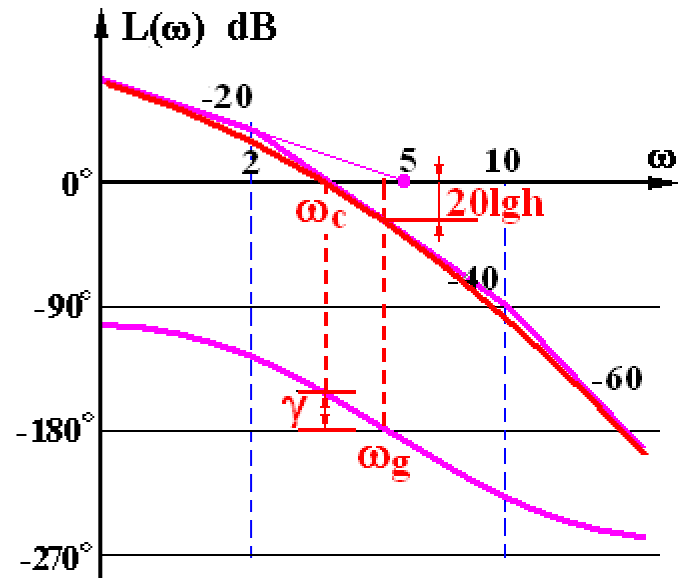
$$|G(\omega_g)|=0.4167$$

$$h=\frac{1}{|G(j\omega_g)|}=\frac{1}{0.4167}=2.4$$



§ 5.5 稳定裕度 (6)





$$\gamma = 180^\circ + \angle G(j\omega_c) = 180^\circ + \varphi(3.16)$$

$$= 180^\circ - 90^\circ - \arctan\frac{3.16}{2} - \arctan\frac{3.16}{10}$$

$$= 90^\circ - 57.67^\circ - 17.541^\circ = 14.8^\circ < 18.5^\circ$$

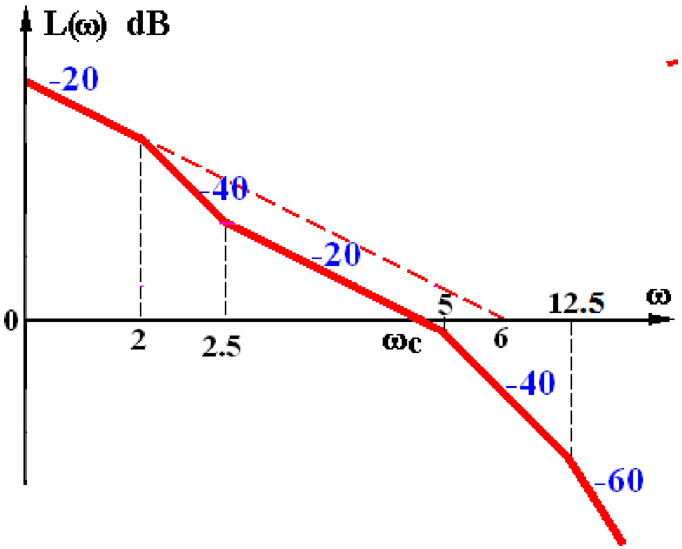
$$\omega_g = \sqrt{2\times 10} = 4.47$$

$$h = \frac{1}{|G(j4.47)|}$$

$$= \frac{1}{0.4167} = 2.4$$

§ 5.5 稳定裕度 (7)

例5 $G(s)=\dfrac{6(\frac{s}{2.5}+1)}{s(\frac{s}{2}+1)(\frac{s}{5}+1)(\frac{s}{12.5}+1)}$，求$\gamma$，$h$。



解. 作$L(\omega)$求 ω_c

法I: $\dfrac{6}{\omega_c}=\dfrac{2.5}{2}$ $\qquad \omega_c=\dfrac{6\times 2}{2.5}=4.8$

法II: $G(j\omega_c)=1=\dfrac{6\times\frac{\omega_c}{2.5}}{\omega_c\cdot\frac{\omega_c}{2}\cdot 1\cdot 1}=\dfrac{6\times 2}{2.5\omega_c}$ $\qquad \omega_c=\dfrac{6\times 2}{2.5}=4.8$

$$\gamma=180^\circ+\angle G(j\omega_c)$$

$$=180^\circ+\arctan\frac{4.8}{2.5}-90^\circ-\arctan\frac{4.8}{2}-\arctan\frac{4.8}{5}-\arctan\frac{4.8}{12.5}$$

