Problem Set 1 **Physics 324 Autumn 2016** **Due Tuesday October 11 in class**

DHC

1. Assume that in a given state of a hydrogen atom an electron (charge $-e$ mass m) is in a circular orbit about a fixed proton (charge $+e$) with quantized angular momentum, $L = n\hbar$ (which comes from assuming $p = h/\lambda$ and $n\lambda = 2\pi r$).

a) Find the total energy of the orbit (kinetic plus potential) relative to that of the free stationary electron.
b) Derive the expression for the wavelength of light emitted when the electron makes a transition from a higher to a lower orbit, $\lambda^{-1} = R_\infty(m^{-2} - n^{-2})$, where n and m are different integers, including getting an expression for R_∞.
c) Calculate R_∞ in units of eV by looking up the values of the constants in the expression, and check it is equal to the Rydberg constant, 1.10×10^7 m^{-1}.
d) Calculate λ for the transition between the lowest energy (ground) state and the next lowest (first excited) states. This is the longest wavelength in the Lyman series in the spectrum of hydrogen.

2. The black-body energy/unit frequency/unit volume is $u_\nu(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}$.

a) Show for small enough ν that $u_\nu \propto \nu^2$, and write down the condition on ν for this to apply.
b) Find an expression for the frequency ν_{max} at which u_ν has its maximum, and hence explain why hotter objects looks bluer and colder objects redder. You will need to use Mathematica to solve a short dimensionless equation to get the numerical coefficient.
c) Derive from u_ν the energy/unit *wavelength*/unit volume, $u_\lambda(\lambda, T)$. (Careful!)

3. Consider a particle moving in one dimension with even-parity wavefunction $\psi(x, t) = Ae^{-x^2/a^2 + i\omega t}$.

a) Determine the normalized probability density $\rho = |\psi(x)|^2$ and the normalization constant A.
b) Sketch both $|\psi|$ and ρ vs x, by hand, with indications of scale.
c) Make the corresponding sketches for an odd-parity wavefunction $\psi_A(x, t) = Bxe^{-x^2/a^2 + i\omega t}$. What do I mean by odd parity?
d) For ψ only, find the expectation values of the position, $\langle x \rangle$, and momentum, $\langle p \rangle = \int dx \psi^*(x, t)\hat{p}\psi(x, t)$, of the particle, where $p = -i\hbar\partial/\partial x$.
e) Find the uncertainty in position, Δx, such that $\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2$, and in momentum, Δp.
f) What constraint does the uncertainly principle place on the constant a?

4. Prove Ehrenfest's second theorem, $d\langle p \rangle/dt = \langle -\partial V/\partial x \rangle$, using Schroedinger's equation. Can you see why this is something like Newton's second law for a quantum particle?