Chapter 4 – Probability

Section 4.1 – Experiments, Sample Spaces, and Events

Definition: An **experiment** is an activity in which there are at least two possible outcomes and the result of the activity cannot be predicted with absolute certainty.

All of the outcomes from an experiment can be determined by constructing a tree diagram, a visual road map of possible outcomes.

Example: Two students are selected at random and asked if they are left-handed or right-handed. Construct a tree diagram depicting all the possible outcomes of this experiment. What changes if a student is ambidextrous?

Example: A student is randomly chosen and asked if s/he is left-handed or right-handed. We continue choosing students until we find the first that is left-handed. Construct a tree diagram depicting all the possible outcomes of this experiment.

Definition: The **sample space** associated with an experiment is a listing of all the possible outcomes, using set notation. It is the collection of all outcomes written mathematically, with curly braces, and denoted by S.

Example: Find the sample space for the experiments in the two examples above.

Definition:

- An **event** is any collection (or set) of outcomes from an experiment (any subset of the sample space).
- A **simple event** is an event consisting of exactly one outcome.
- An event has **occurred** if the resulting outcome is contained in the event.

An event may be given in standard set notation, or it may be denoted in words. If a written definition is given, we need to translate the words into mathematics in order to identify the event outcomes.

Notation:

- Events are denoted with capital letters, for example, A, B, C, …
- Simple events are often denoted by E_1, E_2, E_3, …

It is possible for an event to be empty. An event containing no outcomes is denoted by $\{\ \}$ or $\emptyset$ (the empty set).

Example: From our initial experiment, a student was either left-handed (L) or right-handed (R) and the sample space was $S = \{LL, LR, RL, RR\}$. In this case, we could define four simple events: $E_1 = \{LL\}$, $E_2 = \{LR\}$, $E_3 = \{RL\}$, $E_4 = \{RR\}$. Other examples of events could be:

- Let A be the event that at least one student is left-handed.
  $A = \{LL, LR, RL\}$

- Let B be the event that at most one student is left-handed.
  $B = \{LR, RL, RR\}$
- Let C be the event that both students write with the same hand.
  $C = \{LL, RR\}$

Example: Consider an experiment with sample space $S = \{YYY, YYN, YNY, YNN, NYY, NYN, NNY, NNN\}$. Find the outcomes in each of the following events:

a) $A = \text{exactly one } Y$
b) $B = \text{exactly two } N\text{s}$
c) $C = \text{at least one } Y$
d) $D = \text{at most one } N$

Definition: Let A and B denote two events associated with a sample space S.

- The event $\boldsymbol{A}$ **complement**, denoted $\overline{A}$ (or A' or A^C), consists of all outcomes in the sample space S that are *not* in A.
- The event $\boldsymbol{A}$ **union** $\boldsymbol{B}$, denoted $A \cup B$, consists of all outcomes in A or B or both.
- The event $\boldsymbol{A}$ **intersection** $\boldsymbol{B}$, denoted by $A \cap B$, consists of all outcomes in both A and B.
- If A and B have no elements in common, they are **disjoint** or **mutually exclusive**, written $A \cap B = \{\,\}$.

Example: Consider an experiment with sample space $S = \{YYY, YYN, YNY, YNN, NYY, NYN, NNY, NNN\}$ and events

$A = \text{exactly one } Y$

$B = \text{exactly two } N\text{s}$

$C = \text{at least one } Y$

$D = \text{at most one } N$

Find the outcomes in each event described in words and write each as a combination of the events A, B, C, and D.

a) Exactly one Y or at most one N
b) Two or more Ns
c) Exactly two Ns and at least one Y
d) Two or more Ys

“Clicker Question 26-30”

A **Venn diagram** may be used to visualize a sample space and events, to determine outcomes in combinations of events, and to answer probability questions in later sections.

- Draw a rectangle to represent the sample space.
- Figures (often circles) are drawn inside the rectangle to represent events.
- Plane regions represent events.

Example: Using a Venn diagram, shade each of the operations in the definition above: $\overline{A}$, $A \cup B$, $A \cap B$, $A \cap B = \{\}$.

HW: Section 4.1: # 4.6, 4.7, 4.9 – 4.11, 4.15, 4.22 – 4.26