Image filtering

$$g[m,n] = \sum_{k,l} I(m+k, n+l) * f(k,l)$$

Image I 8x8

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Kernel f 3x3

1	2	3
4	5	6
7	8	9

Output g

28	39	39	39	39	39	39	24
33	45	45	45	45	45	45	27
33	45	45	45	45	45	45	27
16	21	21	21	21	21	21	12
5	6	6	6	6	6	6	3
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Same position

Register

a	b	c
d	e	f
g	h	i

Loop over every pixel

Calculate result = a*1+b*2+…+i*9

Image Convolution

$$g[m,n] = I \otimes f = \sum_{k,l} I(m-k, n-l) * f(k,l)$$

Image I 8x8

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Kernel f 3x3

1	2	3
4	5	6
7	8	9

Output g

12	21	21	21	21	21	21	16
27	45	45	45	45	45	45	33
27	45	45	45	45	45	45	33
24	39	39	39	39	39	39	28
15	24	24	24	24	24	24	17
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Same position

Register

a	b	c
d	e	f
g	h	i

Loop over every pixel

Calculate result = i*1+h*2+…+b*8+a*9

Image Convolution

$$g[m,n] = I \otimes f = \sum_{k,l} I(m-k, n-l) * f(k,l)$$

Image I 8x8

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Kernel f *flipped* 3x3

9	8	7
6	5	4
3	2	1

Output g

12	21	21	21	21	21	21	16
27	45	45	45	45	45	45	33
27	45	45	45	45	45	45	33
24	39	39	39	39	39	39	28
15	24	24	24	24	24	24	17
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Same position

