Data Mining: Data (Chapter 2)

郑子彬　副教授
中山大学 数据科学与计算机学院
zhzibin@mail.sysu.edu.cn
2016年

Project: Crime Classification

- 组队要求：3个小组成员，如有特殊情况（如不够人）可向TA申请
- 组队信息3月12日前发送到邮箱：dm2016sysu@sina.com
- TA统计好组队信息后将邮件通知各小组组号，Kaggle注册账号请使用DM_组号（如DM_001），评分将根据leaderboard上指定的账号名的分数排名

Example

You receive an email from a medical researcher concerning a project that you are eager to work on.

Hi,

I've attached the data file.

Each line contains the information for a single patient and consists of five fields.

We want to predict the last field using the other fields.

Thanks and see you in a couple of days.

Example

The first few rows of the file are as follows:

012	232	33.5	0	10.7
020	121	16.9	2	210.1
027	165	24.0	0	427.6
⋮				

Nothing looks strange. You put your doubts aside and start the analysis.

Two days later you arrive for the meeting, and before the meeting, you strike up a conversation with a statistician who is working on the project.

Example

Statistician: So, you got the data for all the patients?

Data Miner: Yes. I haven't had much time for analysis, but I do have a few interesting results.

Statistician: Amazing. There were so many data issues with this set of patients that I couldn't do much.

Data Miner: Oh? I didn't hear about any possible problems.

Statistician: But surely you heard about what happened to field 4? It's supposed to be measured on a scale from 1 to 10, with 0 indicating a missing value, but because of a data entry error, all 10's were changed into 0's.

Data Miner: Interesting. Were there any other problems?

Statistician: Yes, fields 2 and 3 are basically the same, but I assume that you probably noticed that.

Data Miner: Yes, but these fields were only weak predictors of field 5.

012	232	33.5	0	10.7
020	121	16.9	2	210.1
027	165	24.0	0	427.6
⋮				

Example

Statistician: Anyway, given all those problems, I'm surprised you were able to accomplish anything.

Data Miner: True, but my results are really quite good. Field 1 is a very strong predictor of field 5. I'm surprised that this wasn't noticed before.

Statistician: What? Field 1 is just an identification number.

Data Miner: Nonetheless, my results speak for themselves.

Statistician: Oh, no! I just remembered. We assigned ID numbers after we sorted the records based on field 5. There is a strong connection, but it's meaningless. Sorry.

Lesson: Get to know your data!

012	232	33.5	0	10.7
020	121	16.9	2	210.1
027	165	24.0	0	427.6
.				

What is Data?

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
  - Examples: eye color of a person, temperature, etc.
  - Attribute is also known as variable, field, characteristic, or feature
- A collection of attributes describe an object
  - Object is also known as record, point, case, sample, entity, or instance

Attributes

Objects

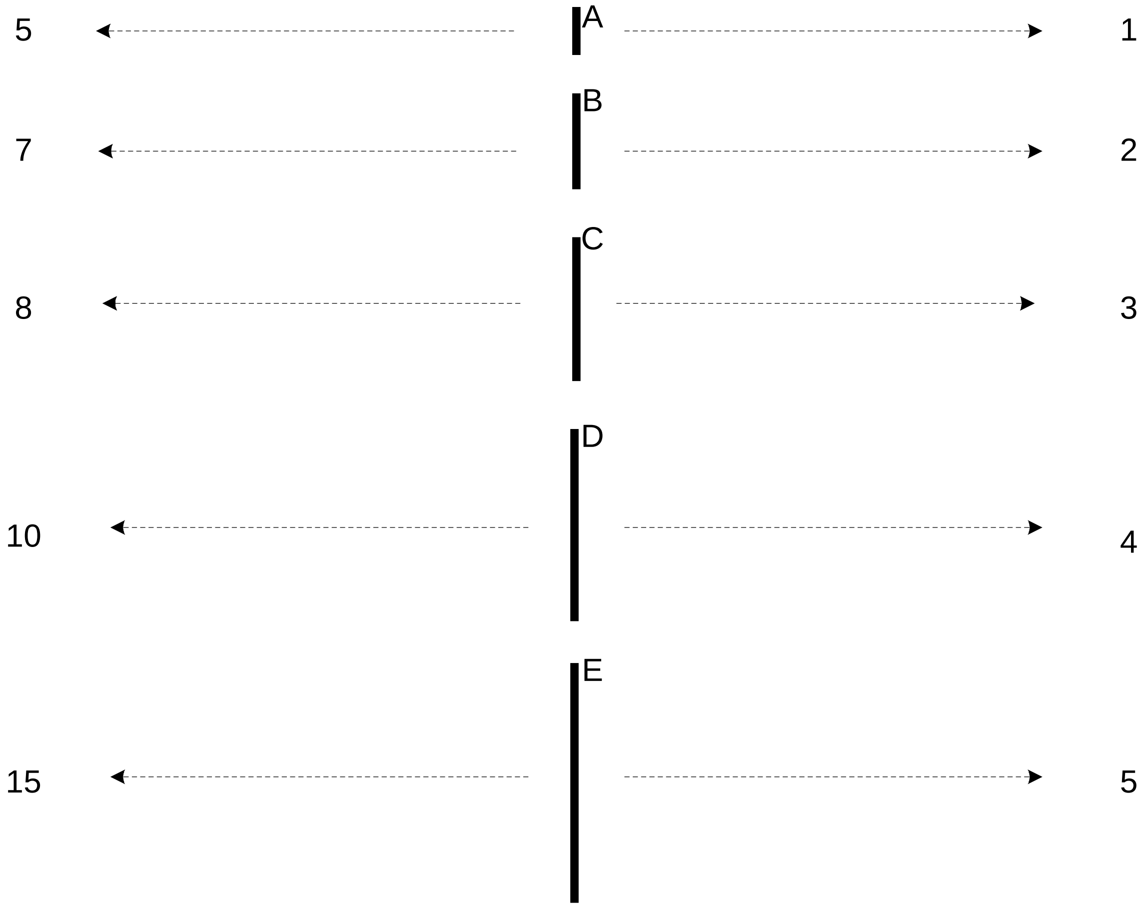
Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Attribute Values

- Attribute values are numbers or symbols assigned to an attribute
- Distinction between attributes and attribute values
  - Same attribute can be mapped to different attribute values
    - Example: height can be measured in feet or meters
  - Different attributes can be mapped to the same set of values
    - Example: Attribute values for ID and age are integers
    - But properties of attribute values can be different
      - ID has no limit but age has a maximum and minimum value

Measurement of Length

- **The way you measure an attribute is somewhat may not match the attributes properties.**



Types of Attributes

- There are different types of attributes
  - Nominal
    - Examples: ID numbers, eye color, zip codes
  - Ordinal
    - Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
  - Interval
    - Examples: calendar dates, temperatures in Celsius or Fahrenheit.
  - Ratio
    - Examples: temperature in Kelvin, length, time, counts

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
  - Distinctness: $=$ $\neq$
  - Order: $<$ $>$
  - Addition: $+$ $-$
  - Multiplication: $*$ $/$

  - Nominal attribute: distinctness
  - Ordinal attribute: distinctness & order
  - Interval attribute: distinctness, order & addition
  - Ratio attribute: all 4 properties

Attribute Type	Description	Examples	Operations
Nominal	The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. (=, ≠)	zip codes, employee ID numbers, eye color, sex: {*male, female*}	mode, entropy, contingency correlation, χ^2 test
Ordinal	The values of an ordinal attribute provide enough information to order objects. (<, >)	hardness of minerals, {*good, better, best*}, grades, street numbers	median, percentiles, rank correlation, run tests, sign tests
Interval	For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. (+, -)	calendar dates, temperature in Celsius or Fahrenheit	mean, standard deviation, Pearson's correlation, *t* and *F* tests
Ratio	For ratio variables, both differences and ratios are meaningful. (*, /)	temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current	geometric mean, harmonic mean, percent variation

Attribute Level	Transformation	Comments
Nominal	Any one-to-one mapping	If all employee ID numbers were reassigned, would it make any difference?
Ordinal	An order preserving change of values, i.e., *new_value = f(old_value)* where *f* is a monotonic function.	An attribute encompassing the notion of good, better best can be represented equally well by the values {1, 2, 3} or by { 0.5, 1, 10}.
Interval	*new_value =a * old_value + b* where a and b are constants	Thus, the Fahrenheit and Celsius temperature scales differ in terms of where their zero value is and the size of a unit (degree).
Ratio	*new_value = a * old_value*	Length can be measured in meters or feet.

Discrete and Continuous Attributes

- Discrete Attribute
  - Has only a finite or countably infinite set of values
  - Examples: zip codes, counts, or the set of words in a collection of documents
  - Often represented as integer variables.
  - Note: binary attributes are a special case of discrete attributes

- Continuous Attribute
  - Has real numbers as attribute values
  - Examples: temperature, height, or weight.
  - Practically, real values can only be measured and represented using a finite number of digits.
  - Continuous attributes are typically represented as floating-point variables.

