LITTORAL MATHEMATICS TEACHERS ASSOCIATION
(LI.M.T.A)

GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION

0775 FURTHER MATHEMATICS 3

APRIL 2021 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**3**
Subject Code No.	**0775**

THREE HOURS

Full marks may be obtained for answers to ALL questions.

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

---

1. Forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are such that $\mathbf{F}_1$ has magnitude $6N$ and acts in the direction of the vector $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ through the point with position vector $\mathbf{r}_1=[3\mathbf{i}+2\mathbf{j}-4\mathbf{k}]m$, $\mathbf{F}_2=[-\mathbf{i}+3\mathbf{j}+\mathbf{k}]N$ acts through the point with position vector $\mathbf{r}_2=[3\mathbf{i}-3\mathbf{j}-3k]m$, and $\mathbf{F}_3$ has magnitude $7N$ and acts along the line $l:\mathbf{r}=[8\mathbf{i}+3\mathbf{j}]+\mu[6\mathbf{i}+3\mathbf{j}+2\mathbf{k}]$.

(i) Calculate the work done by $\mathbf{F}_1$ in moving a particle to the point with position vector $[4\mathbf{i}-3\mathbf{j}-2\mathbf{k}]m$ **(3marks)**

(ii) Show that the forces $\mathbf{F}_2$ and $\mathbf{F}_3$ are concurrent **(3marks)**

This system of three forces is equivalent to a force $\mathbf{F}_4$ acting through the point with position vector $\mathbf{r}_4=[3\mathbf{i}-2\mathbf{j}-2\mathbf{k}]m$, together with a couple **G.**

(iii) Find $\mathbf{F}_4$ and $\mathbf{G}$ **(6marks)**

2.

(a) Given that $4(1+x)\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ and that $y=1$, $\frac{dy}{dx}=0$ when $x=0$.

Find as a series of ascending powers of x up to and including the term in x^3, the solution of this differential equation **(4marks)**

Hence using the approximations $h^2\left(\frac{d^2y}{dx^2}\right)_n \approx y_{n+1}-2y_n+y_n$ and $2h\left(\frac{dy}{dx}\right)=y_{n+1}-y_n$ and a step length of 0.2, find correct to 3 decimal places, the approximated value of y when $x=0.4$ **(5marks)**

(b) The table below gives the values of x and their corresponding values of y for a continuous function $y=f(x)$

x	-1	-0.5	0	0.5	1
y	5.4	4.6	4.2	1.2	6.0

Using Simpson's rule, find the mean value of y for $-1\le x\le 1$ **(4marks)**

3. A particle P of mass m moves in a straight line. At time t seconds, the displacement of P from a fixed point O on the line is x metres and P is moving with velocity $\dot{x}ms^{-1}$. Throughout the motion, two horizontal forces act on P. A force of magnitude $2mn|\dot{x}|$ directed towards O, and a resistance force $3mn^2|x|$, $n>0$.

(i) Show that x satisfies $\frac{d^2x}{dt^2}+2n\frac{dx}{dt}+3n^2x=0$ **(2marks)**

(ii) Given that $x=a$, $\dot{x}=0$ when $t=0$, show that

$$x(t)=ae^{-nt}\left[\cos\left(\sqrt{2}nt\right)+\frac{\sqrt{2}}{2}\sin\left(\sqrt{2}nt\right)\right]$$ **(6marks)**

(iii) Show that P passes through O when $\tan\left(\sqrt{2}nt\right)=-\sqrt{2}$ **(2marks)**

(iv) Find the period of oscillation for $n=\sqrt{2}$ **(2marks)**

4. A truck of mass m starts from rest and moves along a straight level road, with engine working at a constant power rate P. The total resistance to the motion is kv, where v is the speed of the truck. If the maximum speed of the truck on this road is u,

(i) Show that, when the truck is travelling at a speed v, the total resistance to its motion is $\frac{Pv}{u^2}$ **(2marks)**

(ii) Show also that the equation of motion of the truck is $m\left[\frac{v^2}{u^2-v^2}\right]\frac{dv}{dx}=\frac{p}{u^2}$ **(2marks)**

