Homework 3: Root finding

Due May 10.

1. Write a program to find the roots of one-dimensional equations using a) Bisection method and b) Newton-Raphson method.

   Test your implementations on the following equations:

   (a) A simple test case: $x^2 = a$ (choose an a, say `2.0`, and solve for x).

   (b) Kepler's equation, used for determining Solar System orbits:

   $$M = E - e\sin(E).$$

   Here M is the "mean anomoly", an angle that increases linearly in time; E is the "eccentric anomoly", the position of the body from the center (not the focus) of the ellipse, and e is the eccentricity of the ellipse. Choose $M = 1.5$ and $e = 0.5$ and solve for E in the interval $0 \leq E < 2\pi$. Try it again for $M = 1.5$ and $e = 0.9$, but be careful where you start the Newton-Raphson.

   For each attempt, plot the "trajectory" of the iterations by displaying each iteration on a plot of x vs. $f(x)$ (where $f(x)$ is the function for which you want to find the root), and connecting successive points. That is, connect the point $(x_0, f(x_0))$ to $(x_1, f(x_1))$, and so on.

2. It is frequently required that the Kepler problem be solved to the maximum precision available as quickly as possible. Attempt to do this with your Kepler solver. Which method, Bisection or Newton-Raphson, is better? Use your solver to plot $\sin(E)$ vs. $\cos(E)$ for 20 equally spaced values of M between 0 and 2π for an $e = 0.9$ orbit.