$$=180^\circ+62.5^\circ-90^\circ-67.4^\circ-43.8^\circ-21^\circ=20.3^\circ$$

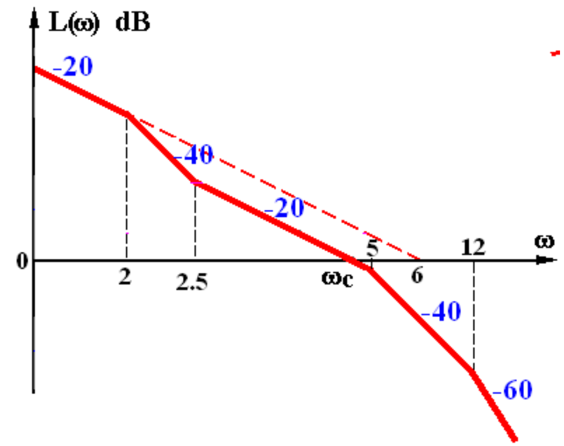
§ 5.5 稳定

$$G(s)=\frac{6(\frac{s}{2.5}+1)}{s(\frac{s}{2}+1)(\frac{s}{5}+1)(\frac{s}{12.5}+1)}=\frac{300(s+2.5)}{s(s+2)(s+5)(s+12.5)}$$

求ωg $\varphi(\omega_g)=\arctan\frac{\omega_g}{2.5}-90^\circ-\arctan\frac{\omega_g}{2}-\arctan\frac{\omega_g}{5}-\arctan\frac{\omega_g}{12.5}=-180^\circ$

$$\arctan\frac{\omega_g}{12.5}+\arctan\frac{\omega_g}{5}+\arctan\frac{\omega_g}{2}-\arctan\frac{\omega_g}{2.5}=90^\circ$$

$$\arctan\left[\frac{\frac{\omega_g}{12.5}+\frac{\omega_g}{5}}{1-\frac{\omega_g^2}{12.5\times5}}\right]+\arctan\left[\frac{\frac{\omega_g}{2}-\frac{\omega_g}{2.5}}{1+\frac{\omega_g^2}{2\times2.5}}\right]=90^\circ$$

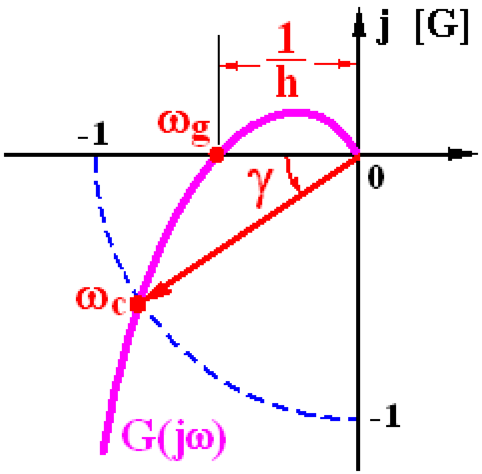


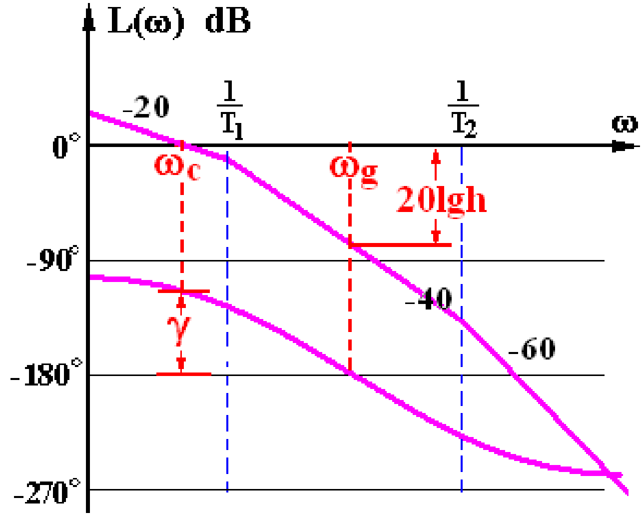
$$\arctan\frac{[A]+[B]}{1-[A]\cdot[B]}=90^\circ \qquad \Rightarrow \qquad [A]\cdot[B]=1$$

整理得 $\omega_g^4-49.75\omega_g^2-312.5=0$ 解出 $\omega_g=7.4\ (\mathrm{rad/s})$

$$h=\frac{1}{|G(j\omega_g)|}=\frac{\omega_g\sqrt{\omega_g^2+2^2}\sqrt{\omega_g^2+5^2}\sqrt{\omega_g^2+12.5^2}}{300\cdot\sqrt{\omega_g^2+2.5^2}}=3.135$$

课程小结





$j\omega_c)| = 1$

$: 180° + \angle G(j\omega_c)$

$(j\omega_g) = -180°$

$\dfrac{1}{|G(j\omega_g)|}$

稳定裕度的意义 $\begin{cases} \gamma,\ h \text{ 的几何意义} \\ \gamma,\ h \text{ 的物理意义} \end{cases}$

稳定裕度计算方法 $\begin{cases} L(\omega) \Rightarrow \omega_c \Rightarrow \gamma = 180° + \varphi(\omega_c) \\ \varphi(\omega) = -180° \Rightarrow \omega_g \Rightarrow h = \dfrac{1}{|G(j\omega_g)|} \end{cases}$