Important Characteristics of Structured Data

– **Dimensionality**
  - **Curse of Dimensionality**

– **Sparsity**
  - **Only presence counts**

– **Resolution**
  - **Patterns depend on the scale**

Types of data sets

- **Record**
  - Data Matrix
  - Document Data
  - Transaction Data
- **Graph**
  - World Wide Web
  - Molecular Structures
- **Ordered**
  - Spatial Data
  - Temporal Data
  - Sequential Data
  - Genetic Sequence Data

Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
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10	No	Single	90K	**Yes**

Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute

- Such data set can be represented by an m by n matrix, where there are m rows, one for each object, and n columns, one for each attribute

Projection of x Load	Projection of y load	Distance	Load	Thickness
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Document Data

- Each document becomes a `term' vector,
  - each term is a component (attribute) of the vector,
  - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	pla y	ball	score	game	wi n	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

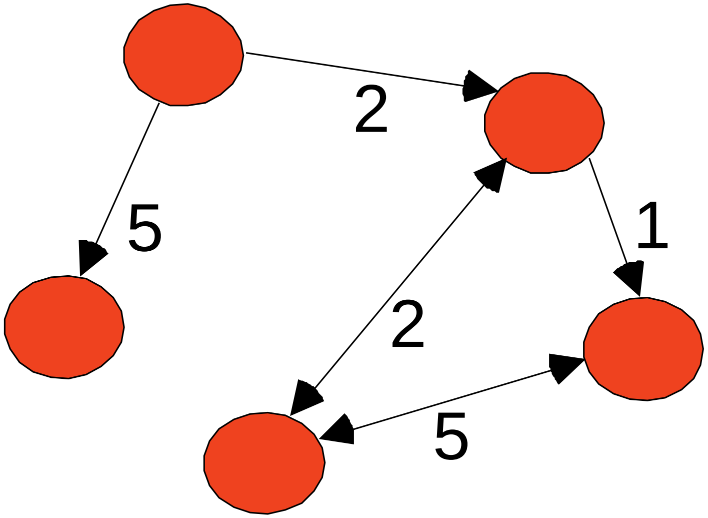
Transaction Data

- A special type of record data, where
  - each record (transaction) involves a set of items.
  - For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Graph Data

- Examples: Generic graph and HTML Links



Data Mining

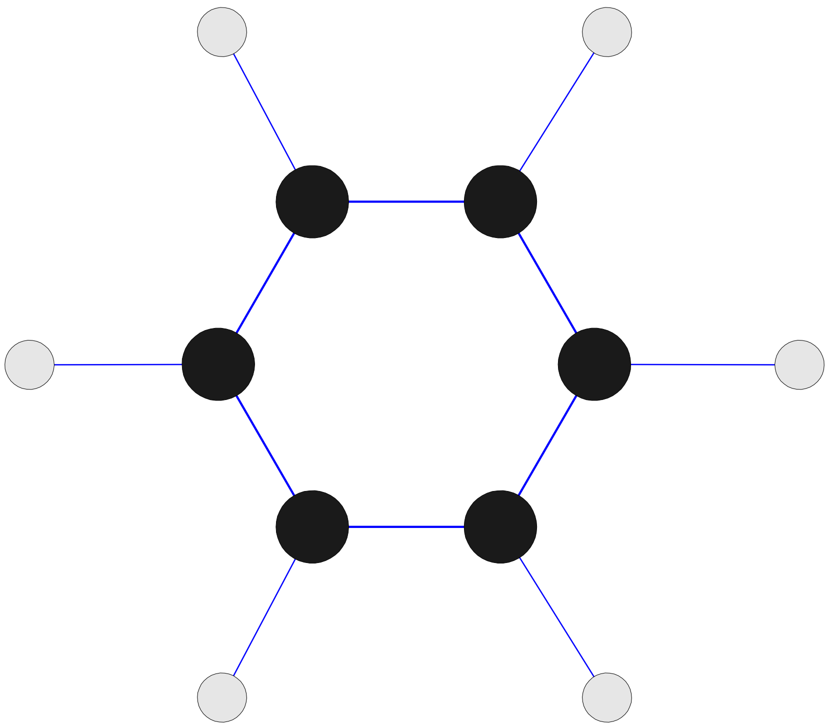
Graph Partitioning

Parallel Solution of Sparse Linear System of Equations

<a href="papers/papers.html#ffff">
N-Body Computation and Dense Linear System Solvers

Chemical Data

- Benzene Molecule（苯分子）: C_6H_6



Ordered Data: Sequential Data

- Sequential Data

Time	Customer	Items Purchased
t1	C1	A, B
t2	C3	A, C
t2	C1	C, D
t3	C2	A, D
t4	C2	E
t5	C1	A, E

Customer	Time and Items Purchased
C1	(t1: A,B) (t2:C,D) (t5:A,E)
C2	(t3: A, D) (t4: E)
C3	(t2: A, C)

