

# Feature Matching and RANSAC

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Recognising Panoramas.

[M. Brown and D. Lowe, ICCV 2003]

[Brown, Szeliski, Winder, CVPR' 2005]

*with a lot of slides stolen from  
Steve Seitz, Rick Szeliski, A. Efros*

# Introduction

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Are you getting the whole picture?

- Compact Camera FOV =  $50 \times 35^\circ$

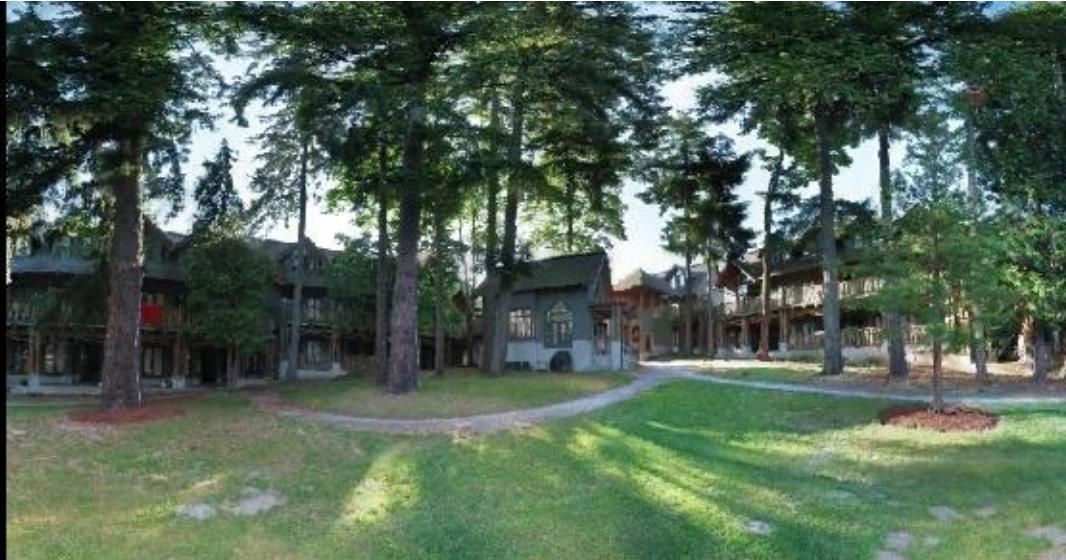


# Introduction

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Are you getting the whole picture?

- Compact Camera FOV =  $50 \times 35^\circ$
- Human FOV =  $200 \times 135^\circ$

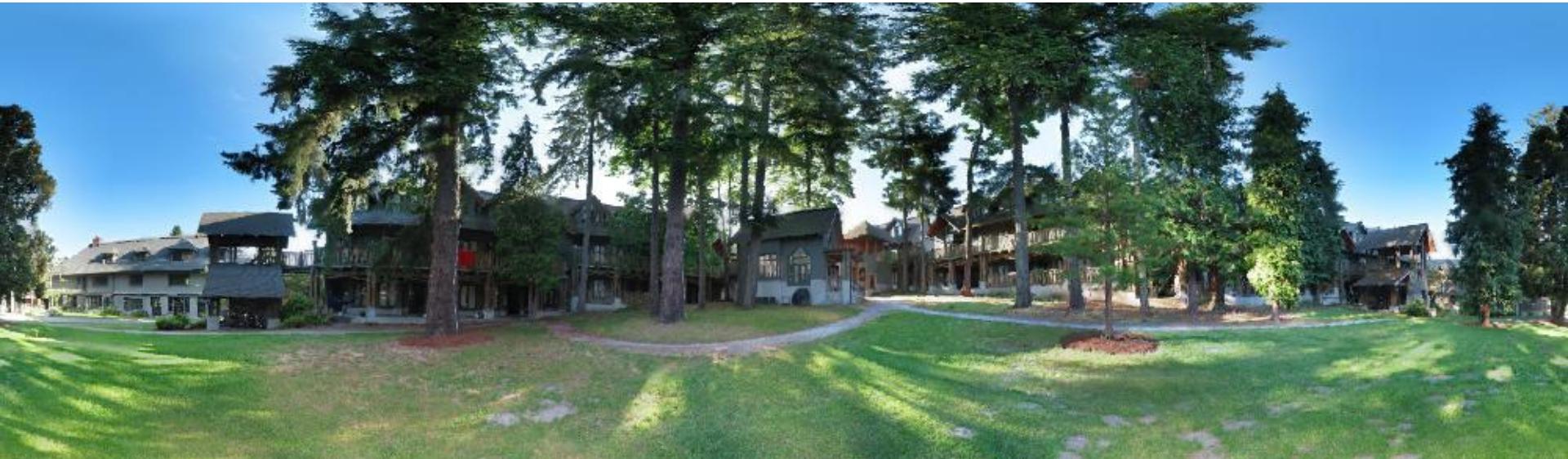


# Introduction

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Are you getting the whole picture?

- Compact Camera FOV =  $50 \times 35^\circ$
- Human FOV =  $200 \times 135^\circ$
- Panoramic Mosaic =  $360 \times 180^\circ$



# Why “Recognising Panoramas”?

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# Why “Recognising Panoramas”?

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## 1D Rotations ( $\theta$ )

- Ordering  $\Rightarrow$  matching images

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## 1D Rotations ( $\theta$ )

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- 2D Rotations ( $\theta, \phi$ )
  - Ordering  $\not\Rightarrow$  matching images

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- Ordering  $\Rightarrow$  matching images



- 2D Rotations ( $\theta, \phi$ )
  - Ordering  $\not\Rightarrow$  matching images



# Why “Recognising Panoramas”?

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## 1D Rotations ( $\theta$ )

- Ordering  $\Rightarrow$  matching images

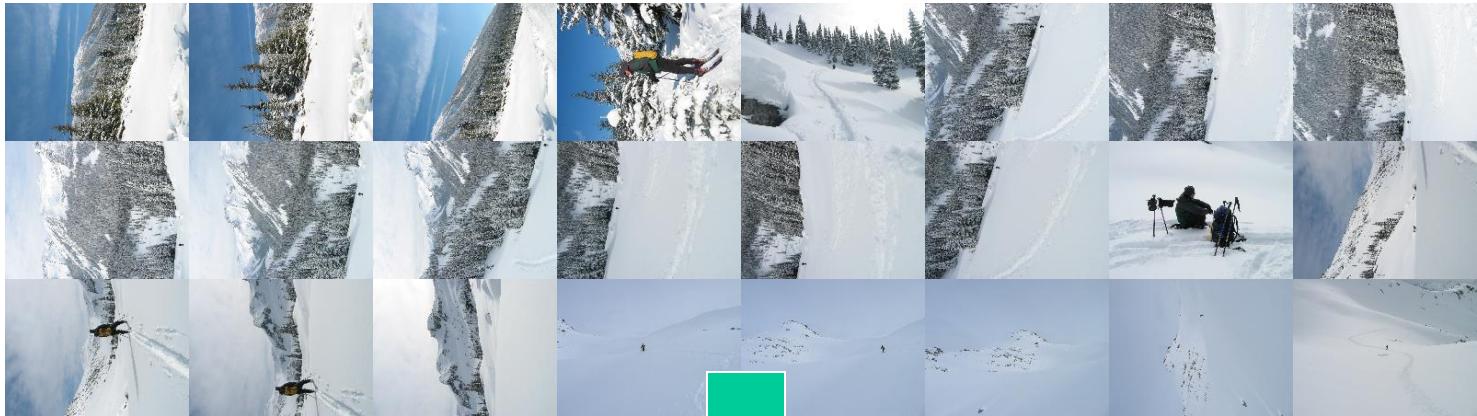


- 2D Rotations ( $\theta, \phi$ )
  - Ordering  $\not\Rightarrow$  matching images



# Why “Recognising Panoramas”?

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# Overview

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Feature Matching

Image Matching

Multi-band Blending

Results

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## Feature Matching

- Corner Features
- Nearest Neighbour Matching

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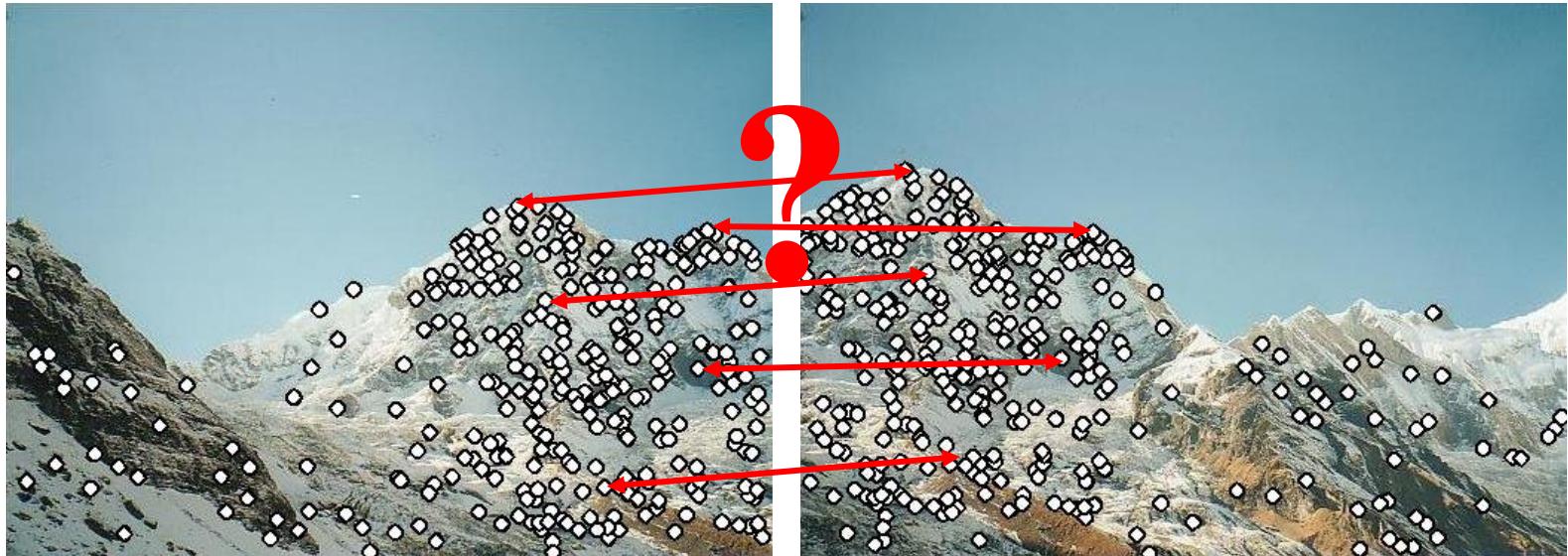
Conclusions

# Feature descriptors

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We know how to detect points

Next question: **How to match them?**



Point descriptor should be:

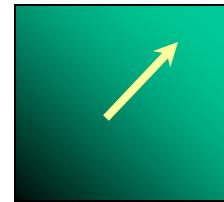
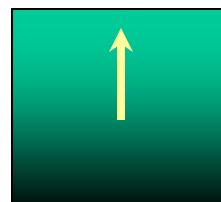
1. Invariant
2. Distinctive

# Descriptors Invariant to Rotation

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Find local orientation

Dominant direction of gradient



- Extract image patches relative to this orientation

# Multi-Scale Oriented Patches

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## Interest points

- Multi-scale Harris corners
- Orientation from blurred gradient
- Geometrically invariant to rotation

## Descriptor vector

- Bias/gain normalized sampling of local patch (8x8)
- Photometrically invariant to affine changes in intensity

[Brown, Szeliski, Winder, CVPR' 2005]

# Descriptor Vector

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Orientation = blurred gradient

