Statistical Inference Course Project

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```
setwd("/Users/yulong/GitHub/Statistical-Inference-Course-Project")
```

Part 1: Simulation Exercise Instructions

Instructions

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should

- Show the sample mean and compare it to the theoretical mean of the distribution.
- Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.

Loading Libraries

```
library("data.table")
library("ggplot2")
```

Task

```
# set seed for reproducability
set.seed(31)
# set lambda to 0.2
lambda <- 0.2
# 40 samples
n <- 40
# 1000 simulations
```

```r
simulations <- 1000
# simulate
simulated_exponentials <- replicate(simulations, rexp(n, lambda))
# calculate mean of exponentials
means_exponentials <- apply(simulated_exponentials, 2, mean)
```

Question 1

Show where the distribution is centered at and compare it to the theoretical center of the distribution.

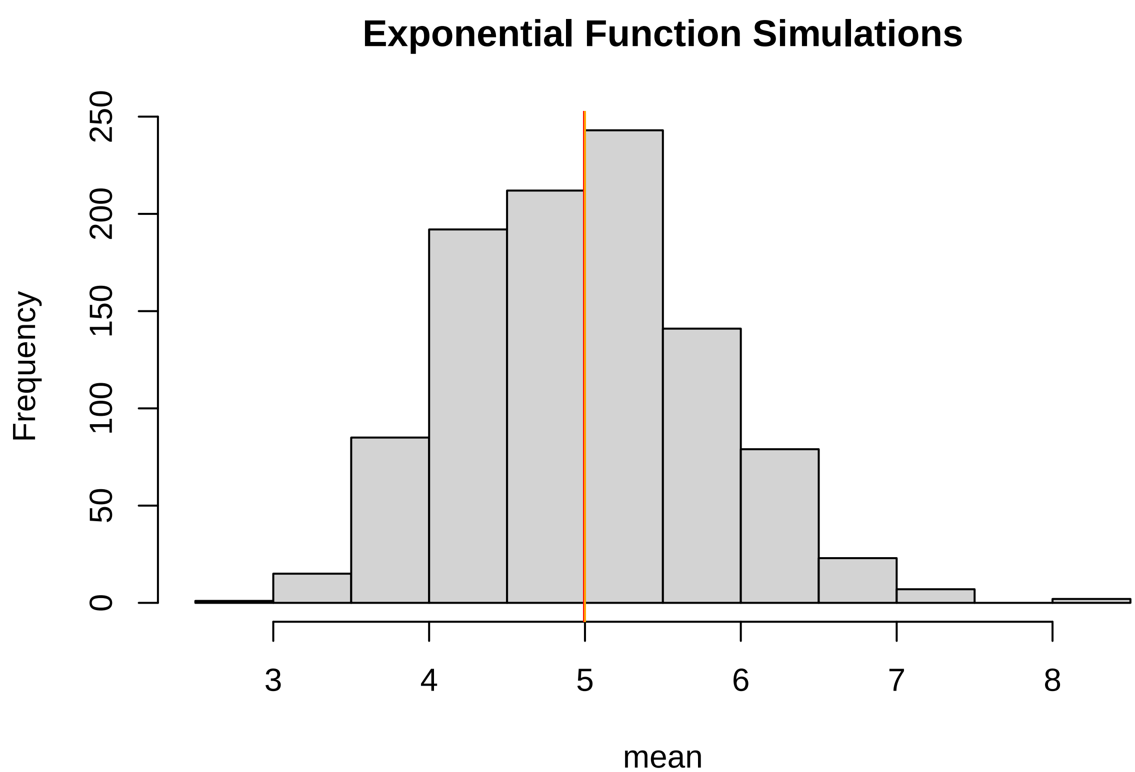
```r
analytical_mean <- mean(means_exponentials)
analytical_mean
```

```
## [1] 4.993867
```

```r
# analytical mean
theory_mean <- 1/lambda
theory_mean
```

```
## [1] 5
```

```r
# visualization
hist(means_exponentials, xlab = "mean", main = "Exponential Function Simulations")
abline(v = analytical_mean, col = "red")
abline(v = theory_mean, col = "orange")
```



The analytics mean is 4.993867 the theoretical mean 5. The center of distribution of averages of 40 exponentials is very close to the theoretical center of the distribution.

Question 2

Show how variable it is and compare it to the theoretical variance of the distribution..