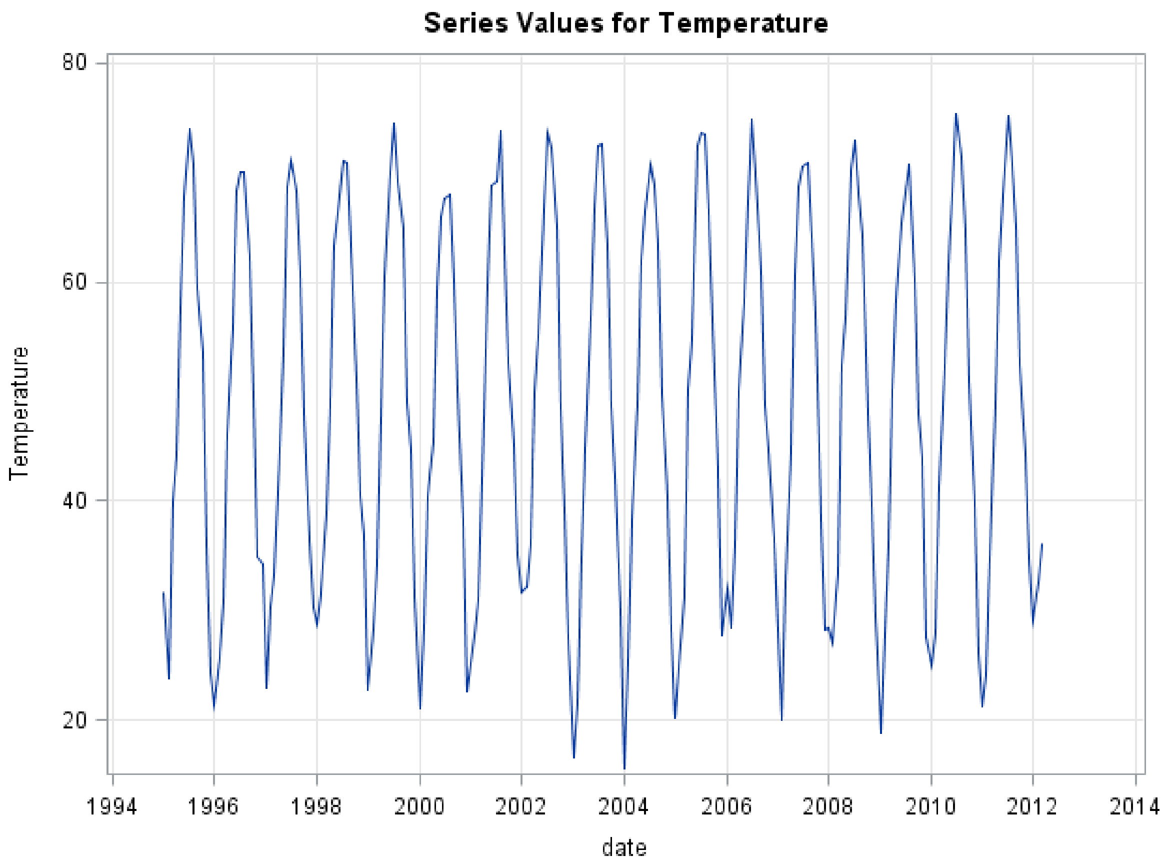
Ordered Data: Sequence Data

- Genomic sequence data

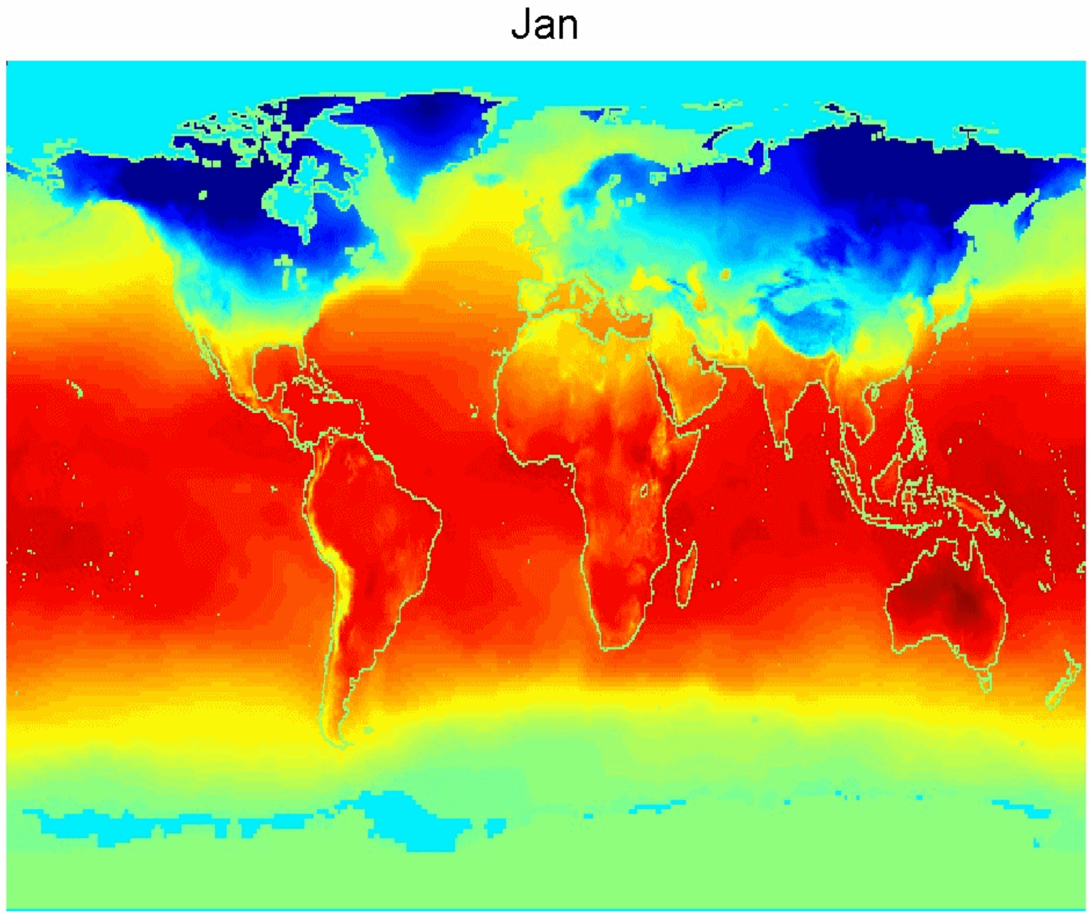
```
GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG
```

Ordered Data: Time Series Data

- Special type of sequential data
- Temporal autocorrelation



Ordered Data: Spatio-Temporal Data



Average Monthly Temperature of land and ocean

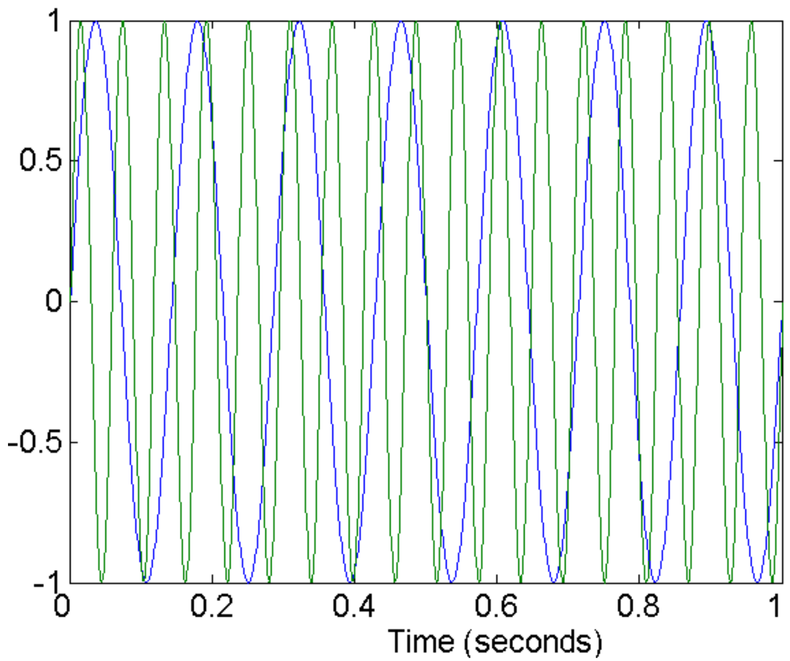
Data Quality

- What kinds of data quality problems?
- How can we detect problems with the data?
- What can we do about these problems?

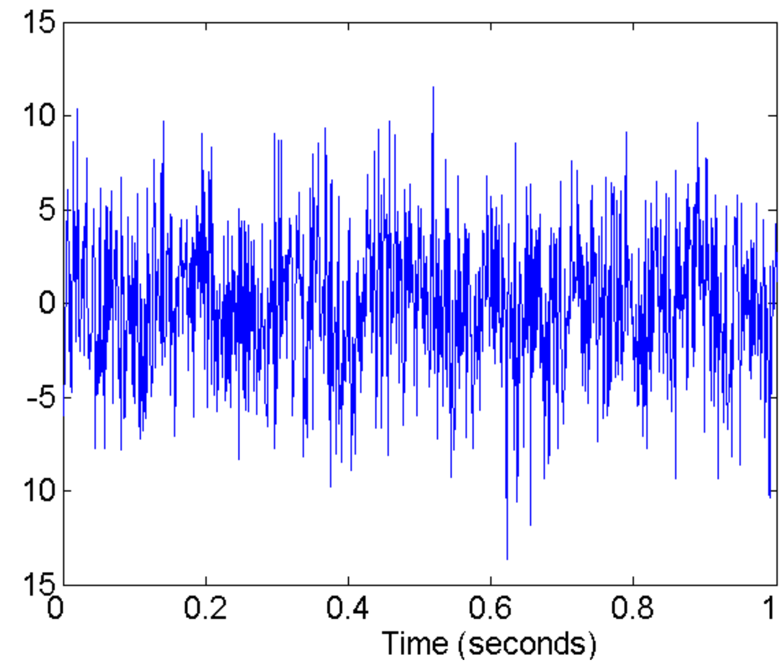
- Examples of data quality problems:
  - Noise and outliers
  - missing values
  - duplicate data

Noise

- Noise refers to modification of original values
  - Examples: distortion of a person's voice when talking on a poor phone and "snow" on television screen



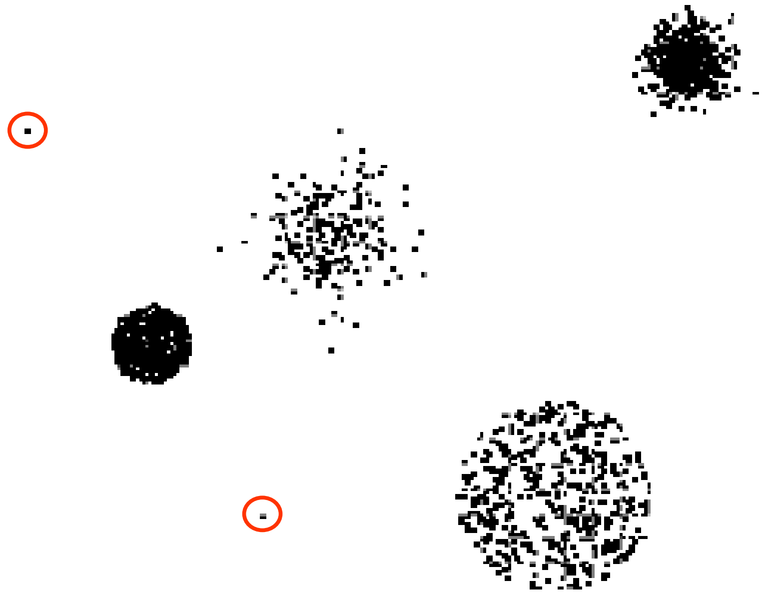
Two Sine Waves



Two Sine Waves + Noise

Outliers

- Outliers are data objects with characteristics that are considerably different than most of the other data objects in the data set



Missing Values

- Reasons for missing values
  - Information is not collected
    (e.g., people decline to give their age and weight)
  - Attributes may not be applicable to all cases
    (e.g., annual income is not applicable to children)

- Handling missing values
  - Eliminate Data Objects
  - Estimate Missing Values
  - Ignore the Missing Value During Analysis
  - Replace with all possible values (weighted by their probabilities)

Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
  - Major issue when merging data from heterogeous sources
- Examples:
  - Same person with multiple email addresses
- Data cleaning
  - Process of dealing with duplicate data issues

Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation

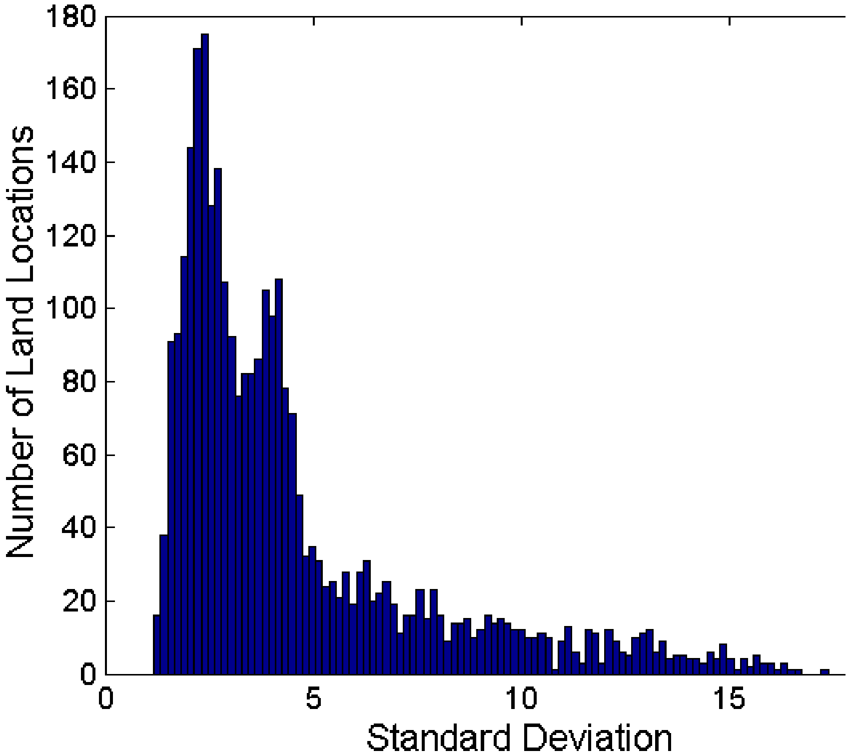
Aggregation

- Combining two or more attributes (or objects) into a single attribute (or object)

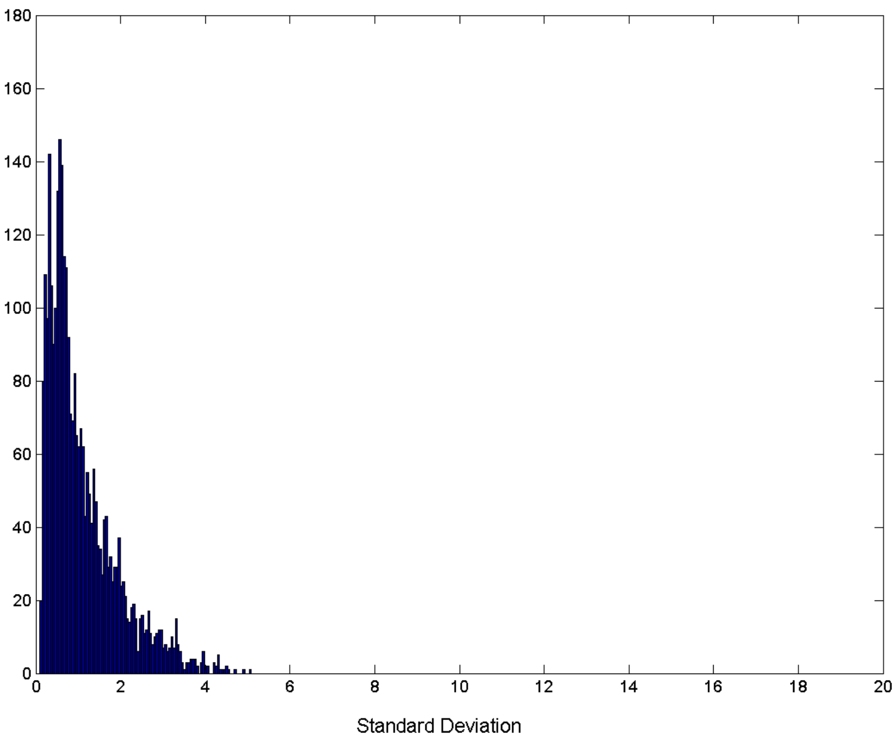
- Purpose
  - Data reduction
    - Reduce the number of attributes or objects
  - Change of scale
    - Cities aggregated into regions, states, countries, etc
  - More “stable” data
    - Aggregated data tends to have less variability

Aggregation

Variation of Precipitation in Australia



Standard Deviation of Average Monthly Precipitation



Standard Deviation of Average Yearly Precipitation

Sampling

- **Sampling is the main technique employed for data selection.**
  - **It is often used for both the preliminary investigation of the data and the final data analysis.**

- **Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.**

- **Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.**

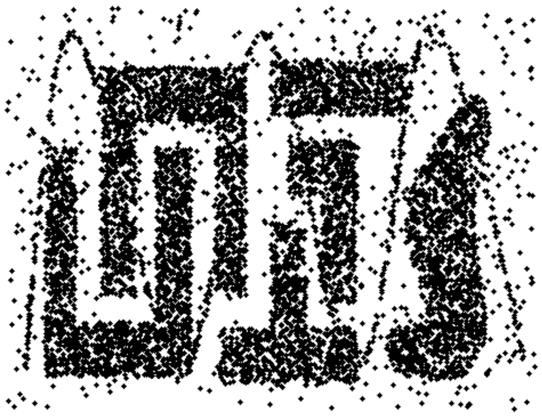
Sampling …

- The key principle for effective sampling is the following:
  - using a sample will work almost as well as using the entire data sets, if the sample is representative
  - A sample is representative if it has approximately the same property (of interest) as the original set of data

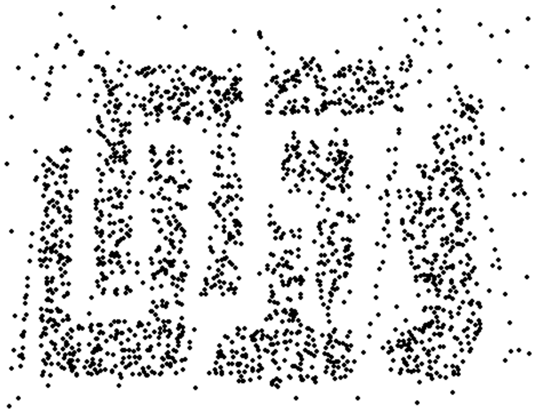
Types of Sampling

- Simple Random Sampling
  - There is an equal probability of selecting any particular item
- Sampling without replacement
  - As each item is selected, it is removed from the population
- Sampling with replacement
  - Objects are not removed from the population as they are selected for the sample.
    - In sampling with replacement, the same object can be picked up more than once
- Stratified sampling
  - Split the data into several partitions; then draw random samples from each partition

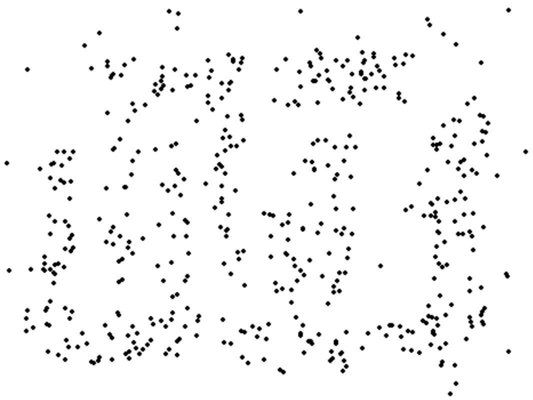
Sample Size



8000 points



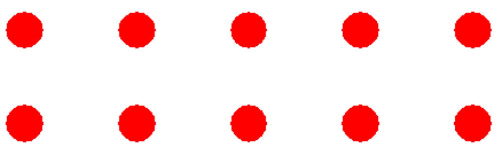
2000 Points

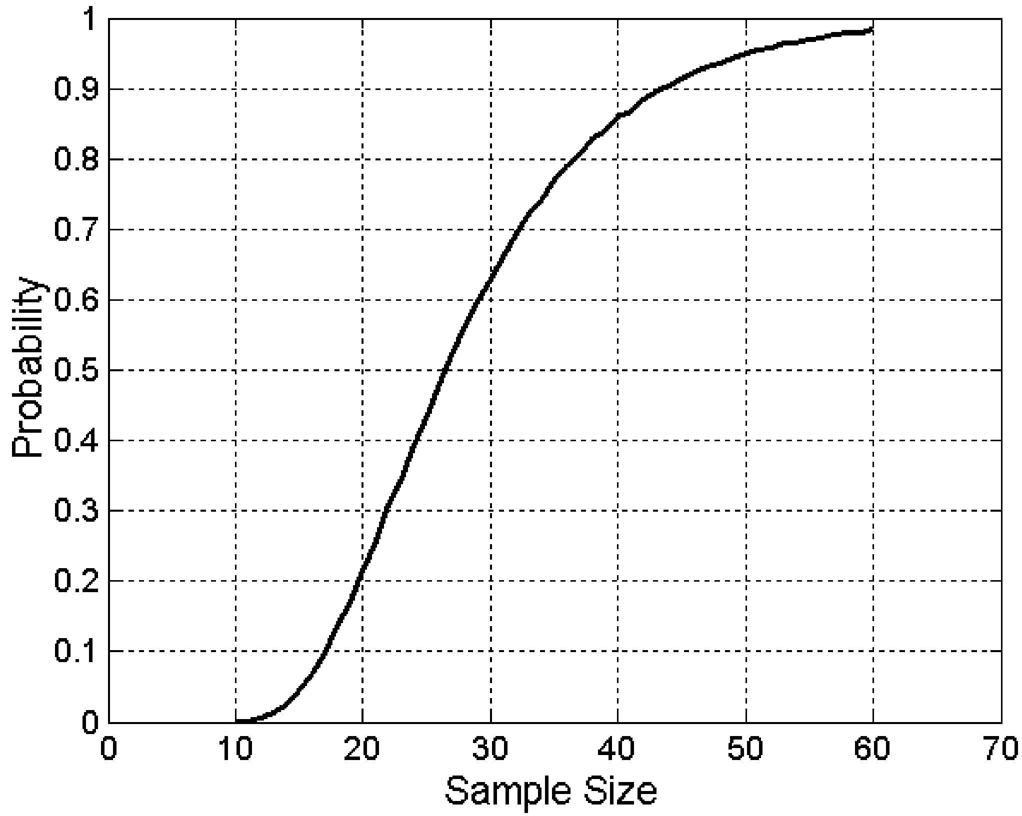


500 Points

Sample Size

- **What sample size is necessary to get at least one object from each of 10 groups.**





