R Code for Mini Project 2

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February 18, 2020

1 Global notation

Unless otherwise specified, we adopt the following conventions:

- n is the number of nodes in the model
- N is the number of iterations

2 General Ising Model code

2.1 IsingDesign.R

Input n. Output a Design Matrix which is 2^n by n

2.2 IsingDesignQ.R

Input n. Output a Design Matrix which is 2^n by $\frac{n(n+1)}{2}$ whose columns correspond to $X_1, X_2, \ldots, X_n, X_1X_2, \ldots, X_1X_n, X_2X_3, \ldots, X_2X_n, \ldots X_{n-1}X_n$.

2.3 IsingCounts.R

Outputs a 'response vector' of length 2^n for use when fitting GLMs (or otherwise) counting each sign pattern, in the usual binary ordering. *Input dataset, though this currently generates one using Gibbs sampling, so input N*

3 Sampling from Ising Models

3.1 DirectSampler.R

Input N, n. Gives N direct samples from the Ising model of size n (i.e. by computing the pmf). Currently lambda, mu inputs are manually changed inside the function, and are set to be normal distributions with a given seed.

3.2 GibbsSampler.R

As above, but sample is produced via Gibbs sampling.

4 Equality constraints

4.1 FlatteningRank4.R

Input a four by four matrix to compute the flattening rank.

4.2 FlatteningRank.R

Work in progress, will compute flattening rank of n by n matrix.

5 Fitting MTP_2 Ising Models

5.1 glmIsing.R

Input n and N. Generates data using IsingCounts.R and regresses this vector against the output of IsingDesignQ.R using a Poisson GLM (log link). Note that on the log scale, using the counts as opposed to the normalized counts only affects the constant.

5.2 IsingIPS.R

Implements the algorithm of [1]. Input sample first and second moments, dimension of model (not really needed) and tolerance. Requires a number of subroutines detailed below.

5.2.1 CanonicalJ.R

Inputs i, j and Y, the current estimate of the mass function. Outputs J_{ij} (as defined in the reference).

5.2.2 Delta.R

Input i, j, Y, x, M and returns Δ_{ij} (as defined in the reference).

5.2.3 LambdaStar.R

Input i, j, Y, x, M and optionally J_{ij} (will probably remove this input). Output λ^* (as defined in the reference).

5.2.4 FirstMoment.R

Finds the first moment μ of an inputted mass function (where element i of the pmf vector corresponds to the i^{th} sign pattern when written in the binary ordering).

5.2.5 SecondMoment.R

As above but computes second moment Ξ.

References

[1] Steffen Lauritzen, Caroline Uhler, and Piotr Zwiernik. Total positivity in structured binary distributions. *arXiv preprint arXiv:1905.00516*, 2019.