```
# standard deviation of distribution
standard_deviation_dist <- sd(means_exponentials)
standard_deviation_dist
```

```
## [1] 0.7931608
```

```
# standard deviation from analytical expression
standard_deviation_theory <- (1/lambda)/sqrt(n)
standard_deviation_theory
```

```
## [1] 0.7905694
```

```
# variance of distribution
variance_dist <- standard_deviation_dist^2
variance_dist
```

```
## [1] 0.6291041
```

```
# variance from analytical expression
variance_theory <- ((1/lambda)*(1/sqrt(n)))^2
variance_theory
```

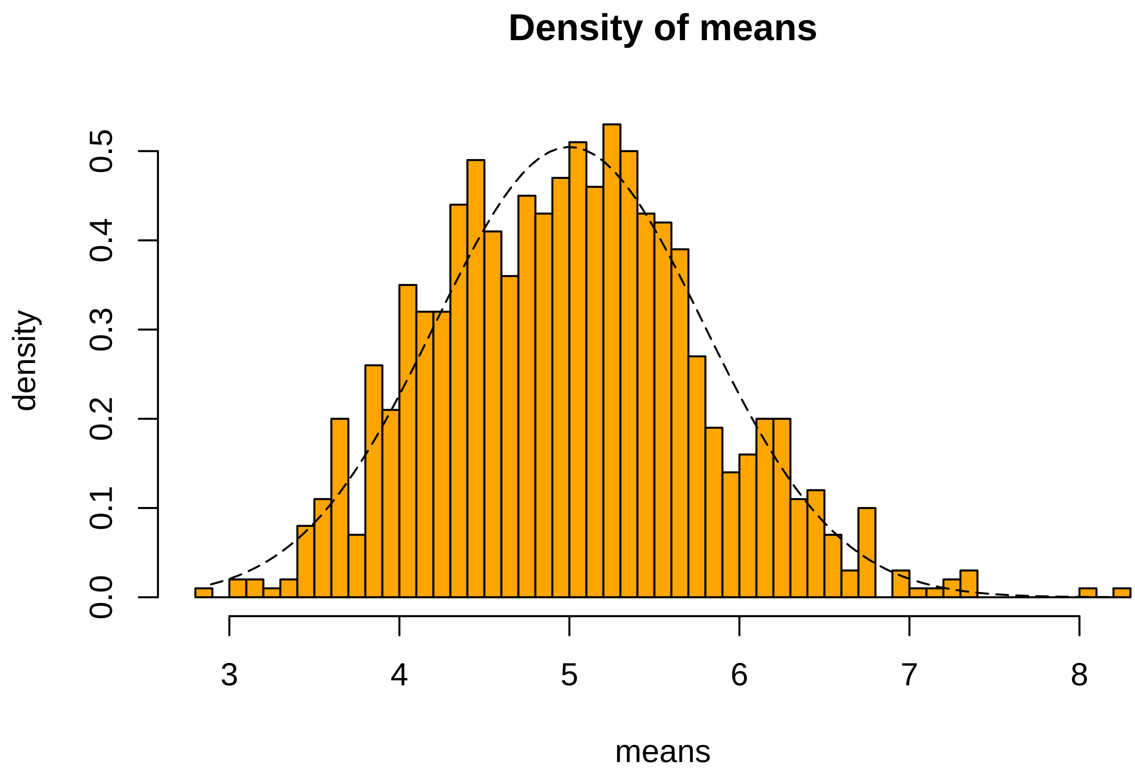
```
## [1] 0.625
```

Standard Deviation of the distribution is 0.7931608 with the theoretical SD calculated as 0.7905694. The Theoretical variance is calculated as ((1 / lambda) * (1/n2))2 = 0.625. The actual variance of the distribution is 0.6291041