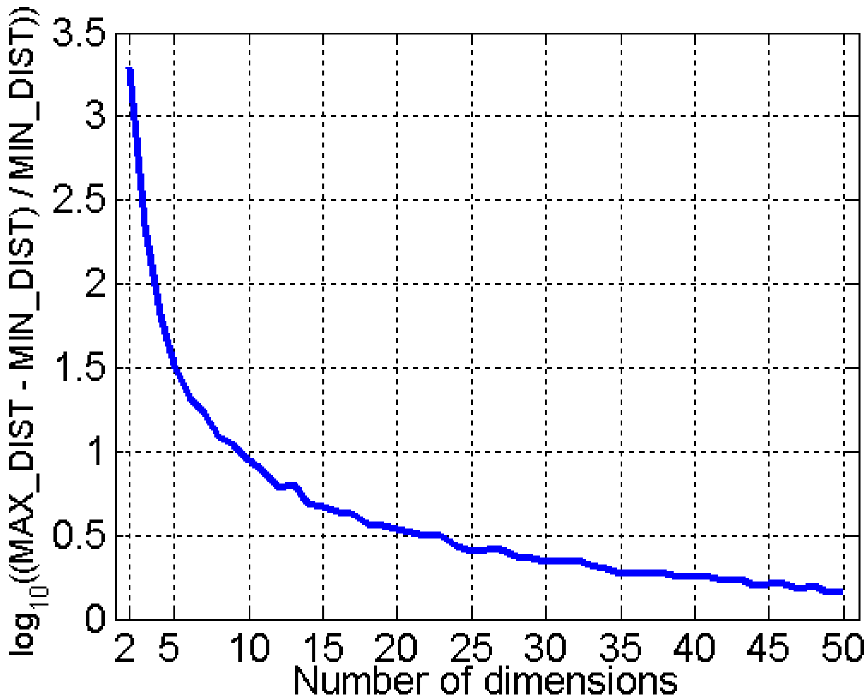
Progressive Sampling

- Start with a small sample
- Increase the sample size
- Need to evaluate the sample to judge if it is large enough
- Marginal effect (边际效应)

Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies

- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful



- **Randomly generate 500 points**
- **Compute difference between max and min distance between any pair of points**

Dimensionality Reduction

- Purpose:
  - Avoid curse of dimensionality
  - Reduce amount of time and memory required by data mining algorithms
  - Allow data to be more easily visualized
  - May help to eliminate irrelevant features or reduce noise

- Techniques
  - Principle Component Analysis
  - Singular Value Decomposition
  - Others: supervised and non-linear techniques

Dimensionality Reduction: PCA

- 变量之间是有一定的相关关系的
- 当两个变量之间有一定相关关系时，可以解释为这两个变量的信息有一定的重叠
- 主成分分析是对于原先提出的所有变量，将重复的变量（关系紧密的变量）删去多余，建立尽可能少的新变量，使得这些新变量是两两不相关的
- 这些新变量在反映信息方面尽可能保持原有的信息

Feature Subset Selection

- Another way to reduce dimensionality of data

- Redundant features
  - duplicate much or all of the information contained in one or more other attributes
  - Example: purchase price of a product and the amount of sales tax paid

- Irrelevant features
  - contain no information that is useful for the data mining task at hand
  - Example: students' ID is often irrelevant to the task of predicting students' GPA

Feature Subset Selection

- Techniques:
  - Brute-force approch:
    - Try all possible feature subsets as input to data mining algorithm
  - Embedded approaches:
    - Feature selection occurs naturally as part of the data mining algorithm
  - Filter approaches:
    - Features are selected before data mining algorithm is run
  - Wrapper approaches:
    - Use the data mining algorithm as a black box to find best subset of attributes

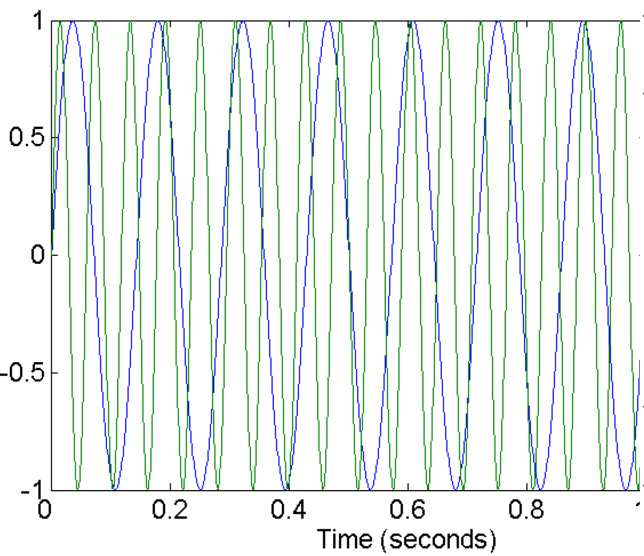
Feature Creation

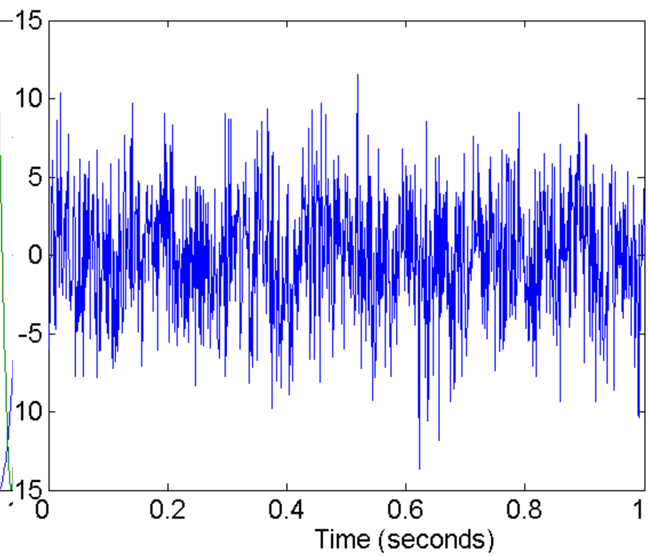
- Create new attributes that can capture the important information in a data set much more efficiently than the original attributes

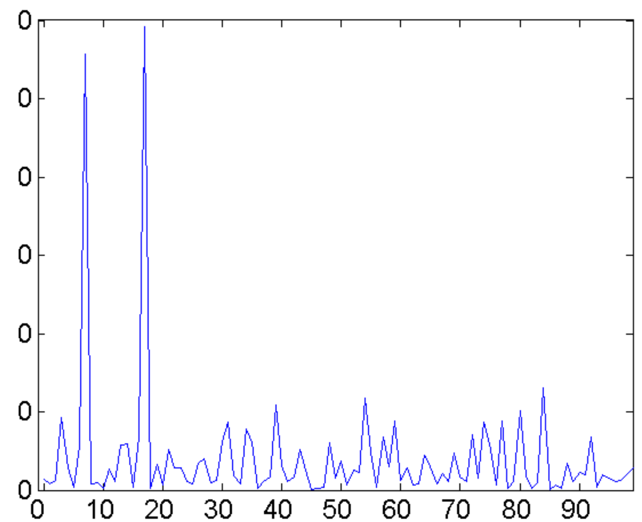
- Three general methodologies:
  - Feature Extraction
    - domain-specific
  - Mapping Data to New Space
  - Feature Construction
    - combining features

Mapping Data to a New Space

- **Fourier transform**
- **Wavelet transform**







Two Sine Waves

Two Sine Waves + Noise

Frequency

Discretization and Binarization

- Discretization: Transform a continuous attribute to categorical attribute
- Binarization: Transform continuous (or discrete) attributes into one or more binary attributes

表 2-5　一个分类属性到三个二元属性的变换

分类值	整数值	x_1	x_2	x_3
awful	0	0	0	0
poor	1	0	0	1
OK	2	0	1	0
good	3	0	1	1
great	4	1	0	0