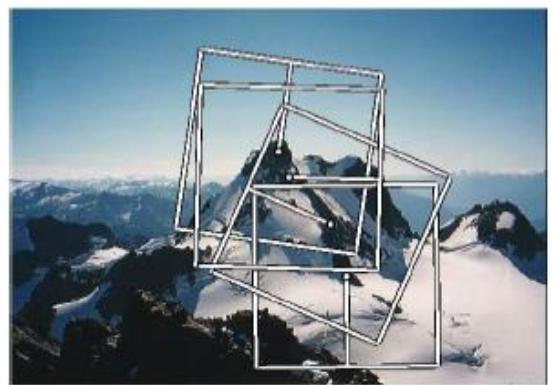
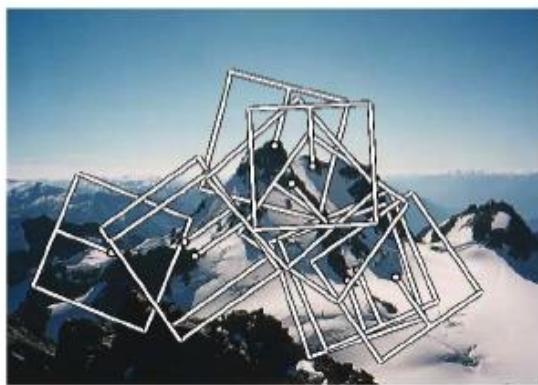
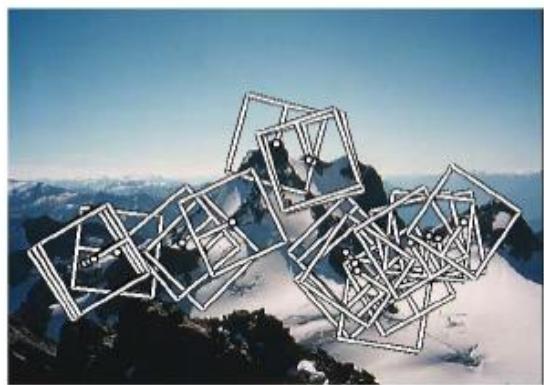
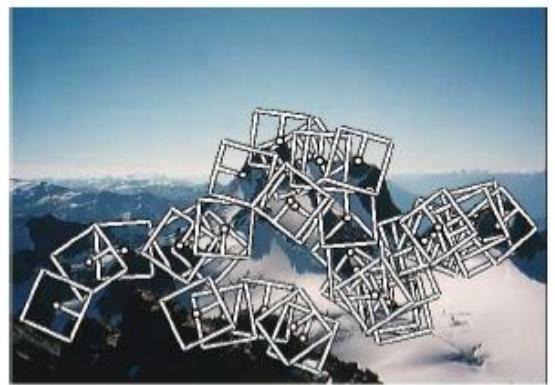
Rotation Invariant Frame

- Scale-space position ( $x, y, s$ ) + orientation ( $\theta$ )



# Detections at multiple scales

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*Figure 1. Multi-scale Oriented Patches (MOPS) extracted at five pyramid levels from one of the Matier images. The boxes show the feature orientation and the region from which the descriptor vector is sampled.*

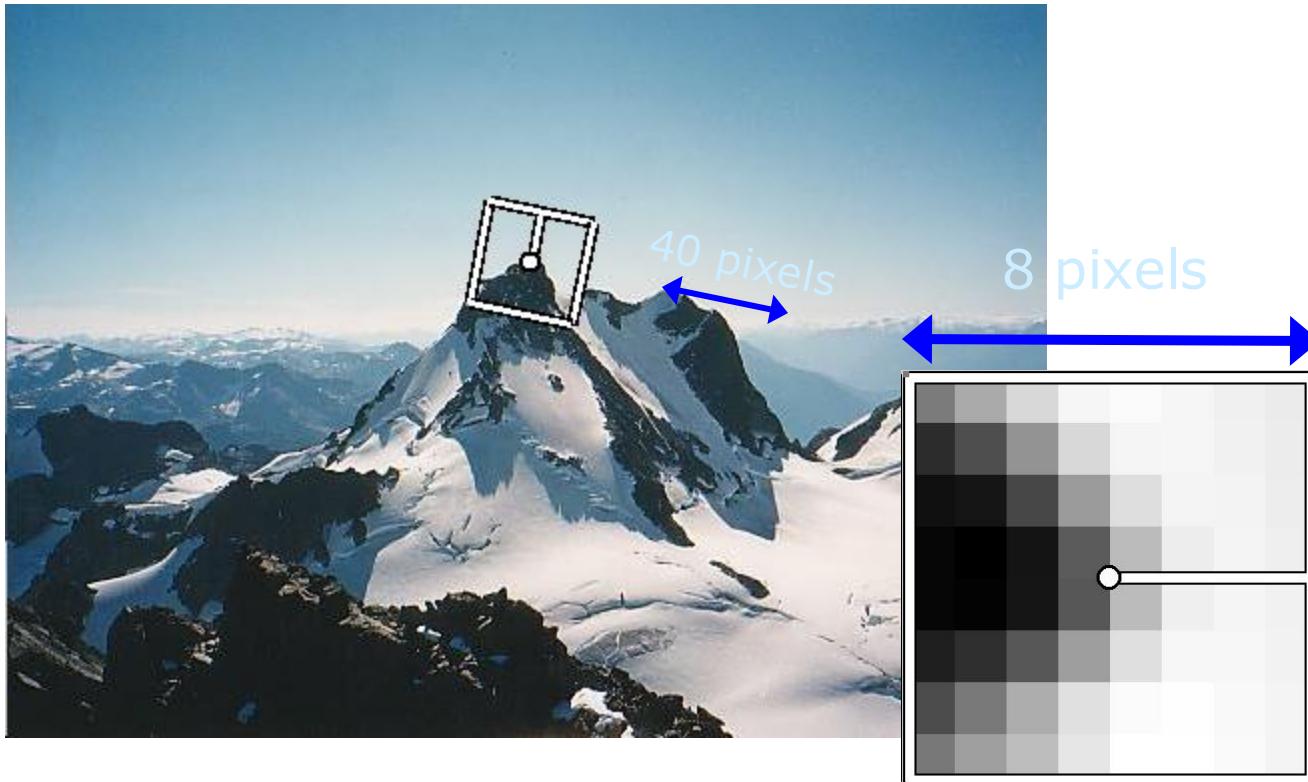
# MOPS descriptor vector

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8x8 oriented patch

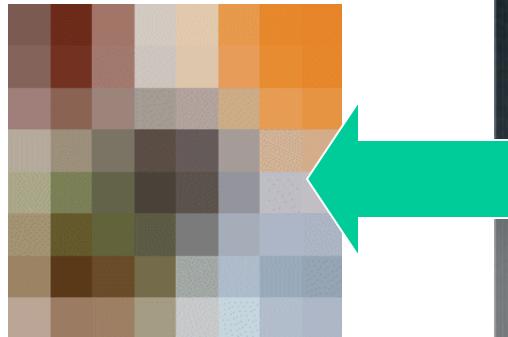
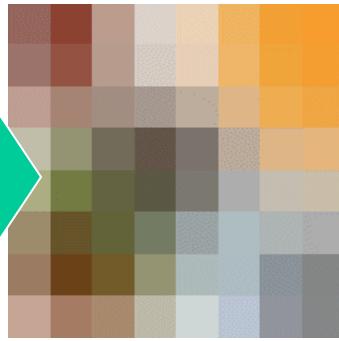
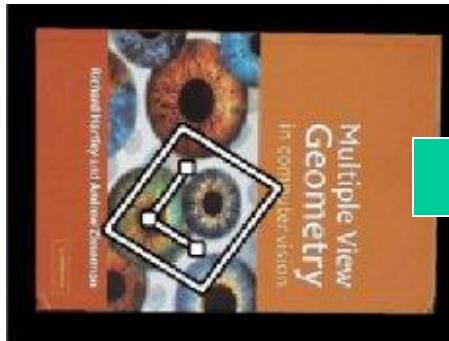
- Sampled at 5 x scale

Bias/gain normalisation:  $I' = (I - \mu)/\sigma$



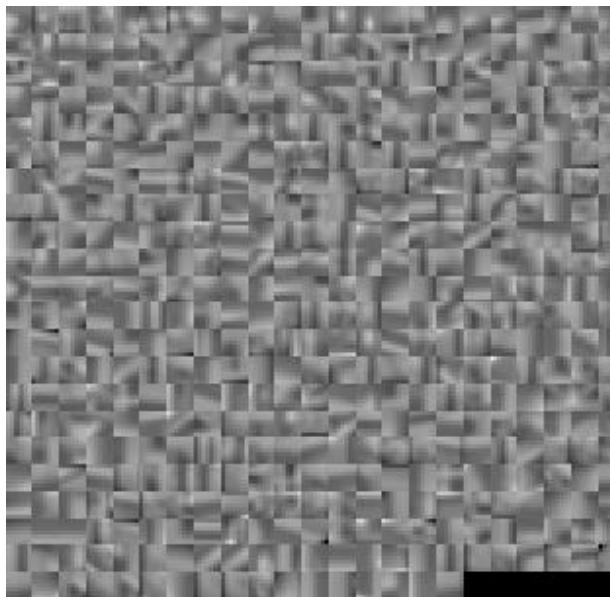
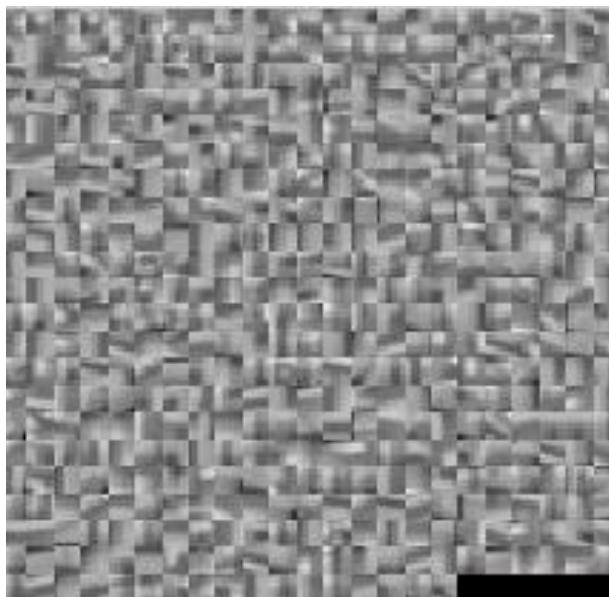
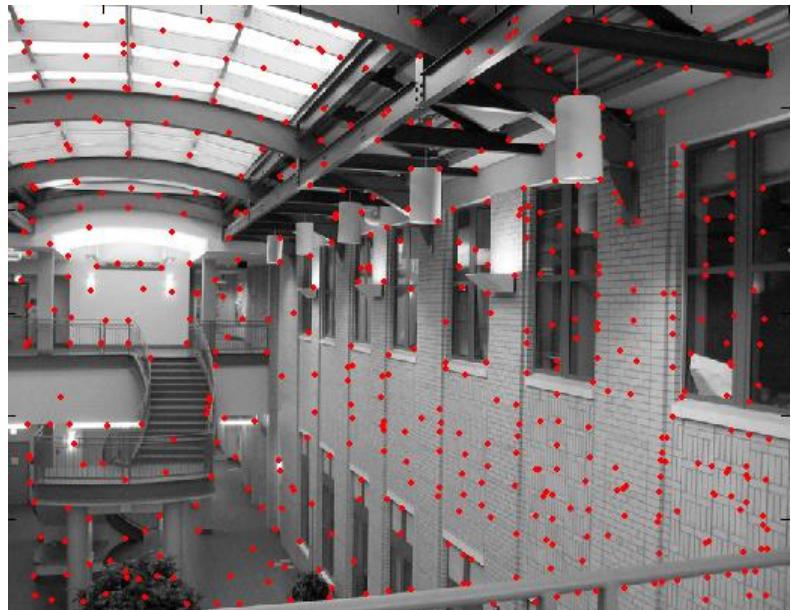
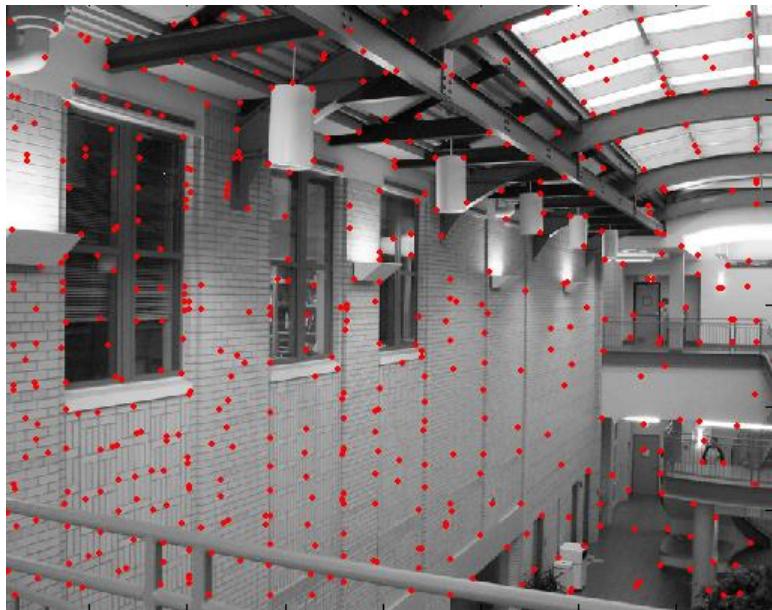
# Invariant Features

