1. Based on Kogut, 4-1. Two events occur at the same place in frame S and are separated by a time interval of 15 s. What is the spatial separation between these two events in frame S' in which the events are separated by a time interval of 23 s? Frame S' moves with constant velocity along the $-x^1$ direction of frame S.

   You can do this problem either using Lorentz tranformations or using invariant separations.

2. Consider a series of $N+1$ different inertial reference frames S_0, ..., S_N where each frame S_k moves with speed v in the x-direction *relative* to frame S_{k-1}. (So frame S_1 moves with velocity $v\,\hat{\boldsymbol{e}}_x$ relative to frame S_0, frame S_2 moves with velocity $v\,\hat{\boldsymbol{e}}_x$ relative to frame S_1, *etc.*) To find the combined effect of multiple boosts along a common direction, the easiest approach is *not* to use velocity addition formulas but rather to characterize boosts using *rapidity* η, defined by $\tanh\eta = v/c$. [See Yaffe's notes for a brief discussion of rapidity, and his Appendix A for a reminder about the properties of sinh, cosh and tanh.]

   (a) Explain why the speed of frame S_N relative to frame S_0 cannot be just Nv.
   (b) Sketch a plot of rapidity *vs.* v/c.
   (c) The Lorentz transformation matrix which converts the coordinates of frame S_k into those of frame S_{k-1} is, as discussed in lecture and in the notes,

   $$\Lambda = \begin{pmatrix} \gamma & \frac{v}{c}\gamma & 0 & 0 \\ \frac{v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   Show that, when expressed in terms of rapidity, this takes the form

   $$\Lambda(\eta) = \begin{pmatrix} \cosh\eta & \sinh\eta & 0 & 0 \\ \sinh\eta & \cosh\eta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   (d) Show that rapidities of collinear boosts are additive: $\Lambda(\eta_1)\Lambda(\eta_2) = \Lambda(\eta_1+\eta_2)$.
   (e) Use this property to solve the original question: find the speed of frame S_N relative to S_0.

3. Based on Kogut, 2-7. At 1:00 a spaceship passes Earth with a velocity of $0.8\,c$. Observers on the ship and on the Earth synchronize their clocks at that moment.

   (a) At 2:30, as recorded in the spaceship's frame, the ship passes another space probe that is fixed relative to the Earth and whose clocks are synchronized with respect to the Earth. What time is it according to the space probe's clock?
   (b) How far from the Earth is the probe, as measured in the Earth's frame?
   (c) At 2:30, spaceship time, the ship sends a light signal back to Earth. According to Earthbound clocks, when is the signal received on Earth?
   (d) Earth immediately sends another light signal back to the spaceship. According to spaceship time, when does the spaceship receive this reply signal?

4. Based on Kogut, 3-2. A police officer aims a stationary radar transmitter backward along the highway toward oncoming traffic. The radar detector picks up the reflected waves and analyzes their frequencies. Suppose that the transmitter generates waves at a frequency ν_0 and detects the waves reflected by an approaching car at frequency ν_r. Recall that radar is an electromagnetic wave and assume that its speed in air is c (whereas infact it is very slightly slower).

(a) Draw a spacetime diagram showing the world lines of the stationary police officer, the approaching speeding car, and at least two transmitted and reflected radar wave-crests. (Recall that such crests occur once every period, with the period being $T = 1/\nu_0$.)

(b) From the geometry of your spacetime diagram, relate the time between transmitted radar wave-crests to the time between reflected wave-crests as detected by the stationary police officer, and derive the relation $\nu_r = \nu_0\,(1{+}v/c)/(1{-}v/c)$.

(c) Let ν' denote the frequency of the radar wave-crests as measured by an observer in the car. Explain why $\nu'/\nu_0 = \nu_r/\nu'$, and derive the Doppler shift formula $\nu'/\nu_0 = \sqrt{(1{+}v/c)/(1{-}v/c)}$.

(d) Suppose that the car's speed is 120 mph and $\nu_0 = 10^{10}$ Hz. Find the fractional difference between ν_0 and ν_r by linearizing the formula in part (b). (Since the car's speed is much smaller than c, this is a excellent approximation.)

(e) **Extra credit.** Handheld radar equipment commonly carried by law enforcement personnel can detect such a small fractional frequency shift. How is this possible?