Discretization and Binarization

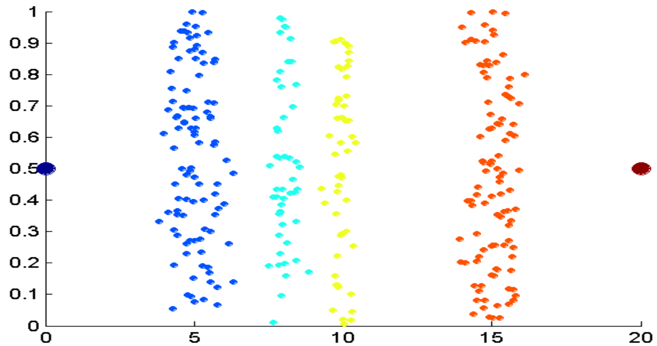
表 2-6　一个分类属性到五个非对称二元属性的转换

分类值	整数值	x_1	x_2	x_3	x_4	x_5
awful	0	1	0	0	0	0
poor	1	0	1	0	0	0
OK	2	0	0	1	0	0
good	3	0	0	0	1	0
great	4	0	0	0	0	1

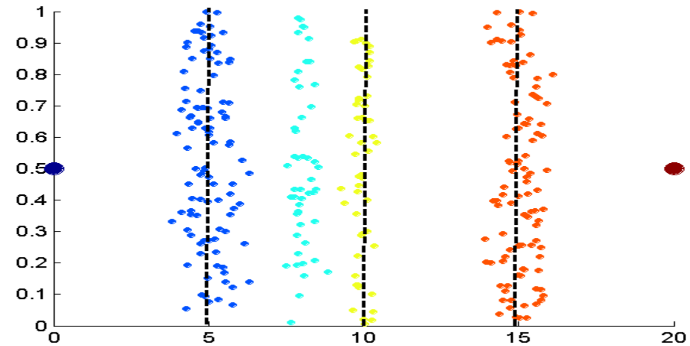
Discretization

- The best discretization depends on the algorithm being used
- How many categories?
- How to map the values of continuous attributes to these categories?
- How many split points to choose and where to place them?
- Solutions
  - Unsupervised discretization
  - Supervised discretization

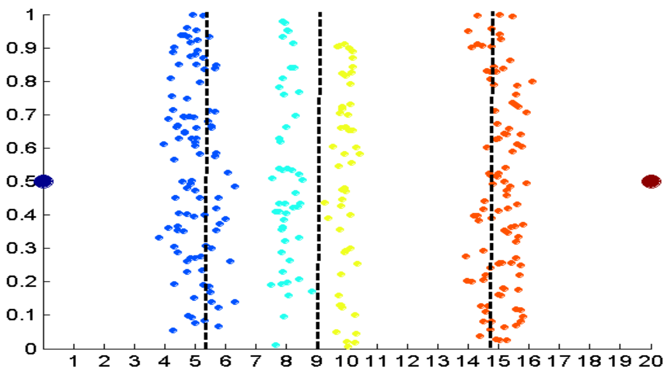
Discretization Without Using Class Labels



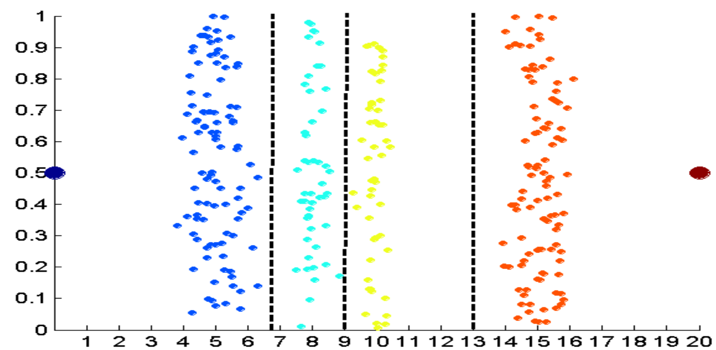
Data



Equal interval width



Equal frequency



K-means

Supervised Discretization

- 基于熵的离散化方法
  - 最大化区间的纯度

$$e_i = -\sum_{j=1}^{k} p_{ij} \log_2 p_{ij}$$

首先，需要定义熵（entropy）。设 k 是不同的类标号数，m_i 是某划分的第 i 个区间中值的个数，而 m_{ij} 是区间 i 中类 j 的值的个数。第 i 个区间的熵 e_i 由如下等式给出

$p_{ij} = m_{ij}/m_i$ 是第 i 个区间中类 j 的概率（值的比例）。

Supervised Discretization：Entropy (熵)

- Entropy (熵)
  - 熵的概念是由德国物理学家克劳修斯于1865年所提出。熵最初是被用在热力学方面的
  - 香农1948年的一篇论文《A Mathematical Theory of Communication》提出了**信息熵**的概念，解决了对信息的量化度量问题，并且以后信息论也被作为一门单独的学科
- 要搞清楚一件非常非常不确定的事，就需要了解大量的信息。相反，如果我们对某件事已经有了较多的了解，我们不需要太多的信息就能把它搞清楚。
- 对于任意一个随机变量 X，熵定义如下：“变量的不确定性越大，熵也就越大，把它搞清楚所需要的信息量也就越大。”

Entropy (熵)

- 世界杯谁是冠军?
- 世界杯赛后问一个知道结果的观众"哪支球队是冠军"？ 他不愿意直接告诉我，而要让我猜，并且我每猜一次，他要收一元钱才肯告诉我是否猜对了，那么我需要付给他多少钱才能知道谁是冠军呢?
- 我可以把球队编上号，从 1 到 32， 然后提问： "冠军的球队在 1-16 号中吗?" 假如他告诉我猜对了， 我会接着问： "冠军在 1-8 号中吗?" 假如他告诉我猜错了， 我自然知道冠军队在 9-16 中。 这样最多只需要五次， 我就能知道哪支球队是冠军
- 谁是世界杯冠军这条消息的信息量值五块钱

Entropy (熵)

- 不需要猜五次就能猜出谁是冠军，巴西、德国、意大利这样的球队得冠军的可能性比美国、韩国等队大的多。
- 第一次猜测时不需要把 32 个球队等分成两个组，而可以把少数几个最可能的球队分成一组，把其它队分成另一组。然后我们猜冠军球队是否在那几只热门队中。
- 重复这样的过程，根据夺冠概率对剩下的候选球队分组，直到找到冠军队。也许三次或四次就猜出结果。
- 当每个球队夺冠的可能性（概率）不等时，“谁世界杯冠军”的信息量的信息量比五比特少。
  – “谁是世界杯冠军”的信息量：

    = -（p1*log p1 + p2 * log p2 + . . . +p32 *log p32),

  – P1,…p32是32个球队各自夺冠的概率
- **课外阅读：《数学之美》第六章“信息的度量与作用”**

Supervised Discretization

- 基于熵的离散化方法
  - 最大化区间的纯度

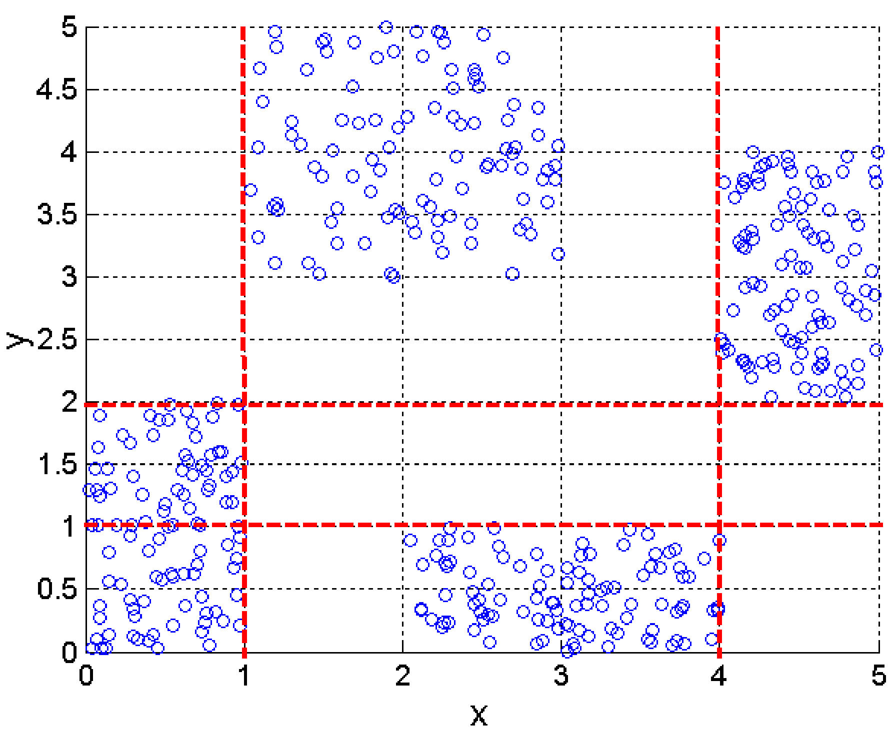
$$e_i = -\sum_{j=1}^{k} p_{ij} \log_2 p_{ij}$$

首先，需要定义熵（entropy）。设 k 是不同的类标号数，m_i 是某划分的第 i 个区间中值的个数，而 m_{ij} 是区间 i 中类 j 的值的个数。第 i 个区间的熵 e_i 由如下等式给出