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# Feature matching

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# Overview

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## Feature Matching

- corner Features
- Nearest Neighbour Matching

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# Nearest Neighbour Matching

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Find k-NN for each feature

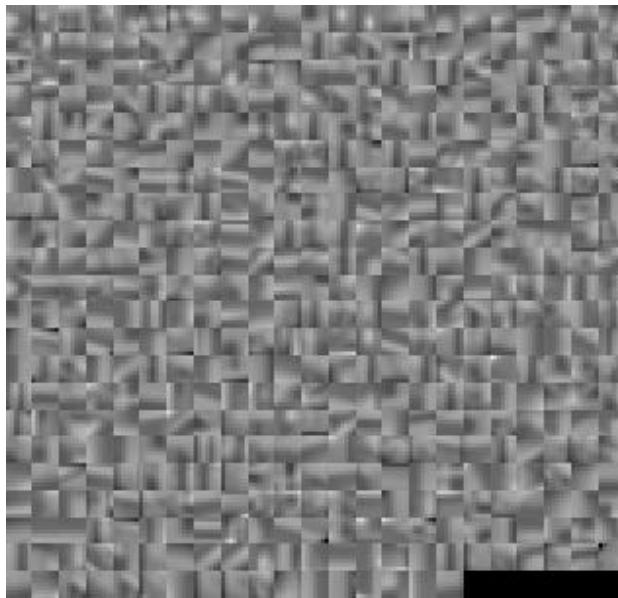
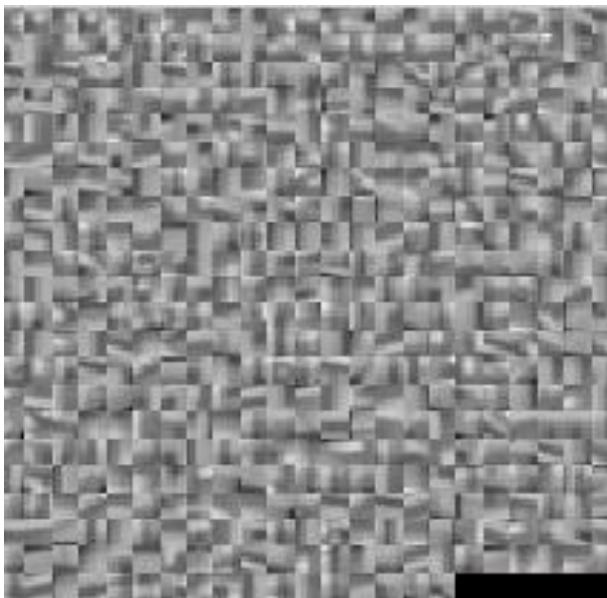
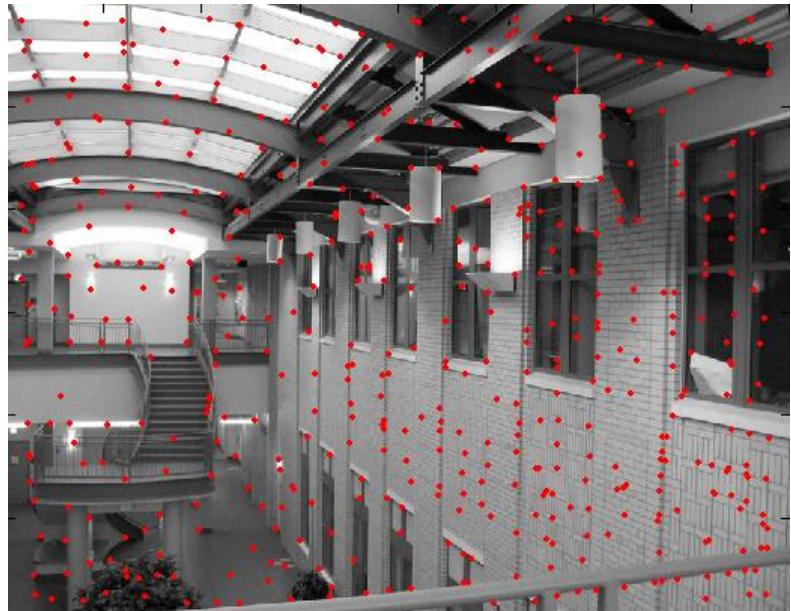
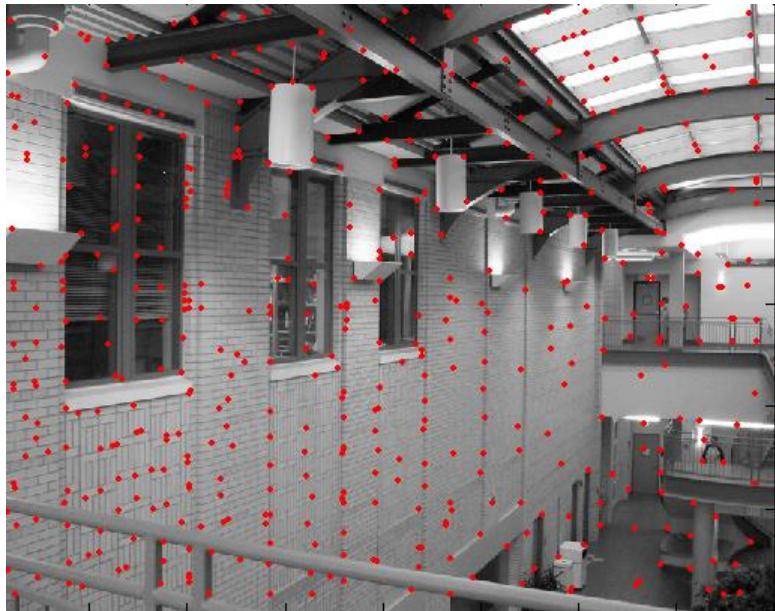
- $k \approx$  number of overlapping images (we use  $k = 4$ )

Use k-d tree

- k-d tree recursively bi-partitions data at mean in the dimension of maximum variance
- Approximate nearest neighbours found in  $O(n \log n)$

# What about outliers?

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# Feature-space outlier rejection

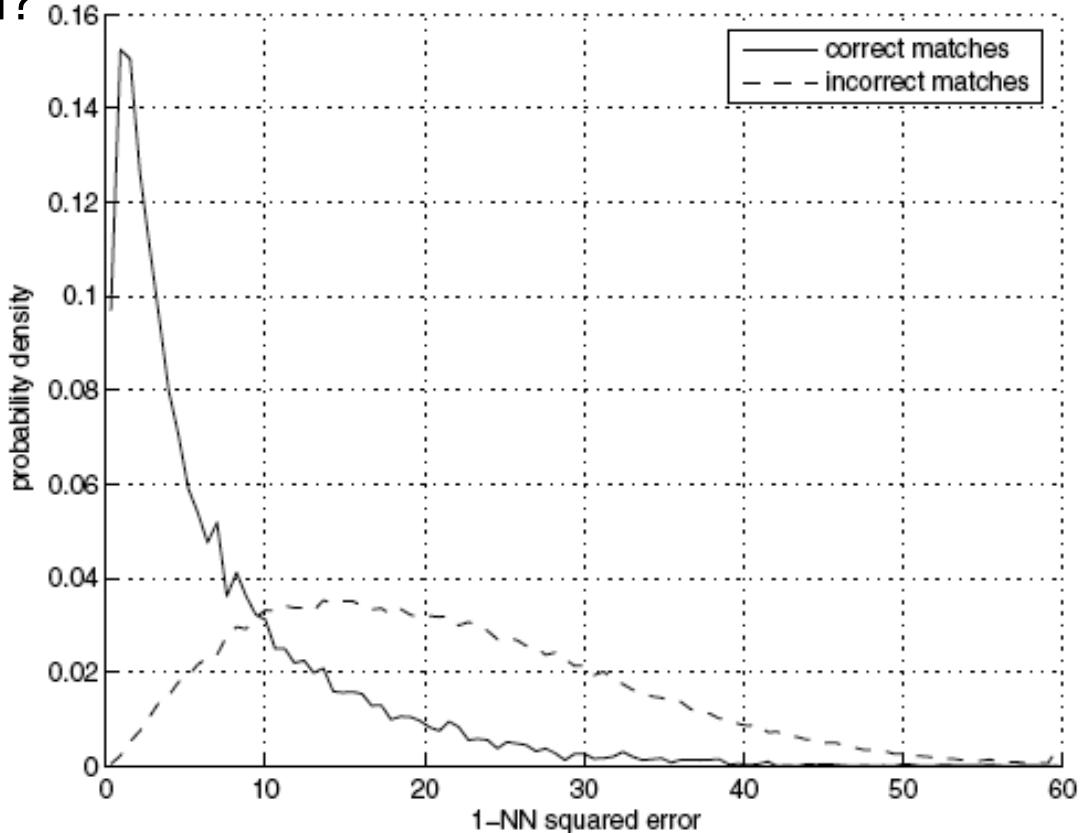
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Let's not match all features, but only these that have “similar enough” matches?

How can we do it?

$$\text{SSD}(\text{patch1}, \text{patch2}) < \text{threshold}$$

How to set threshold?