Register

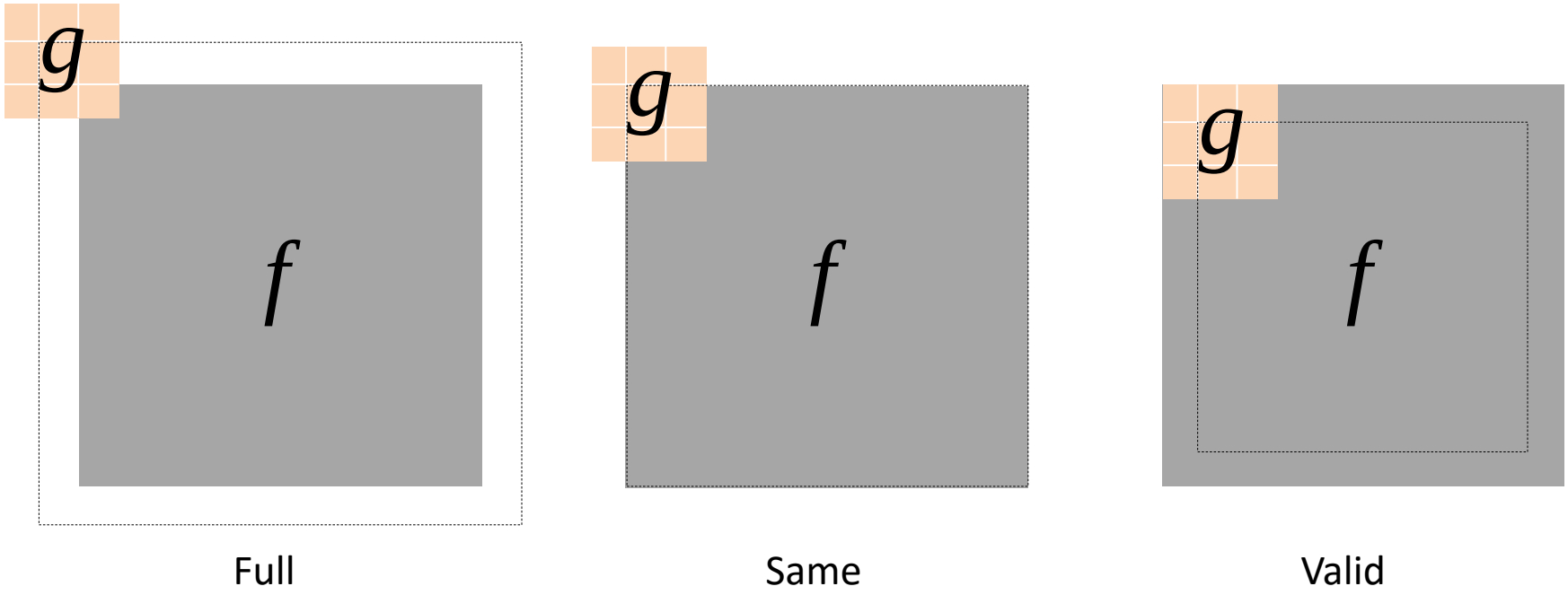
a	b	c
d	e	f
g	h	i

Loop over every pixel

Calculate result = i*1+h*2+…+b*8+a*9

Output Size of Image Convolution

$f \otimes g$



Full Same Valid

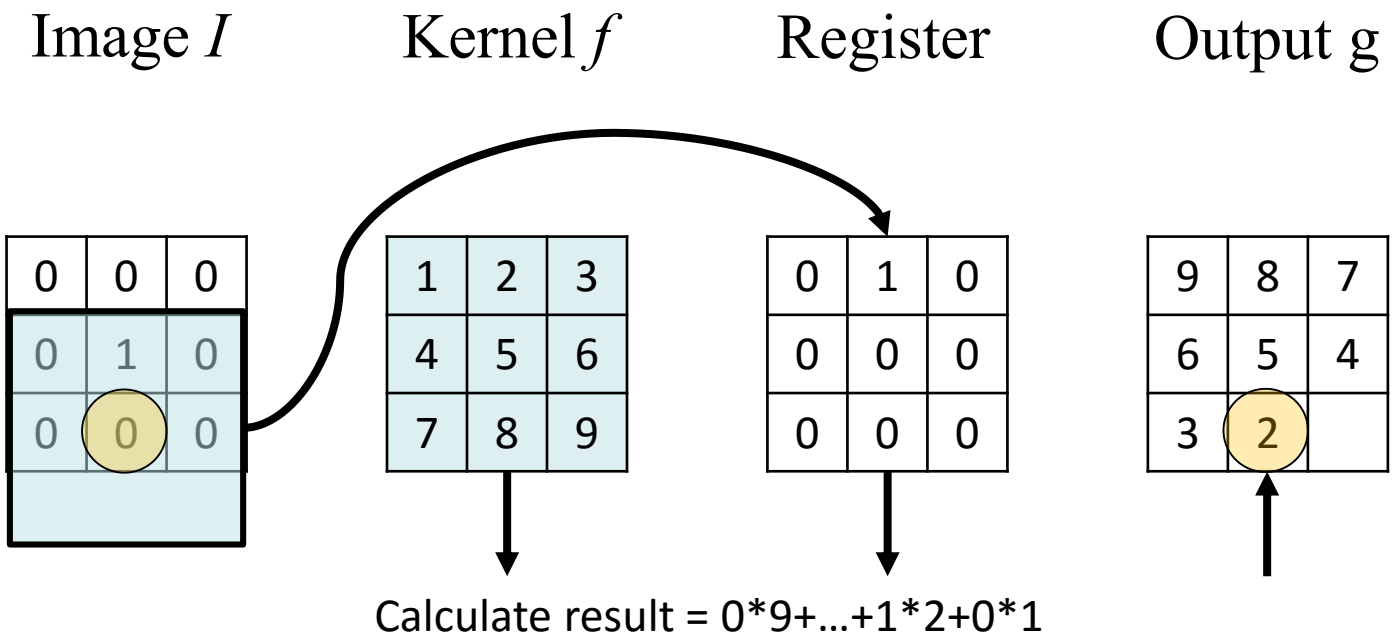
filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

