Problem Set 4 **Physics 324 Autumn 2016** **Due Tuesday November 8 in class**

DHC

Consider a particle of mass m in a potential $V(x)$ in 1D such that $V \to 0$ as $|x| \to \infty$. (Hints/instructions for most of these can be found in Griffiths.)

1. Show that there are no *degenerate* bound states: that is, if $\phi_{\mathrm{n}}(x)$ and $\phi_{\mathrm{m}}(x)$ are distinct bound states ($E_n < 0$ and $E_m < 0$) then their energies must be different, $E_n \neq E_m$.
2. Show that if the potential is symmetric, $V(x) = V(-x)$, then every bound state is either symmetric (of *even parity*), $\phi_n(x) = \phi_n(-x)$ or antisymmetric (of *odd parity*), $\phi_n(x) = -\phi_n(-x)$.
3. Consider the case of a finite square well with $V(x) = -V_0$ for $|x| < a$ and $V(x) = 0$ otherwise. Define $k^2 = (2m/\hbar^2)(E + V_0)$ and $\alpha^2 = -2mE/\hbar^2$. By analyzing the odd-parity bound states, matching up the exponential tails outside to sine functions inside, find a transendental equation whose solutions give the odd energy levels E_n.
4. Normalize the bound-state wavefunctions.
5. Solve the transendental equation graphically and find an expression the minimum magnitude of V_0 below which there are no bound states.
6. Consider now the limit of a narrow, deep finite square well: $a \to 0,\ V_0 a = \text{const}.$ Show that this behaves as a δ-function potential, and hence show that a δ-function well has no odd bound states.
7. For the scattering states ($E > 0$), find the energy dependence of the transmission coefficient $T(E)$ (Eq. 2.169) in this limit.
8. Construct the S-matrix (defined in Griffiths problem 2.52) for this δ-function limit.