1. To keep you practiced with four-vectors...
   The components (in a common frame) of six different spacetime vectors are:

$$A^\mu = \{1,\,2,\,0,\,0\}\,,\qquad B^\mu = \{3,\,1,\,2,\,2\}\,,\qquad C^\mu = \{3,\,0,\,2,\,-2\}\,,$$
$$D^\mu = \{2,\,-1,\,0,\,3\}\,.\qquad E^\mu = \{5,\,0,\,3,\,4\}\,,\qquad F^\mu = \{2,\,0,\,0,\,1\}\,.$$

   (a) Indicate which of these vectors are spacelike, which are timelike, and which are lightlike.
   (b) Which vectors, when multiplied by mc, could be the 4-momentum of an object of mass m?
   (c) Which of the above vectors, if any, are orthogonal to the vector $G^\mu = \{2,\,0,\,0,\,3\}$? (Recall that two four-vectors are orthogonal if their dot product vanishes.)
   (d) Which vectors, if any, could be the 4-acceleration of an object with 4-velocity $u^\mu = \{2,\,1,\,-1,\,1\}$? Hint: remember that u^2 is constant, and that $a = du/d\tau$.

2. In this question assume that $m_u = m_d$, so that the strong interactions exactly conserve isospin, implying that the initial and final states of each process must have the same total isospin and the same total I_z.

   For each of the following processes, what quantum numbers, if any, are violated? Recall that our list now includes P^μ, $\vec{J}$, B, Q, L, S, U and D as well as parity P, charge conjugation C, isospin and G parity. Explain whether the process is allowed at all, and, if so, whether it will be dominantly an (isospin-symmetric) strong, electromagnetic or weak process. For each process that is possible, draw a Feynman diagram involving some combination of quarks, leptons, photons, gluons and W-bosons that can give rise to the reaction. (There are many examples of such diagrams to look at in the lecture notes, but beware that in some cases it is quite tricky to figure out a diagram that works.)

   (a) $\Lambda^0 \to p + e^- + \bar{\nu}_e$
   (b) $K^- + p \to K^+ + \Xi^-$
   (c) $K^+ + p \to K^+ + \Sigma^+ + \bar{K}^0$
   (d) $\eta' \to \rho^+ + \pi^-$
   (e) $\bar{p} + n \to \pi^- + \pi^0$
   (f) $\Sigma^+ \to \pi^+\pi^0$

3. In 1987, a supernova (star explosion) occurred in the Large Magellanic Cloud, a dwarf galaxy about 50 kilo-parsecs from the Earth. This is usually called SN1987A. Eleven antineutrinos were detected by the Kamiokande-II experiment (an experiment designed to look for proton decay, and neutrinos from the sun, that turned out serendipitously to be a supernova detector as well). These arrived in a ~ 13 second interval just before the arrival of visible light from the supernova. The detected antineutrinos had energies of about 15 MeV. These events are believed to be due to thermal emission of antineutrinos from the supernova. The Kamiokande-II detector measured Čerenkov light produced by antineutrino-proton scattering, $\bar{\nu}_e + p^+ \to n + e^+$, or antineutrino-electron scattering, $\bar{\nu} + e^- \to \bar{\nu} + e^-$, in about 700 tons of water. The produced positron (or scattered electron) has is sufficient energetic that its velocity is greater than the speed of light in water. Charged particles moving faster than light in a medium emit light (Čerenkov radiation) in a characteristic pattern that can be detected.

   As discussed in lectures, the rate Γ of scattering events is given by the flux of projectiles times the number of targets times the cross section, $\Gamma = (\text{flux}) \cdot N_{\text{targets}} \cdot \sigma$. The cross section for antineutrino-proton scattering is approximately

$$\sigma \approx 10^{-47}\ \mathrm{m}^2 \left(\frac{E_\nu}{1\ \mathrm{MeV}}\right)^2 \left(1 - \frac{Q}{E_\nu}\right) \sqrt{1 - \frac{2Q}{E_\nu} + \frac{Q^2 - m_e^2}{E_\nu^2}}\,,$$

where E_ν is the antineutrino energy and $Q = 1.3\,\mathrm{MeV}$ is the neutron-proton mass difference. (The cross section for antineutrino-electron scattering is about 100 times smaller at these energies.)

Use the above information to estimate the total energy emitted by the supernova in antineutrinos, assuming that this emission is isotropic. Hints: Evaluate the number of protons in 700 tons of water. Then find the time-integrated flux (or total number of antineutrinos per unit area) that, when combined with the cross section and number of protons, would yield the observed number of events. Assuming that this flux is emitted isotropically from the supernovae, find the total number of emitted antineutrinos and convert this to a total neutrino energy assuming that 15 MeV is the typical antineutrino energy.

4. At `http://pdg.lbl.gov/2011/reviews/rpp2011-rev-cross-section-plots.pdf` , one can find plots of total cross sections for pp, pd, πp, πd, Kp and Kd scattering (where d is the deuteron). These are in Figs. 41.11 through 41.15. In each of the reactions, the first particle is moving (with 3-momentum of magnitude p_{lab}) and we refer to it as the "projectile", while the second particle is at rest. We focus in this problem on the momentum range 10 GeV $< p_{\mathrm{lab}} <$ 300 GeV, where these total cross sections are rather slowly varying. Our aim is to see if we can understand the relative size of the different scattering cross sections using simple arguments.

   [Aside: note that these are *total* cross sections, and that, at these energies the dominant processes are inelastic, e.g. $\pi p \to X$, where $X \neq \pi p$. Typically, many additional particles are produced. You can see this from some of the plots where the elastic cross section is also shown. In elastic scattering the particles in the initial and final states are the same, e.g. $\pi p \to \pi p$, although their directions will be changed.]

   (a) You recall from QM that the de Broglie wavelength is $\lambda_{dB} = h/p = 2\pi\hbar/p$. This holds also relativistically. What range of λ_{dB} does the momentum range given above correspond to? You should find a range that is much smaller than the typical size of the target (which is about 1 fm).

   Now, the de Broglie wavelength is a measure of the resolution with which the projectile matter-wave can discern features in the target (just as for light waves). The fact that $\lambda_{dB} \ll 1\ fm$ suggests that we should think of these scattering processes approximately as the scattering of constituent quarks off each other.

   (b) Make a table of the approximate values of the total cross-sections in the desired momentum range (read off from the plots). What is the fractional variation in these values across the momentum range? If you write $\sigma = L^2$, what is the range of the corresponding values of L?

   (c) Determine the values of the ratios

   $$R(X) = \frac{\sigma_{\mathrm{tot}}(Xd)}{\sigma_{\mathrm{tot}}(Xp)}\,,$$

   for $X = p$, $\pi^\pm$ and K^-. Suggest a simple explanation for the results.

   (d) Determine the ratios

   $$R_\pi = \frac{\sigma_{\mathrm{tot}}(\pi^\pm p)}{\sigma_{\mathrm{tot}}(pp)} \quad \text{and} \quad R_K = \frac{\sigma_{\mathrm{tot}}(K^- p)}{\sigma_{\mathrm{tot}}(pp)}\,,$$

   and suggest a simple explanation for the results.