Problem Set 2 **Physics 324 Autumn 2016** **Due Tuesday October 18 in class**

DHC

Consider a particle of mass m confined in one dimension in a potential well of shape $V(x)$.

1. Prove that the time-independent wave function $\phi_n(x)$, with energy E_n, can always be taken to be real, but the time-dependent wavefunction $\psi(x,t)$ cannot. (You can look it up, or use the hint in Griffiths.)

2. Show that all E_n must be higher than the mimimum of the potential well (again, hint in Griffiths.)

3. Consider from this point on the case $V(x) = 0$ for $0 < x < L$ and $V(x) = \infty$ otherwise.
   a. Go through the procedure of finding the normalized eigenfunctions $\phi_n(x)$ and eigenvalues E_n $(n = 1,2,\ldots)$ of the time-independent Schroedinger equation with this potential. Show that the eigenfunctions for different n are orthogonal.
   b. From there, obtain the full series expansion of a general time-dependent wavefunction $\psi(x,t)$ in this potential, define the coefficients, and show how to determine them from the inner product of ψ and the ϕ_n (yes, it's in the book.)
   c. Sketch the probability density in the ground state, and for $n = 10$. Which one is more like the classical probability?

4. Now assume the wavefunction at $t = 0$ is $\psi(x,0) = A[\phi_m(x) + \phi_{m+1}(x)]$ where m is a large, even integer.
   a. Find A, σ_x and σ_p for this state at $t = 0$. Make use of orthonormality and symmetry where possible in the calculations. Check that the uncertainty principle holds, $\sigma_x\sigma_p \geq \hbar/2$. (TOO HARD - WILL NOT BE GRADED!)
   b. Find $\langle x \rangle$ at time t, and its oscillation frequency, ω_n. (WILL NOT BE GRADED)
   c. Find the time-dependent probability density $\rho(x,t)$ and identify the scillation frequency, ω_n.
   d. Consider instead a classical particle in the same box with kinetic energy E. Show that for large m, ω_m approaches ω_c, the fundamental frequency of the classical particle's bouncing back and forth.
   e. At time t_0 a measurement is made and the result is E_m. What is $\psi(x,t)$ at later times?

5. Now assume that at $t = 0$ the particle is in the left quarter of the well, equally likely to be found anywhere in the range $0 < x < L/4$.
   a. What is $\psi(x,0)$, assuming it is real and normalized?
   b. What is the probability that a subsequent measurement of energy yields the result E_1?
   c. If the first measurement yields result E_1, what is the probability that a second subsequent measurement of the energy will yield result E_2?