# Feature-space outlier rejection

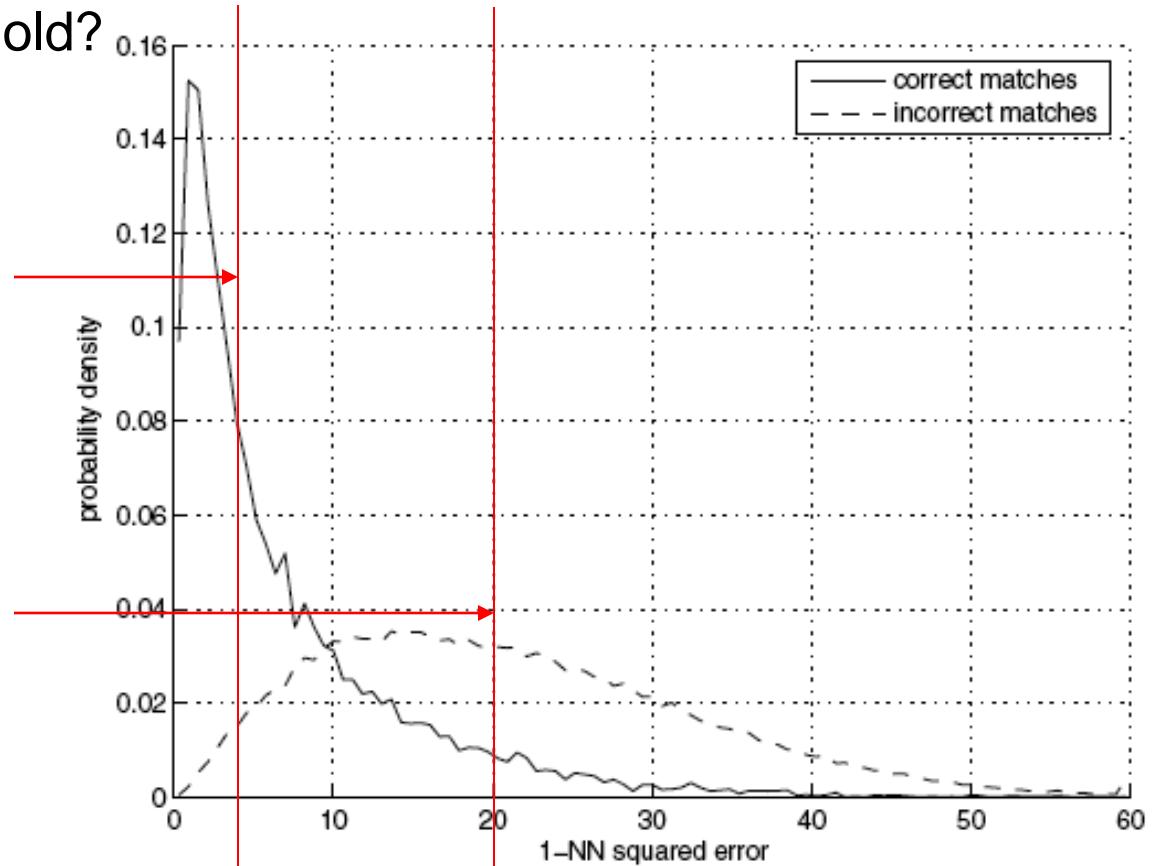
Let's not match all features, but only these that have “similar enough” matches?

How can we do it?

$$\text{SSD}(\text{patch1}, \text{patch2}) < \text{threshold}$$

How to set threshold?

**Too low, miss many good matches**



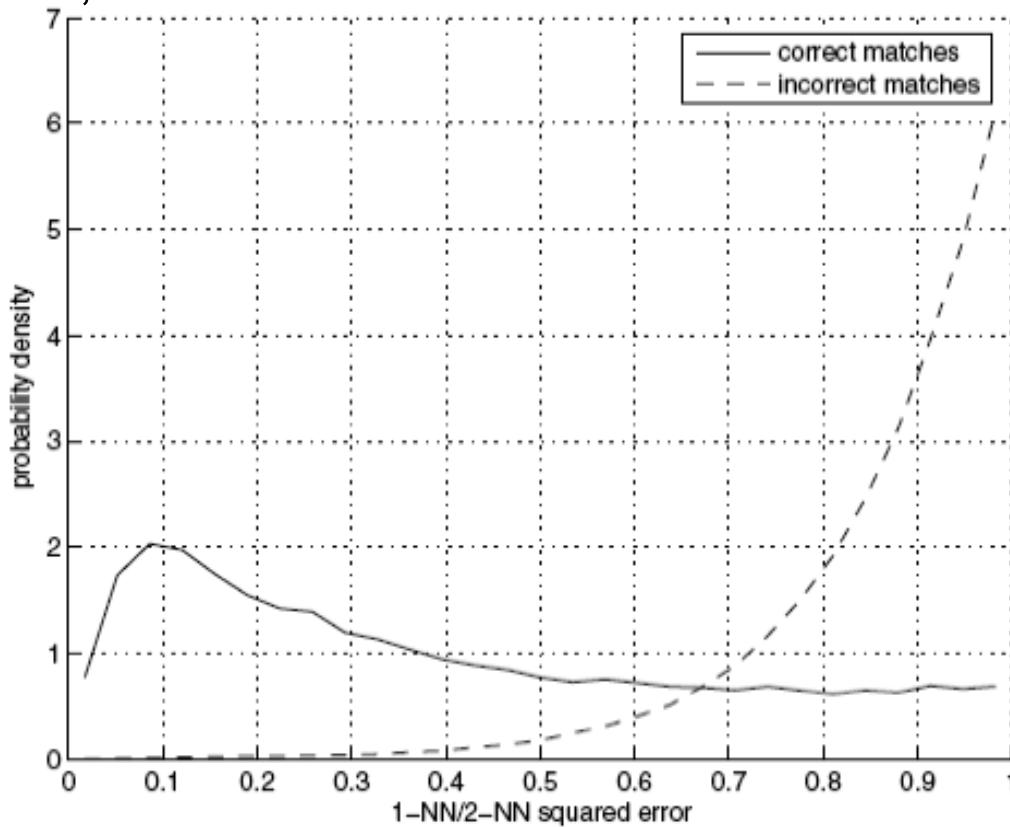
**Too high, too many false matches**

# Feature-space outlier rejection

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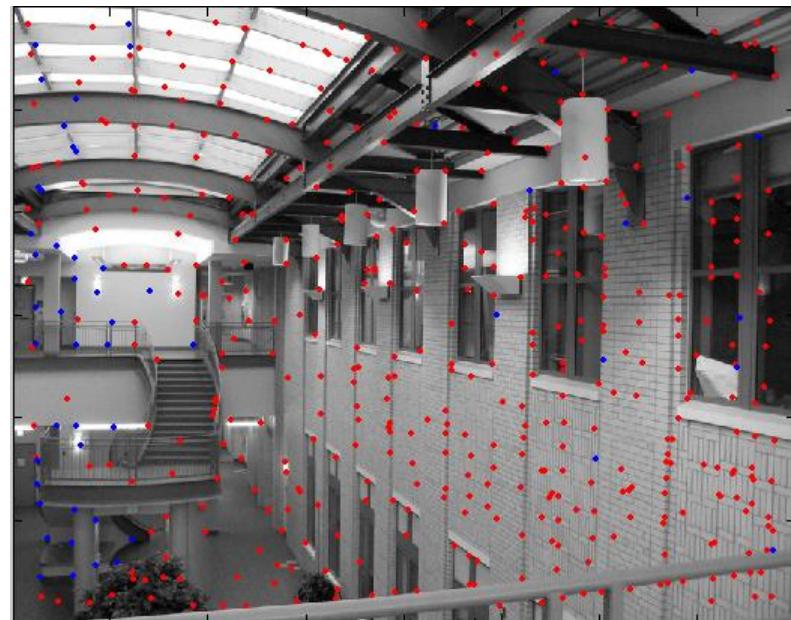
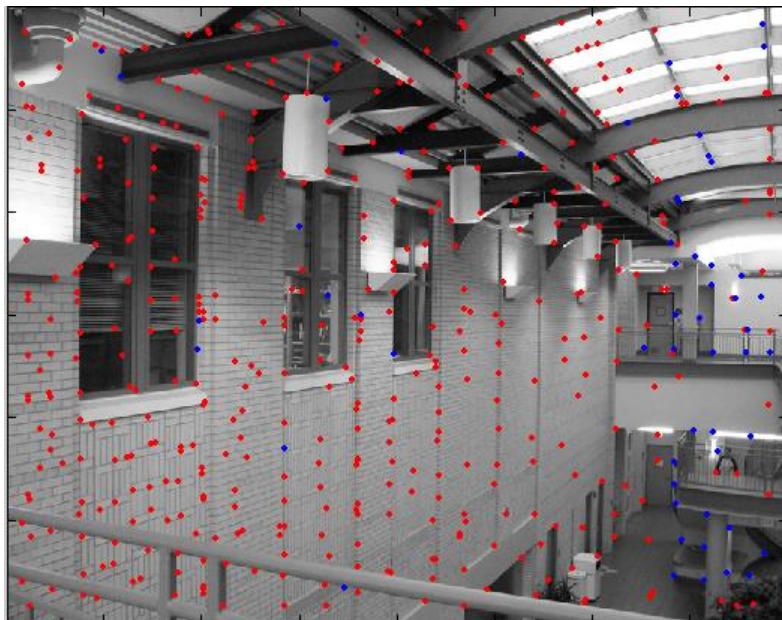
A better way [Lowe, 1999]:

- 1-NN: SSD of the closest match
- 2-NN: SSD of the second-closest match
- Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
- That is, is our best match so much better than the rest?



# Feature-space outlier rejection

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Can we now compute  $H$  from the blue points?

- No! Still too many outliers...
- What can we do?

# Overview

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## Feature Matching

- Corner Features
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Feature Matching

[Image Matching](#)

Multi-band Blending

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Feature Matching

## Image Matching

- RANSAC for Homography
- Probabilistic model for verification

Multi-band Blending

Results

# Overview

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Feature Matching

Image Matching

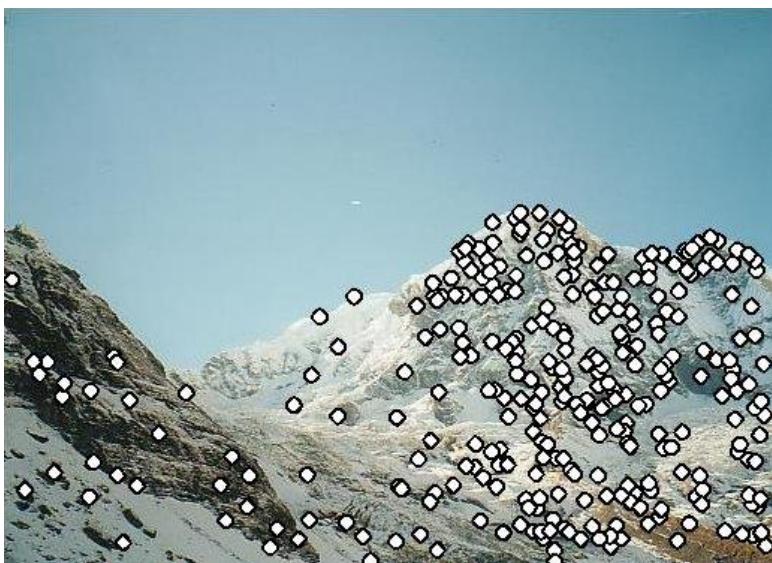
- RANSAC for Homography

Multi-band Blending

Results

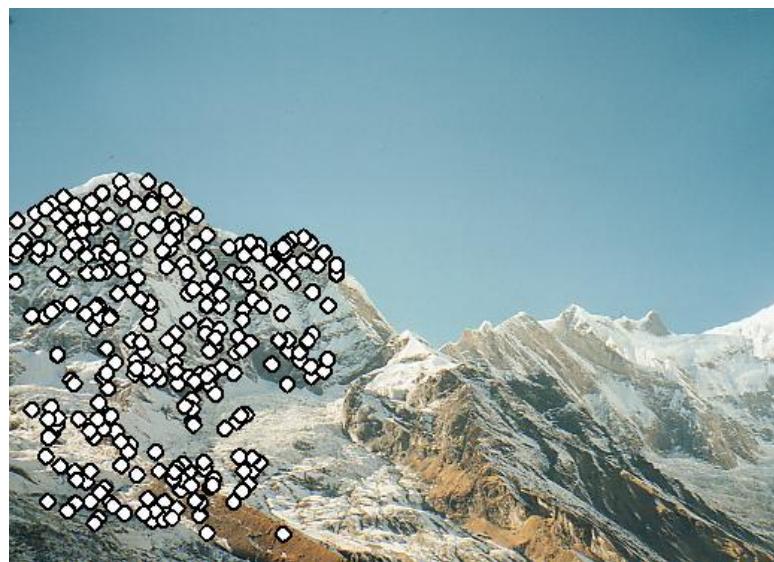
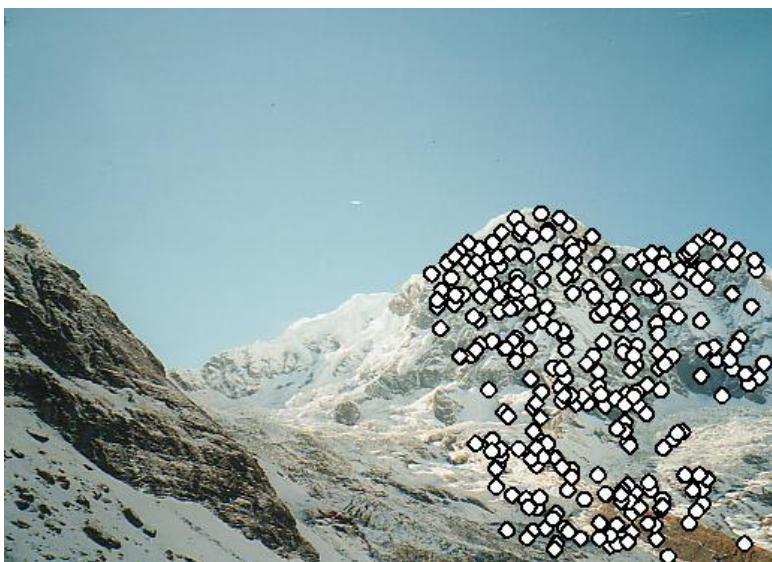
# RANSAC for Homography

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# RANSAC for Homography

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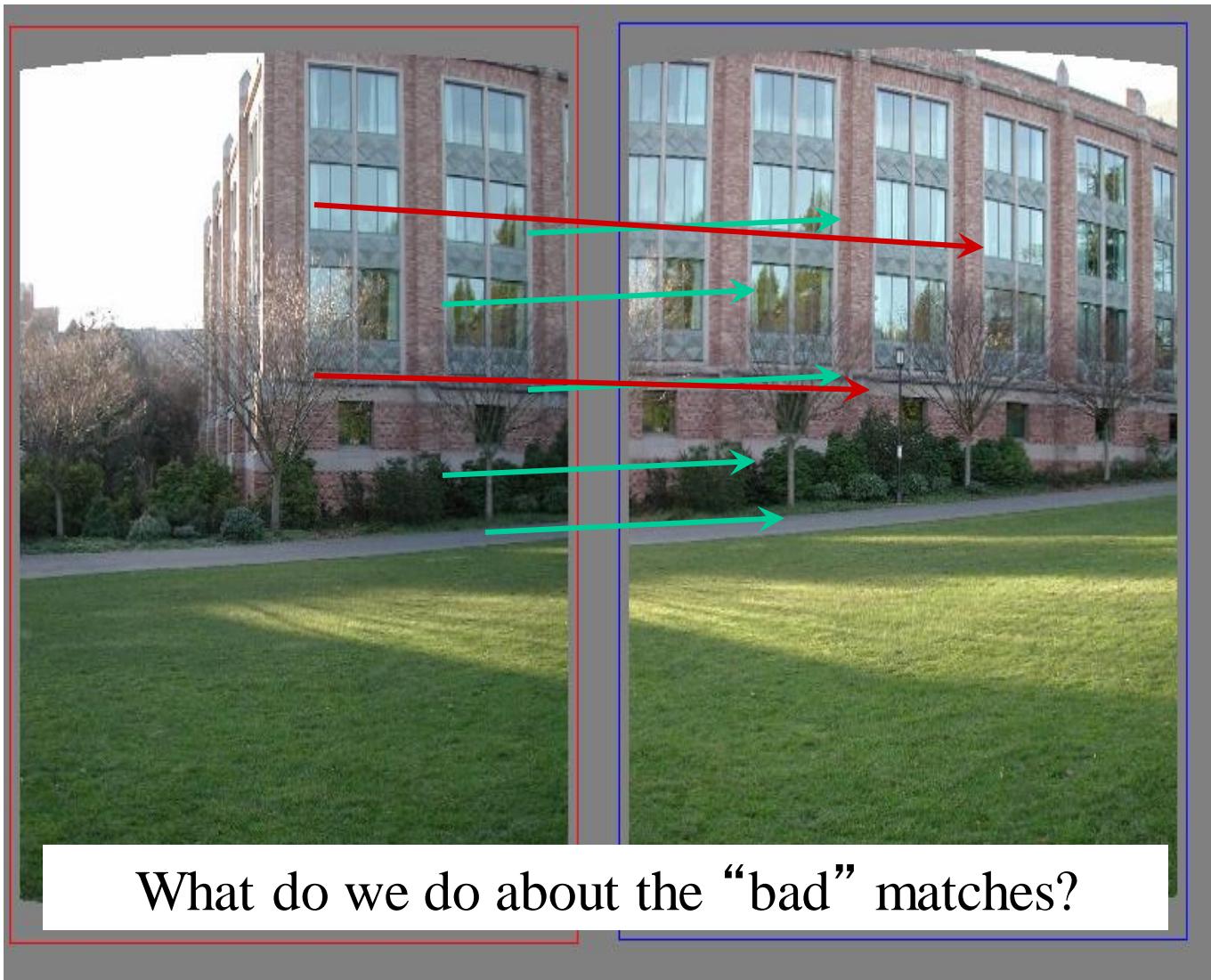
# RANSAC for Homography

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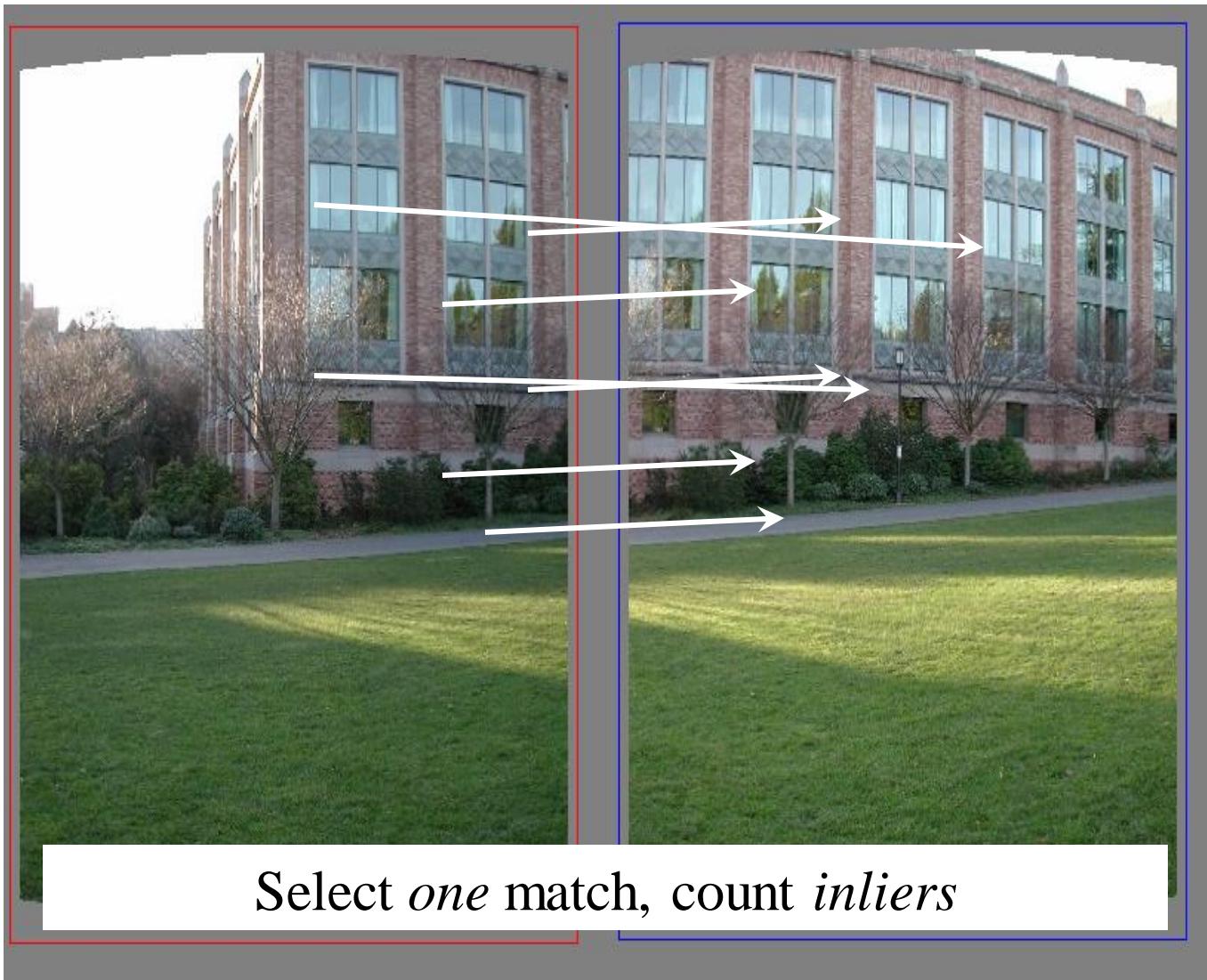


# Matching features

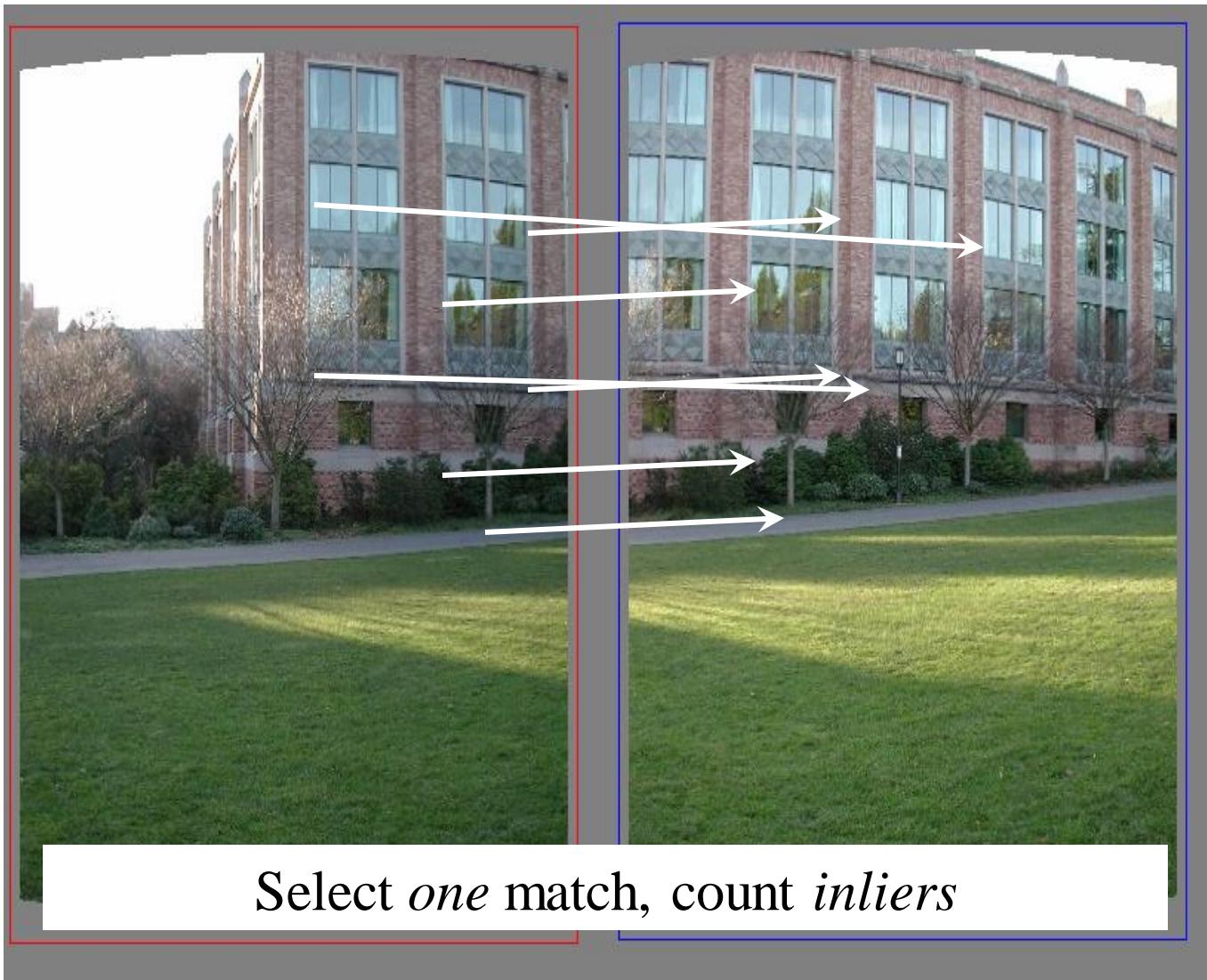
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# Random Sample Consensus