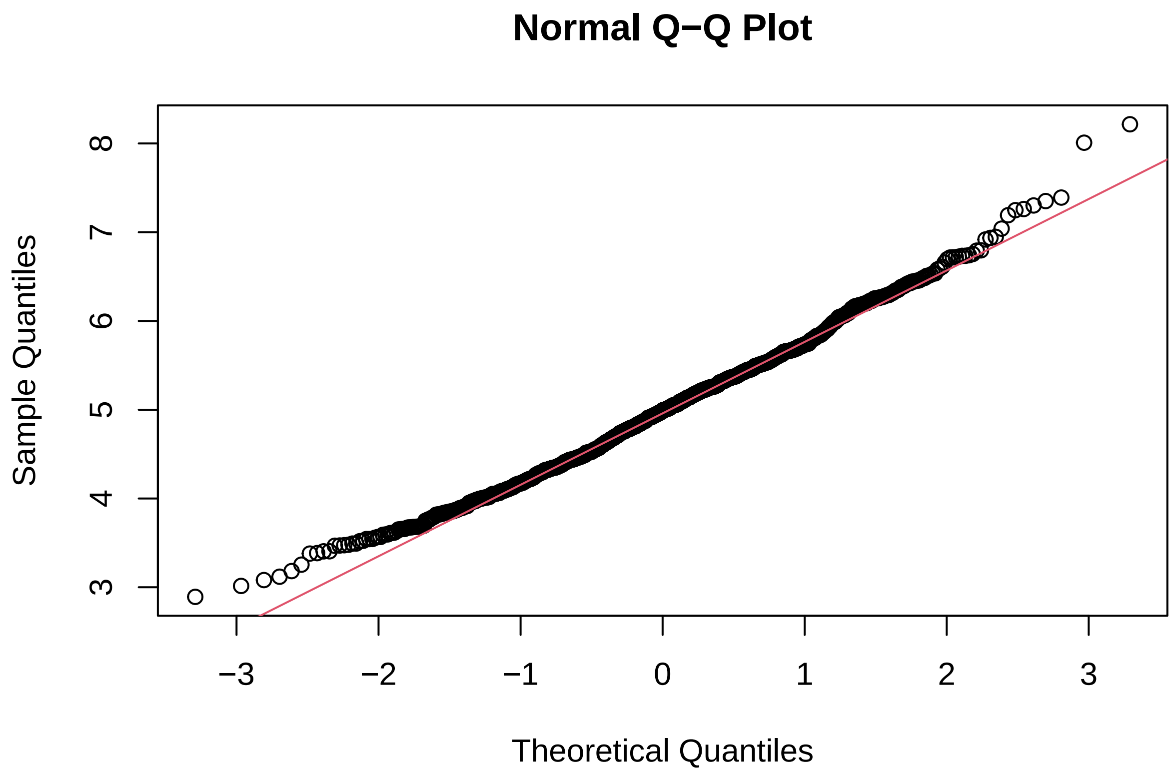
Question 3

Show that the distribution is approximately normal.

```
xfit <- seq(min(means_exponentials), max(means_exponentials), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(n)))
hist(means_exponentials,breaks=n,prob=T,col="orange",
     xlab = "means",main="Density of means",ylab="density")
lines(xfit, yfit, pch=22, col="black", lty=5)
```




```r
# compare the distribution of averages of 40 exponentials to a normal distribution
qqnorm(means_exponentials)
qqline(means_exponentials, col = 2)
```



Due to Due to the central limit theorem (CLT), the distribution of averages of 40 exponentials is very close to a normal distribution.