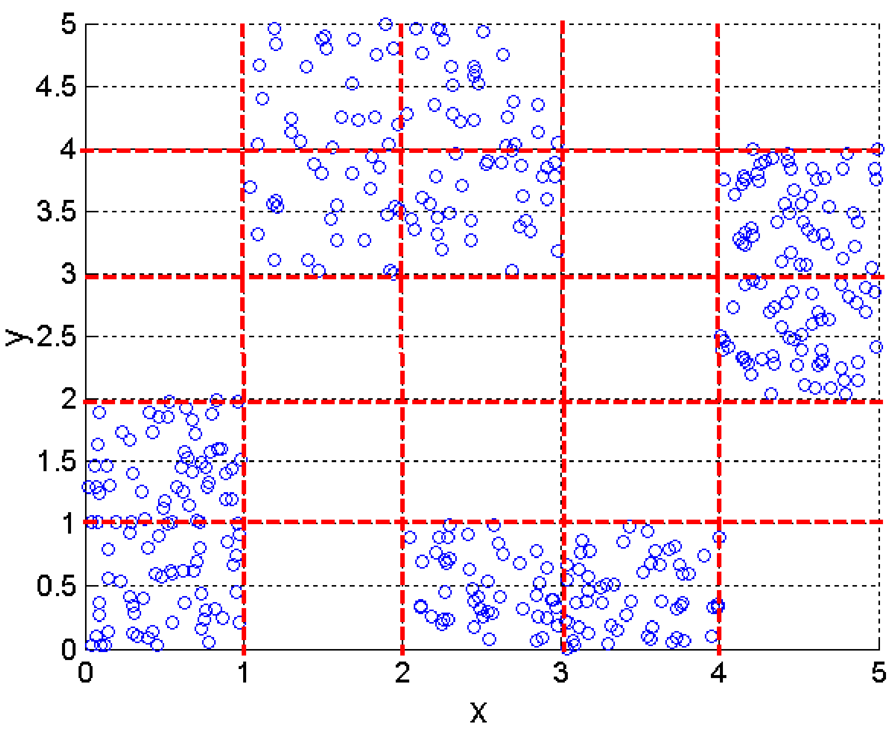
$p_{ij} = m_{ij}/m_i$ 是第 i 个区间中类 j 的概率（值的比例）。

Supervised Discretization

- 熵：区间纯度的度量
  - 只包含一个类：熵为0
  - 包含所有类，并且每类出现的概率相等：熵最大



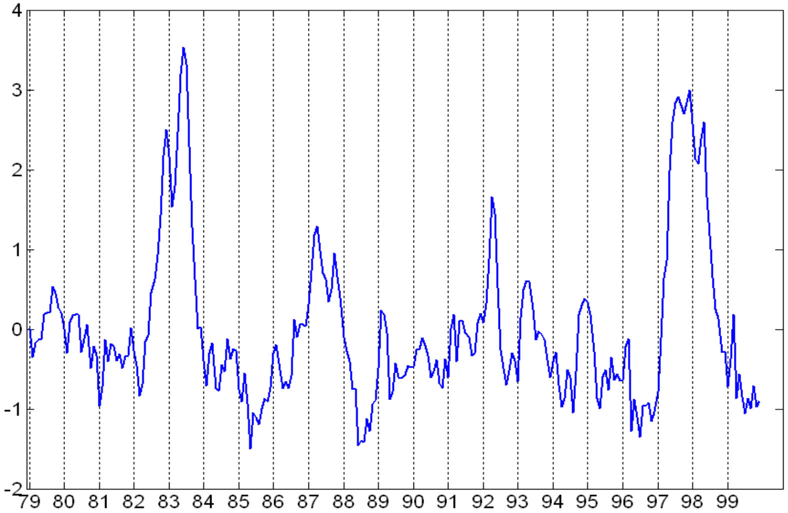
3 categories for both x and y



5 categories for both x and y

Attribute Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
  - Simple functions: x^k, $\log(x)$, e^x, $|x|$
  - Standardization and Normalization



Similarity and Dissimilarity

- Similarity
  - Numerical measure of how alike two data objects are.
  - Is higher when objects are more alike.
  - Often falls in the range [0,1]
- Dissimilarity
  - Numerical measure of how different are two data objects
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies

Similarity/Dissimilarity for Simple Attributes

p and *q* are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{\lvert p-q \rvert}{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{\lvert p-q \rvert}{n-1}$
Interval or Ratio	$d = \lvert p - q \rvert$	$s = -d$, $s = \frac{1}{1+d}$ or $s = 1 - \frac{d - min\_d}{max\_d - min\_d}$

Table 5.1. Similarity and dissimilarity for simple attributes

Euclidean Distance

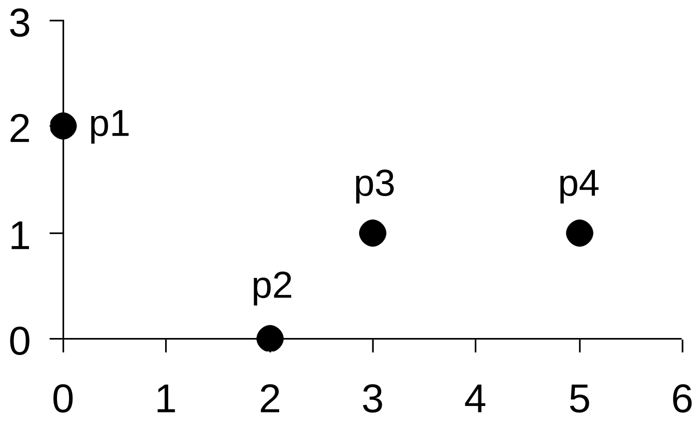
- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the k^th attributes (components) or data objects p and q.

- Standardization is necessary, if scales differ.

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$dist = \left(\sum_{k=1}^{n} | p_k - q_k |^r \right)^{\frac{1}{r}}$$

Where *r* is a parameter, *n* is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects *p* and *q*.

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Minkowski Distance: Examples

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance.
  - A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors
- $r = 2$. Euclidean distance
- $r \to \infty$. "supremum" (L_{max} norm, L_∞ norm) distance.
  - This is the maximum difference between any component of the vectors
- Do not confuse *r* with *n*, i.e., all these distances are defined for all numbers of dimensions.

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

Distance Matrix

Common Properties of a Distance

- Distances, such as the Euclidean distance, have some well known properties.

  1. $d(p, q) \geq 0$ for all p and q and $d(p, q) = 0$ only if $p = q$. (Positive definiteness，非负性)
  2. $d(p, q) = d(q, p)$ for all p and q. (Symmetry，对称性)
  3. $d(p, r) \leq d(p, q) + d(q, r)$ for all points p, q, and r. (Triangle Inequality，三角不等式)

  where $d(p, q)$ is the distance (dissimilarity) between points (data objects), p and q.

- A distance that satisfies these properties is a metric （度量）

Example: Non-metric dissimilarities

- A={1,2,3,4}   B={2,3,4}
- A-B = {1}   B-A=∅
- dis(A,B) = size(A - B) = 1
- dis(B,A) = size(B - A) = 0
- **dis(A,B) = size(A-B) + size (B-A)**

Example: Non-metric dissimilarities

- Distance between time of the day:
- d(t1,t2)=t2-t1   if t1<=t2
- d(t1,t2)=24+(t2-t1)   if t1>=t2
- d(1PM,2PM) = 1   d(2PM, 1PM) = 23

Common Properties of a Similarity

- Similarities, also have some well known properties.

1. $s(p, q) = 1$ (or maximum similarity) only if $p = q$.
2. $s(p, q) = s(q, p)$ for all p and q. (Symmetry)

where $s(p, q)$ is the similarity between points (data objects), p and q.

Similarity Between Binary Vectors

- Common situation is that objects, *p* and *q*, have only binary attributes

- Compute similarities using the following quantities

  M_{01} = the number of attributes where p was 0 and q was 1

  M_{10} = the number of attributes where p was 1 and q was 0

  M_{00} = the number of attributes where p was 0 and q was 0

  M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

  SMC = number of matches / number of attributes

  $= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$

  J = number of 11 matches / number of not-both-zero attributes values

  $= (M_{11}) / (M_{01} + M_{10} + M_{11})$

SMC versus Jaccard: Example

$p =$ 1 0 0 0 0 0 0 0 0 0

$q =$ 0 0 0 0 0 0 1 0 0 1

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)

$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)

$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)

$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$SMC = (M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$

$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$

Extended Jaccard Coefficient (Tanimoto)

- Variation of Jaccard for continuous or count attributes
  - Reduces to Jaccard for binary attributes

两个向量的交集

$$T(p,q) = \frac{p \bullet q}{\|p\|^2 + \|q\|^2 - p \bullet q}$$

两个向量的并集

$J = (M_{11}) / (M_{01} + M_{10} + M_{11})$

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1||\ ||d_2|| ,$$

where $\bullet$ indicates vector dot product and $|| d ||$ is the length of vector d.