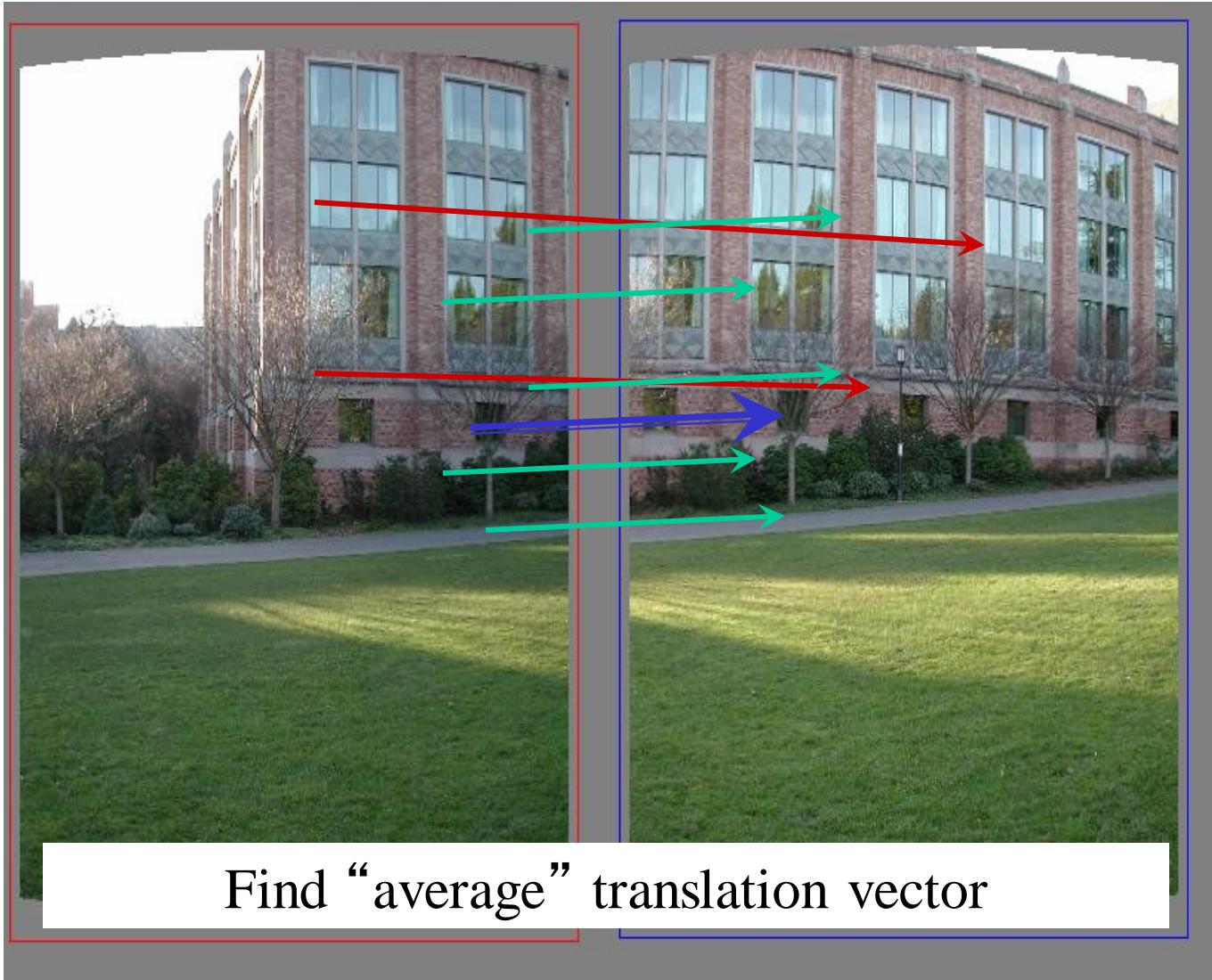


# Random Sample Consensus



# Least squares fit

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# RANSAC for estimating homography

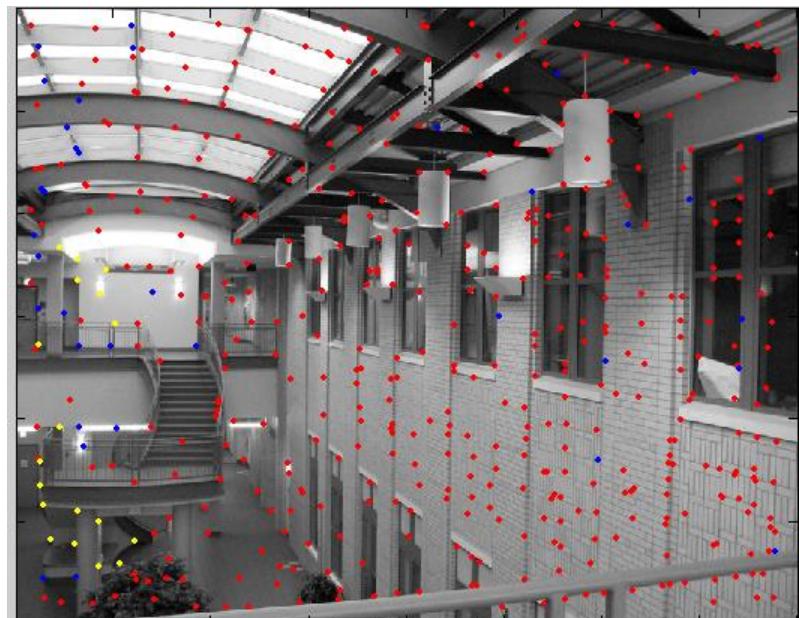
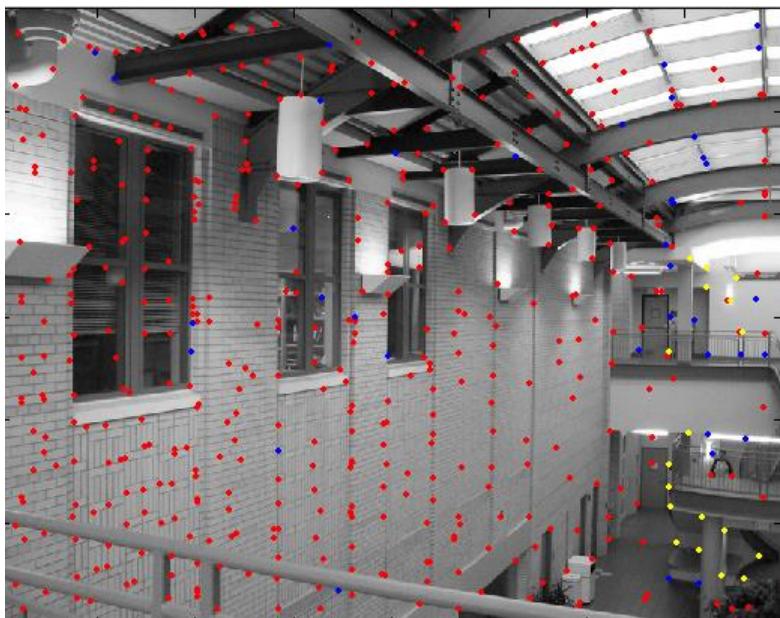
---

RANSAC loop:

- 
1. Select four feature pairs (at random)
  2. Compute homography  $H$  (exact)
  3. Compute *inliers* where  $SSD(p_i', H p_i) < thresh$
  4. Keep largest set of inliers
  5. Re-compute least-squares  $H$  estimate on all of the inliers

# RANSAC

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# RANSAC for estimating homography

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RANSAC loop:

1. Select four feature pairs (at random)
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# Image warping with homographies

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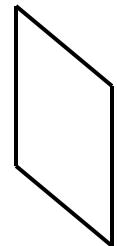
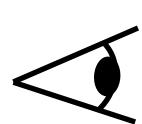
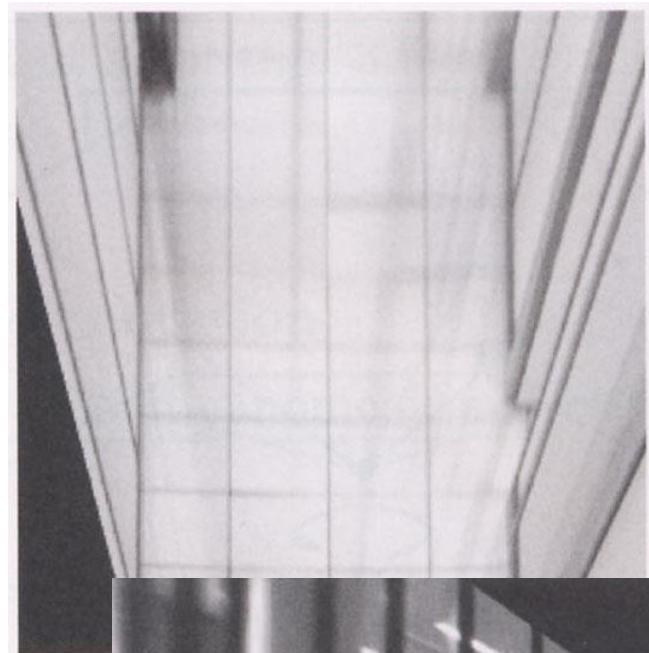
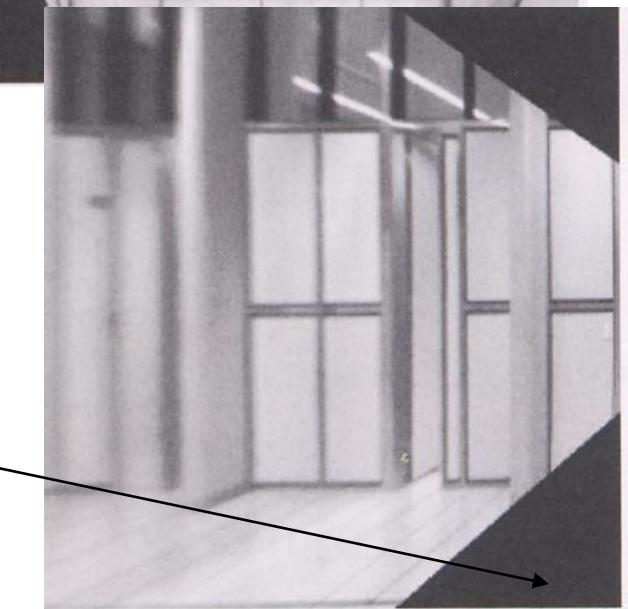