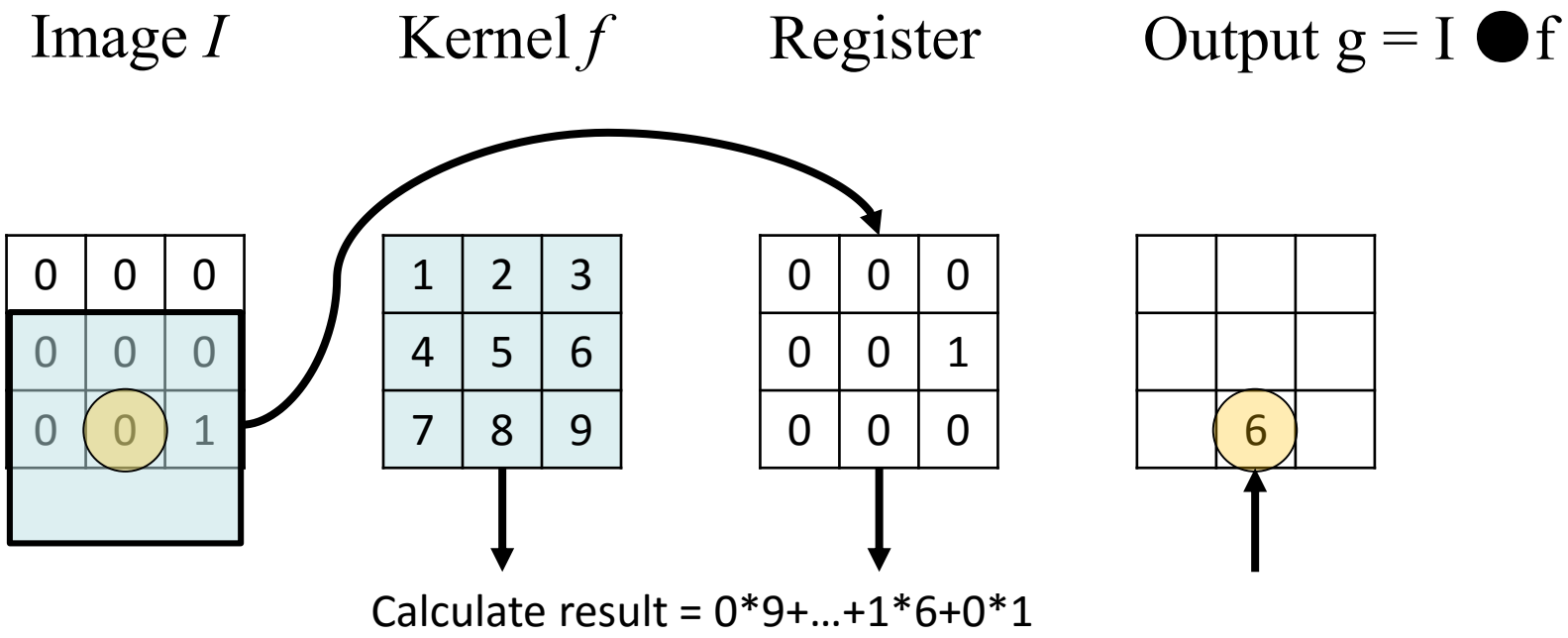
Same: output_size = f_size

Valid: output_size = f_size – (g_size – 1)

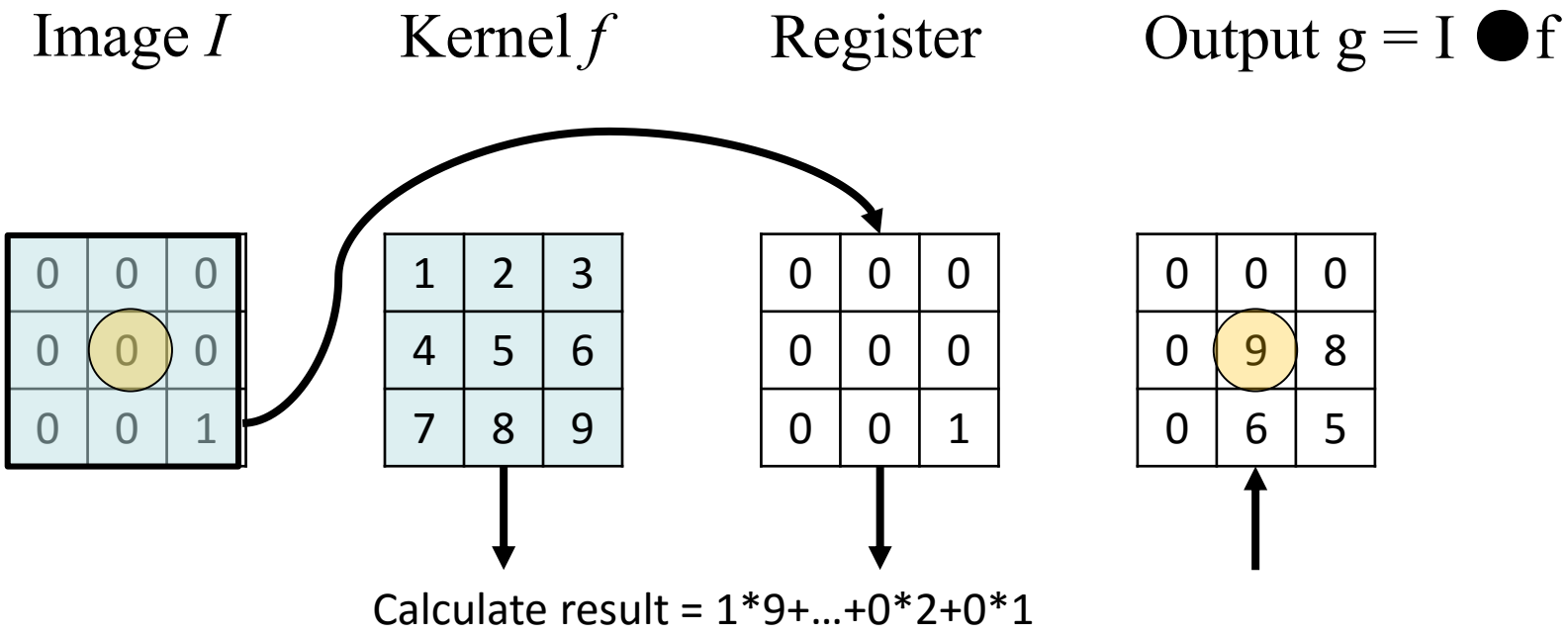
Filtering Special case: impulse function



Filtering Special case: impulse function



Filtering Special case: impulse function



Filtering Impulse functions shift images

Image I

1	2	3
4	5	6
7	8	9

Kernel f

0	0	0
0	0	0
0	0	1

Result g = I ●f

5	6	0
8	9	0
0	0	0

- In this case the resulting filtered image is shifted to the upper right

Filtering is not associative (flipped)

I

1	2	3
4	5	6
7	8	9

f

0	0	0
0	0	0
0	0	1

g = I ● f

5	6	0
8	9	0
0	0	0

f

0	0	0
0	0	0
0	0	1

I

1	2	3
4	5	6
7	8	9

g = f ● I

0	0	0
0	9	8
0	6	5

Image Convolution

$$g[m,n] = I \otimes f = \sum_{k,l} I(m-k, n-l) * f(k,l)$$

Image I 8x8

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Kernel f 3x3

1	2	3
4	5	6
7	8	9

Output g

12	21	21	21	21	21	21	16
27	45	45	45	45	45	45	33
27	45	45	45	45	45	45	33
24	39	39	39	39	39	39	28
15	24	24	24	24	24	24	17
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Same position

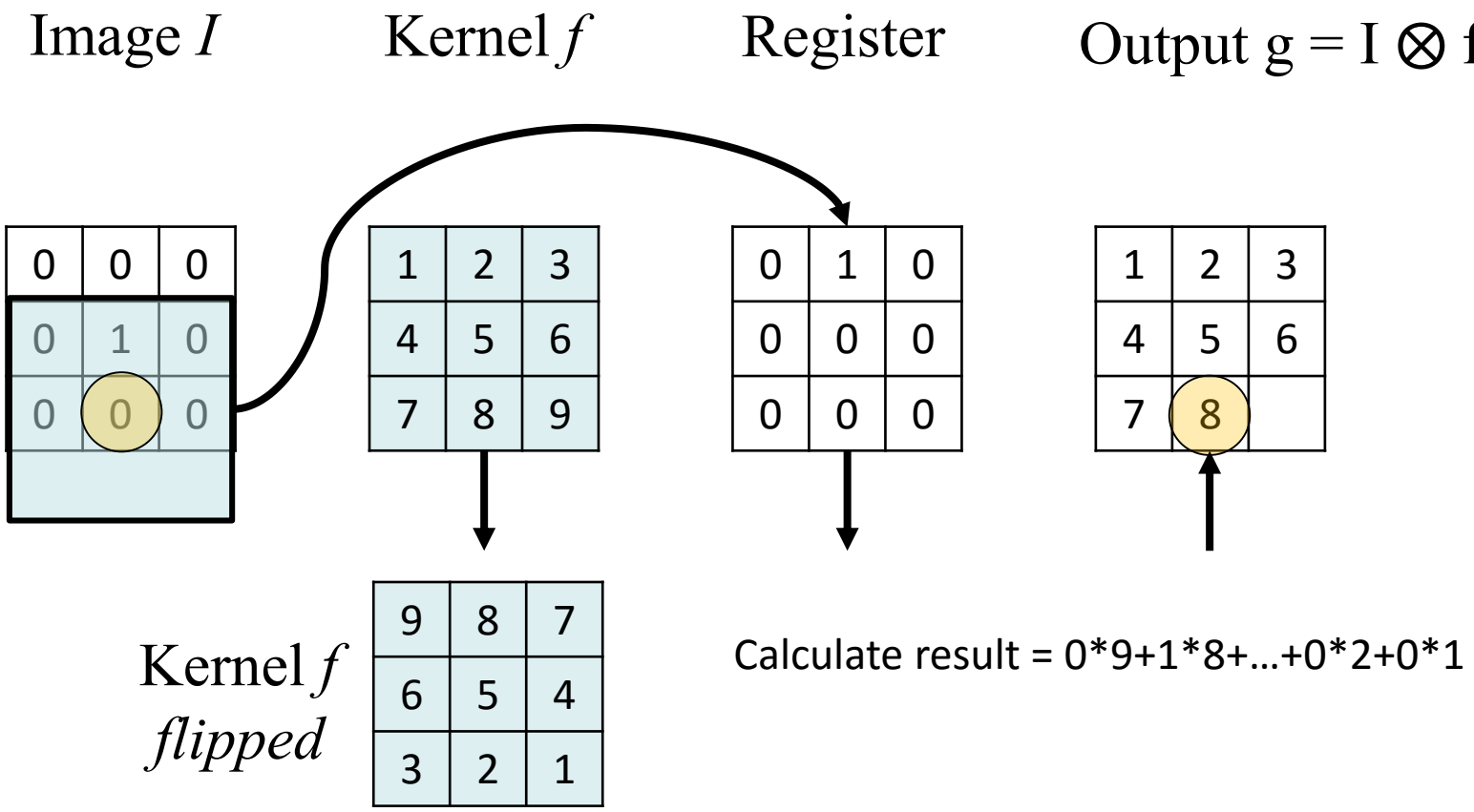
Register

a	b	c
d	e	f
g	h	i

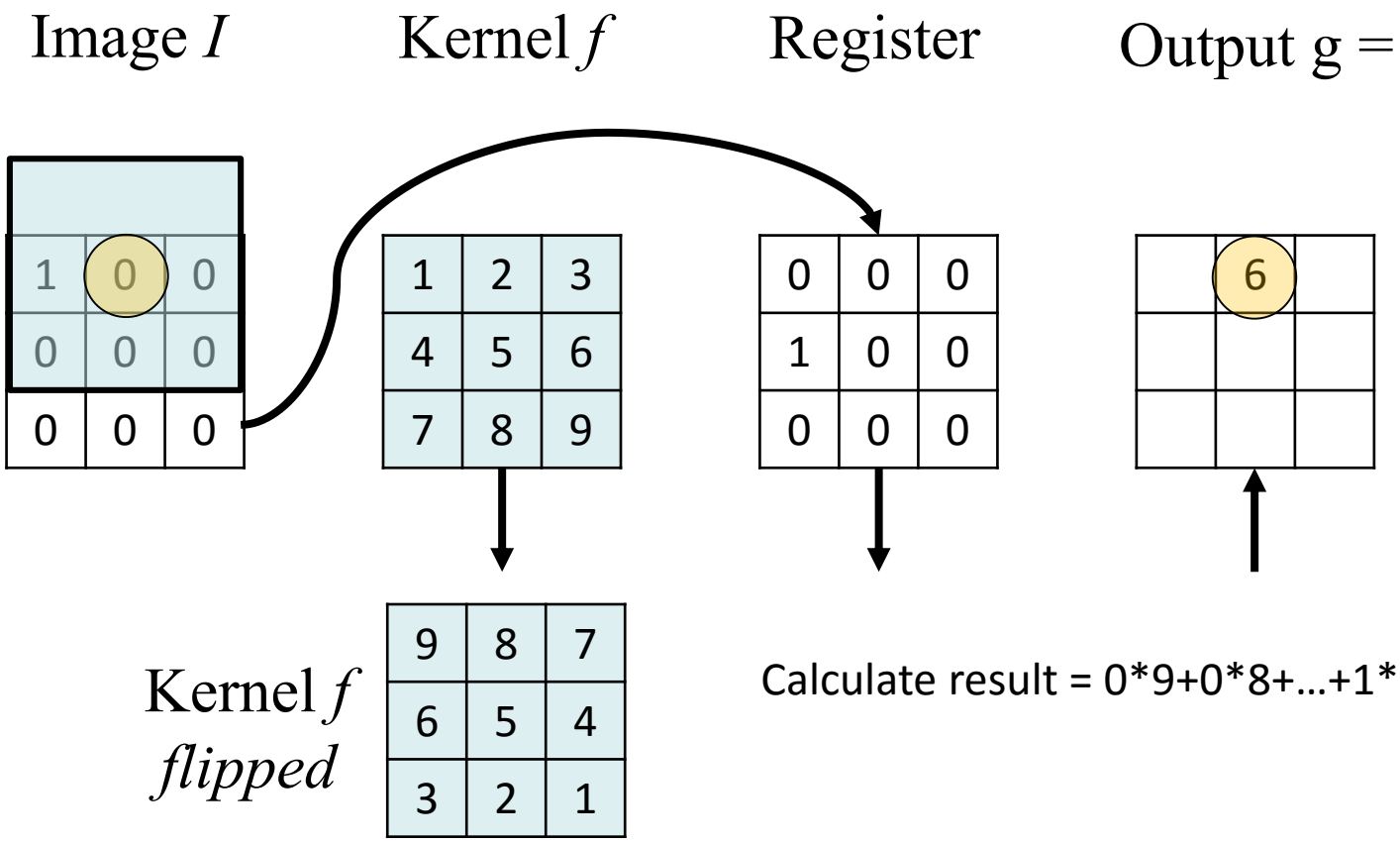
Loop over every pixel

Calculate result = i*1+h*2+…+b*8+a*9

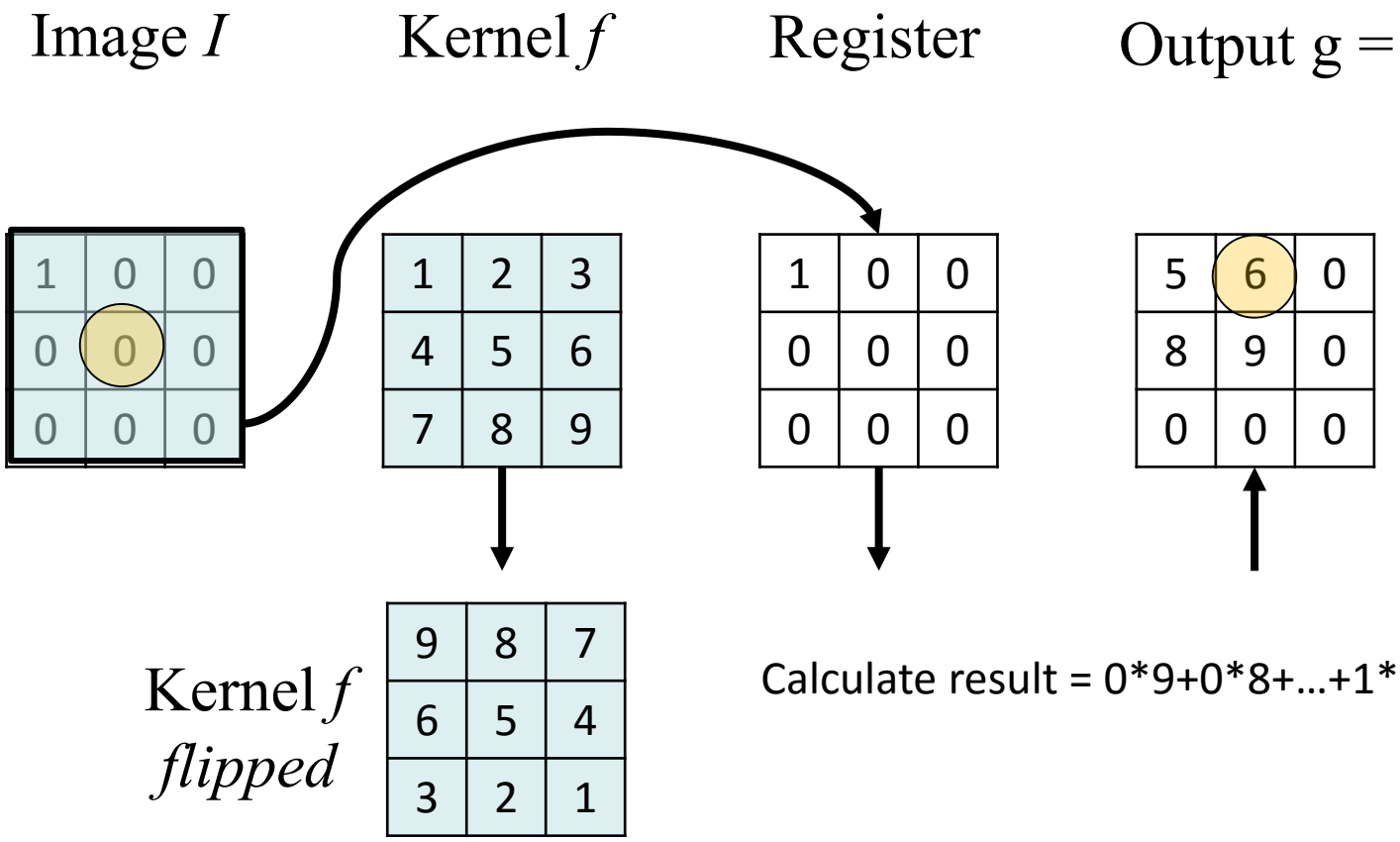
Convolution Special case: impulse function



Convolution Special case: impulse function



Convolution Special case: impulse function



Convolution Impulse functions shift images

Image I

1	2	3
4	5	6
7	8	9

Kernel f

1	0	0
0	0	0
0	0	0

Kernel f'

