Problem Set 3 **Physics 324 Autumn 2016** **Due Tuesday November 1 in class**

DHC

1. Watch video no. 8 in the MIT OCW 8.04 Quantum Physics 1 course by Barton Zweibach. Then write out his proof of why $E_{n+1} > E_n$, for the time-independent Schroedinger equation, where n is the number of nodes.

2. Harmonic oscillator:
   a) Show that, for a particle in 1D, $[\hat{x}, \hat{p}] = i\hbar = -[\hat{p}, \hat{x}]$. Is this true for any $V(x)$?
   b) In state $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$, what is the probability to find the particle within the classically allowed region? Use Mathematica or similar to do the integral.
   c) Given $\psi_n(x) = \frac{1}{\sqrt{n!}}(a_+)^n \psi_0(x)$, find $\psi_2(x)$ and $\psi_3(x)$ and check that they contain the Hermite polynomials $H_2(\zeta)$ and $H_3(\zeta)$ where $\zeta = \left(\frac{m\omega}{\hbar}\right)^{1/2} x$.
   d) Check that ψ_2 and ψ_3 are orthogonal.
   e) Write both x and p in terms of a_+ and a_-. Using this, and orthonormality, find $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$, $\langle T \rangle$ and $\langle V \rangle$ where T and V are the kinetic and potential energy for for ψ_n.
   f) At $t = 0$ a particle has wavefunction $\psi(x, 0) = A(\psi_0 + \psi_1)$. If the energy is measured, what are its possible values and their probabilities?
   g) Find the probability density at later times, correctly normalized. Describe what is happening.

3. Free particle:
   a) Let P_{ab} be the probability of finding a particle in the range (a, b) at time t. Show, using the time-dependent Schroedinger equation, that $\frac{dP_{ab}}{dt} = J(a, t) - J(b, t)$, where $J(x, t) \stackrel{\text{def}}{=} \frac{i\hbar}{2m}\left(\psi \frac{\partial \psi^*}{\partial x} - \psi^* \frac{\partial \psi}{\partial x}\right)$ is the **probability current**. (This works for any $V(x)$).
   b) Find $J(x, t)$ for a free particle momentum eigenstate $\psi = Ae^{i\left(kx - \frac{\hbar k^2}{2m}t\right)}$. Can you find A?
   c) At $t = 0$ free particle has a gaussian wavepacket, $\psi(x, 0) = Ae^{-ax^2}$. Find A.
   d) Find $\psi(x, t)$. [Hints in Griffiths if needed.]
   e) Find the probability density at time t in terms of the quantity $\xi = \sqrt{\frac{a}{1+(2\hbar a t/m)^2}}$. Sketch it at $t = 0$ and again for some very large t. Qualitatively what happens as time goes on?
   f) At what time does the system come closest to the limit set by the uncertainty principle?