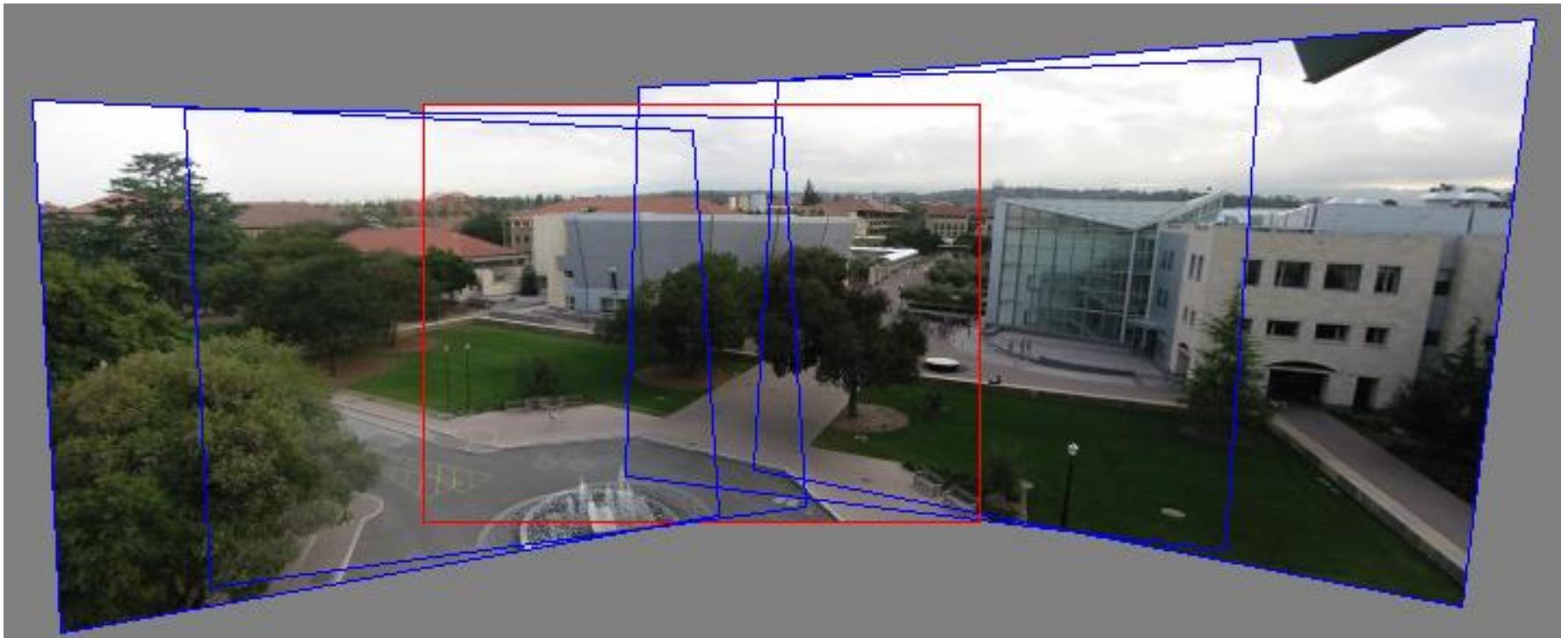
image plane in front

black area  
where no pixel  
maps to



# Panoramas

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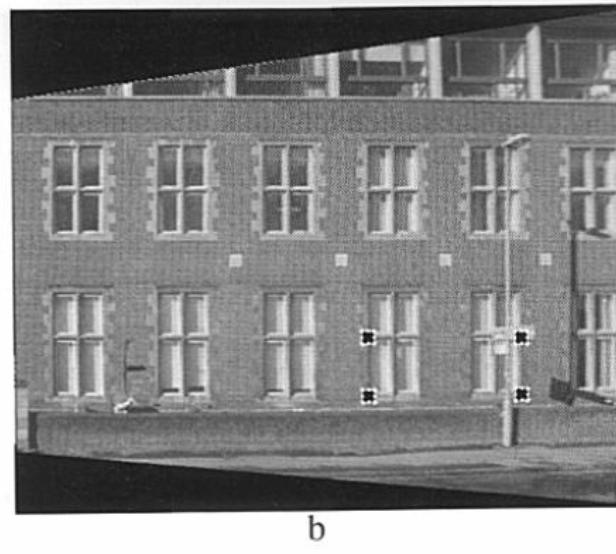
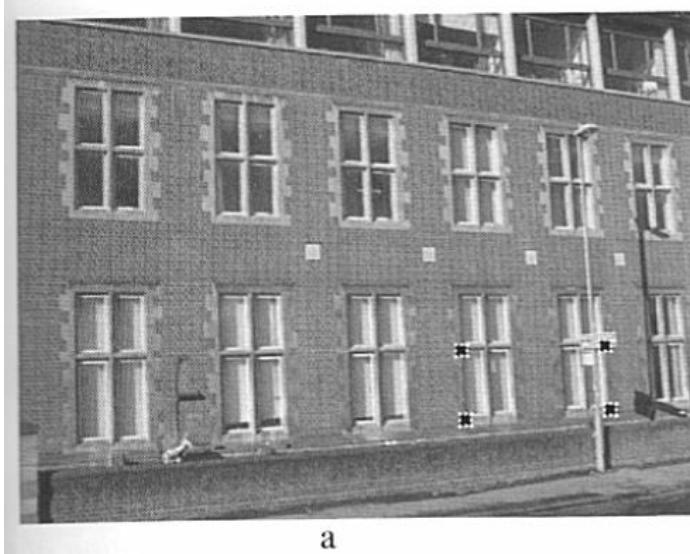
Pick one image (red)

Warp the other images towards it (usually, one by one)

blend

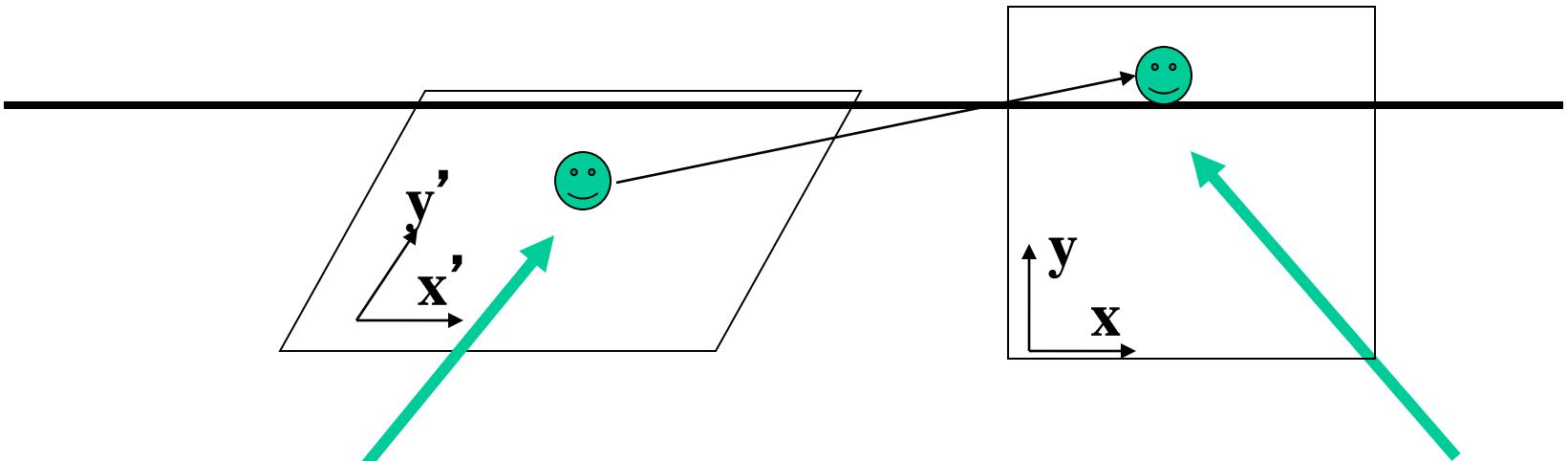
# 4 point algorithm

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$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$$\mathbf{x}' = \mathbf{Hx}$$



$$\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x' = \frac{h_{11}x_1 + h_{12}x_2 + h_{13}x_3}{h_{31}x_1 + h_{32}x_2 + h_{33}x_3}$$

**How many independent para? Can we always set  $h_{33} = 1$ ?**

# 4 points direct solution

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For each point  $\mathbf{x}_i$ , we have

$$\mathbf{H}\mathbf{x}_i = \begin{bmatrix} h^{1T} \mathbf{x}_i \\ h^{2T} \mathbf{x}_i \\ h^{3T} \mathbf{x}_i \end{bmatrix}$$

Since  $\mathbf{x}'_i = (x'_i, y'_i, w'_i) = \mathbf{H}\mathbf{x}_i$  Satisfies:

$$\mathbf{x}'_i \times \mathbf{H}\mathbf{x}_i = 0$$

and

$$\mathbf{x}'_i \times \mathbf{H}\mathbf{x}_i = \begin{bmatrix} y'_i h^{3T} \mathbf{x}_i - w'_i h^{2T} \mathbf{x}_i \\ w'_i h^{1T} \mathbf{x}_i - x'_i h^{3T} \mathbf{x}_i \\ x'_i h^{2T} \mathbf{x}_i - y'_i h^{1T} \mathbf{x}_i \end{bmatrix}$$

# 4 points algorithm

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Rewrite the equation as:

$$\begin{bmatrix} 0^T & -w_i' \mathbf{x}_i^T & y_i' \mathbf{x}_i^T \\ -w_i' \mathbf{x}_i^T & 0^T & -x_i' \mathbf{x}_i^T \\ -y_i' \mathbf{x}_i^T & x_i' \mathbf{x}_i^T & 0^T \end{bmatrix} \begin{bmatrix} h^1 \\ h^2 \\ h^3 \end{bmatrix} = 0$$

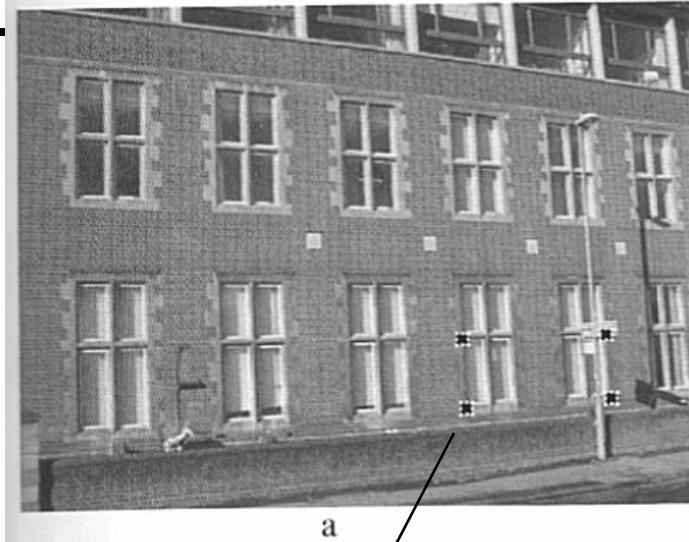
$A_i \mathbf{h} = 0$



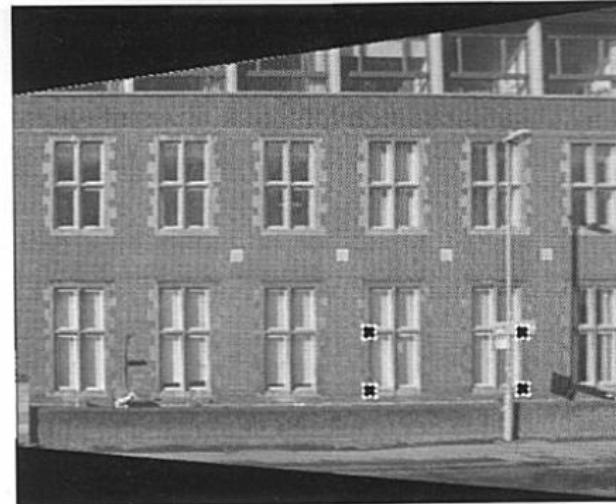
1 point gives two independent equations,