0	0	0
0	0	0
0	0	1

Result $I \otimes f$

5	6	0
8	9	0
0	0	0

- In this case the resulting convolution image is shifted to the upper left

Convolution is associative

I

1	2	3
4	5	6
7	8	9

f

1	0	0
0	0	0
0	0	0

$I \otimes f$

5	6	0
8	9	0
0	0	0

f

1	0	0
0	0	0
0	0	0

I

1	2	3
4	5	6
7	8	9

f$\otimes I$

5	6	0
8	9	0
0	0	0

Linear independence

Image I

0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0

5x5

Kernel f

1	0	0
0	1	0
0	0	0

Output $g=g_1+g_2$

1	1	1	1	0
1	1	1	1	0
1	1	1	1	0
1	1	1	1	0
0	1	0	1	0

Kernel f_1

1	0	0
0	0	0
0	0	0

Output g_1

1	0	1	0	0
1	0	1	0	0
1	0	1	0	0
1	0	1	0	0
0	0	0	0	0

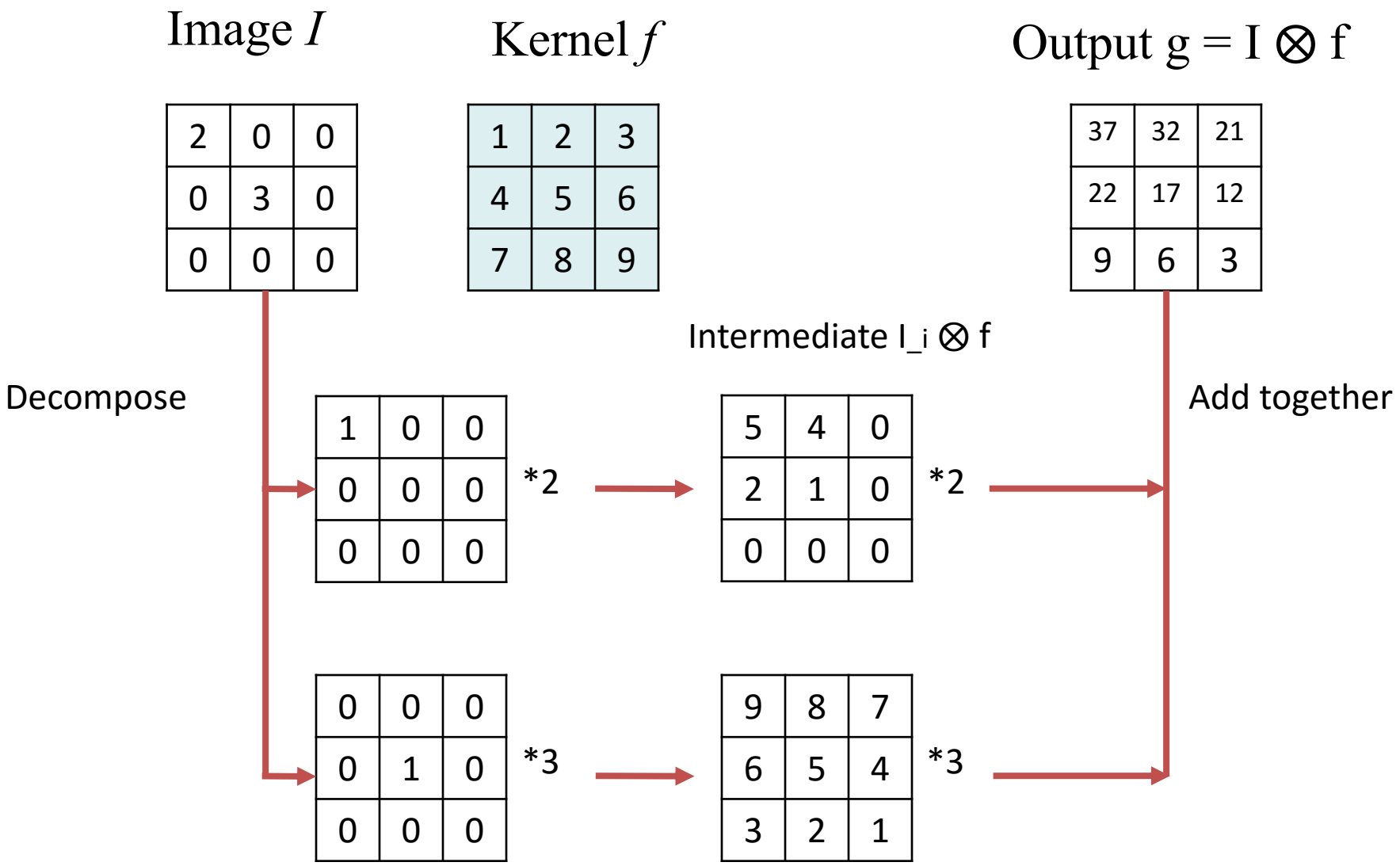
Kernel f_2

0	0	0
0	1	0
0	0	0

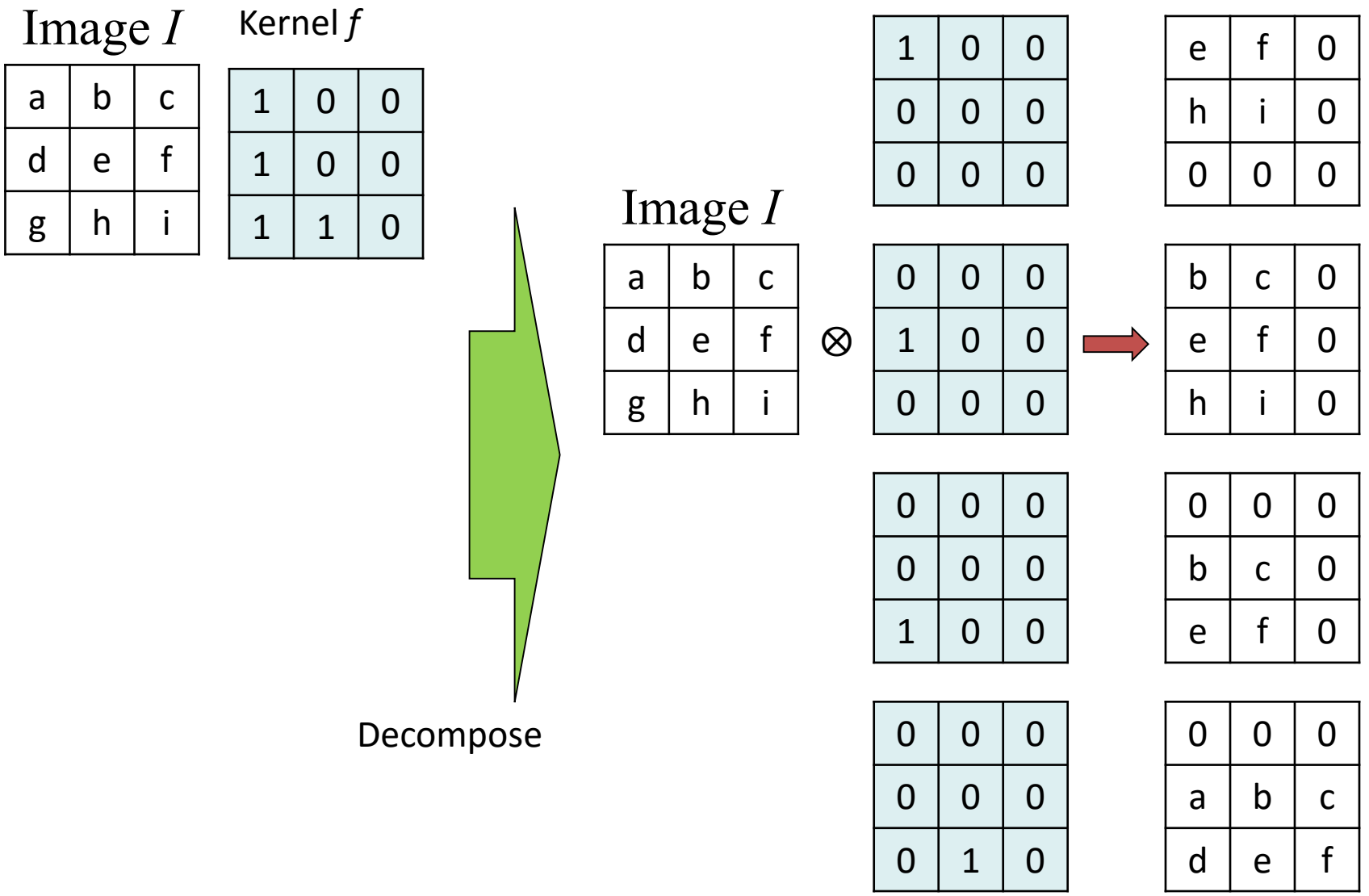
Output g_2

0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0

Linear independence



- Convolution has commutative property $I \otimes f$



- Convolution has commutative property

$$f \otimes I$$