(iii) Show further that in order to raise the speed from rest to $\frac{u}{2}$, the truck travels a distance

$$\frac{mu^3}{2P}\left[\ln 3-1\right]$$ **(5marks)**

(iv) Show that the time taken by the truck to cover this distance is $\frac{mu^2}{2P}\ln\left(\frac{4}{3}\right)$ **(4marks)**

---

5. A smooth sphere A of mass $2m$ moving on a smooth horizontal floor with velocity $(5\mathbf{i}+10\mathbf{j})ms^{-1}$ impinges obliquely on a smooth stationary sphere B of mass m. At the instant of collision, the line joining the centres of the spheres is parallel to the vector $4\mathbf{i}+3\mathbf{j}$. Given that the coefficient of restitution between the two spheres is e and that the direction of motion of sphere A after impact, makes an angle α with the centres of the spheres, show that

(a)

(i) $(2-e)\tan\alpha=3$ **(7marks)**

(ii) $\frac{3}{2}\le\tan\alpha\le 3$ **(3marks)**

(b) Given that $\tan\alpha=2$, find the value of e and hence the velocity of A after impact. **(3marks)**

6.

(a) A particle moves in a plane so that at time t, its polar coordinates (r,θ) are given by $r=2+\sin\theta$, $\theta=4t$

Find the speed of the particle when $\theta=\frac{\pi}{2}$ **(4marks)**

(b) **The** polar coordinates of a particle P moving in a plane are such that at time t,

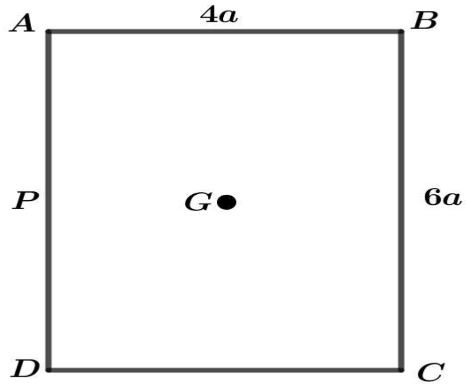
$$\frac{dr}{dt}=8\sin\theta \quad \textbf{and} \quad \frac{d\theta}{dt}=(1-4\cos\theta)^2$$

If $r=2$ when $\theta=\frac{\pi}{3}$, show that the polar equation of the curve is given by

$$r=\frac{2}{4\cos\theta-1}$$

Hence or otherwise, find the transverse component of the acceleration of P when $\theta=\frac{\pi}{2}$, **(7marks)**

7. **A** uniform rectangular lamina ABCD has mass M , with $AB = 4A$ and $BC = 6A$. The centre of mass of the lamina is G, and the midpoint of AD is P



(a)

(i) Show that the moment of inertia of the rectangular lamina about an axis perpendicular to its plane and passing through G is $\frac{13Ma^2}{3}$ **(2marks)**

(ii) Hence, find the moment of inertia of the lamina about an axis perpendicular to its plane and passing through P **(2marks)**

(b) The lamina is smoothly hinged at P and set free to rotate in a vertical plane about a fixed horizontal axis which is perpendicular to its plane and passing through P. Initially the lamina is held with PG horizontal and then released. At time t after its release, PG makes an angle θ with the horizontal

(i) Show that $\dot{\theta}^2 = \frac{12g\sin\theta}{25a}$ **(3marks)**

(ii) Hence or otherwise, determine an expression for $\ddot{\theta}$, in terms of a , g and θ . **(2marks)**

(c) Show that the magnitude of the component of the force at P in the direction PA which the hinge exerts on the lamina is $\frac{13Mg\cos\theta}{25}$. **(3marks)**

8.

(a) A continuous random variable X has probability density function

$$f(x) = \begin{cases} 6x(1-x), & \text{if } 0 \le x \le k \\ 0, & \text{elsewhere} \end{cases}$$

Find :

(i) the value of the constant k **(4marks)**

(ii) the cumulative distribution function, $F(X)$ **(2marks)**

(b) The masses of articles produced in a particular factory are normally distributed with mean μ and standard deviation σ . Given that 10% of the articles have a mass greater than 82grams and 5% have a mass less than 36grams .

(a) Find the values of μ and σ , correct to 2 significant figures **(5marks)**

Hence or otherwise,

(b) Find the probability that any selected article, have a mass in the interval $[50, 75]$ **(3marks)**

END!