H has 8 independent parameters => need 4 points

# 4 point algorithm



a



b

$$A = \left( \begin{array}{ccc} 0^T & -w_i' x_i^T & y_i' x_i^T \\ -w_i' x_i^T & 0^T & -x_i' x_i^T \\ -y_i' x_i^T & x_i' x_i^T & 0^T \end{array} \right)$$

$$\begin{bmatrix} h^1 \\ h^2 \\ h^3 \end{bmatrix}$$

**Compute**  
 $[v, d] = \text{Eig}(A^T A),$   
**set  $h = \text{eigenvector}$**   
**with the smallest**  
**eigenvalue**

# Overview

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Feature Matching

## Image Matching

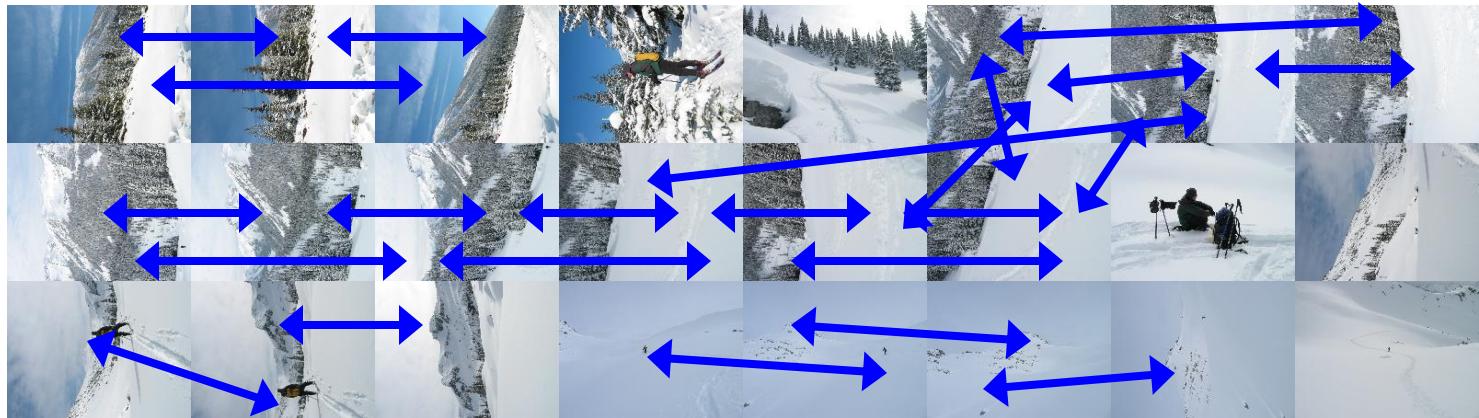
- RANSAC for Homography

Multi-band Blending

Results

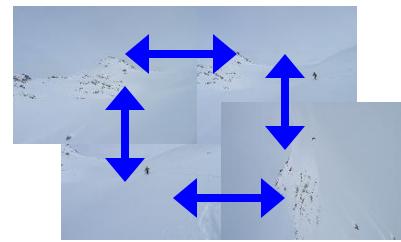
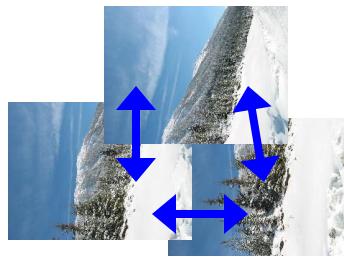
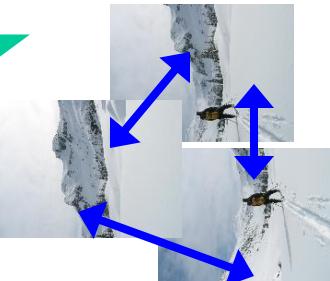
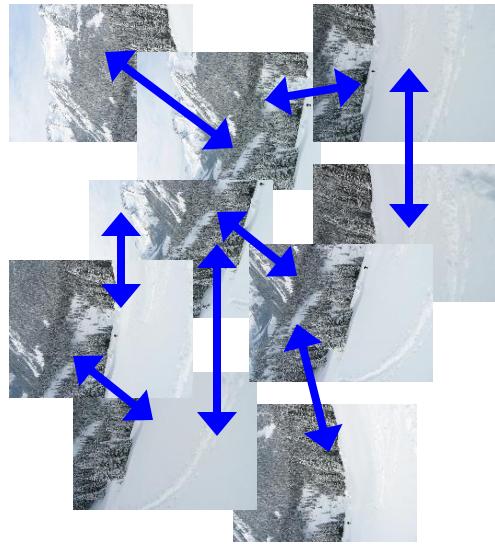
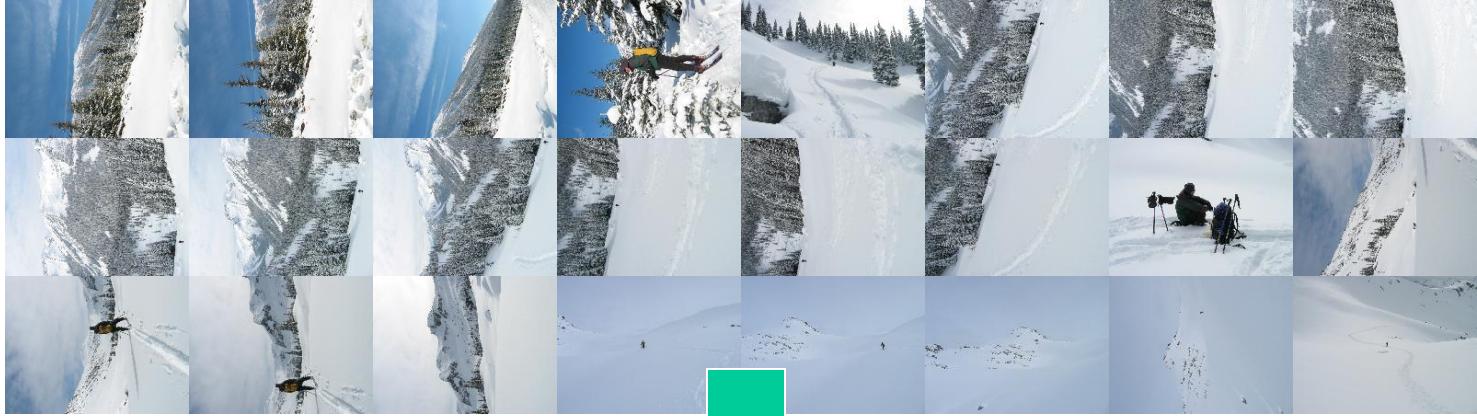
# Finding the panoramas

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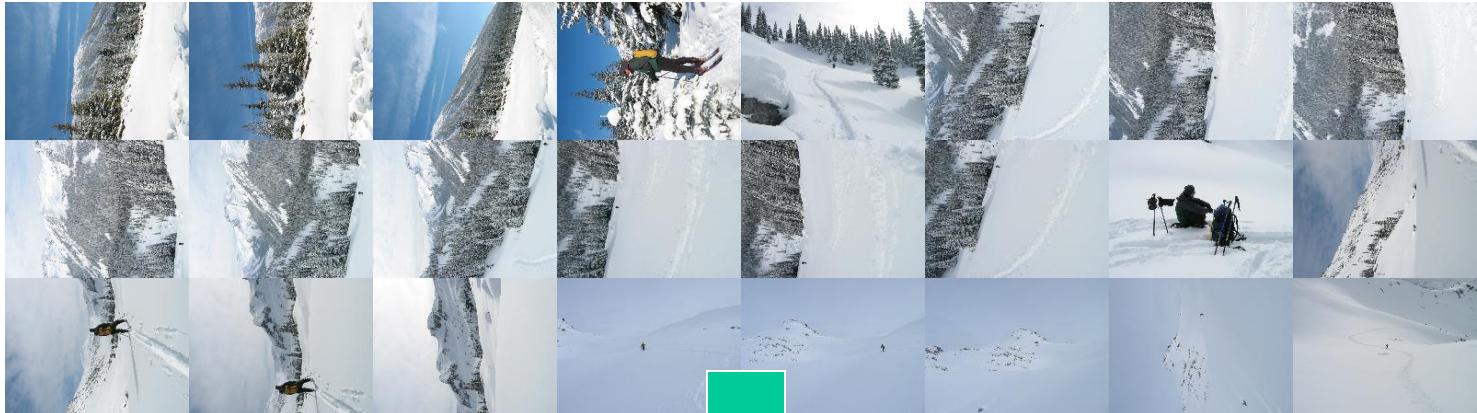
# Finding the panoramas

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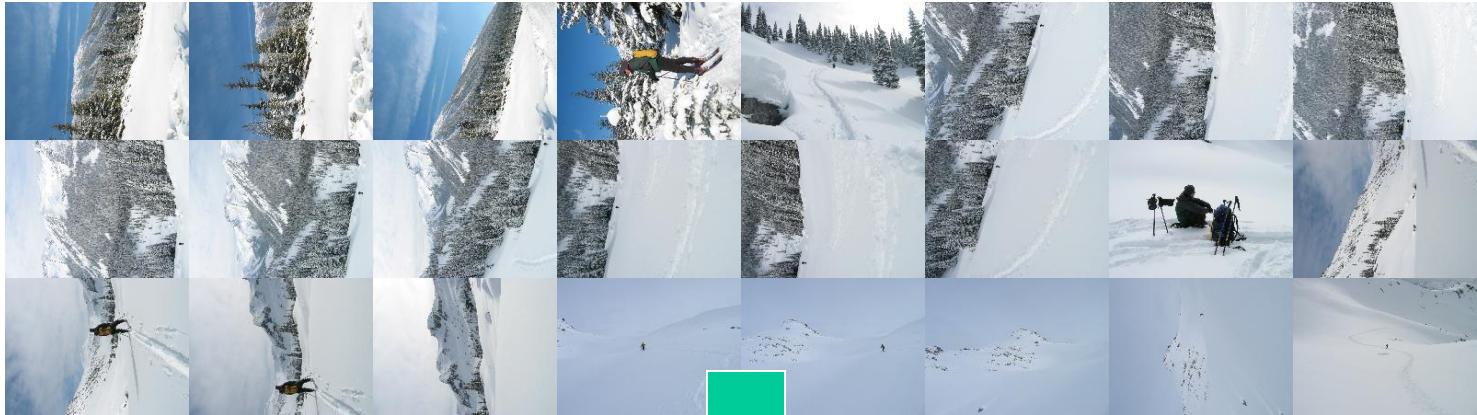
# Finding the panoramas

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# Finding the panoramas

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# Multi-band Blending

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Burt & Adelson 1983

- Blend frequency bands over range  $\propto \lambda$



# 2-band Blending

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Low frequency ( $\lambda > 2$  pixels)



High frequency ( $\lambda < 2$  pixels)

# Linear Blending



# 2-band Blending









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# Results

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