- Example:

d_1 = **3 2 0 5 0 0 0 2 0 0**

d_2 = **1 0 0 0 0 0 0 1 0 2**

$d_1 \bullet d_2$= 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5

$||d_1||$ = $(3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)^{0.5}$ = $(42)^{0.5}$ = 6.481

$||d_2||$ = $(1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2)^{0.5}$ = $(6)^{0.5}$ = 2.245

$\cos(d_1, d_2)$ = .3150

A . B = ||A|| *||B|| * cos (A,B); A = d1/||d1||, B=d2/||d2||

Correlation（PCC皮尔森相关性）

- Correlation measures the linear relationship between objects
- To compute correlation, we standardize data objects, p and q, and then take their dot product

$$p'_k = (p_k - mean(p)) / std(p)$$

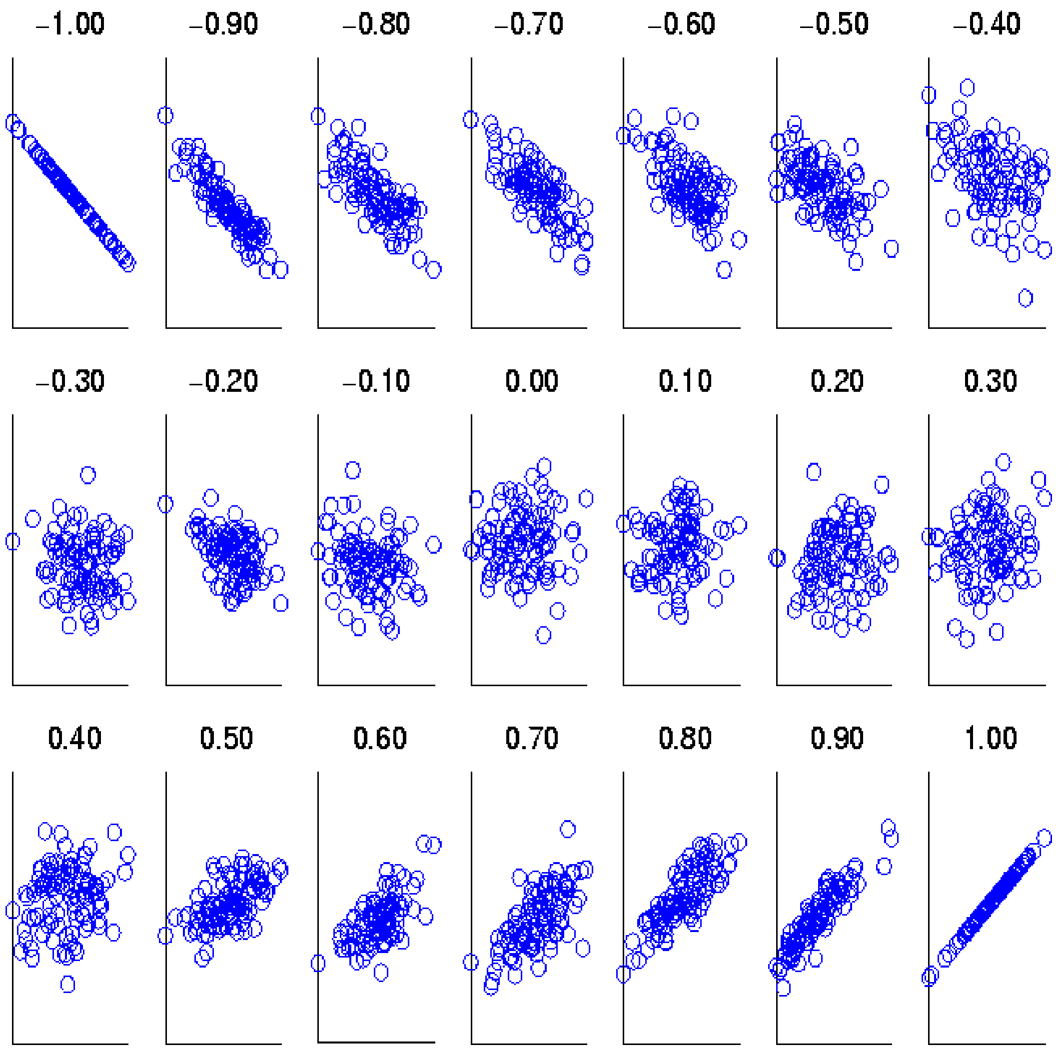
$$q'_k = (q_k - mean(q)) / std(q)$$

$$correlation(p, q) = p' \bullet q'$$

先标准化，再内积；内积代表相似性

Cosine vs PCC：标准化的过程不同

Visually Evaluating Correlation



Scatter plots showing the similarity from –1 to 1.

General Approach for Combining Similarities

- Sometimes attributes are of many different types, but an overall similarity is needed.

1. For the k^{th} attribute, compute a similarity, s_k, in the range $[0, 1]$.
2. Define an indicator variable, δ_k, for the k_{th} attribute as follows:

$$\delta_k = \begin{cases} 0 & \text{if the } k^{th} \text{ attribute is a binary asymmetric attribute and both objects have} \\ & \text{a value of 0, or if one of the objects has a missing values for the } k^{th} \text{ attribute} \\ 1 & \text{otherwise} \end{cases}$$

3. Compute the overall similarity between the two objects using the following formula:

$$similarity(p, q) = \frac{\sum_{k=1}^{n} \delta_k s_k}{\sum_{k=1}^{n} \delta_k}$$

Using Weights to Combine Similarities

- May not want to treat all attributes the same.
  - Use weights w_k which are between 0 and 1 and sum to 1.

$$similarity(p, q) = \frac{\sum_{k=1}^{n} w_k \delta_k s_k}{\sum_{k=1}^{n} \delta_k}$$

$$distance(p, q) = \left(\sum_{k=1}^{n} w_k |p_k - q_k|^r \right)^{1/r}.$$

Density

- Density-based clustering require a notion of density

- Examples:
  - Euclidean density
    - Euclidean density = number of points per unit volume

  - Probability density

  - Graph-based density

Euclidean Density – Cell-based

- Simplest approach is to divide region into a number of rectangular cells of equal volume and define density as # of points the cell contains

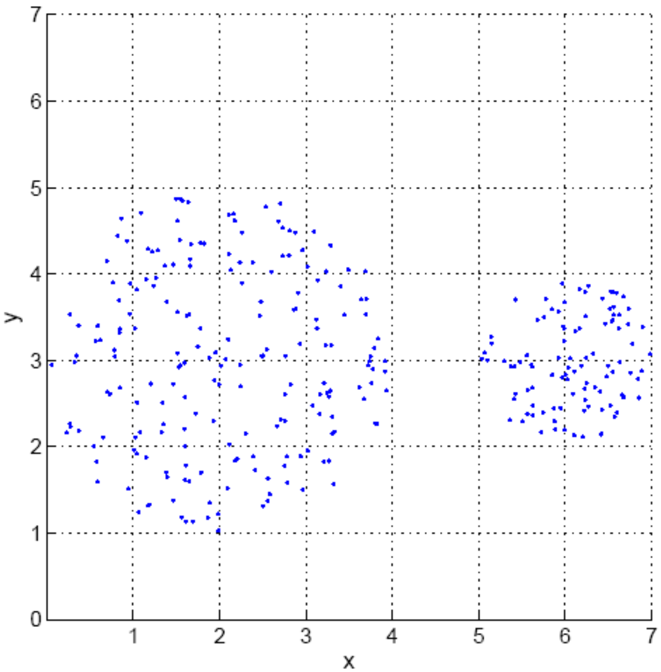


Figure 7.13. Cell-based density.

0	0	0	0	0	0	0
0	0	0	0	0	0	0
4	17	18	6	0	0	0
14	14	13	13	0	18	27
11	18	10	21	0	24	31
3	20	14	4	0	0	0
0	0	0	0	0	0	0

Table 7.6. Point counts for each grid cell.

Euclidean Density – Center-based

- Euclidean density is the number of points within a specified radius of the point

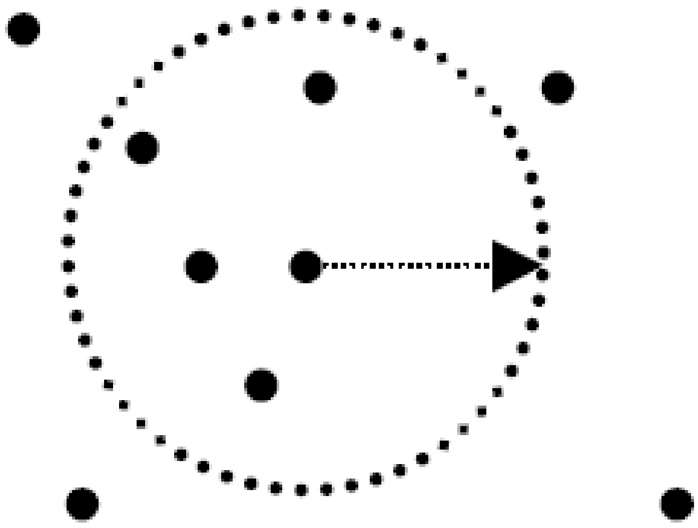


Figure 7.14. Illustration of center-based density.

Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation